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Research Article

An Inventory Lot Sizing Model Of Deteriorating Items with Time and Price Dependent Demand, by Considering the Time Value of Money

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Abstract

In this paper, an inventory lot sizing model is proposed for a single deteriorating product with time and price dependent demand, by considering the time value of money.it is assumed the rate of deterioration is constant, the interest is compounded continuously, and the shortage happens in the form of partial backorder. The product is purchased from several suppliers at different prices and sold at a unique price. The closed form solution is presented for a special case with no shortage. A numerical example is solved and analyzed in the GAMS software. It is shown that with an increase in the rate of deterioration, the model decreases the selling price in order to stimulate the demand and deplete the positive inventory faster to avoid extensive deteriorations. In addition, the fraction of time with positive inventory level is reduced. The sensitivity analysis of the interest rate showed that as the interest rate increases, the model increases the economic order size while reduces the selling price in order to get higher positive net cash flows as soon as possible. With the increase of the shortage costs, the model tried to expose less shortage by increasing the fraction of time with positive inventory level. This resulted in more deterioration in the inventory and required larger order size.

Keywords: Lot sizing, Deterioration; Time value of money; Price dependent demand; Time dependent demand; Partial backorder.

1. Introduction

The management of inventory items is an important issue for the efficient performance of the supply chain. The main issue is the determination of the optimal order quantities by considering the existing constraints, in order to minimize the total costs associated with ordering, purchasing, holding and shortages. In many studies, inventory items are considered to be durable which means they preserve their initial condition in the entire cycle. But a group of inventory items are subject to change through the time. The first category includes items whose value is lost over time due to new technology or introduction of alternative, such as computer chips, mobile phones, fashion and seasonal products and so on. These items have a short life span in the market. After a period of popularity in the market, they lose their economic value due to changes in consumer preference; product upgrades and other similar reasons. The second category of items are deteriorated, damaged, evaporated or expired over time, such as meat, vegetables, fruits, medicines and flowers. Both categories have a short life span and are exposed with more complicated issues in comparison to durable items.

In the literature, the rate of deterioration is of a variety of types, including a constant rate of deterioration, which is a linear function of time. The modeling of constant considered two-parameter or three-parameter Weibull distribution deterioration rate.

Another key issue in the inventory modeling is the way that considers the capital opportunity cost. In most studies and approximate method that considers a percentage of the product marginal cost is utilized. Time value of money is a more precise method that considers the net present value (NPV) of all the inventory related cash flows. The time value of money was first examined by Hadley in 1964. He compared the order values calculated using the average annual cost and discounted price in the case of a lack of available inventory, and concluded with numerical examples that the total in cost in the two models are almost equal. Ghiami (2023) compares classic and NPV methods in some the well-known inventory control problems and concludes that, the classic approach underestimates the optimal profit function in comparison to the NPV framework.

Since deteriorating items lose their initial condition through the time, their demand is very dependent on their age and the retailers might provide price discounts to stimulate the demand. Time and price dependent demand adds to the complexity of the inventory decisions. The integrated approach in deteriorating items with price and time dependent demand is the main subject of our research. In this regard, in the following we provide an overview of the research that has been done so far on deterioration items and time value of money.

