Journal of Industrial Engineering 3(2009)1-13

A Hybrid Meta-Heuristic Method to Optimize Bi-Objective Single Period Newsboy Problem with Fuzzy Cost and Incremental Discount

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Received 3 May., 2009; Revised 2 Jun., 2009; Accepted 10 Jun. 2009

Abstract

In this paper the real-world occurrence of the multiple-product multiple-constraint single period newsboy problem with two objectives, in which there is incremental discounts on the purchasing prices, is investigated. The constraints are the warehouse capacity and the batch forms of the order placements. The first objective of this problem is to find the order quantities such that the expected profit is maximized and the second objective is maximizing the service rate. It is assumed that holding and shortage costs, modeled by a quadratic function, occur at the end of the period, and that the decision variables are integer. A formulation to the problem is presented and shown to be an integer nonlinear programming model. Finally, an efficient hybrid algorithm of harmony search, goal programming, and fuzzy simulation is provided to solve the model. The results are illustrated by a numerical example.

Keywords: Single period newsboy problem; fuzzy variables; mixed integer nonlinear programming; goal programming; harmony search; fuzzy simulation.

1. Introduction and literature review

The single period newsboy problem simply deals with situations where the demand for a commodity is uncertain (random) and the ordered items that remain unsold or unused at the end of the cycle become obsolete. Hence, on one hand the buyer may incur a disposal cost and on the other if he initially decides to buy smaller amounts of these commodities, shortages may occur, causing loss of revenue. In this problem, the commodity has the most important characteristic of a "single-period" product, and the question becomes how to determine the quantity to be ordered to minimize (maximize) the costs (profit). The answer to this question is the main objective of the classical single period newsboy problem.

In real world situations, many products have a limited selling period, so the single period problem is often used to aid decision-making in fashion, sporting, Service industries, etc. to manage capacity or evaluate advanced booking of orders. Abdel-Malek and Montanari [2], Abdel-Malek and Areeratchakul [3], and Vairaktarakis [43] considered a multi-product newsboy problem with budget constraint. Matsuyama [31] analyzed the single period problem in which a fraction of the shortage was backordered. Moreover, Alfares and Elmorra [4] analyzed the single period problem in both single-periodic and multi-periodic frames in which random yield and fixed order cost were considered.

Reyes [34] used the newsboy model in a supply chain in which both sides had incomplete information on the demand. Mostard and Teunter [32] considered a single-period model in which a percentage of the sold products were returned, assuming that these products could be returned in a specific range of time and could be sold again if not damaged. Keren and Pliskin [23] presented the newsboy problem as a risk-averse model and calculated the optimal order quantity by using the utility theory. Chen and Chuang [7] analyzed the single period newsboy problem along with the shortage level constraint. Abdel-Malek and Montanari [2] presented the single period problem with budget constraint and proposed different formulae to obtain the order quantity for three ranges of the budget quantity. Furthermore, Abdel-Malek and Areeratchakul [3] used the quadratic programming approach in a multi-product single period problem with budget, capacity, and order constraints. Taleizadeh et al. [37] developed a multi-

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product newsboy model with incremental discounts and batch orders in which the service level and warehouse space assumed constraints. Then, they proposed a genetic algorithm to solve the obtained non-linear integer model.

Dutta et al. [10] presented a single-period inventory problem in an imprecise and uncertain mixed environment. The aim of Dutta et al.'s [10] research was to introduce demand as a fuzzy random variable. Ji and Shao [20] considered the model for the single period problem with fuzzy demands and quantity discounts in hierarchical decision system by manufacturer and retailer. Shao and Ji [35] investigated the multi-product single period problem with fuzzy demands under budget constraint.

In this paper, a multi-inventory single-period newsboy problem is considered in which the uncertain demand follows a Poisson distribution; batch orders and warehouse capacity are considered constraints; the incremental discount policy is used to purchase the items; and the holding and shortage cost are fuzzy variables. The overall goal is to establish the optimal order quantity for each product that serves the dual purpose of maximizing expected profits and service level.

The rest of the paper is organized as follows. In section 2 some required definitions in fuzzy environment to model the problem are given. In section 3 the problem is defined in details. The mathematical formulation of the problem comes in section 4 and the proposed hybrid solution algorithm is explained in section 5. In order to demonstrate the application of the proposed methodology in real world environment, a numerical example is given in section 6. Finally, the conclusion and recommendations for future research comes in section 7.

2. Fuzzy environment

Let us first present some definitions in fuzzy environment that will be used to model the problem at hand. We adopt the concepts of the credibility, possibility and necessity theory, as well as credibility of fuzzy event and the expected value of a fuzzy variable based on Liu [29].

Definition 1: A Fuzzy number is of LR-Type, if there exist reference functions L (for the left), R (for the right), and scalars $\alpha > 0, \beta > 0$ with

$$\mu(\tilde{\xi}) = \begin{cases} 1 & \tilde{\xi} \in [m,n] \\ L(\frac{m-\tilde{\xi}}{\alpha}) & \tilde{\xi} \le m \\ R(\frac{\tilde{\xi}-n}{\beta}) & \tilde{\xi} \ge n \end{cases} \quad (1)$$

Where $\tilde{\xi}$ is abbreviated form of $\tilde{\xi} = (m,n,\alpha,\beta)_{L-R}$. The Triangular and trapezoidal fuzzy variable are specific kind of LR-Type.

