Journal of Optimization in Industrial Engineering
Vol. 16, Issue 2, Summer & Autumn 2023, 295-302
DOI: 10.22094/JOIE.2024.1973228.2014

Research Article

Sizing Optimization of Truss Structures with Discrete Design Variables Using Combined PSO Algorithm with Special Particles Method

Ali Gheibi [a], Reza SojoudiZadeh [a,*], Hadi Azizian[a], Mahdi Gheibi [b]

[a] *Department of Civil Engineering, Mahabad Branch, Islamic Azad University, Mahabad, Iran*
[b] *Department of Civil Engineering, West Tehran Branch, Islamic Azad University, Tehran, Iran*

Received 13 November 2022; Revised 15 December 2023; Accepted 25 December 2023

Abstract

This paper proposes a modified particle swarm optimization (MPSO) algorithm for discrete sizing optimization of truss structures. The original particle swarm optimization (PSO) is a population-based metaheuristic that fluctuates the search agents about the best solution based on Eberhart functions. The efficiency of the PSO in solving standard optimization problems of well-known problems of truss structures has been demonstrated in The literature. However, its performance in tackling the discrete optimization problems of truss structures is not competitive compared with the recent existing metaheuristic algorithms. In the framework of the proposed MPSO a number of worst solutions of the current population is replaced by some variants of the global best solution found so far. Moreover, an efficient mutation operator is added to the algorithm that reduces the probability of getting stuck in local optima. The efficiency of the proposed MPSO is illustrated through two benchmark optimization problems of truss structures.

***Keywords*:** Discrete optimization; Sizing optimization; Truss structures; Metaheuristic; PSO.

1. Introduction

Saving in energy and material consumption is an important factor in the field of green engineering and usually from an economical viewpoint, the structure with minimum weight is defined as the best structure. In order to find such designs, structural optimization techniques can be effectively used. In the last decade, many optimization techniques have been developed and successfully applied to a wide range of structural optimization problems including sizing, layout and topology optimization problems (Kaveh & Talatahari, 2009),(Gholizadeh, 2013), (Zhu et al., 2018) . Metaheuristics are the most general kinds of stochastic optimization algorithms and they are now recognized as one of the most practical approaches for solving a wide range of optimization problems. The main idea behind designing these metaheuristic algorithms is to solve complex optimization problems where other optimization methods have failed to be effective. The practical advantage of metaheuristics lies in both their effectiveness and general applicability. In recent years, metaheuristic algorithms are emerged as the global search approaches, which are responsible to tackle the complex optimization problems.

Most of the metaheuristic algorithms are developed based on natural phenomena. Every metaheuristic method consists of a group of search agents that explore the design space based on randomization and some specified rules inspired the laws of natural phenomena. For example, Genetic Algorithms (GA) (Holland & Reitman, 1997), Biogeography-Based Optimization (BBO) (Simon, 2008), and Differential Evolution (DE) (Storn & Price , 1997) are developed based on the Darwin's principle of survival of the fittest. Gravitational Search Algorithm (GSA) (Rashedi et al, 2009), Colliding Bodies Optimization (CBO) (Kaveh & Mahdavi , 2014) and Center of Mass Optimization (CMO) (Gholizadeh & Ebadijalal, 2018) are Physics-based metaheuristic algorithms. Particle Swarm Optimization (Eberhart & Kennedy, 1995) (PSO), Ant Colony Optimization (Dorigo & Birattari, 2010) (ACO), Bat algorithm (Yang, 2010) (BA) and Dolphin Echolocation Algorithm (DEA) (Kaveh & Farhoudi, 2016) are recognized as popular Swarm intelligence metaheuristics.

One of the newly developed metaheuristic algorithms is the Sine Cosine Algorithm (SCA), which is proposed by Mirjalili (Mirjalili, 2016). The SCA requires that the generated solutions fluctuate outwards or towards the best solution found so far using sine and cosine functions. It was demonstrated in (Mirjalili, 2016) that the SCA is able to solve the continuous optimization problems effectively.