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Ghare and Schrader (1963) first introduced the issue of product deterioration into inventory control issues. After that, Eilon and Mallaya (1966) expanded the model and introduced demand for product-related prices. Cohen (1977) presented a model for the inventory system, taking into account the shortage of deteriorating items with a deteriorating rate that follows exponential distribution. Kang and Kim (1983) expanded Cohen's model and developed the inventory system with a limited production rate and no shortage. Aggarwal and Jaggi (1989) pointed out in their paper the existence of a defect in the Cohen model and provided a simpler method for calculating product prices and optimal production policies. Subsequently, Wee (1997) presented their model for corrupted items which failure rates comply with the Weibull distribution. Ghosh and Chaudhuri (2006) developed an economic order quantity model for deteriorating items with the assumption that the shortage is allowed in the case of time-dependent demand. In their model, the horizons are considered unlimited. Mukhopadhyay et al. (2005) developed the model of the economic order quantity for deteriorating items in situations where demand is price dependent. In their model, the rate of deterioration follows the Weibull distribution. Dye (2007) developed a deterministic inventory model for deteriorating items with two warehouses. A rented warehouse is used when the ordering quantity exceeds the limited capacity of the owned warehouse, and it is assumed that deterioration rates of items in the two warehouses may be different. In addition, shortage is allowed in the warehouse and assumed that the backlogging demand rate is dependent on the duration of the stock out. Sankar Sana (2010) developed a model with an optimal selling price and lot size with time varying deterioration and partial backlogging. He developed an EOQ (economic order quantity) model over an infinite time horizon for perishable items where demand is price dependent and partial backorder is permitted. The rate of deterioration is taken to be time proportional and it is assumed that shortage occurs at starting of the inventory cycle. Valliathal and Uthayakumar (2011) conducted a new study of an EOQ model for deteriorating items with shortages under inflation. Wee (1997) discuss the effects of inflation on an EOQ model for deteriorating items under stock-dependent demand and time-dependent partial backlogging. The inventory model is studied under the replenishment policy starting with no shortages. Begum et al. (2012) examined a replenishment policy for items with price-dependent demand, time-proportional deterioration and no shortages. In their article, an order up to level inventory system for deteriorating items has been developed with demand rate as a function of selling price. The demand and the deterioration rate were price dependent and time proportional, respectively. They considered a perishable item that follows three-parameter Weibull distribution deterioration. Tan and Weng (2012) investigated a discrete-time deteriorating inventory model with time-varying demand, variable deterioration rate and waiting-time-dependent partial backlogging. A new inventory system is considered for deteriorating items in which unsatisfied demands are partially backlogged depending on the waiting time until the next replenishment while deterioration rates are constant, but different from period to period. Maihami and Nakhai Kamalabadi (2012) developed joint pricing and inventory control for non-instantaneous deteriorating items with partial backlogging and time and price dependent demand. They adopted a price and time dependent demand function. Shortages was allowed and partially backlogged. The major objective was to determine the optimal selling price, the optimal replenishment schedule and the optimal order quantity simultaneously such that, the total profit is maximized. Ghiami et al. (2013) developed a two-echelon supply chain model for deteriorating inventory in which the retailer’s warehouse has a limited capacity. The system includes one wholesaler and one retailer and aims to minimize the total cost. The demand rate in retailer is stock-dependent and in case of any shortages, the demand is partially backlogged. Sarkar and Sarkar (2013) examined an inventory model for deteriorating items with inventory dependent demand function. Most of the inventory models are considered with constant rate of deterioration. They consider time varying deterioration rate. Based on the demand and inventory, the model was considered with three possible cases. Panda et al. (2013) provided an optimal pricing and lot-sizing for perishable inventory with price and time dependent demand. They proposed a dynamic pre- and post-deterioration cumulative discount policy to enhance inventory depletion rate. It was assumed that demand is a price and time dependent ramp-type function and the product starts to deteriorate after certain amount of time. Unlike the conventional inventory models with pricing strategies, which are restricted to a fixed number of price changes and to a fixed cycle length, they allowed the number of price changes before as well as after the start of deterioration and the replenishment cycle length to be the decision variables. Sarkar et al. (2013) examined an inventory model with variable demand, component cost and selling price for deteriorating items. They developed an economic order quantity model for finite production rate and deteriorating items with time dependent increasing demand. The component cost and the selling price were considered at a continuous rate of time. Sicilia et al. (2014) reviewed an inventory model for deteriorating items with shortages and time-varying demand. A deterministic inventory system for items with a constant deterioration rate was studied. Demand varied in time and it was assumed that it follows a power pattern. Shortages were allowed and backlogged. The ordering cost, the holding cost, the backlogging cost, the deteriorating cost, and the purchasing cost were considered in the inventory management. Chaudhary and Sharma (2015) reviewed a model based on Weibull deteriorating rate with price dependent demand and inflation. Commodities such as fruits, vegetables and foodstuffs suffer from depletion by direct spoilage while kept in store. The deterioration rate follows Weibull distribution with two parameters. Demand rate is assumed as price dependent in linear form. It is an ordinary fact that unique price of items attracts more customers. Pal et al. (2015) presented a production inventory model for deteriorating items with ramp type demand allowing inflation and shortages under fuzziness. The deterioration rate was represented by a two-parameter Weibull distribution. As inflation erodes the value of money so they also considered the effect of inflation when there was shortage in the stock under finite time horizon. Wee and Law (1999) presented an inventory model for a system with price dependent demand and deteriorating items whose failure rates follow a Weibull distribution and the shortage is allowed, taking into account the time value of money. Taleizadeh and Nematollahi (2014) reviewed an inventory control problem for deteriorating items with backordering and financial considerations. They examine the effects of time value of money and inflation on the optimal ordering policy in an inventory control system. The demand and deterioration rates are constant. Ouyang et al. (2003) reviewed an inventory model for deteriorating items with stock-dependent demand under the conditions of inflation and time value of money. They incorporated the effects of inflation and time-value of money in inventory decision making when demand, at each time moment rather than being constant, is considered to be dependent upon current stock level. In addition, the shortages are neither completely backlogged nor completely lost assuming the backlogging rate to be linearly dependent on the amount of demand backlogged. Vikas et al. (2016) presented an EOQ models with optimal replenishment policy for perishable items taking into account the time value of money. They followed the Discounted Cash Flow (DCF) approach to investigate inventory replenishment problem over a fixed planning horizon.

Based on the literature study of deteriorating items, the time value of money is less considered in the modeling while it seems that sometimes yields meaningful differences in comparison to classic modeling. In addition, since deteriorating items lose their initial condition through the time, the time and price-dependent demand should be considered in the modeling. Therefore, in this paper, an inventory lot Sizing model of deteriorating items with time and price dependent demand is developed which considers the time value of money.

In this regard consider a retailer which buys a product from a few suppliers at different prices and sells at the same price in the market as shown in Figure (1). The product is deteriorated with a constant rate. Demand is dependent on time and price, and the time value of money is included in the model. The net present value (NPV) is the utilized method in considering the time value of money and the interest is compounded continuously. The total order quantity is received from suppliers at the beginning of each replenishment cycle. The shortage is allowed, and happens in the form of partial backorder, that is, a fraction of orders are backlogged and another fraction of the order is lost. The objective is to maximize the net present value of the total profit. The main variables are the amount of economic order to each supplier in each replenishment cycle, the optimal cycle time, maximum positive inventory level and maximum backorder in each replenishment cycle. In the next section the formulation of the problem is presented.