Definition 2: Let $\tilde{\xi}$ be a fuzzy number with the membership function $\mu(\tilde{\xi})$. Then the possibility, necessity, and credibility measure of the fuzzy event $\tilde{\xi} \ge r$ can be represented, respectively, by:

$$Pos\left\{\tilde{\xi} > r\right\} = \sup_{\tilde{\xi} \ge r} \mu(\tilde{\xi}) \quad (2)$$

$$Nec\left\{\tilde{\xi} \ge r\right\} = 1 - \sup_{\tilde{\xi} < r} \mu(\tilde{\xi}) \quad (3)$$

$$Cr\left\{\tilde{\xi} \ge r\right\} = \frac{1}{2}\left[Pos\left\{\tilde{\xi} \ge r\right\} + Nec\left\{\tilde{\xi} \ge r\right\}\right] \quad (4)$$

Definition 3: The expected value of a fuzzy variable $\tilde{\xi}$ is defined as:

$$E\left[\tilde{\xi}\right] = \int_0^\infty Cr\left\{\tilde{\xi} \ge r\right\} dr - \int_{-\infty}^0 Cr\left\{\tilde{\xi} \le r\right\} dr \quad (5)$$

the expected value of a triangular fuzzy variable $\tilde{\xi} = (\xi_1, \xi_2, \xi_3)$ is:

$$E[\tilde{\xi}] = \frac{1}{4}(\xi_1 + 2\xi_2 + \xi_3) \quad (6)$$

Definition 4: Let $\tilde{\xi}$ be a fuzzy variable. Then the optimistic function of α is defined as:

$$\tilde{\xi}_{\sup}(\alpha) = \sup\left[r \middle| Cr\left\{\tilde{\xi} \ge r\right\} \ge \alpha\right] \quad , \quad \alpha \in (0,1] \quad (7)$$

Definition 5: Assume $C_1, C_2, \cdots, C_k$ are real constants and $G_1(\tilde{\xi}), G_2(\tilde{\xi}), \cdots G_k(\tilde{\xi})$ are functions of fuzzy variable, then

$$E\left[\sum_{k=1}^{K} C_k G_k(\tilde{\xi})\right] = \sum_{k=1}^{K} C_k E(G_k(\tilde{\xi})) \quad (8)$$

3. Problem definition

Consider a company that orders products to a supplier with the following rules of placing the orders that take place only once and only at the start of a period. The customer demand for each product (j) follows a Poisson distribution with parameter λ_j. The order quantity of each product should only be integer multiples of packets each with n_j items. There is no enforced constraint on the supplier to supply an order. The entire capacity of the warehouse is assigned to the products. Shortage is licensable and takes the lost sale condition. The shortage and holding costs are fuzzy and deployed at the end of the period and increase in quadratic fashion. The transportation cost is deployed to carry the products and has two components as fixed cost for each shipment and variable cost for each unit of the products. Discount for purchasing items is allowed and follows incremental discount rule. Since the transportation and the order-processing times are relatively very small to the cycle length, we assume that the lead-time is equal to zero, which is the common practice in the single period problems. The goal is to determine the order quantity of each product such that the constraints are satisfied and both the expected profit and service rate are maximized.

4. Problem modeling

Since the orders are placed only once and at the beginning of each period, one may take advantage of the classical single period problem and develop a mathematical model for the problem at hand. To do this, the variables and the parameters of the model are first defined. Then, the costs and the constraints are determined. Finally, the model is presented.

4.1. The parameters and the variables

For $i=1,2,...,n$ and $j=1,2,...,T$ the parameters and the variables of the model are:

T : Total number of products.

X_j : The stochastic demand of the j^{th} product.

λ_j : The expected demand of the j^{th} product.

$f_{X_j}(x_j)$: The probability mass function of the j^{th} product demand.

V_j: The number of items in the packets of the j^{th} product.

$\hat{H}_j(x)$: The fuzzy holding cost function of the j^{th} product at the end of a period.

$\hat{h}_{1_j}$: The fuzzy linear coefficient of the quadratic holding cost function of the j^{th} product.

$\hat{h}_{2_j}$: The fuzzy quadratic coefficient of the quadratic holding cost function of the j^{th} product.

$\hat{\pi}_j(x)$: The shortage cost function of the j^{th} product at the end of a period.

$\hat{\pi}_{1_j}$: The fuzzy linear coefficient of the quadratic shortage cost function of the j^{th} product.

$\hat{\pi}_{2_j}$: The fuzzy quadratic coefficient of the quadratic shortage cost function of the j^{th} product.

A : The fixed transportation cost of each shipment.

K_j : The variable transportation cost of each unit of the j^{th} product.

m : Number of shipments.

Q_j: A decision variable representing the order quantity of the j^{th} product.

B_j : A decision variable representing the number of packets that have been ordered for the j^{th} product.

f_j : The space required for each packet of the j^{th} product.

$\hat{f}$: The capacity of a shipment.

α_j : The minimum service level of the j^{th} product.

q_{ij} : The i^{th} discount break point of the j^{th} product.

C_{ij} : The purchase cost of the j^{th} product in the i^{th} break point.

F : The total available warehouse space.

C_{H_j} : The expected holding cost of the j^{th} product at the end of a period.

C_{B_j} : The expected shortage cost of the j^{th} product at the end of a period.

C_{P_j} : The expected purchasing cost of the j^{th} product.

C_T : The transportation cost of the product(s).

P_j : The selling price of the j^{th} product.

R_j : The expected revenue of the j^{th} product.

Z_P : The expected profit.

Z_{SR} : The expected service rate.

In the next section, a single-product problem is first modeled and then it is extended it to the multi-product case.

4.2. Modeling the first objective (profit)

In order to model the profit associated with a single-product problem, first the revenue and then the costs (holding, shortage, transportation and purchase) are modeled.

4.2.1. Revenue

To calculate the revenue obtained from selling the j^{th} product in a period, let us assume that if the total demand quantity is more than the order quantity, then the sold quantity is Q_j. Otherwise, it is X_j. In other words:

Sold quantity of the j^{th} product =

$$\begin{cases} Q_j & \text{if} \quad X_j \ge Q_j \\ X_j & \text{if} \quad X_j < Q_j \end{cases}$$

(9)

Since the probability mass function of the demand for product j is $f_{X_j}(x_j)$, the expected sold quantity of the j^{th} product at the end of the period is determined as:

$$\sum_{X_j=0}^{Q_j-1} X_j.f_{X_j}(x_j) + \sum_{X_j=Q_j}^{+\infty} Q_j.f_{X_j}(x_j)$$

(10)

Hence, the expected revenue is obtained by:

$$R_j = \sum_{X_j=0}^{Q_j-1} P_j.X_j.f_{X_j}(x_j) + \sum_{X_j=Q_j}^{+\infty} P_j.Q_j.f_{X_j}(x_j) \tag{11}$$

4.2.2. Costs

The costs of the problem are holding, shortage, transportation and purchasing that are determined as follows.