Optimization of truss structures is very popular in the area of structural optimization and over the last decades, various algorithms have been proposed for solving these problems. There is a significant number of metaheuristics employed for truss optimization with discrete variables in the literature such as: Discrete Heuristic Particle Swarm Ant Colony Optimization (DHPSACO) (Kaveh & Talatahari, 2009b) Improved Dolphin Echolocation Algorithm (IDEA) (Gholizadeh, 2016), Improved Mine Blast Algorithm (IMBA) (Sadollah & Eskandar, 2015), Adaptive Elitist Differential Evolution (AEDE) (Gholizadeh & Milany , 2016), and Improved Fireworks Algorithm (IFWA) (Ho-

* Corresponding author Email address: Reza.SojoudiZadeh@iau.ac.ir

Huu, 2016). In the present study, PSO is focused and a modified particle swarm optimization (MPSO) is proposed to handle the truss structures optimization with discrete design variables. In the MPSO two main strategies are followed for the exploration and exploitation of the design space. In the first strategy; which is obtained exactly from lion behavior in wild life toward its deeply ill new born lion cub; some of worst solutions in each iteration are removed and the same number of variants of the best solution is added to the population. In the second strategy, a mutation operator is added to the algorithm. Four benchmark optimization problems of truss structures with discrete variables are presented and the results of MPSO are compared with literature.

2. Truss Optimization Problem

For the optimization problem of trusses, objective function is the structural weight and some limitations are usually considered on nodal displacements and element stress as the design constraints. The formulation of truss structures optimization problem is as follows:

$$Minimize\ W = \sum_{i=1}^{n} \gamma_i\ L_i\ X_i \tag{1}$$

$Subject\ to:$

$$g_j^d = \frac{d_j}{\bar{d}_j} - 1 < 0\ , j = 1,2,..,m \qquad g_k^s = \frac{\sigma_k}{\bar{\sigma}_k} - 1 < 0\ , k = 1,2,..,n \tag{2}$$

$$X_i^L \le X_i \le X_i^U \tag{3}$$

where W is structural weight; γ_i, l_i and X_i are the density of material, element length and cross-sectional area of i^{th} element, respectively; displacement and stress constraints are represented by g^d and g^s , respectively; dj and σ_k are j^{th} node displacement and k^{th} element stress, respectively; and are their allowable values; n and m are numbers of elements and nodes, respectively. The following exterior penalty function (EPF) is employed to handle the constraints of the above-constrained optimization problem.

$$\varphi = W \times \left[1 + r_p \sum_{j=1}^{m} \left(\max\{0, g_j^d\}\right)^2 + r_p \sum_{k=1}^{n} (\max\{0, g_k^s\})^2\right] \tag{4}$$

Where Φ is pseudo unconstrained objective function; and r_p is a penalty parameter. In this study, r_p is linearly increased from 1.0 at the first iteration to 10^6 at the last one during the optimization process.

3. Particle Swarm Optimization Algorithm

In essence, all population-based metaheuristic algorithms explore the design space using a number of search agents, which follow a set of updating rules. These updating rules play an important role in performance of the metaheuristic algorithms. In the particle swarm optimization Algorithm (PSO) [10] the following equation is used as the updating rule of position of population in the design space:

$$X_i^t = X_i^{t-1} + V_i^t \tag{5}$$

$$V_i^t = \theta \times V_i^{t-1} + C_1 r_1 [P_i^{best} - X_i^{t-1}] + C_2 r_2 [G_i^{best} - X_i^{t-1}] \tag{6}$$

$$\theta = \theta_{max} - \frac{\theta_{max-}\theta_{min}}{t_{max}} \times t \tag{7}$$

Where X_i^t and X_i^{t-1} are the i^{th} design variable at iterations *t* and *t-1*, respectively. P_i^{best} is the best solution in the *i*th iteration; G_i^{best} is the best solution encountered in all the previous iterations ; C_1 and C_2 which are individual and social learning rates, respectively, usually are assumed to be 2 . r_1 and r_2 are uniformly distributed random numbers in the range 0 and 1; tmax is the maximum number of iterations. θ which balances between global and local exploration is obtained The values of $\theta_{max} = 0.9$ and $\theta_{min} = 0.4$ are used in this study.

It was demonstrated that the original PSO performs properly in solving standard optimization problems of well-known optimization problems. The computational experience of the present study however reveals that its efficiency as metaheuristic algorithm can be improved for discrete sizing optimization of truss structures.

4. Modified Particle Swarm Optimization Algorithm

In order to improve the performance of the PSO in dealing with the discrete sizing optimization problems of truss structures two computational strategies are implemented and the improved metaheuristic is named as modified particle swarm optimization algorithm (MPSO). The proposed strategies, termed here as Regeneration and Mutation, are described below.

