Sensitivity Analysis of TOPSIS Technique: The Results of Change in the Weight of One Attribute on the Final Ranking of Alternatives

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Abstract

Most of data in Multi-attribute decision making (MADM) problems are changeable rather than constant and stable. Therefore, sensitivity analysis after problem solving can effectively contribute to making accurate decisions. In this paper, we offer a new method for sensitivity analysis in multi-attribute decision making problems in which if the weights of one attribute changes, then we can determine changes in the results of the problem. These changes involve changes in the weight of other attributes and the change in the final rank of alternatives. This analysis was conducted for Technique for order-preference by similarity to ideal solution (TOPSIS) technique, one of the most frequently used multi-attribute decision making techniques, and the formulas were obtained. The paper continues with a numerical example and at last conclusions and suggestions for future researches are offered.
Keywords: Multi-attribute decision making (MADM), TOPSIS technique, Sensitivity analysis.

1. Introduction

Multi-attribute decision making (MADM) models are selector models that are used for evaluating, ranking and selecting the most appropriate alternative from among several alternatives.
Alternatives of a MADM problem are evaluated by k attributes and the most appropriate alternative is selected or they are ranked in accordance with attribute's value for each alternative and the importance of each attribute for decision maker.
A MADM model is formulated as a decision making matrix as follow:

$$\begin{array}{c} \\ A_1 \\ A_2 \\ \cdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_k \end{array} \\ \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1k} \\ d_{21} & d_{22} & \cdots & d_{2k} \\ \cdots & \cdots & \cdots & \cdots \\ d_{m1} & d_{m2} & \cdots & d_{mk} \end{bmatrix} \end{array}$$

In this matrix A1, A2, A3, …, Am are available and C1, C2, C3, …, Ck predetermined m alternatives and are effective k attributes in decision making that are used for measuring utility of each alternative and dij are special Value of attribute jth for alternative ith, in other words the efficiency of the alternative ith against the attribute jth.
The most important in MADM models is that the data used are unstable and changeable. Hence, sensitivity analysis after problem solving can effectively contribute to making accurate decisions.
Sensitivity analysis for MADM models is one of the prevalent issues in MADM field on which researches have been conducted for the last decades. The first researches in this field are the works of Evans [3], Fishburn, Isaacs [4] and Schneller, Sphicas [9] that focused on determining decision sensitivity to probabilistic estimation errors.
Soofi [10] and Barron, Schmidt [1] suggested a sensitivity analysis for additive MADM models. They assumed a set of weights for attributes and obtained a new set of weights for them, so that the efficiency of alternatives has become equal or their order has changed.
Ma et al. [8] studied the structure of weights' set and conditions that result in special ranking or priority of one alternative to another, in additive decision making models.
Insua & French [7] offering a method at the frame of algorithms in sensitivity analysis studied the result of changes in attributes' weights on the final score of alternatives in MADM models and calculated the required change in attributes' weights for changing the optimal

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solution. These algorithms and methods were revised by Insua, Salhi, Proll [6].
Sanchez and Triantaphyllou [11] studied two types of sensitivity analysis for three methods of MADM. The first type determines the most sensitive attribute and calculates the change in attributes' weights that leads to change in the ranking of alternatives and the second type measures the sensitivity of decision making matrix elements. Zavadskas et al. [13] proposed a model for determining sensitivity to changes of separate parameters that enable to increase the reliability of the applied methods. Eshlaghy et al. [2] studied sensitivity analysis approach to produce complementary information by determination of criteria values domain in decision making matrix.
Yeh [12] presented a new approach to the selection of compensatory MADM methods for a specific cardinal ranking problem via sensitivity analysis of attribute weights. In line with the context-dependent concept of informational importance, the approach examines the consistency degree between the relative degree of sensitivity of individual attributes using an MADM method and the relative degree of influence of the corresponding attributes indicated by Shannon's entropy concept.
Memariani et al. [9] provided a new method for sensitivity analysis of MADM problems so that by using it and changing the weights of attributes, one can determine changes in the final results of a decision making problem. This analysis was applied for SAW technique.
In this paper, we offer a new method for sensitivity analysis of multi-attribute decision making problems so that by using it and changing one element of decision making matrix, we can determine changes in the results of a decision making problem. This analysis is performed on the TOPSIS technique and the formulae are obtained.
Since this method has a robust mathematical infrastructure that is suitable for most multi-attribute decision making problems, we applied sensitivity analysis for it.
The rest of the paper is organized as follows:
In the next section, the TOPSIS technique is reviewed and formulae and relations are mentioned. In the third section, the most important part of the article, a new method for sensitivity analysis of MADM models is developed. To do this, we first study the result of change in the weight of one attribute on the weights of other attributes. Then we study the results of change in the weight of one attribute on the final score of all attributes. In section 4, by presenting a numerical example the obtained relations and formulae are tested and their accuracy is confirmed. Finally, the article is summarized and conclusions and suggestions for future researches are cited.