4.2.2.1. Holding cost

Since it is assumed that the holding cost of the j^{th} product occurs at the end of the period, the determination of the end-point expected inventory at the start of the period is needed. If the total demand quantity is more than the order quantity, i.e., $X_j \ge Q_j$, then the inventory quantity at the end of the period is zero. However, if the total demand quantity is less than the order quantity, then the inventory quantity at the end of the period is $Q_j - X_j$. In other words

End-period inventory level of the j^{th} product =

$$\begin{cases} 0 & \text{if} \quad X_j \ge Q_j \\ (Q_j - X_j) & \text{if} \quad X_j < Q_j \end{cases}$$

(12)

Since the probability mass function of the demand for product j is $f_{X_j}(x_j)$, then the expected inventory at the end of the period is determined as:

$$\sum_{X_j=0}^{Q_j-1} (Q_j - X_j) f_{X_j}(x_j)$$

(13)

Finally, considering the quadratic increase of the holding cost [28], the expected holding cost at the end of the period which is calculated at the start of the period is

$$C_{H_j} = \sum_{X_j=0}^{Q_j-1} \left(\hat{h}_{1_j} (Q_j - X_j) + \hat{h}_{2_j} (Q_j - X_j)^2 \right) f_{X_j}(x_j)$$

(14)

4.2.2.2. Shortage cost

If shortage occurs during a period, it will take the lost sale condition. Since the holding cost is calculated at the end of the period, the expected shortage is calculated at the same time. In this case, if the total demand quantity is more than the ordered quantity, i.e., $X_j > Q_j$, then at the end of the period the shortage quantity will be $X_j - Q_j$. However, if the total demand quantity at the end of the period is less than the order quantity, the shortage quantity at the end of the period will be zero. In other words,

Shortage quantity at the end of the period=

$$\begin{cases} (X_j - Q_j) & \text{if} \quad X_j > Q_j \\ 0 & \text{if} \quad X_j \le Q_j \end{cases} \tag{15}$$

Accordingly, the expected shortage at the end of the period is:

$$\sum_{X_j=Q_j+1}^{+\infty} (X_j - Q_j) f_{X_j}(x_j) \tag{16}$$

Taking into account the quadratic shortage cost function of Lin and Tsai [28], we have

$$C_{B_j} = \sum_{X_j=Q_j+1}^{+\infty} \begin{pmatrix} \hat{\pi}_{1_j}(X_j - Q_j) \\ +\hat{\pi}_{2_j}(X_j - Q_j)^2 \end{pmatrix} f_{X_j}(x_j) \qquad (17)$$

4.2.2.3. Transportation cost

The transportation cost is calculated based on equation (18), in which $f_j B_j$ is the required space to ship the order from the supplier.

$$C_T = \begin{cases} A + K_j Q_j & ; \quad 0 < f_j B_j \le \hat{f} \\ 2A + K_j Q_j & ; \quad \hat{f} < f_j B_j \le 2\hat{f} \\ \vdots & \qquad \vdots \\ mA + K_j Q_j & ; (m-1)\hat{f} < f_j B_j \le m\hat{f} \end{cases}$$

(18)

By introducing the binary variables $Y_k ; k = 1,2,...,m$, the transportation cost can be incorporated with the mathematical model of the problem as

$$\begin{aligned} & C_T = K_j Q_j + \sum_{k=1}^{m} kAY_k \\ & 0 < f_j B_j \le \hat{f} Y_1 \\ & \hat{f} Y_2 < f_j B_j \le 2\hat{f} Y_2 \\ & \vdots \\ & (m-1)\hat{f} Y_m < f_j B_j \le m\hat{f} Y_m \\ & Y_1 + Y_2 + \cdots + Y_m = 1 \\ & Y_k = 0,1 \end{aligned} \qquad (19)$$

4.2.2.4. Purchasing cost under incremental discount

The purchasing cost of the company for the j^{th} product at the beginning of a period can be calculated using the incremental discount policy. Let the incremental discount policy be

$$C_{P_j} = \begin{cases} C_{1j} Q_j & ; 0 < Q_j \le q_{1j} \\ C_{1j} q_{1j} + C_{2j}(Q_j - q_{2j}) & ; q_{1j} < Q_j \le q_{2j} \\ \vdots & \\ C_{1j} q_{1j} + C_{2j}(q_{2j} - q_{1j}) & \\ +\ldots + C_{nj}(Q_j - q_{n-1,j}) & ; \ Q_j \ge q_{nj} \end{cases}$$

(20)

Where q_{l_j} and C_{l_j} ; $l = 1,2,...,n$ are the discount points and the purchasing costs for each unit of the j^{th} product that corresponds to its l^{th} discount break point, respectively.

In order to include the incremental discount policy in the inventory model, equation (21) is used to model the incremental discount policy.

$$\begin{aligned} & C_{P_j} = C_{1j} W_{1j} + C_{2j} W_{2j} + \ldots + C_{nj} W_{nj} \\ & Q_j = W_{1j} + W_{2j} + \ldots + W_{nj} \\ & q_{1j}\lambda_{2j} \le W_{1j} \le q_{1j}\lambda_{1j} \\ & \left(q_{2j} - q_{1j}\right)\lambda_{3j} \le W_{2j} \le \left(q_{2j} - q_{1j}\right)\lambda_{2j} \\ & \qquad \vdots \end{aligned}$$

(21)

$$\begin{aligned} & 0 \le W_{nj} \le M\lambda_{nj} \\ & \lambda_{1j} \ge \lambda_{2j} \ge \cdots \ge \lambda_{nj} \\ & \lambda_{ij} = 0,1 \qquad \forall i, \qquad i = 1,2,\cdots,n \end{aligned}$$

in which $W_{ij}; i = 1,2,...,n$ and $j = 1,2,...,T$ are the modeling variables to convert equation (20) to (21) and M is a very big number.