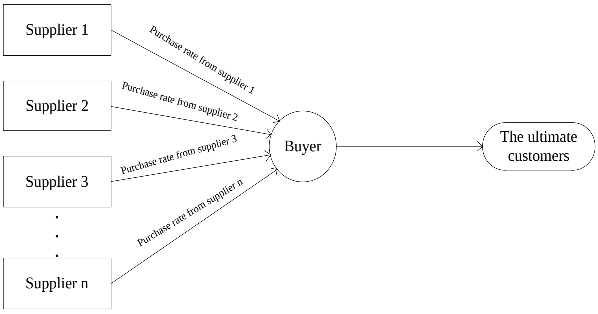


Fig. 1. Schematic of the supply chain problem

2. Model formulation

Consider a single buyer that orders a deteriorating product to multiple suppliers which propose different prices. The retailer sells the product with a constant price in the market and the shortage is allowed and happens in the form of partial backorder. The problem is to determine the optimal order size, replenishment cycle, maximum positive inventory level and maximum backorder in each cycle. We use the NPV approach and the interest is compounded continuously. The following assumptions and notation are considered in the model:

2.1. Assumptions

- The inventory items are deteriorated with a constant rate.
- All parameters of the model are definite.
- The demand is linearly depended to price and exponentially depended to time which is presented by $d(p,t) = (a - bp)e^{-\lambda t}$ (a and b are constant and positive values).
- The time value of money is included in the model and the interest is continuously compounded.
- The shortage is allowed and is in the form of partial backorder.
- There is a single deteriorating product with several suppliers.

2.2. Notation

In the following, we describe the indices, parameters and variables.

Index

$j \in \{1, .., J\}$ Index of Suppliers

Parameters

$d(p,t)$ The demand which is dependent on time and price

h	Unit holding cost
ρ	Nominal interest rate per unit time
A_j	Ordering cost to Supplier j
$\hat{\pi}$	Unit backorder cost
π	Unit lost sale cost
c_j	Unit purchasing cost from Supplier j
β	The fraction of demand that is backordered in the period of shortage
θ	The deterioration rate
cap_j	The capacity of Supplier j

Variables

Q	Order quantity
I_{max}	Maximum positive inventory level
b_{max}	Maximum backordered demand in a cycle
$I(t)$	Positive inventory level at the moment t
$b(t)$	Backordered shortage at the moment t
t_1	The length of time with positive inventory level
t_2	The length of time with shortage
T	Cycle time
x_j	The fraction of order purchased from Supplier j
p	Unit selling price on the market

The zero and one variable

$$Z_j : \begin{cases} 0 & if\ Supplier\ j\ is\ not\ selected\ (x_j = 0) \\ 1 & if\ Supplier\ j\ is\ selected\ (x_j > 0) \end{cases}$$

2.3 Formulation of the model

As it is shown in Figure (2), at the beginning of each period, Q items are received where b_{max} units are allocated to backordered demand and I_{max} units enter the warehouse. The positive inventory of the warehouse gets to zero in the time period of 0 to t_1, then shortage occurs at intervals t_1 to T.

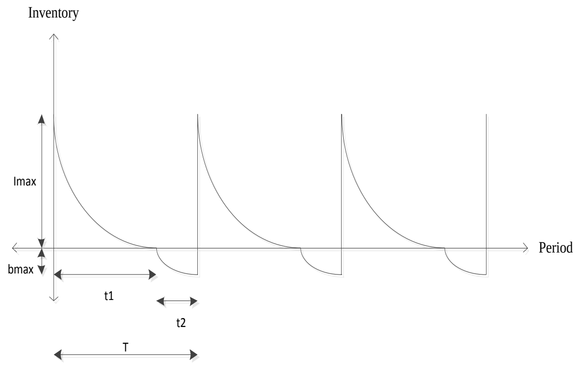


Fig. 2. The inventory level of an item in a cycle

The differential equation of inventory level with respect to time is expressed by a non-linear function as follows:

$$\frac{dI(t)}{dt} + \theta I(t) = -d(p,t) \quad 0 \le t \le t_1 \tag{1}$$

In this regard, θ is the rate of deterioration, $I(t)$ is the positive inventory level, and $d(p,t)$ is time and price dependent demand. In fact, Equation (1) shows that the inventory level decreases in time due to the demand and deterioration. By solving differential equation (1) with respect to $I(t)$ the positive inventory level is presented by Equation (2).

$$I(t) = \frac{(a-bp)}{(\theta-\lambda)}\left[e^{(\theta-\lambda)t_1-\theta t} - e^{-\lambda t}\right] \quad 0 \le t \le t_1 \tag{2}$$

Given the boundary condition $I(0) = I_{max}$, the maximum positive inventory level (I_{max}) is obtained as follows.

$$I_{max} = I(0) = \frac{(a-bp)}{(\theta-\lambda)}\left[-1 + e^{(\theta-\lambda)t_1}\right] \tag{3}$$

Since the change of inventory level in the period of shortage is only due to demand, the negative inventory level is calculated by Equation (4).

$$b(t) = \frac{(a-bp)\beta}{\lambda}\left[e^{-\lambda t} - e^{-\lambda t_1}\right] \quad t_1 \le t \le T \tag{4}$$

Given the boundary condition $b(T) = -b_{max}$, the maximum backorder level (b_{max}) is obtained from Equation (5).

$$b_{max} = -\frac{(a-bp)\beta}{\lambda}\left[e^{-\lambda T} - e^{-\lambda t_1}\right] \tag{5}$$

Now we can calculate inventory related costs as follows.