2. A Review on TOPSIS Technique

In a MADM model, the ideal solution such A^* is the one that has the greatest utility on all of the attributes. That is

$$A^* = \{C_1^*, C_2^*, \dots, C_k^*\}\ ;\ C_j^* = \max_i U_j(r_{ij})$$

i=1,2,…,m j=1,2,….,k (1)

And the worst or the anti-ideal alternative such A is the one that has the least utility on all of the attributes. That is

$$A^- = \{C_1^-, C_2^-, \dots, C_K^-\}\ ;\ C_j^- = \min_i U_j(r_{ij})$$

i=1,2,…,m j=1,2,….,k (2)

The TOPSIS technique by considering the difference of alternatives from ideal and anti-ideal solution, selects the one that has the least difference from ideal and the greatest difference from anti-ideal solution. So, TOPSIS technique has the following steps for solving MADM models.
Step1. Transform decision making matrix to a normalized matrix by using the Euclidean norm, defined as:

$$r_{ij} = \frac{d_{ij}}{\sqrt{\sum_{i=1}^{m} d_{ij}^2}} \quad i = 1,2,\dots,m \quad j = 1,2,\dots,k \tag{3}$$

Note. If there are qualitative attributes, we can use scales for quantifying them in order to solve by TOPSIS technique.
Step 2. Calculate weighted normalized matrix V= $(V_{ij})_{m\times k}$ by considering the normalized matrix from step 1 and the vector of attributes' weights from Decision Maker (DM), that is a m×k matrix and its elements are:

$V_{ij}= r_{ij}.w_j$ i=1,2,..,m , j=1,2,…,k (4)

Step 3. Determine the ideal and anti-ideal solutions by considering the weighted normalized matrix, as:

$$A^+ = \{(max_i\ v_{ij}|j \in J), (min_i\ v_{ij}|j \in J')|i = 1,2,..,m\}$$
$$= \{v_1^+, v_2^+, \dots, v_j^+, \dots, v_k^+\} \tag{5}$$
$$A^- = \{(min_i\ v_{ij}|j \in J), (max_i\ v_{ij}|j \in J')|i = 1,2,..,m\}$$
$$= \{v_1^-, v_2^-, \dots, v_j^-, \dots, v_k^-\} \tag{6}$$

Wherein J's profit attributes and J's cost ones.
Step 4. Calculate the distance of alternatives from ideal and anti-ideal solutions. For this, we usually use the Euclidean norm as follow:

$$d_i^+ = \{\textstyle\sum_j (v_{ij} - v_j^+)^2\}^{1/2} \quad ;i=1,2,..,m \tag{7}$$
$$d_i^- = \{\textstyle\sum_j (v_{ij} - v_j^-)^2\}^{1/2} \quad i=1,2,..,m \tag{8}$$

Wherein, d_i^+ is the distance of the ith alternative from the ideal solution and d_i^- is that of anti-ideal solution.
Step 5. Calculate the relative distance of alternatives such A_i from ideal solution as

$$cl_i^+ = \frac{d_i^-}{d_i^- + d_i^+}, \quad i=1,2,..,m \tag{9}$$

Then, sort them by cl_i^+ descending.