4.2.3. Modeling the second objective (service rate)

Let the service rate of the j^{th} product (Z_{SR}) be the ratio of the expected customers' demands that are satisfied in a period to the average costumers' demand. Since the expected satisfied demand at the end of a period is determined by:

$$\sum_{X_j=0}^{Q_j-1} (Q_j - X_j) f_{X_j}(x_j)$$

(22)

Then the second objective is obtained as:

$$Z_{SR} = \frac{\sum_{X_j=0}^{Q_j-1}(Q_j - X_j) f_{X_j}(x_j)}{\lambda_j}$$
(23)

4.2.4. Constraints

The constraints of the problem at hand are the warehouse space and ordering in batch form. Since the space required for each packet of the j^{th} product is f_j square meters, the number of packets that have been ordered for the j^{th} product is B_j, and the total available warehouse space is F square meters, the warehouse space constraint becomes

$$f_j B_j \le F$$
(24)

However, we need the orders to be placed in packets of size V_j. In this case, we have

$$Q_j = V_j B_j$$
(25)

Finally, the multi-product model with incremental discount and fuzzy costs will be:

$$Max: Z_P = \sum_{j=1}^{T}\sum_{X_j=0}^{Q_j-1} P_j . X_j . f_{X_j}(x_j)$$

$$+\sum_{j=1}^{T}\sum_{X_j=Q_j}^{+\infty} P_j . Q_j . f_{X_j}(x_j)$$

$$-\sum_{j=1}^{T}\sum_{X_j=0}^{Q_j-1}\left(\hat{h}_{1j}(Q_j - X_j) + \hat{h}_{2j}(Q_j - X_j)^2\right)\frac{e^{-\lambda_j}\lambda_j^{X_j}}{X_j!}$$

$$-\sum_{j=1}^{T}\sum_{X_j=Q_j+1}^{\infty}\left(\hat{\pi}_{1j}(X_j - Q_j) + \hat{\pi}_{2j}(X_j - Q_j)^2\right)\frac{e^{-\lambda_j}\lambda_j^{X_j}}{X_j!}$$

$$-\sum_{j=1}^{T} K_j Q_j - \sum_{k=1}^{m} kAY_k - \sum_{j=1}^{T}\sum_{i=1}^{n} C_{ij} W_{ij}$$

$$Max: Z_{SR} = \frac{1}{T}\sum_{j=1}^{T}\sum_{X_j=0}^{Q_j-1}\frac{(Q_j - X_j) f_{X_j}(x_j)}{\lambda_j}$$

s.t:

$$\sum_{j=1}^{T} f_j B_j \le F$$

$$Q_j = V_j B_j \quad \forall j, \quad j = 1,2,...,T$$

$$q_{1j}\lambda_{2j} \le W_{1j} \le q_{1j}\lambda_{1j} \quad \forall j, \quad j = 1,2,...,T$$

$$\vdots$$

$$\left(q_{ij} - q_{i-1,j}\right)\lambda_{ij} \le W_{ij} \le \left(q_{ij} - q_{i-1,j}\right)\lambda_{i-1,j}$$

$$\forall i, \quad i = 2,...,n_j - 1 \text{ and } \forall j, \quad j = 1,2,...,T$$

$$0 \le W_{n_j j} \le M\lambda_{n_j j} \quad \forall j,\ j = 1,2,...,T, \text{M is a big number}$$

$$0 < \sum_{j=1}^{T} f_j B_j \le \hat{f} Y_1$$

$$(k-1)\hat{f} Y_i < \sum_{j=1}^{T} f_j B_j \le k\hat{f} Y_k \ ;\forall\, k = 2,3,\cdots,m$$

$$\sum_{k=1}^{m} Y_k = 1 \quad , \quad Y_k = 0,1 \ ; \ \forall\ k = 1,2,...,m$$

$$\lambda_{1j} \ge \lambda_{2j} \ge \cdots \ge \lambda_{n_j j} \quad \forall j, \quad j = 1,2,\cdots,T$$

$$\lambda_{ij} = 0,1 \quad \forall j,\ j = 1,2,\cdots T \text{ and } \forall i, i = 1,2,\cdots,n$$

$$B_j \ge 0 \text{ and integer} \quad \forall j, \quad j = 1,2,\cdots,T$$
(26)

5. A solution algorithm

Since the model in (26) is mixed-integer-nonlinear in nature, reaching an analytical solution (if any) to the problem is difficult (Gen and Cheng [12]). As a result, in this section a stochastic search algorithm is used to solve the model. However, since the models have two objectives, a goal programming framework is first applied to formulate them and then a Harmony search algorithm is employed to solve it.

5.1. Goal programming modeling

The scope of this research is limited to the application of goal programming (GP) approach to real life manufacturing situations in a multi-objective environment. For a rigorous mathematical analysis of multi-objective programming approach, the reader is referred to Steuer [36].

The multi-objective models in the context of manufacturing were formulated and solved in the recent past (a few sample studies include Kalpic, et al. [22]; Nagarur, et al. [33]) to provide information on the trade-off among multi-objectives. However, although it represents a viable approach to production planning, multi-objective goal programming (MOGP) is not as widespread among manufacturing companies as desired.