Regeneration: in each iteration of the optimization process, the population, including np particles, is sorted in an ascending order based on the objective function values of particles as represented below:

$$sort\ (X^t) = \left[\ X_1^t\ ...\ X_k^t\ ...\ X_{np-1}^t ... X_{np}^t\ \right] \tag{8}$$

Where *sort* (X^t) is the sorted current population; and X_k^t to X_{np}^t are the worst solutions at iteration *t* that should be regenerated. Then, a number of $\lambda \times np$ worst particles (i.e. X_k^t to X_{np}^t) are removed from the population and instead, the best solution found so far, $X^* = [X_1^*\ X_2^* ... X_j^* ... X_n^*]^T$, is copied $\lambda \times np$ times in the population. In these solutions, except the last one, one randomly selected design variable is regenerated in the design space on a random basis as follows:

$$X_l^k \rightarrow \left[X_1^*, X_2^*, ..., X_j^*, X_n^*\right]^T \quad , \ l = k, ..., np-1 \tag{9}$$

$$X_j^* = round\left(X_j^L + r \times \left(X_j^U - X_j^L\right)\right) \quad , \; j \in [1,2,\ldots,n] \tag{10}$$

Where X_j^L and X_j^U are lower and upper bounds of the jth design variable; and r is a random number in [0,1].

The regenerated design variables of particles X_k^t to X_{np-1}^t are substituted in the last particle (X_{np}^t). This strategy will increase the probability of finding the promising regions of the design space.

Mutation: in the framework of MPSO, a mutation operation is implemented to reduce the probability of trapping into local optima. In this way, a mutation rate of *mr* is considered and for each particle (X_i , i=1,2,…,np) a random number in [0 ,1] is selected in each iteration. If for the i^{th} particle, the selected random number is less than *mr*, X_i will be regenerated in the design space as follows:

$$X_i^{t+1} = round\left[X_i^t + \left(\frac{t}{t_{Max}}\right) \times R \times (X_{best}^t - X_r^t)\right] \tag{11}$$

Where in iteration t, R^t is a vector of random numbers in [0, 1]; X_{best}^t is the best particle of the current population; and X_r^t is a randomly selected particle from the current population.

In the framework of MPSO, a simple mechanism is employed to return into the feasible region the agents that violate side constraints. During the optimization process, if a design variable violates the side constraints, it will be replaced by the lower/upper bound as follows:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^L & if\ X_{i,j}^{t+1} < X_{i,j}^L \\ X_{i,j}^U & if\ X_{i,j}^{t+1} > X_{i,j}^U \end{cases} \tag{12}$$

Where $X_{i,j}^L$ and $X_{i,j}^U$ are respectively the lower and upper bounds of the jth design variable of the ith solution.

The best combination of internal parameters λ and *mr*, is determined by performing sensitivity analysis. In this way, $\lambda \in \{0.1, 0.2, 0.3\}$ and $mr \in \{0.01, 0.05, 0.10\}$ are considered and for each combination of these two parameters, 20 independent optimization runs are conducted. The results of this study demonstrate that the best combination is $\lambda=0.2$ and $mr=0.05$. The flowchart of MPSO is depicted in Fig. 1.

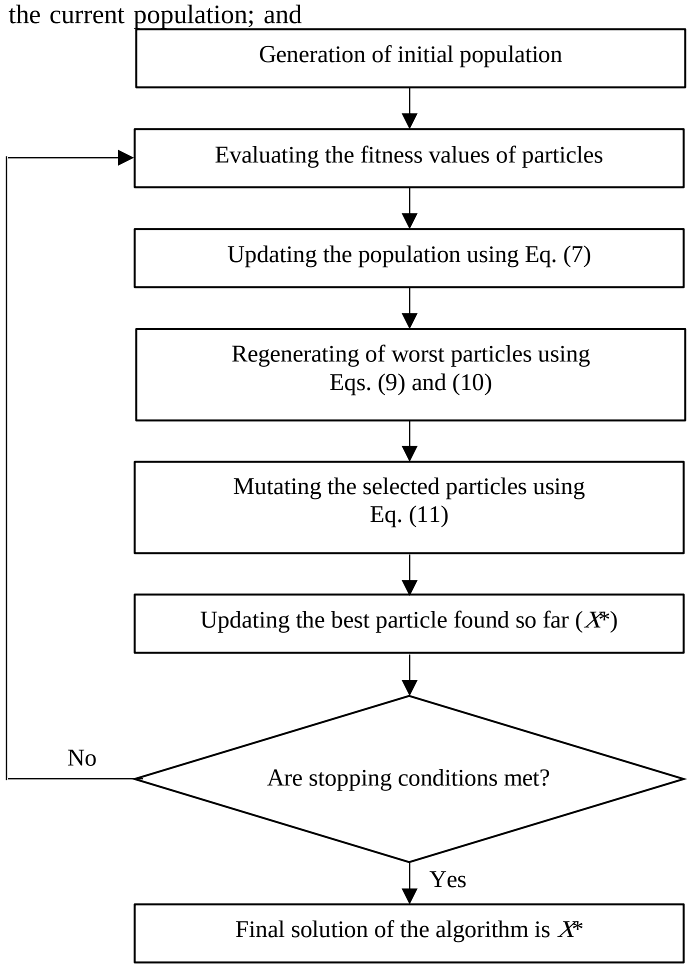

Fig. 1. Flowchart of MPSO

5. Numerical results

In order to illustrate the merit of the proposed MPSO, two popular discrete benchmark truss optimization problems are presented and the obtained results are compared with those of literature. For the presented examples, 10 independent optimization runs are performed and the best weight (Best), average weight (Average) and the standard deviation (SD) of optimal weights are reported.