3. Developing a New Method for Sensitivity Analysis of MADM Problems

Earlier researches on the sensitivity analysis of MADM problems often focused on determining the most sensitive attribute. They also focused on finding the least value of the change. However, a new method for sensitivity analysis of MADM problems is considered in this article that calculates the changing in the final score of alternatives when a change occurs in the weight of one attribute.

3. 1. The effect of change in the weight of one attribute on the weight of other attributes

The vector for weights of attributes is $W^t = (w_1, w_2, \dots, w_k)$ wherein weights are normalized with a sum of 1, that is:

$$\sum_{j=1}^{k} w_j = 1 \qquad (10)$$

With these assumptions, if the weight of one attribute changes, then the weight of other attributes change accordingly, and the new vector of weights transformed into $W'^t = (w'_1, w'_2, \dots, w'_k)$

The next theorem depicts changes in the weight of attributes.

Theorem 3.1.1. In the MADM model, if the weight of the Pth attribute, changes by Δ_p, then the weight of other attributes change by Δ_j, where:

$$\Delta_j = \frac{\Delta_p . w_j}{w_p - 1} \; ; j = 1,2,..,k \quad , j \neq p \qquad (11)$$

Proof. If the new weight of the attribute is w'_j and the new weight of the Pth attribute changes as:

$$w'_p = w_p + \Delta_p \qquad (12)$$

Then, the new weight of the other attributes would change as

$$w'_j = w_j + \Delta_j \; ; j = 1,2,..,k \, , j \neq p \qquad (13)$$

And because the sum of weights must be 1 then:

$$\sum_{j=1}^{k} w'_j = \sum_{j=1}^{k} w_j + \sum_{j=1}^{k} \Delta_j \Rightarrow \sum_{j=1}^{k} \Delta_j = 0 \qquad (14)$$

Therefore:

$$\Delta_p = -\sum_{\substack{j=1 \\ j \neq p}}^{k} \Delta_j \qquad (15)$$

Where:

$$\Delta_j = \frac{\Delta_p . w_j}{w_p - 1} \; ; j = 1,2,..,k \quad , j \neq p \qquad (16)$$

Since:

$$-\Delta_p = \sum_{\substack{j=1 \\ j \neq p}}^{k} \Delta_j = \sum_{\substack{j=1 \\ j \neq p}}^{k} \frac{\Delta_p . w_j}{w_p - 1} = \frac{\Delta_p}{w_p - 1} \sum_{\substack{j=1 \\ j \neq p}}^{k} w_j = \frac{\Delta_p}{w_p - 1}\left(1 - w_p\right) = -\Delta_p \qquad (17)$$

Main result. In a MADM problem, if the weight of the Pth attribute changes from w_p to w'_p as:

$$w'_p = w_p + \Delta_p \qquad (18)$$

Then, the weight of other attributes would change as:

$$w'_j = \frac{1 - w_p - \Delta_p}{1 - w_p} . w_j = \frac{1 - w'_p}{1 - w_p} . w_j$$
$$j = 1,2,..,k \quad , j \neq p \qquad (19)$$

Since, for $j = 1,2,..,k \quad , j \neq p$ we have:

$$w'_j = w_j + \Delta_j = w_j + \frac{\Delta_p . w_j}{w_p - 1} = \frac{w_j(w_p - 1) + \Delta_p . w_j}{w_p - 1} \qquad (20)$$

$$\Rightarrow w'_j = \frac{\left(1 - w_p - \Delta_p\right). w_j}{1 - w_p} = \frac{1 - w'_p}{1 - w_p} . w_j \; ;$$
$$j = 1,2,..,k \, , j \neq p \qquad (21)$$

Then, new vector for weights of attributes would be $W'^t = (w'_1, w'_2, \dots, w'_k)$, that is:

$$w'_j = \begin{cases} w_j + \Delta_p & j = p \\ \frac{1 - w'_p}{1 - w_p} . w_j & j \neq p \, , j = 1,2,..,k \end{cases} \qquad (22)$$

$$w'_p = w_p + \Delta_p \quad \Rightarrow \begin{cases} if \; w'_p > w_p & \Rightarrow w'_j < w_j \\ if \; w'_p < w_p & \Rightarrow w'_j > w_j \end{cases}$$
$$j = 1,2,..,k \, , j \neq p \qquad (23)$$