The GP appears to be an appropriate, powerful and flexible technique for decision analysis of the troubled modern decision maker who is burdened with achieving multiple conflicting objectives under complex environmental constraints. The extensive surveys of the GP by Tamiz, et al. [42], and Aouni and Kettani [6] have reflected this. The modeling approach of GP does not attempt to maximize or minimize the objective function directly as in the case of conventional linear programming. Instead it seeks to minimize the deviations between the desired goals and the actual results to be obtained according to the assigned priorities. A commonly used generalized model for goal programming is as follows (Kwak et al. [25]):

$$Min: z = \sum w_i p_i (d_i^+ + d_i^-)$$

s.t.:

$$\sum_j a_{ij} x_{ij} + d_i^- - d_i^+ = b_i \qquad \forall i, \ \ i = 1,2,...,m$$

$$x_{ij}, d_i^-, d_i^+ \geq 0$$

$$d_i^- . d_i^+ = 0 \quad , \ (i = 1,2,...,m; j = 1,2,...n) \qquad (27)$$

where p_i is the preemptive factor/priority level assigned to each relevant goal in rank order $(p_1 \geq p_2 \geq ... \geq p_n)$ and w_i are non-negative constants representing the relative weights assigned within a priority level to the deviational variables, d_i^+, d_i^- for each i^{th} corresponding goal, b_i. The x_{ij} represents the decision variables and a_{ij} represents the decision variable coefficients.

Based upon the generalized GP model in (27), in this section, the model in (26) is changed into a goal programming form. To determine the goal (b_1,b_2) this model is first solved as single objective ones. However, $b_2 = 1$ is considered by default. The new model for incremental discount policy will become:

$$Min: Z = p_1 W_1 d_1^- + p_2 W_2 d_2^-$$

s.t.:

$$\sum_{j=1}^{T} \sum_{X_j=0}^{Q_j-1} P_j . X_j . f_{X_j}(x_j) + \sum_{j=1}^{T} \sum_{X_j=Q_j}^{+\infty} P_j . Q_j . f_{X_j}(x_j)$$

$$-\sum_{j=1}^{T} \sum_{X_j=0}^{Q_j-1} \left(\hat{h}_{1j}(Q_j - X_j) + \hat{h}_{2j}(Q_j - X_j)^2 \right) \frac{e^{-\lambda_j} \lambda_j^{X_j}}{X_j!}$$

$$-\sum_{j=1}^{T} \sum_{X_j=Q_j+1}^{\infty} \left(\hat{\pi}_{1j}(X_j - Q_j) + \hat{\pi}_{2j}(X_j - Q_j)^2 \right) \frac{e^{-\lambda_j} \lambda_j^{X_j}}{X_j!}$$

$$-\sum_{j=1}^{T} K_j Q_j - \sum_{k=1}^{m} kAY_k - \sum_{j=1}^{T} \sum_{i=1}^{n} C_{ij} W_{ij} + d_1^- - d_1^+ = b_1$$

$$\frac{1}{T} \sum_{j=1}^{T} \sum_{X_j=0}^{Q_j-1} \frac{(Q_j - X_j) f_{X_j}(x_j)}{\lambda_j} + d_2^- - d^+_2 = b_2$$

$$\sum_{j=1}^{T} f_j B_j \leq F$$

$$Q_j = V_j B_j \qquad \forall j, \quad j = 1,2,...,T$$

$$q_{1j} \lambda_{2j} \leq W_{1j} \leq q_{1j} \lambda_{1j} \qquad \forall j, \qquad j = 1,2,...,T$$

$$\vdots$$

$$\left(q_{ij} - q_{i-1,j} \right) \lambda_{ij} \leq W_{ij} \leq \left(q_{ij} - q_{i-1,j} \right) \lambda_{i-1,j}$$

$$\forall i, \ \ i = 2,...,n_j - 1 \ \ \text{and} \ \ \forall j, \ \ j = 1,2,...,T$$

$$0 \leq W_{n_j j} \leq M \lambda_{n_j j} \ \ \forall j, j = 1,2,...,T, \text{M is a big number}$$

$$0 < \sum_{j=1}^{T} f_j B_j \leq \hat{f} Y_1$$

$$(k-1) \hat{f} Y_i < \sum_{j=1}^{T} f_j B_j \leq k \hat{f} Y_k \qquad \forall k = 2,3,\cdots,m$$

$$\sum_{k=1}^{m} Y_k = 1 \quad , \quad Y_k = 0,1 \ ; \ \forall \ k = 1,2,...,m$$

$$\lambda_{1j} \geq \lambda_{2j} \geq \cdots \geq \lambda_{n_j j} \qquad \forall j, \qquad j = 1,2,\cdots,T$$

$$\lambda_{ij} = 0,1 \ \ \forall j, j = 1,2,\cdots T \ \text{and} \forall i, i = 1,2,\cdots,n$$

$$B_j \geq 0 \text{ and integer} \quad \forall j, \qquad j = 1,2,\cdots,T$$

$$d_2^-.d_2^+ = 0,$$
$$d_1^-.d_1^+ = 0$$
$$d_1^-, d_1^+, d_2^-, d_2^+ \geq 0,\ B_j \geq 0 \text{ and integer}$$
$$\forall j,\ j = 1,2,\cdots,T \qquad (28)$$

5.2. Harmony search

New ways have been found to optimize problems for less than a century, but nature has used various ways of optimization for millions of years. Recently scientists mimicked nature to solve different kinds of complex optimization problems. Most of these problems are so complicated and time consuming that we cannot use an exact algorithm to solve them. Thus, typically some non-precise algorithms are used to find a near optimum solution in a shorter period. We call these algorithms meta-heuristic (Dorigo and Stutzle [8]).