Example 1: 72-bar spatial truss

The 72-bar spatial truss is shown in Fig. 2. In this example, there are 16 groups of elements as follows:
(1) A_1–A_4, (2) A_5–A_{12}, (3) A_{13}–A_{16}, (4) A_{17}–A_{18}, (5) A_{19}–A_{22}, (6) A_{23}–A_{30} (7) A_{31}–A_{34}, (8) A_{35}– A_{36}, (9)A_{37}–A_{40}, (10) A_{41}–A_{48}, (11) A_{49}–A_{52}, (12) A_{53}–A_{54}, (13) A_{55}– A_{58}, (14) A_{59}–A_{66} (15) A_{67}–A_{70}, (16) A_{71}–A_{72}.

The modulus of elasticity and material density are 10^4 ksi and 0.1 lb/in^3, respectively. During the optimization process, the design variables are selected from the data base of Table 1. The allowable stress in elements is ±25 ksi and the allowable horizontal displacement is ±0.25 in. In addition, there are two loading conditions given in Table 2.

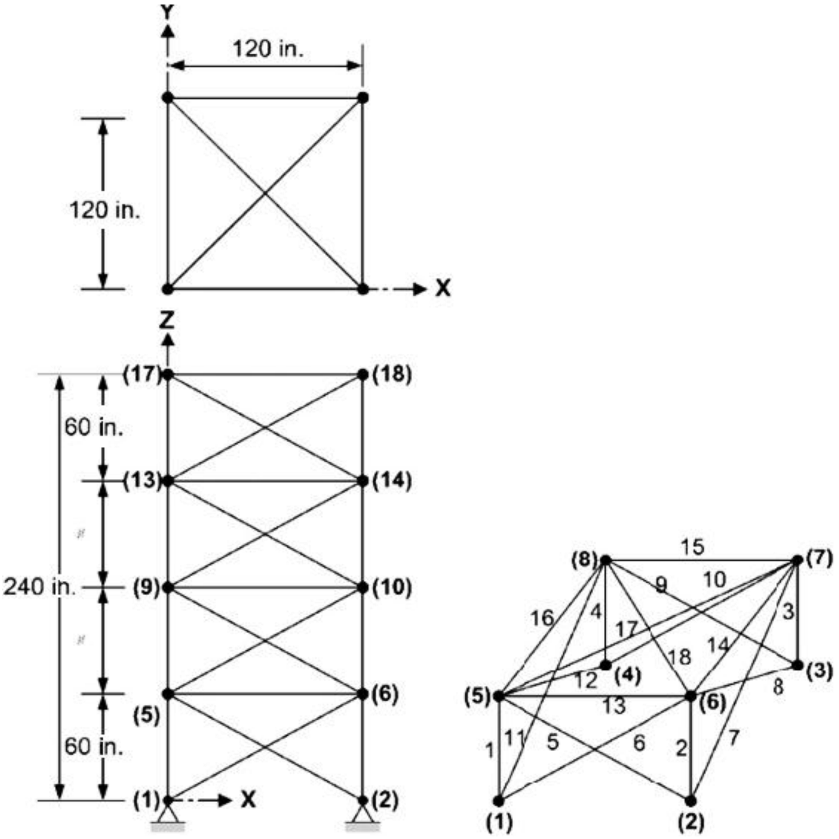

Fig. 2. 72-bar truss

Table 1
Available cross-sectional areas of the AISC

No.	mm^2	in^2	No.	mm^2	in^2	No.	mm^2	in^2	No.	mm^2	in^2
1	71.613	0.111	17	1008.385	1.563	33	2477.414	3.84	49	7419.340	11.5
2	90.968	0.141	18	1045.159	1.62	34	2496.769	3.87	50	8709.660	13.5
3	126.451	0.196	19	1161.288	1.80	35	2503.221	3.88	51	8967.724	13.9
4	161.290	0.250	20	1283.868	1.99	36	2696.769	4.18	52	9161.272	14.2
5	198.064	0.307	21	1374.191	2.13	37	2722.575	4.22	53	9999.980	15.5
6	252.258	0.391	22	1535.481	2.38	38	2896.768	4.49	54	10322.560	16.0
7	285.161	0.442	23	1690.319	2.62	39	2961.284	4.59	55	10903.204	16.9
8	363.225	0.563	24	1696.771	2.63	40	3096.768	4.80	56	12129.008	18.8
9	388.386	0.602	25	1858.061	2.88	41	3206.445	4.97	57	12838.684	19.9
10	494.193	0.766	26	1890.319	2.93	42	3303.219	5.12	58	14193.520	22.0
11	506.451	0.785	27	1993.544	3.09	43	3703.218	5.74	59	14774.164	22.9
12	641.289	0.994	28	2019.351	3.13	44	4658.055	7.22	60	15806.420	24.5
13	645.160	1.0	29	2180.641	3.38	45	5141.925	7.97	61	17096.740	26.5
14	792.256	1.228	30	2238.705	3.47	46	5503.215	8.53	62	18064.480	28.0
15	816.773	1.266	31	2290.318	3.55	47	5999.988	9.30	63	19354.800	30.0