The sum of new weights of attributes that are obtained in (22) is 1, because:

$$\sum_{j=1}^{k} w'_j = \sum_{\substack{j=1 \\ j \neq p}}^{k} w'_j + w'_p = \sum_{\substack{j=1 \\ j \neq p}}^{k} \frac{w_j\left(1 - w_p - \Delta_p\right)}{1 - w_p} + w_p + \Delta_p$$
$$= \frac{\left(1 - w_p - \Delta_p\right)}{1 - w_p} \sum_{\substack{j=1 \\ j \neq p}}^{k} w_j + w_p + \Delta_p$$
$$= \frac{\left(1 - w_p - \Delta_p\right)}{1 - w_p} . \left(1 - w_p\right) + w_p + \Delta_p$$
$$= 1 - w_p + w_p + \Delta_p = 1 \qquad (24)$$

Corollary. In the new vector of weights that is obtained by (22), the weight's ratio is the same (exception of the Pth attribute) because new weights for attributes (exception of the Pth attribute) is obtained by multiplying the constant $\frac{(1 - w_p - \Delta_p)}{1 - w_p}$ to the old weight. Then, the ratio of new weight of attribute C_i to new weight of attribute C_j for i,j=1,2,…,k , i, j≠p is the same to ratio of old ones. That is

$$\frac{w'_i}{w'_j} = \frac{w_i}{w_j} ; i, j = 1,..,k \, , j \neq p \qquad (25)$$

3.2. The effect of change in the weight of one attribute on the final score of alternatives in TOPSIS technique In a decision making problem solved by TOPSIS, if the weight of one attribute changes, then the final score of alternatives will change. The next theorem calculates this change.

Theorem 3.2.1 In the MADM model of TOPSIS, if the weight of the Pth attribute changes by Δ_p, then the final score of the ith alternative, i=1,2,…,m would change as below:

$$cl'^{+}_i = \frac{d'^{-}_i}{d'^{+}_i + d'^{-}_i} \qquad (26)$$

Where d'^{+}_i, d'^{-}_i, are calculated as follow:

$$d'^{+}_i = \{\gamma^2 . d_i^{+2} + (1 - \gamma^2)\left(v_{ip} - v_p^{+}\right)^2 + \Delta_p^2 (r_{ip} - r_{lp})^2 + 2\Delta_p\left(v_{ip} - v_{lp}^{+}\right)\left(r_{ip} - r_{lp}\right)\}^{1/2} \qquad (27)$$

$$d_i'^- = \{\gamma^2.d_i^{-2} + (1-\gamma^2)(v_{ip} - v_p^-)^2 + \Delta_p^2(r_{ip} - r_{lp})^2 + 2\Delta_p(v_{ip} - v_{lp}^-)(r_{ip} - r_{lp})\}^{1/2} \quad (28)$$

For simplicity, we perform the following changes:

$$\gamma = \frac{1-w_p-\Delta_p}{1-w_p} = \frac{1-w_p'}{1-w_p} \quad (29)$$

$$w_p' = w_p + \Delta_p \Rightarrow \begin{cases} if\ 0<\gamma<1 & \Rightarrow w_p' > w_p \\ if\ \gamma > 1 & \Rightarrow w_p' < w_p \end{cases} \quad (30)$$

$$l = \begin{cases} max_i\, v_{ip}\ if\ p \in J,\ i = 1,2,\dots,m \\ min_i\, v_{ip}\ if p \in J',\ i = 1,2,\dots,m \end{cases} \quad (31)$$

$$l' = \begin{cases} min_i\, v_{ip}\ if\ p \in J,\ i = 1,2,\dots,m \\ max_i\, v_{ip}\ if p \in J',\ i = 1,2,\dots,m \end{cases} \quad (32)$$

Proof. By considering equation (30), if the weight of the Pth attribute changes by Δ_p, then the weights of other attributes would change by:

$$w_j' = \frac{(1-w_p-\Delta_p).w_j}{1-w_p} = \frac{1-w_p'}{1-w_p}.w_j = \gamma w_j$$
$$j = 1,2,\dots,k\ , j \neq p \quad (33)$$

To prove equations (27) and (28), we consider these changes in all steps of TOPSIS technique.