Ordering cost per cycle (OC)

$$OC = -\sum_{j=1}^{J} A_j Z_j \tag{6}$$

Purchase cost per cycle (PC)

The purchasing quantity in each period is obtained by the summation of maximum positive and negative inventory levels. Therefore the purchasing cost in each cycle is presented by Equation (7)

$$PC = -\sum_{j=1}^{J} Qx_j c_j$$

$$= -\sum_{j=1}^{J}\left[\left(\frac{a-bp}{\theta-\lambda}\left[-1+e^{(\theta-\lambda)t_1}\right]\right)-\left(\frac{(a-bp)\beta}{\lambda}\left[e^{-\lambda T}-e^{-\lambda t_1}\right]\right)\right]x_j c_j \quad (7)$$

Holding cost per cycle (HC)

$$HC = -\int_0^{t_1} hI(t)e^{-\rho t}\,dt$$

$$= -\int_0^{t_1} h\left(\frac{a-bp}{\theta-\lambda}\right)\left[-e^{-\lambda t}+e^{(\theta-\lambda)t_1-\theta t}\right]e^{-\rho t}dt$$

$$= -h\left(\frac{a-bp}{\theta-\lambda}\right)\left[\frac{1}{\lambda+\rho}\left[e^{-(\rho+\lambda)t_1}-1\right]-\frac{1}{(\theta+\rho)}\left[e^{-(\rho+\lambda)t_1}-e^{(\theta-\lambda)t_1}\right]\right] \quad (8)$$

Backorder shortage cost per cycle (BC)

$$BC = -\int_{t_1}^{T}\hat{\pi}b(t)e^{-\rho t}dt$$

$$= -\int_{t_1}^{T}\hat{\pi}\left(\frac{(a-bp)\beta}{\lambda}\right)\left[e^{-\lambda t}-e^{-\lambda t_1}\right]e^{-\rho t}dt$$

$$= -\frac{\hat{\pi}\beta(a-bp)}{\lambda}\left[-\frac{1}{(\lambda+\rho)}e^{-(\lambda+\rho)T}+\frac{1}{\rho}e^{-\rho T-\lambda t_1}+\frac{1}{(\lambda+\rho)}e^{-(\lambda+\rho)t_1}-\frac{1}{\rho}e^{-(\rho+\lambda)t_1}\right] \quad (9)$$

Lost sale cost per cycle (LS)

$$LS = -\int_{t_1}^{T}\pi d(p,t)(1-\beta)e^{-\rho t}dt$$

$$= -\int_{t_1}^{T}\pi(a-bp)e^{-\lambda t}(1-\beta)e^{-\rho t}dt = \frac{1}{\lambda+\rho}\pi(a-bp)(1-\beta)\left[e^{-(\lambda+\rho)T}-e^{-(\lambda+\rho)t_1}\right] \quad (10)$$

Revenue per cycle (RV)

$$RV = \int_0^{t_1} pd(p,t)e^{-\rho t}dt+\int_{t_1}^{T} pd(p,t)\beta e^{-\rho t}dt = \int_0^{t_1} p(a-bp)e^{-\lambda t}e^{-\rho t}dt+\int_{t_1}^{T} p(a-bp)e^{-\lambda t}\beta e^{-\rho t}dt =$$

$$= p(a-bp)\left[-\frac{1}{(\lambda+\rho)}\left(e^{-(\lambda+\rho)t_1}-1\right)\right]+p\beta(a-bp)\left[-\frac{1}{(\lambda+\rho)}\left(e^{-(\lambda+\rho)T}-e^{-(\lambda+\rho)t_1}\right)\right] \quad (11)$$

In order to determine the optimal value of the decision variables, the objective function must be calculated which is difference between revenue and costs, and the cost function includes the costs of ordering, purchasing, holding and shortage. Ultimately the net present value in a cycle that is represented by NPV^c it is obtained as follows:

$$NPV^c = -\sum_{j=1}^{J} A_j Z_j - \sum_{j=1}^{J}\left[\left(\frac{a-bp}{\theta-\lambda}\left[-1+e^{(\theta-\lambda)t_1}\right]\right)-\left(\frac{(a-bp)\beta}{\lambda}\left[e^{-\lambda T}-e^{-\lambda t_1}\right]\right)\right]x_j c_j - h\left(\frac{a-bp}{\theta-\lambda}\right)\left[\frac{1}{\lambda+\rho}\left[e^{-(\rho+\lambda)t_1}-1\right]-\frac{1}{(\theta+\rho)}\left[e^{-(\rho+\lambda)t_1}-e^{(\theta-\lambda)t_1}\right]\right]-\frac{\hat{\pi}\beta(a-bp)}{\lambda}\left[-\frac{1}{(\lambda+\rho)}e^{-(\lambda+\rho)T}+\frac{1}{\rho}e^{-\rho T-\lambda t_1}+\frac{1}{(\lambda+\rho)}e^{-(\lambda+\rho)t_1}-\frac{1}{\rho}e^{-(\rho+\lambda)t_1}\right]+\frac{1}{\lambda+\rho}\pi(a-bp)(1-\beta)\left[e^{-(\lambda+\rho)T}-e^{-(\lambda+\rho)t_1}\right]+p(a-bp)\left[-\frac{1}{(\lambda+\rho)}\left(e^{-(\lambda+\rho)t_1}-1\right)\right]+p\beta(a-bp)\left[-\frac{1}{(\lambda+\rho)}\left(e^{-(\lambda+\rho)T}-e^{-(\lambda+\rho)t_1}\right)\right] \quad (12)$$