16	939.998	1.457	32	2341.931	3.63	48	6999.986	10.85	64	21612.860	33.5

Table 2
Loading conditions for the 72-bar truss

Node	Loading condition 1 (kips)			Loading condition 2 (kips)		
	F_x	F_y	F_z	F_x	F_y	F_z
17	5.0	5.0	–5.0	0.0	0.0	–5.0
18	0.0	0.0	0.0	0.0	0.0	–5.0
19	0.0	0.0	0.0	0.0	0.0	–5.0
20	0.0	0.0	0.0	0.0	0.0	–5.0

In the optimization process the population size and maximum number of iterations are considered to be 50 and 200, respectively. The results obtained in the present study are compared with those of HPSO (Gholizadeh& Milany, IMBA (Sadollah et al.2016) and AEDE (Ho-Huu et al.,2015) in Table 3. Furthermore, convergence curves of PSO and MPSO are compared in Fig. 2.These results reveal that, MPSO is competitive in comparison with other algorithms of literature. The statistical results of IMBA are slightly better than those of MPSO however at very high computational effort. In addition, it is demonstrated that the performance of MPSO is better than that of PSO.

Table 3
Results of optimization for the 72-bar truss

Design variables	HPSO	IMBA	AEDE	This Study	
				PSO	MPSO
A_1–A_4	4.97	1.990	1.990	1.990	1.990
A_5–A_{12}	1.228	0.442	0.563	0.563	0.563
A_{13}–A_{16}	0.111	0.111	0.111	0.111	0.111
A_{17}–A_{18}	0.111	0.111	0.111	0.111	0.111
A_{19}–A_{22}	2.88	1.228	1.228	1.228	1.228
A_{23}–A_{30}	1.457	0.563	0.442	0.442	0.442
A_{31}–A_{34}	0.141	0.111	0.111	0.111	0.111
A_{35}–A_{36}	0.111	0.111	0.111	0.111	0.111
A_{37}–A_{40}	1.563	0.563	0.563	0.563	0.563
A_{41}–A_{48}	1.228	0.563	0.563	0.563	0.563
A_{49}–A_{52}	0.111	0.111	0.111	0.111	0.111
A_{53}–A_{54}	0.196	0.111	0.111	0.111	0.111
A_{55}–A_{58}	0.391	0.196	0.196	0.196	0.196
A_{59}–A_{66}	1.457	0.563	0.563	0.563	0.563
A_{67}–A_{70}	0.766	0.391	0.391	0.391	0.391
A_{71}–A_{72}	1.563	0.563	0.563	0.563	0.563
Best (lb)	933.09	389.33	389.33	389.33	389.33
Average (lb)	N/A	389.82	390.91	406.62	389.75
SD (lb)	N/A	0.84	1.161	12.21	0.928
Analyses	50000	50000	4160	10000	5000