With regard to the changes in the weights, the weighted normalized matrix $V = (v_{ij})_{m\times k}$ in TOPSIS is transformed to $V' = (v'_{ij})_{m\times k}$as:

$$v_{ij}' = w_j'.r_{ij} = \left(\frac{1-w_p-\Delta_p}{1-w_p}\right).w_j.r_{ij} = \left(\frac{1-w_p-\Delta_p}{1-w_p}\right).v_{ij}$$
$$i = 1,2,\dots,m \quad j = 1,2,\dots,k, \quad j \neq p \quad (34)$$

$$v_{ip}' = w_p'.r_{ip} = (w_p+\Delta_p).r_{ip} = v_{ip} + \Delta_p.r_{ip}\ ,$$
$$i = 1,2,\dots,m \quad (35)$$

Since the ideal and anti-ideal solutions are calculated from weighted normalized decision matrix and in both $(j = p\ , j \neq p)$the values of V_{ip}'s at each column change similarly, therefore no change would occur in calculating the ideal and anti-ideal solutions and only their value changes as follow:

If $j = p$, then:

$$v_p'^+ = v_p^+ + \Delta_p.r_{lp} \qquad v_p'^- = v_p^- + \Delta_p.r_{lp} \quad (36)$$

Where:

$$l = \begin{cases} max_i\, v_{ip}\ if\ p \in J,\ i = 1,2,\dots,m \\ min_i\, v_{ip}\ if p \in J',\ i = 1,2,\dots,m \end{cases} \quad (37)$$

$$l' = \begin{cases} min_i\, v_{ip}\ if\ p \in J,\ i = 1,2,\dots,m \\ max_i\, v_{ip}\ if p \in J',\ i = 1,2,\dots,m \end{cases} \quad (38)$$

And if $j \neq p$, then:

$$v_j'^+ = v_j^+\left(\frac{1-w_p-\Delta_p}{1-w_p}\right) = v_j^+\frac{1-w_p'}{1-w_p} = v_j^+.\gamma$$
$$j = 1,2,\dots,k \quad (39)$$

$$v_j'^- = v_j^-\left(\frac{1-w_p-\Delta_p}{1-w_p}\right) = v_j^-\frac{1-w_p'}{1-w_p} = v_j^-.\gamma$$
$$j = 1,2,\dots,k \quad (40)$$

By performing these changes, the distance of alternatives from the ideal and anti-ideal solutions would change as:

$$d_i'^+ = \left\{\sum_{j=1}^{k}(v_{ij}' - v_j'^+)^2\right\}^{\frac{1}{2}} = \left\{\sum_{\substack{j=1\\ j\neq p}}^{k}(v_{ij} - v_j'^+)^2.\gamma^2 + (v_{ip} + \Delta_p.r_{ip} - v_p^+ - \Delta_p.r_{lp})^2\right\}^{1/2} \quad (41)$$

$$d_i'^- = \left\{\sum_{j=1}^{p}(v_{ij}' - v_j'^-)^2\right\}^{\frac{1}{2}} = \left\{\sum_{\substack{j=1\\ j\neq p}}^{k}(v_{ij} - v_j'^-)^2.\gamma^2 + (v_{ip} + \Delta_p.r_{ip} - v_p^+ - \Delta_p.r_{lp})^2\right\}^{1/2} \quad (42)$$

By solving and simplifying (41) and (42), equations (27), (28) are acquired.

The values $d_i'^+, d'^-_i$in equations (27), (28) are calculated by their older values d_i^+, d_i^- ,the value of change in the weight of the Pth attribute, Δ_p, and other available information in the model. These equations can be used in the software that use TOPSIS technique for solving MADM problems to obtain new results in light of change in the weight of one attribute.