When we have a stable cash flow that is equal to f in each T period, its NPV is obtained from Equation (13).

$$NPV = f(1+e^{-\rho T}+e^{-2\rho T}+e^{-3\rho T}+\cdots) = \frac{f}{1-e^{-\rho T}} \quad (13)$$

Therefore, the final modeling of the problem is as follows, and NPV of the objective function for the whole periods is obtained from the following equations.

$$Max\ NPV = NPV^c(1+e^{-\rho T}+e^{-2\rho T}+e^{-3\rho T}+\cdots) = \frac{NPV^c}{1-e^{-\rho T}} \quad (14)$$

Subject to

$$\sum x_j = 1 \qquad \forall j \in \{1,..,J\} \quad (15)$$

$$z_j \geq x_j \qquad \forall j \in \{1,..,J\} \quad (16)$$

$$\left(\frac{a-bp}{\theta-\lambda}\left[-1+e^{(\theta-\lambda)t_1}\right]\right)-\left(\frac{(a-bp)\beta}{\lambda}\left[e^{-\lambda T}-e^{-\lambda t_1}\right]\right)x_j/T \leq cap_j \quad \forall j \in \{1,..,J\} \quad (17)$$

$$x_j \geq 0 \quad \forall j \in \{1,..,J\} \quad (18)$$

$$z_j \epsilon \{0,1\} \quad \forall j \in \{1,..,J\} \quad (19)$$

Constraint (15) ensures that the total accumulated demand in each cycle is purchased from suppliers. Constraint (16) guarantees that if no supplier is selected, no order will be assigned to it. Constraint (17) ensures that supplier's capacity is not violated. Constraint (18) and (19) specify the types of variables.

The cash flow chart is shown in Figure (3). Since the inventory level is decreasing over time and the cost of storage is a percentage of the inventory level, the cash flow of storage cost is decreasing. Due to the increase in the level of shortages during the period, cash flow of shortage cost increases. Revenue is time-dependent, and demand is declining over time, thus the revenue is declining. In the period when shortage occurs and is in the form of partial backorder, the slope of revenue is decreasing further.

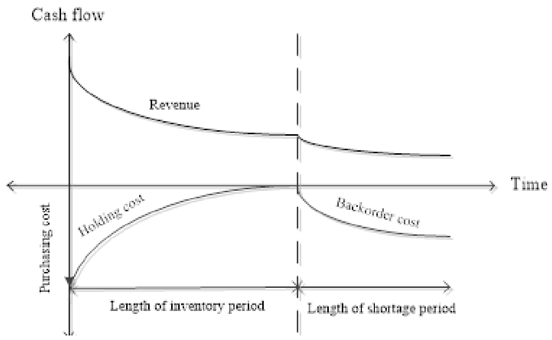


Fig. 3. Cash flow in each period

Analysis of a special case

Theorem: In the first scenario, if we have a supplier and we do not consider the capacity constraint and the shortage is not allowed, the optimal answer to the problem is obtained from the following equation:

$$T^* = \sqrt{\frac{2\,A}{\frac{1}{6}A\rho^2+(a-bp)(\theta-\lambda+\rho)c+h(\frac{a-bp}{\theta-\lambda})(3\,\lambda-\theta)}} \quad (20)$$

Proof:

According to the assumptions of the theorem, the backorder and loss sales are eliminated and $t_1 = T$, and the new NPV is as follows:

$$NPV = -\frac{A}{1-e^{-\rho T}} - (\frac{a-bp}{\theta-\lambda})\left[\frac{e^{(\theta-\lambda)T}-1}{1-e^{-\rho T}}\right]c - h(\frac{a-bp}{\theta-\lambda})[\frac{1}{\lambda+\rho}(\frac{e^{-(\rho+\lambda)}-1}{1-e^{-\rho T}}) - \frac{1}{\theta+\rho}\left[\frac{e^{-(\rho+\lambda)T}-e^{(\theta-\lambda)T}}{1-e^{-\rho T}}\right] - p(a-bp)\left[\frac{1}{\lambda+\rho}\left(\frac{e^{-(\lambda+\rho)T}-1}{1-e^{-\rho T}}\right)\right] \quad (21)$$

By placing the following estimates in the equation above, the Equation (27) is obtained:

$$\frac{1}{1-e^{-\rho T}} = \frac{1}{\rho T} + \frac{1}{2} + \frac{\rho T}{12} \quad (22)$$

$$\frac{e^{-\rho T}}{1-e^{-\rho T}} = \frac{1}{\rho T} + \frac{1}{2} - 1 + \frac{1}{12}T\rho = \frac{1}{\rho T} - \frac{1}{2} + \frac{\rho T}{12} \quad (23)$$

$$\frac{e^{ax}}{1-e^{bx}} = -\frac{1}{bx} + \frac{1}{2} - \frac{a}{b} + \frac{1}{12}x(\frac{-6a^2}{b} + 6\,a - b) \quad (24)$$

$$\frac{1-e^{ax}}{1-e^{bx}} = \frac{a}{b} + \frac{a(a-b)}{2\,b}x \quad (25)$$

$$\frac{e^{ax}-e^{bx}}{1-e^{cx}} = \frac{b-a}{c} - \frac{(a-b)(a+b-c)}{2\,c}x \quad (26)$$