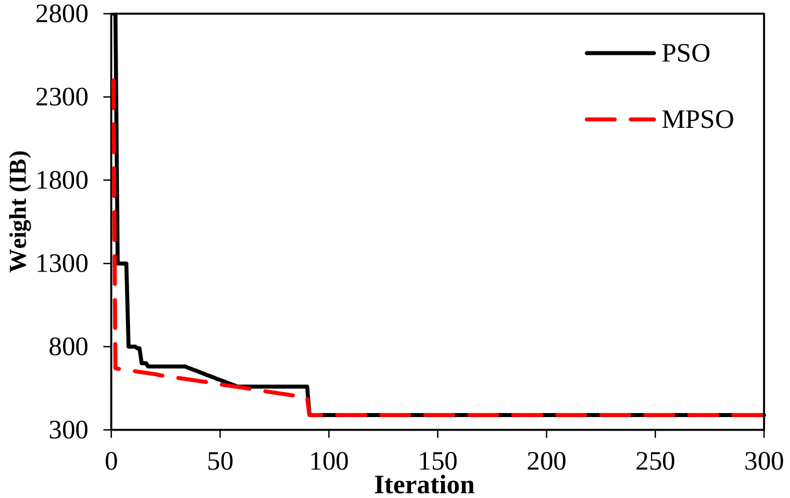

Fig. 2. Convergence histories of PSO and MPSO for 72-bar truss

Example 2: 200-bar planar truss

An 11-level planar truss structure consisting of 200 bars, shown in Fig. 3, was optimized in the above-mentioned papers. The material density is 0.283 lb/in^3 whereas the modulus of elasticity is 30,000 psi. The stress limit was ±10,000 psi. The structure is subjected to the two load cases:

(1) One kip is applied in positive X direction at nodes group N1;

N1 = {1, 6, 15, 20, 29, 34, 43, 48, 57, 62, 71}

(2) 10 kips is applied in negative Y direction at nodes group N2;

N2 ={1, 2, 3, 4, 5, 6, 8, 10, 12, 14, 15, 16, 17, 18, 19, 20, 22, 24, 26, 28, 29, 30, 31, 32, 33, 34, 36, 38, 40, 42, 43, 44, 45, 46, 47, 48, 50, 52, 54, 56, 57, 58, 59, 60, 61, 62, 64, 66, 68, 70, 71, 72, 73, 74, 75 }

(3) Cases 1 and 2 are combined together.

The 200 structural members of this spatial truss are categorized into 29 groups described in Table 4. Discrete values of cross-sectional areas were selected from the following set:

S={0.100, 0.347, 0.440, 0.539, 0.954, 1.081, 1.174, 1.333, 1.488, 1.764, 2.142, 2.697, 2.800, 3.131, 3.565, 3.813, 4.805, 5.952, 6.572, 7.192, 8.525, 9.300, 10.850, 13.330, 14.290, 17.170, 19.180, 23.680, 28.080, 33.700}in.^{2}

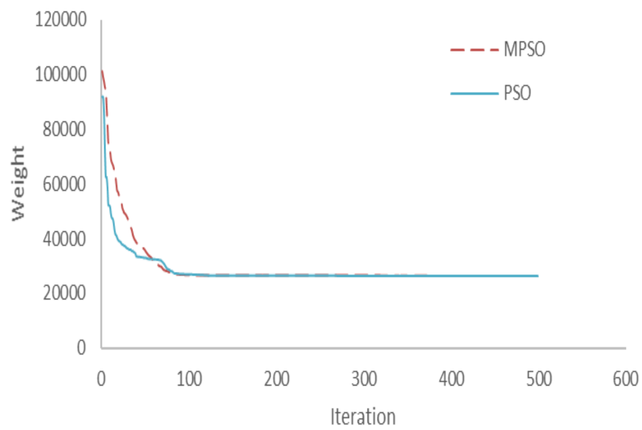

Fig. 4. Convergence histories of PSO and MPSO for 200-bar truss

Table 4 compares the results obtained by the MPSO algorithm and other optimization methods. Fig. 4 compares the corresponding convergence rates. It reveals that the convergence rate of the MPSO is very better than that of the original PSO. From table 4 it can be seen that IGA, DE, AEDE, PSO and MPSO converge to the different best solutions and the MPSO is not competitive with the other algorithms. In this example, MSPO is the best algorithm in terms of Average and SD and the second best algorithm is PSO.