4. Numerical Example

We assume a MADM problem that has three alternatives and four attributes wherein attributes C_1, C_4 are of cost type and attributes C_2, C_3 are of profit type.

$W^t = (0.4, 0.2, 0.3, 0.1)$

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} 13 & 9 & 9 & 8 \\ 5 & 3 & 5 & 12 \\ 7 & 5 & 7 & 6 \end{bmatrix} \end{array}$$

For solving it by TOPSIS technique, normalized matrix by using Euclidean norm is calculated as (43):

$$r_{ij} = \frac{d_{ij}}{\sqrt{\sum_{i=1}^{m} d_{ij}^2}} \qquad i = 1,2,3 \quad j = 1,2,3,4 \quad (43)$$

Then:

$$R=\begin{array}{c} \\ A_1\\ A_2\\ A_3\end{array}\begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4\end{array}\\ \begin{bmatrix}0.83 & 0.84 & 0.72 & 0.51\\ 0.38 & 0.28 & 0.40 & 0.77\\ 0.41 & 0.47 & 0.56 & 0.38\end{bmatrix}\end{array}$$

From the equation $v_{ij}=r_{ij}.w_j \quad i=1,2,3\ ,j=1,2,3,4$, the weighted normalized matrix is:

$$\overline{V}=\begin{array}{c} \\ A_1\\ A_2\\ A_3\end{array}\begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4\end{array}\\ \begin{bmatrix}0.33 & 0.17 & 0.22 & 0.05\\ 0.15 & 0.06 & 0.12 & 0.08\\ 0.17 & 0.09 & 0.17 & 0.04\end{bmatrix}\end{array}$$

Since $J=\{2,3\}, J'=\{1,4\}$ then, ideal and anti-ideal solutions would be:

$A^+=\{0.15,0.17,0.22,0.04\}$

$A^-=\{0.33,0.06,0.12,0.08\}$

By using the Euclidean norm, distance of alternatives from ideal and anti-ideal solutions are:

$$d_1^+=0.18\quad,\quad d_2^+=0.152\quad,\quad d_3^+=0.09$$

$$d_1^-=0.15\quad,\quad d_2^-=0.179\quad,\quad d_3^-=0.8$$

Then, the final score of alternatives are calculated by (44):

$$cl_i^+=\frac{d_i^-}{d_i^-+d_i^+}\quad,\quad i=1,2,3 \tag{44}$$

As $cl_1^+=0.454\quad,\quad cl_2^+=0.54\quad,cl_3^+=0.668$

Therefore, alternatives are ranked as $A_3>A_2>A_1$.

Now we assume that the weight of the 2nd attribute increased by $\Delta_2=0.2$ and be $w'_2=w_2+\Delta_2=0.2+0.2=0.4$. Then by equation (22), the weight of other attributes change as (45):

$$w'_j=\frac{1-w'_2}{1-w_2}.w_j; j=1,3,4$$

$$\Rightarrow\ w'_j=0.75w_j$$

$$\Rightarrow W'^t=(0.3,0.4,0.225,0.075) \tag{45}$$

In TOPSIS technique, this change in the weights affects the weighted normalized matrix, and then we have:

$$V'=\begin{array}{c} \\ A_1\\ A_2\\ A_3\end{array}\begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4\end{array}\\ \begin{bmatrix}0.248 & 0.336 & 0.163 & 0.038\\ 0.114 & 0.112 & 0.090 & 0.057\\ 0.124 & 0.186 & 0.126 & 0.029\end{bmatrix}\end{array}$$

Since $J=\{2,3\}, J'=\{1,4\}$, then, ideal and anti-ideal solutions are calculated and cl'^+_i for each alternative are:

$$cl'^+_1=0.636\quad,\quad cl'^+_2=0.362\quad,cl'^+_3=0.497$$

So $A_1>A_3>A_2$. It is obvious that, the ranking of alternatives has changed because of changing in the weight of the second attribute.