$$NPV = -A\left(\frac{1}{\rho T} + \frac{1}{2} + \frac{\rho T}{12}\right) + \left(\frac{a-bp}{\theta-\lambda}\right)\left(\frac{\theta-\lambda}{-\rho} + \frac{(\theta-\lambda)(\theta-\lambda+\rho)}{-2\,\rho}T\right)c + h\left(\frac{a-bp}{\theta-\lambda}\right)\frac{1}{\lambda+\rho}\left(\frac{-(\rho+\lambda)}{-\rho} + \frac{-(\lambda+\rho)(-\rho-\lambda+\rho)}{-2\,\rho}T\right) + \frac{h}{\theta+\rho}\left(\frac{a-bp}{\theta-\lambda}\right)\left(\frac{\theta-\lambda+\rho+\lambda}{-\,\rho} - \frac{(-\rho-\lambda-\theta+\lambda)(-\rho-\lambda+\theta-\lambda+\rho)}{-2\,\rho}T\right) + p(a-bp)\frac{1}{\lambda+\rho}(\frac{-\lambda-\rho}{-\rho} + \frac{-(\lambda+\rho)(-\lambda-\rho+\rho)}{-2\,\rho}T) \quad (27)$$

We simplify the relations and obtain the Equation (28):

$$NPV = \frac{1}{\rho T}[-A] - \frac{A}{2} - \left(\frac{a-bp}{\rho}\right)c + h\left(\frac{a-bp}{\theta-\lambda}\right)\frac{1}{\rho} - \frac{h}{\rho}\left(\frac{a-bp}{\theta-\lambda}\right) + p(a-bp)\frac{1}{p} + T[\frac{-Ap}{12} + \frac{(a-bp)(\theta-\lambda+\rho)c}{-2\,\rho} + h\left(\frac{a-bp}{\theta-\lambda}\right)\frac{\lambda}{-2\,\rho} + \frac{h(\theta-2\,\lambda)}{2\,\rho}\left(\frac{a-bp}{\theta-\lambda}\right) - p(a-bp)\frac{\lambda}{2\,\rho}] \quad (28)$$

By taking a derivative of Equation (28) with respect to T and solving through Equation (29) and (30), the optimal cycle time is presented by Equation (31):

$$\frac{A}{\rho T^2} = \frac{A\rho}{12} + \frac{(a-bp)(\theta-\lambda+p)c}{2\,\rho} + h\left(\frac{a-bp}{\theta-\lambda}\right)\frac{\lambda}{2\,\rho} - h\left(\frac{a-bp}{\theta-\lambda}\right)\left(\frac{\theta-2\,\lambda}{2\,\rho}\right) = \frac{A\rho}{12} + \frac{(a-bp)(\theta-\lambda+\rho)c}{2\,\rho} + h\left(\frac{a-bp}{\theta-\lambda}\right)\left(\frac{\lambda}{2\,\rho} - \frac{\theta-2\,\lambda}{2\,\rho}\right) = \frac{A\rho}{12} + \frac{(a-bp)(\theta-\lambda+\rho)c}{2\,\rho} + h\left(\frac{a-bp}{\theta-\lambda}\right)(\frac{3\,\lambda-\theta}{2\,\rho}) \quad (29)$$

$$\frac{A}{T^2} = \frac{A\rho^2}{12} + \frac{1}{2}(a-bp)(\theta-\lambda+\rho)c + \frac{1}{2}h(\frac{a-bp}{\theta-\lambda})(3\,\lambda-\theta) \quad (30)$$

$$T^* = \sqrt{\frac{2\,A}{\frac{1}{6}A\rho^2+(a-bp)(\theta-\lambda+\rho)c+h(\frac{a-bp}{\theta-\lambda})(3\,\lambda-\theta)}} \quad (31)$$

3 . Numerical Example

In order to analyze the model presented in this paper, a numerical example is designed and analyzed. This numerical example is such that there is a single item and three suppliers (m, n, p). This example is encoded using the GAMS software and solved with the BONMIN solution tool. The parameters of the numerical example are shown in Tables (1) and (2).

Table 1
Numerical Example Parameters

h	ρ	$\hat{\pi}$	π	β	λ	a	b	θ
0.9	0.0003	0.1	10	0.1	0.005	1300	8	0.01

Table 2
Numerical example parameters for suppliers

Suppliers	cap_j	c_j	A_j
m	50	95	100000
n	40	96	80000
p	60	100	100000

The results of the numerical example are shown in Tables (3) and (4).

Table 3
The selected suppliers and fraction of order allocated to each one

Suppliers	z_j	x_j
m	1	0.333
n	1	0.267
p	1	0.4

Table 4
The optimal value of the variables in the numerical example

T	t_1	p	I_{max}	b_{max}	Q
47.505	32.69	138.252	6890.643	235.126	7125.77

4. Sensitivity Analysis

In this section we will analyze the model by changing the parameters. In fact, with the help of changing a parameter, we examine its effect on the variables of the model and analyze the results.

4.1. Analysis on deteriorating rate

At the beginning we study the effect of deteriorating rate on the model. When the rate of deterioration increases, it results in more deterioration costs and since the model tries to have less inventory levels and try to stimulate the demand to increase the speed of selling to customers. As we expected, this is also shown in the sensitivity analysis of θ with respect to the fraction of $(\frac{t_1}{T})$ and selling price (p).

Here we look at the changes in the rate of deterioration (θ) on the ratio of the time period that with positive inventory levels $(\frac{t_1}{T})$. The initial value of θ is changed as is shown in Table (7) to see its effect on the fraction of $(\frac{t_1}{T})$.