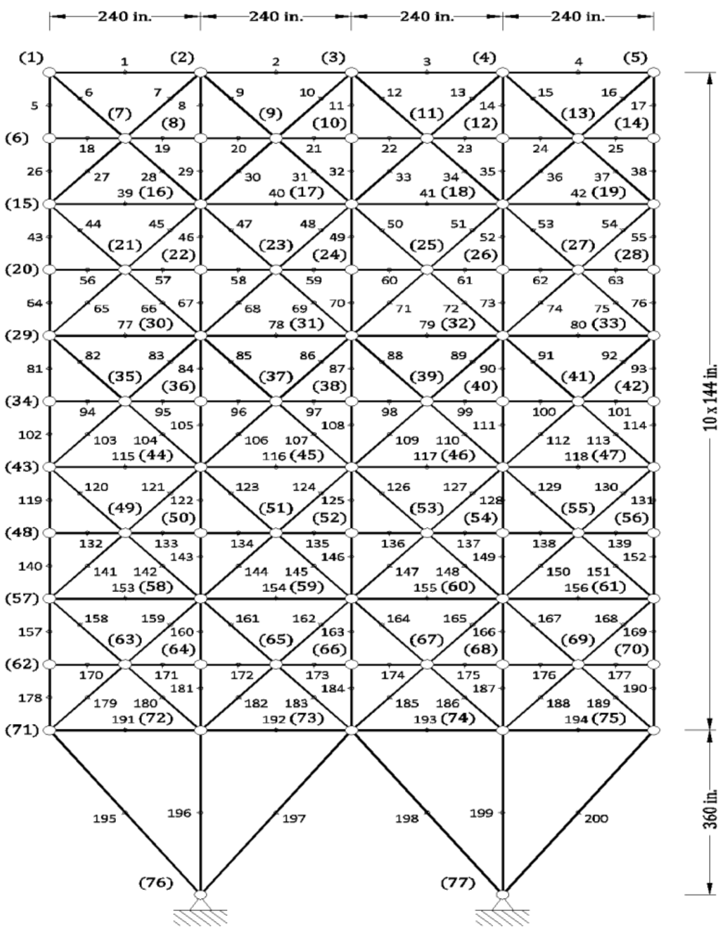

Fig. 3. 200-bar planar truss

Table 4
Results of optimization for the 200-bar truss

Design variables	Members in the group	IGA	DE	AEDE	PSO	MPSO
1	1, 2, 3, 4	0.347	0.1	0.1	0.1	0.1
2	5, 8, 11, 14, 17	1.081	0.954	0.954	1.2445	1.2735
3	19, 20, 21, 22, 23, 24	0.1	0.1	0.347	0.57049	0.1
4	18, 25, 56, 63, 94, 101, 132, 139, 170, 177	0.1	0.1	0.1	0.1	0.1
5	26, 29, 32, 35, 38	2.142	2.142	2.142	2.1642	2.3378
6	6, 7, 9, 10, 12, 13, 15, 16, 27, 28, 30, 31, 33,34,36,37	0.347	0.347	0.347	0.41584	0.1958
7	39, 40, 41, 42	0.1	0.1	0.1	0.4921	2.2189
8	43, 46, 49, 52, 55	3.565	3.131	3.131	2.2283	2.5287
9	57, 58, 59, 60, 61, 62	0.347	0.1	0.347	0.45338	0.79887
10	64, 67, 70, 73, 76	4.805	4.805	4.805	3.2283	3.5287
11	44, 45, 47, 48, 50, 51, 53, 54, 65, 66, 68, 69, 71, 72, 74, 75	0.44	0.347	0.539	2.0204	1.0185
12	77, 78, 79, 80	0.44	0.1	0.347	0.8516	3.8102
13	81, 84, 87, 90, 93	5.952	5.952	5.952	5.445	2.2816
14	95,96, 97, 98, 99, 100	0.347	0.1	0.1	0.1	0.75627
15	102, 105, 108, 111, 114	6.572	6.572	6.572	6.445	3.2816
16	82, 83, 85, 86, 88, 89, 91, 92, 103, 104, 106, 107, 109, 110, 112, 113	0.954	0.44	0.954	0.54992	6.4454
17	115, 116, 117, 118	0.347	0.539	0.44	0.15365	1.7625
18	119, 122, 125, 128, 131	8.525	7.192	8.525	7.9306	8.4743
19	133, 134, 135, 136, 137, 138	0.1	0.44	0.1	0.1	0.38379
20	140, 143, 146, 149, 152	9.3	8.525	9.3	8.9307	10.312
21	120, 121, 123, 124, 126, 127, 129, 130, 141, 142, 144, 145, 147, 148, 150, 151	0.954	0.954	0.954	0.62417	0.95099
22	153, 154, 155, 156	1.764	1.174	1.081	0.1	4.6951
23	157, 160, 163, 166, 169	13.3	10.85	13.33	10.446	14.033
24	171, 172, 173, 174, 175, 176	0.347	0.44	0.539	0.1	0.1
25	178, 181, 184, 187, 190	13.3	10.85	14.29	11.446	16.721
26	158, 159, 161, 162, 164, 165, 167, 168, 179, 180, 182, 183, 185, 186, 188, 189	2.142	1.764	2.142	0.68744	8.6523
27	191, 192, 193, 194	4.805	8.525	3.813	8.65	1.4829
28	195, 197, 198, 200	9.3	13.33	8.525	12.557	4.5291
29	196, 199	17.17	13.33	17.17	12.996	28.638
Best (lb)		28544.01	28075.49	27858.50	26420.25	26257.09
Average (lb)		-	-	28425.87	27880.51	27308.55
SD (lb)		-	-	481.590	1172.26	402.55
Analyses		51360	-	11644	75000	60000