Many researchers have successfully used meta-heuristic methods to solve complicated optimization problems in different fields of scientific and engineering disciplines. Some of these meta-heuristic algorithms are simulating annealing (Aarts and Korst [1], Kirkpatrick et al. [24], Taleizadeh et al. [38],), threshold accepting (Dueck and Scheuer [9]), Tabu search (Joo and Bong [21]), genetic algorithms (Goldberg [19], Al-Tabtabai and Alex [5], Taleizadeh et.al [40]), neural networks (Gaidock et al. [11]), ant colony optimization (Dorigo and Stutzle [8]), fuzzy simulation (Taleizadeh et al. [39]), evolutionary algorithm (Laumanns et al. [26]), harmony search (Lee et al. [27], Geem et al. [13]), and particle swarm optimization (Taleizadeh et.al. [41])

Lee et al. [27] and Vasebi et al. [44] showed that the Harmony Search (HS) algorithm outperforms Genetic Algorithm (GA) because of its multi-vector consideration and fast computation. One of the main advantages of HS versus GA is its simple implementation. Unlike GA that has genetic operators like crossover and mutation, the HS algorithm does not have these types of operators for new generations. This causes an iteration to be faster in HS than that in GA. Therefore, we employ a HS algorithm to solve the models under different criteria. The HS algorithm (Geem et al. [13]), which is inspired from the act of musician groups, was introduced in an analogy with music improvisation process where musicians in an ensemble continue to polish their pitches in order to obtain better harmony. Similar to musician groups when several notes from different musical instruments are played simultaneously by set of the pitch adjusting on a random basis to achieve pleasant harmony in several practices, this algorithm seeks the optimum solution by generating random vector solutions in a Harmony Memory (HM) that are improved iteration by iteration with some pitch adjusting and updating methods to reach global optimum. In summary, according to the analogy of improvisation and optimization, fantastic harmony is considered as global optimum, aesthetic standard is determined by the objective function, pitches of instruments are desired values of the variables, and each practice is the same at each iteration.

The HS optimization method has been applied successfully to various engineering problems such as satellite heat pipe design (Geem and Hwangbo [14]), vehicle routing (Geem et al. [16]), water network design (Geem et al. [15] and [17]) and structural design (Lee and Geem 2004). Mahdavi et al. [30] described an Improved Harmony Search (IHS) algorithm for solving optimization problems. IHS employs a novel method for generating new solution vectors that enhances accuracy and convergence rate of HS algorithm. They discussed the impacts of constant parameters on HS algorithm and presented a strategy for tuning these parameters.

Although HS algorithm has proven its ability of finding near global regions within a reasonable amount of time, it is comparatively inefficient in performing local search. Fesanghari et al. (2008) proposed a Hybrid Harmony Search (HHS) algorithm to solve engineering optimization problems with continuous design variables and employed a Sequential Quadratic Programming (SQP) model to speed up local search and improve precision of the HS solutions.

The HS optimization algorithm applied in this paper is performed by the following steps.

5.2.1. Initialization

The process of initialization has two parts; parameter initialization and HM initialization as described below.

5.2.2. Parameter initialization

The constant parameters of the HS algorithm include Harmony Memory Size (HMS), Harmony Memory Considering Rate ($HMCR$), Pitch Adjusting Rate (PAR), Number of decision variables (N), and the maximum Number of Improvisations (NI). The HMS is the number of simultaneous solution vectors in HM. Based on the frequently used HMS values in other HS applications available in the literature (Geem [18], Geem and Hwangbo [14], Geem et al. [13, 14, 16], it seems that using a small HMS is a good and logical choice with the added advantage of reducing space requirements. Furthermore, since HM resembles the short-term memory of a musician and since the short-term memory of the human is known to be small, it is logical to use a small HMS. In this paper, the numbers 10, 20 and 30 are chosen as different values of HMS.

The $HMCR$ is the probability of choosing HM. Choosing a very small $HMCR$ decreases the algorithm efficiency and the HS algorithm behaves like a pure random search, with less assistance from the HM. Hence, it is generally better to use a large value for the $HMCR$ (i.e. ≥ 0.9). In this research 0.93, 0.95 and 0.99 have been used for $HMCR$. The pitch adjustment is similar to the adjustment of each musical instrument in a jazz so that pleasing harmony can be achieved. The efficiency of the algorithm lies within this pitch adjustment because of the fact that once a feasible design is determined, it searches new solution vectors around this design vector rather than generating arbitrary design vectors. Thus, this operation prevents stagnation and improves the HM for diversity with a greater chance of reaching the global optimum. The PAR is the probability of pitch adjustment where its typical value ranges from 0.3 to 0.99. In this research, 0.3, 0.7 and 0.9 have been utilized for PAR.
The value of N, the number of variables for optimization, is fully depended on the characteristics of the problem. Finally, NI is the maximum number of iterations that the objective function is evaluated In this research, 100, 500 and 1000 are chosen as different iteration numbers.

5.2.3. Harmony memory initialization

The HM is a two-dimensional matrix with HMS rows and $N+1$ columns. The last column of HM is specified by the value of the objective function for each solution vector. Figure (1) shows a sample HM in which X_i^j is one of the decision variables used in HS algorithm, $f(x^j)$ is the value of the objective function for j^{th} vector solution, *i* indicates the index of the decision variable in vector *X*, and *j* is used as an index for the vector solution in HM. The HM is initialized with randomly generated solutions in a specific range limited by upper and lower bounds determined by the problem at hand. However, because of the constraints described in section 4.2.4, only those solution vectors that satisfy the constraints are included in HM.

5.2.4. New harmony generation

New Harmony improvisation is based on three rules: (i) random selection (ii) HM consideration, and (iii) pitch adjustment. In random selection rule, the new value of each decision variable x_i' is randomly chosen within the allowable range of the vector solution $\mathbf{X^j}$. Then, $\mathbf{X}' = \left[x'_1, x'_2, \ldots, x'_N\right]$ will represent the new vector solution. In HM algorithm, the random choosing from HM occurs with probability $HMCR$ and the random selection is performed with probability $1-HMCR$. Algorithm (1) shows the choosing and the selection processes.