Table 7
The effect of changes in the rate of deterioration on the fraction of the time period that with positive inventory levels

	75%	80%	85%	90%	95%	initial number	105%	110%	115%	120%	125%
θ	0.0075	0.008	0.0085	0.009	0.0095	0.01	0.0105	0.011	0.0115	0.01 2	0.01 25
t_1/T	0.861	0.812	0.773	0.74	0.712	0.688	0.667	0.648	0.631	0.61 5	0.60 4

Given the numbers obtained, which is more explicit in Figure (4), by increasing the rate of deterioration, the fraction of the time period that with positive inventory levels is reduced, as the system tries to deplete the storage as soon as possible and have shorter length of time with positive inventory.

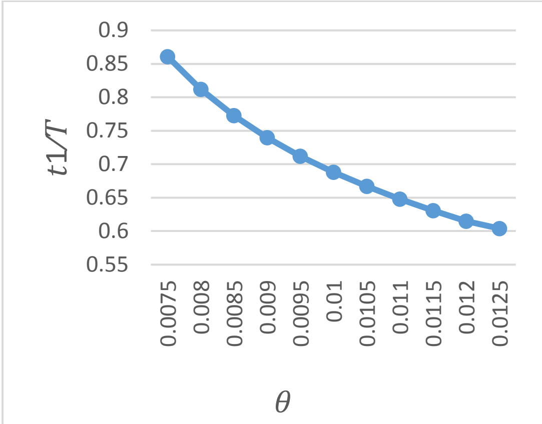


Fig. 4. The effect of changes in the rate of deterioration on the ratio of the time period with positive inventory level

Figure (5) depicts the effect of deterioration rate on the selling price. As it is expected, with the rise in the rate of deterioration, the selling price has fallen, to stimulate the demand and faster selling of the items.

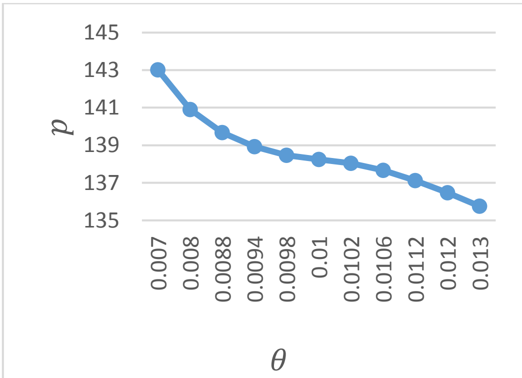


Fig. 5. The effect of changes in the rate of deterioration on sales prices

5.2. Analysis on the interest rate

The selling price is calculated for different values of interest rate which is shown in Table 8.

Table 8
The effect of interest rate changes on sales prices

	25%	50%	70%	85%	95%	initial number	105%	115%	130%	150%
ρ	0.000075	0.00015	0.00021	0.000255	0.000285	0.0003	0.000315	0.000345	0.00039	0.00045
p	138.564	138.46	138.377	138.314	138.272	138.252	138.231	138.189	138.127	138.044

Given the numbers obtained, and as it is more explicit in Figure (6), with the rise in interest rates, the selling price has fallen. It looks like the retailer prefers to even propose more discounts and stimulate the demand in order to get the cash sooner which has a high time value.

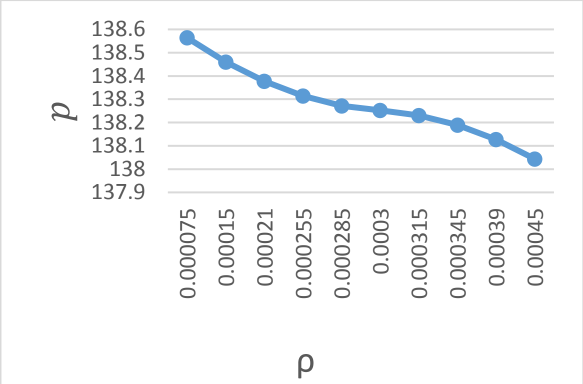
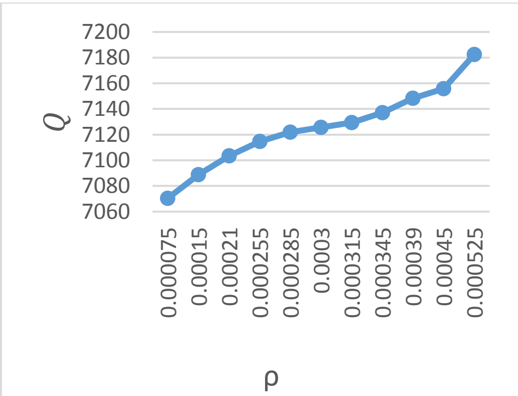


Fig. 6. The effect of interest rate changes on sales prices

It is also worth to mention that we assumed a fixed selling price of the product in the market. Therefore the model tries to get the cash and sell the inventory as soon as possible, as the fixed selling price loses its value through the time. This is also seen in Figure (7) in which the larger quantities are ordered in larger interest rates.



Figu. 7. The effect of interest rate changes on the economic order quantity

5.3. Analysis of the shortage costs

The effect of changes in the lost sales cost ($\boldsymbol{\pi}$) on the fraction of the time with positive inventory level ($\frac{t_1}{T}$) is shown in Table 9.