If we use equations (27), (28), then without resolving the problem, we can calculate the final score of alternatives by considering the change in the weight of second attribute as (46):

$$d'^+_i=\{\gamma^2\cdot d_i^{+2}+(1-\gamma^2)(v_{i2}-v_2^+)^2+\Delta_2^2(r_{i2}-r_{l2})^2+2\Delta_2(v_{i2}-v_2^+)(r_{i2}-r_{l2})\}^{1/2}$$

$$d'^-_i=\{\gamma^2\cdot d_i^{-2}+(1-\gamma^2)(v_{i2}-v_2^-)^2+\Delta_2^2(r_{i2}-r_{l'2})^2+2\Delta_2(v_{i2}-v_2^-)(r_{i2}-r_{l'2})\}^{1/2} \tag{46}$$

Where:

$$\gamma=\frac{1-w'_p}{1-w_p}=\frac{1-w'_2}{1-w_2}=0.75$$

With regard to the matrixes R, V in primal model (before changing the weight of second attribute), we have:

$$V_2^+=0.17,\quad V_2^-=0.06,\quad r_{l2}=0.84,\quad r_{l'2}=0.28$$

By replacing these values in above equations, we have:

$$d'^+_1=0.135\quad,\quad d'^+_2=0.237\quad,\quad d'^+_3=0.154$$

$$d'^-_1=0.236\quad,\quad d'^-_2=0.134\quad,\quad d'^-_3=0.152$$

And from the equation $cl'^+_i=\frac{d'^-_i}{d'^+_i+d'^-_i}$ can calculate cl'^+_i as:

$$cl'^+_1=0.636\quad,\quad cl'^+_2=0.362\quad,cl'^+_3=0.497$$

So the final rank of alternatives would be $A_1>A_3>A_2$ that is exactly the same result obtained by resolving problem.

Note. Accordingly said at corollary of theorem (3.1.1). The ratio of new and old weights of all attributes except attribute 2 will not change, that is:

$$\frac{w'_i}{w'_j}=\frac{w_i}{w_j}\qquad ;\ i,j=1,3,4$$

For example, for attributes 1st and 4th we have:

$$\frac{w'_1}{w'_4}=\frac{w_1}{w_4}\Rightarrow\frac{0.3}{0.075}=\frac{0.4}{0.1}=4 \tag{47}$$

This example demonstrates that:

First, the change in the weight of one attribute affects the weight of other attributes. The value of this change is calculated by equation (22). Second, the final score of all alternatives will change after this change; however, there is no need for resolving the problem. The change in the final score of alternatives is calculated by equations (27), (28).

5. Conclusions and Future Research

In the classic techniques of MADM, often, it is assumed that all the used data (such as weight of attributes, efficiency of alternatives against attributes,…) are deterministic and the final score or utility of alternatives are obtained by MADM solving techniques. However, in reality, data of decision making problem are changing so that, after solving decision making problems, usually a sensitivity analysis is carried out.

The studies done on sensitivity analysis for MADM problems often focused on determining the most sensitive attribute in the model. This attribute is one that, the least change in its weight relative to others, leads to change in ranking of alternatives. Also, they found the value of changing in the weight of one attribute that leads to the change in ranking of alternatives. These researches frequently focused on attributes' sensitivity.

The other type of sensitivity analysis not addressed in the existing literature is calculating the change in the final score of alternatives in light of changes in the weight of a particular attribute. In this sensitivity analysis, for a given change in the weight of one attribute, the change in the score of alternatives is calculated.

The type of sensitivity analysis presented in this paper can be applied in MADM related software for solving decision making problems so by adding it to this software and by utilizing graphical capability of computers, we can change the weight of one attribute arbitrarily and observe its effect on the final score and rank of alternatives, immediately. The following suggestions are proposed for future researches.

Studying the effect of change in one element of decision making matrix on the final score of alternatives in TOPSIS technique.

Studying the effect of simultaneous change in the weight of one attribute and in one element of decision making matrix on the final score of alternatives in TOPSIS technique.

Applying this type of sensitivity analysis for other techniques of MADM including Simple additive weighting method (SAW), Preference ranking organization method for enrichment evaluations (PROMEETHE) and Analytical hierarchy process (AHP).

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