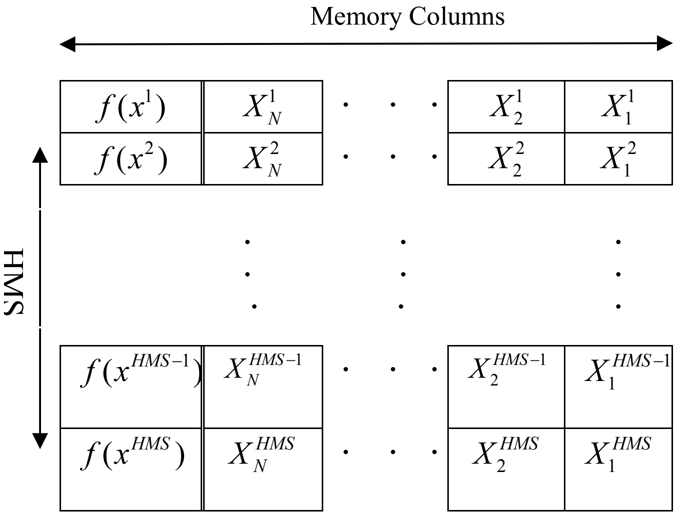

Fig. 1. The sample of harmony memory

For $i = 1:N$
  If
$Rand_i < HMCR$; $Rand_i \sim Uni(0,1)$
   $x_i' \leftarrow x_i' \in [x_i^1, x_i^2, \cdots, x_i^{HMS}]$
  Else
$x_i' \leftarrow$ generate a new one within the allowable range
  End If
End For

Algorithm (1): The choosing and selection processes of HM algorithm

In pitch adjustment, every component obtained by the memory consideration, is examined to determine whether it should be pitch adjusted or not. The value of the decision variable is changed by equation (29) with probability of PAR, and this value is kept without any change with probability $1-PAR$. In equation (29), BW stands for band width and denotes the amount of change for pitch adjustment, and *rand* is a uniform random number between 0 and 1. In this equation, for each component of the vector the selection for increasing or decreasing are carried out with the same probability.

$$\mathbf{X}' = \mathbf{X}' \pm (rand)(BW) \; ; \quad rand \sim U\left[0,1\right] \quad (29)$$

5.2.5. Harmony memory update

The constraint handling part of the algorithm is performed before the HM update and checks whether the constraints of model (28) are satisfied or not. If they are satisfied, then the HM update action occurs. In this stage, by the objective function evaluation, if the new fitness value is better than the worst case in the HM, the worst harmony vector is replaced by the new solution vector. The remaining steps of the HM algorithm are performed after the HM update.

5.2.6. Stopping criterion

The last step in a HS method is to check if the algorithm has found a solution that is good enough to meet the user's expectations. Stopping criteria is a set of conditions such that when satisfied a good solution is obtained. Different criteria used in literature are: 1) stopping the algorithm after a specific number of iterations, 2) no improvement in the objective function, and 3) reaching a specific value of the objective function. In this research, we stop when a predetermined number of consecutive iteration is reached. The number of sequential iterations depends on the specified problem and the expectations of the user.

5.3. Fuzzy simulation

In order to estimate the uncertain costs of the fuzzy model, we employ a simulation technique. Denoting $\hat{h}_{ij}$ by $\hat{h}_{ij}$ = ($\hat{h}_{i1}, \hat{h}_{i2}, \ldots, \hat{h}_{iT}$), μ as the membership function of $\hat{h}$, and μ_{ij} are the membership functions of $\hat{h}_{ij}$, we randomly generate h_{ijk} from the α-level sets of fuzzy variables $\hat{h}_{ij}$, $i = 1,2,...,n$, $j = 1,2,\cdots,T$ and $k = 1,2,...,K$ as $h_k = (h_{i1k}, h_{i2k}, \cdots, h_{iTk})$ and $\mu(h_{ijk}) = \mu_1(h_{i1k}) \wedge \mu_2(h_{i2k}) \wedge, \cdots, \wedge \mu_t(h_{iTk})$, where α is a sufficiently small positive number.
Based on the definition the expected value of the fuzzy variable is:

$$E\left[Z(\hat{h},\hat{\pi},Q)\right] = \int_0^{+\infty} Cr\left\{Z(\hat{h},\hat{\pi},Q) \geq r\right\} dr - \int_{-\infty}^{0} Cr\left\{Z(\hat{h},\hat{\pi},Q) \leq r\right\} dr \quad (30)$$

Then, provided N_{FS} is sufficiently large, for any number $r \geq 0$, $Cr\left\{Z(\tilde{\xi},Q) \geq r\right\}$ can be estimated by:

$$Cr\left\{Z(\hat{h},\hat{\pi},Q) \geq r\right\} = \frac{1}{2}\left(\max_{k=1,2,...,N}\left\{\mu_k \left| Z(\hat{h},\hat{\pi},Q) \geq r\right.\right\} + 1 - \max_{k=1,2,...,N}\left\{\mu_k \left| Z(\hat{h},\hat{\pi},Q) < r\right.\right\} \right) \quad (31)$$

And for any number $r < 0$, $Cr\left\{Z(\hat{h},\hat{\pi},Q) \leq r\right\}$ can be estimated by:

$$Cr\left\{Z(\hat{h},\hat{\pi},Q) \leq r\right\} = \frac{1}{2}\left(\max_{k=1,2,...,N}\left\{\mu_k \left| Z(\hat{h},\hat{\pi},Q) \leq r\right.\right\} + 1 - \max_{k=1,2,...,N}\left\{\mu_k \left| Z(\hat{h},\hat{\pi},Q) > r\right.\right\} \right) \quad (32)$$

5.4. The solution procedure

In summary, the hybrid procedure of HS, GP and FS involves the following steps:

Step0. Initialize both the HM of the HS algorithm and FS parameters.

Step1. Set $E_{ij} = 0, G_j = 0$

Step2. *Randomly generate* h_{ijk} *and* π_{jk} *from α-level sets of fuzzy variables* $\hat{h}_{ij}, \hat{\pi}_j$, *and set* $h_k = (h_{i1k}, h_{i2k}, \cdots, h_{iTk}), \pi_k = (\pi_{1k}, \pi_{2k}, \cdots, \pi_{Tk})$

Step3. Set $a_{ij}^1 = h_{j1} \wedge h_{j2} \wedge \cdots \wedge h_{jk}$, $a_j^2 = \pi_{j1} \wedge \pi_{j2} \wedge \cdots \wedge \pi_{jk}$.