Table 9
The effect of changes in the lost sale cost on the fraction of the time with positive inventory level

	70%	80%	88%	94%	98%	initial number	102%	106%	112%	120%	130%
π	7	8	8.8	9.4	9.8	10	10.2	10.6	11.2	12	13
t_1/T	0.656	0.667	0.675	0.682	0.686	0.688	0.69	0.695	0.701	0.709	0.72

Given the numbers obtained, and is it is more explicit in Figure (8), with the increase in the lost sale cost; the fraction of the time with positive inventory level is increased. Therefore the model tries to face less shortage,

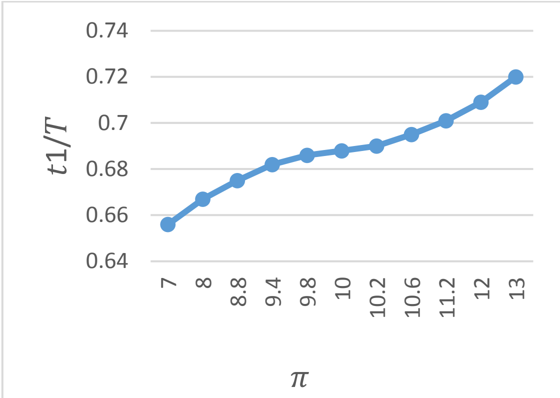


Figure 8. The effect of changes in the cost of sales lost on the ratio of the time period that we do not have shortage on the total duration of the period

By the increase in shortage costs, the model prefers larger cycles with positive inventory levels. This leads to more deterioration and thus larger order quantities are needed. This is shown in Figure (9).

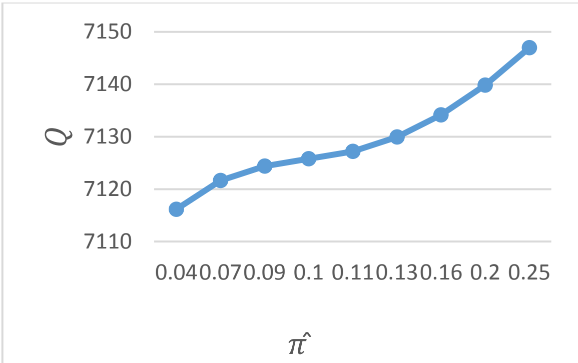


Figure 9. The effect of Changes in the backorder cost on the order quantity

5.4. Analysis of the purchase price

We examined the effect of changes in the purchasing cost from the first supplier (c_1) on the sales price (p) which is shown in Table 10.

Table 10
Effect of Changes in Purchase Cost from First Supplier on Sales Price

	70%	80%	88%	94%	98%	initial number	102%	106%	112%	120%	130%
c_1	66.5	76	83.6	89.3	93.1	95	96.9	100.7	106.4	114	123.5
p	138.236	138.241	138.245	138.248	138.251	138.252	138.253	138.255	138.258	138.262	138.268

Figure (10) depicts the effect of the purchasing price on the selling price. As it is expected and seen in Figure (10) the sales price has risen as the purchase price increases.

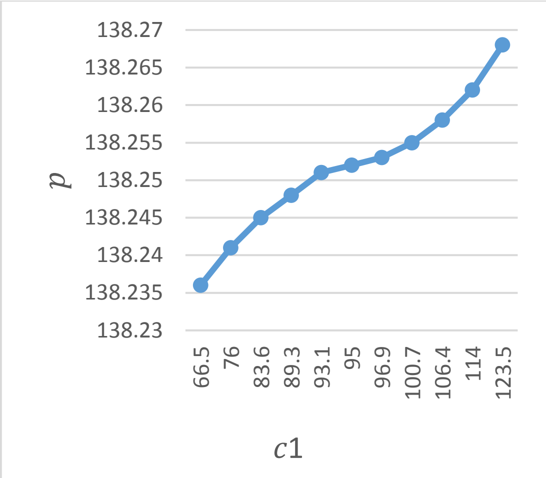


Fig. 10. The effect of purchasing price of the first supplier to the sales price

4. Conclusion

In this paper, we presented modeling of lot sizing problem with deteriorating items with time and price-dependent demand by considering the time value of money. The single-product model was purchased from several suppliers at a different price and sold at a constant price. The time value of money was included and interest was compounded continuously by the net present value method; the shortage was allowed and happened in the form of partial backorder. The optimal cycle time of the model in a particular case with a single supplier, and without shortage was presented. The numerical example was solved using the GAMS software, and sensitivity analysis was performed to study the effect of some model parameters on the results. These results indicated that as the rate of deterioration increases, fraction of time with positive inventory level is decreased, since the system tries to avoid extensive deterioration of the product. In addition, as the rate of deterioration increases, the sales price of the product is reduced to stimulate the demand in order to deplete the storage as soon as possible. Also, as the rate of deterioration increases, the economic order quantity decreases. The sensitivity analysis of the interest rate showed that as the interest rate rises, the economic order quantity increases, and the sales price of the product decreases. This happens since the model tries to get more cash flows from the selling of the product in the earlier times which based on NPV approach have more value. With increasing cost of shortage, order quantity increases since the shortage cycles are decreased and therefore more deterioration happens in the longer cycles with positive inventory level.

As a managerial insight we can mention that in the cases of fast deteriorating items or high interest rates, it is important to stimulate the demand and have a faster rate in selling the product.

For future studies, we can mention the extension of metaheuristics methods to be able to solve the problem in large cases. Use of time-dependent deterioration rates is also an interesting issue to study and compare the results.

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