$b_{ij}^1 = h_{j1} \vee h_{j2} \vee \cdots \vee h_{jk}$, $b_j^2 = \pi_{j1} \vee \pi_{j2} \vee \cdots \vee \pi_{jk}$

Step4. *Randomly generate* r_{j1}, r_{j2} *from Uniform* $[a_{j1}, b_{j1}], [a_{j2}, b_{j2}]$ *respectively.*

Step5. If $r_{j1} \geq 0$, *then* $E_{ij} \leftarrow E_{ij} + Cr\left\{\hat{h}_{ij} \geq r_{j1}\right\}$.

Step6. If $r_{j1} < 0$, *then* $E_{ij} \leftarrow E_{ij} - Cr\left\{\hat{h}_{ij} \leq r_{j1}\right\}$.

Step7. If $r_{j2} \geq 0$, *then* $G_j \leftarrow G_j + Cr\left\{\hat{\pi}_j \geq r_{j2}\right\}$.

Step8. If $r_{j2} < 0$, *then* $G_j \leftarrow G_j - Cr\left\{\hat{\pi}_j \leq r_{j2}\right\}$.

Step9. *Repeat the four to nine steps for* N_{FS} *times.*

Step10. $E\left[\hat{h}_{ij}\right] = a_{ij}^{1} \vee 0 + b_{ij}^{1} \wedge 0 + E_{ij} \times \dfrac{b_{ij}^{1} - a_{ij}^{1}}{N_{FS}}$.

Step11.

$$E\left[\hat{\pi}_{j}\right] = a_{j}^{2} \vee 0 + b_{j}^{2} \wedge 0 + G_{j} \times \frac{b_{j}^{2} - a_{j}^{2}}{N_{FS}}$$

Step12. Make a new vector $\mathbf{X}'$. For each component x_i' :

- With probability $HMCR$ pick the component from memory,
- With probability $1-HMCR$ pick a new random value in the allowed range.

Step13. Pitch adjustment: For each component x_i' :

- With probability PAR , a small change is made to x_i' .
- With probability $1-PAR$ do nothing.

Step14. If $\mathbf{X}'$ is better than the worst $\mathbf{X}^{\mathbf{j}}$ in the memory, then replace $\mathbf{X}^{\mathbf{j}}$ by $\mathbf{X}'$.
Step15. Go to step 2 until a maximum number of iterations has been reached.

6. A numerical example

Consider a multi-product single period newsboy problem with fifteen products and deterministic and fuzzy general data given in Tables (1) and (2), respectively. The total available warehouse space is 1750 square meters; the fixed cost for each shipment is 500, $K_j = 0.1C_j$ in incremental discount. Moreover, $W_1 = W_2 = 1, p_1 = 1, p_2 = 10000$, $b_1 = 10349$, and $b_2 = 1$. Table (3) shows different values of the HS parameters used to obtain the solution. In this research all of the possible combinations of the HS parameters (C_2, C_1 and N) are employed and using the *max(max)* criterion the best combination of the parameters has been selected. Table (4) shows the best results of the algorithm. The best combinations of the parameters of HS algorithm are $HMCR = 0.95$, $HMS = 10$, $PR = 0.7$ and $NI = 1000$.
Furthermore, in fuzzy simulation we employed $\alpha = 0.9$ and $N_{FS} = 10$.

Table 1
Deterministic general data

(j)	1	2	3	4	5	6	7	8	9	10
V_j	3	6	5	1	10	3	10	1	6	5
λ_j	62	89	102	66	19	91	123	52	83	95
f_j	4	5	3	1	5	6	2	4	4	3
C_{1_j}	10	30	18	100	35	28	30	100	25	20
C_{2_j}	8	27	15	90	30	25	25	90	21	18
C_{3_j}	7	24	12	80	28	22	23	80	18	15
C_{4_j}	5	21	10	75	20	20	18	60	16	13
q_{1_j}	15	40	30	30	40	40	40	20	15	30
q_{2_j}	30	70	90	45	70	70	70	40	35	90
q_{3_j}	50	100	100	70	140	100	110	60	70	100

Table 2
Fuzzy general data

(j)	h_{1_j}	h_{2_j}	π_{1_j}	π_{2_j}
1	[1,2,3]	[2,3,4]	[25,30,35]	[30,35,40]
2	[1,3,5]	[3,4,5]	[8,10,12]	[13,15,17]
3	[1,2,3]	[2,3,4]	[20,21,22]	[24,26,28]
4	[3,4,5]	[4,5,6]	[25,30,35]	[30,35,40]
5	[3,5,7]	[4,6,8]	[10,15,20]	[15,20,25]
6	[1,2,3]	[2,3,4]	[40,45,50]	[50,52,54]
7	[0.5,1,1.5]	[1.5,2,2.5]	[5,7,9]	[10,12,14]
8	[4,5,6]	[5,6,7]	[40,45,50]	[45,50,55]
9	[2,4,6]	[3,5,7]	[28,30,32]	[40,45,50]
10	[4,5,6]	[5,6,7]	[32,34,36]	[35,40,45]

Table 3
The parameters of the HS algorithm

HMS	HMCR	PAR	NI
10	0.93	0.30	100
20	0.95	0.70	500
30	0.99	0.90	1000

Table 4
The best values of variables

(j)	1	2	3	4	5	6	7	8	9	10
Q_j	93	72	85	49	60	78	80	57	54	95
Z	3250									
Z_{SR}	0.8726									

7. Conclusion and recommendations for future research

In this paper, a multi-product single period newsboy problem with incremental discount policy in which the batch orders and warehouse space are constraints, was developed. Then, the meta-heuristic solution algorithm of FS + GP + HS has been proposed to solve the obtained non-linear integer model. At the end, a numerical example was given to demonstrate the application of the proposed method. Some of the future works of this research are:

- The holding and shortage costs may be considered to occur during a period.
- Emergency order can be deployed to the model to overcome the shortage.

8. References

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