6. Concluding Remarks

The present study focuses on a firstly developed PSO algorithm and proposes a modified PSO (MPSO). As the original version of this metaheuristic seriously suffers from the slow convergence rate when dealing with the discrete truss optimization problems. The proposed MPSO integrates two computational strategies during its search process. In the first strategy, named as Regeneration, a kind of elitism is utilized by substituting a number of worst solutions of the current population with some variants of the global best solution. In the second strategy, named as Mutation, a mutation operation is performed to increase the probability of finding the global optimum or near global optima.

In order to illustrate the efficiency of the MPSO, two well-known discrete benchmark truss optimization problems, including 72 and 200-bar trusses, are presented and the results of MPSO are compared with those of HPSO, HHS, AEDE, ECBO, IMBA and PSO. The numerical results demonstrate that the original PSO is not competitive with the mentioned algorithms and consequently some modifications are needed. In contrast, in the both cases, the proposed MPSO outperforms other algorithms and presents an appropriate performance.

References

Cheng MY, Prayogo D, Wu YW, Lukito MM. (2016).A hybrid harmony search algorithm for discrete sizing optimization of truss structure. Automat Constr; 69: 21-33.

Dorigo M, Birattari M. (2010).Ant colony optimization, Encyclopedia of Machine Learning, Springer, pp. 36–39.

Eberhart RC, Kennedy J. (1995).A new optimizer using particle swarm theory, in: Proceedings of the Sixth International Symposium on Micro Machine and Human Science, pp. 39–43.

Gholizadeh S, Ebadijalal M. (2018).Performance based discrete topology optimization of steel braced frames by a new metaheuristic. Adv Eng Softw, 123: 77–92.

Gholizadeh S, Milany A. (2018).An improved fireworks algorithm for discrete sizing optimization of steel skeletal structures. Eng Optim. doi.org/10.1080/0305215X.2017.1417402.

Gholizadeh S, Poorhoseini H. (2016). Seismic layout optimization of steel braced frames by an improved dolphin echolocation algorithm. Struct Multidisc Optim, 54:1011–29..

Gholizadeh S.(2013). Layout optimization of truss structures by hybridizing cellular automata and particle swarm optimization. Comput Struct, 125: 86-99.

Ho-Huu V, Nguyen-Thoi T, Vo-Duy T, Nguyen-Trang T.(2016). An adaptive elitist differential evolution for optimization of truss structures with discrete design variables. Comput Struct 2016; 165: 59-75.

Holland J,Reitman J.Cognitive (1997). Systems based on adaptive algorithms, AC SIGART Bull 1977;63:49–49.

Kaveh A, Farhoudi N.A (2013).New optimization method: Dolphin echolocation.Adv Eng Softw, 59:53–70.

Kaveh A, Ilchi Ghazaan M. (2015).A comparative study of CBO and ECBO for optimal design of skeletal structures. Comput Struct, 153:137-47.

Kaveh A, Mahdavi VR.(2014). Colliding Bodies Optimization method for optimum discrete design of truss structures, Comput Struct, 139: 43–53.

Kaveh A, Talatahari S. (2009a).A particle swarm ant colony optimization for truss structures with discrete variables. J Constr Steel Res, 65:1558-68.

Kaveh A, Talatahari S.(2009b) Size optimization of space trusses using Big Bang-Big Crunch algorithm. Comput Struct, 87:1129-40.

Li LJ, Huang ZB, Liu F.(2009). A heuristic particle swarm optimization method for truss structures with discrete variables. Comput Struct, 87:435-43.

Mirjalili S. SCA: A Sine Cosine Algorithm for solving optimization problems, Knowl-Based Syst 2016; 96: 120-133.

Rashedi E, Nezamabadi-Pour H, Saryazdi S. (2009).GSA: a gravitational search algorithm. Inf Sci, 179: 2232–2248.

Sadollah A, Eskandar H, Bahreininejad A, Kim JH. (2015).Water cycle, mine blast and improved mine blast algorithms for discrete sizing optimization of truss structures. Comput Struct, 149:1-16.

Simon D. (2008).Biogeography-based optimization, IEEE Trans Evol Comput,12: 702–713.

Storn R, Price K. (1997).Differential evolution–a simple and efficient heuristic for global optimization over continuous spaces. J Global Optim, 11: 341–359.

Yang XS. (2010).A new metaheuristic bat-inspired algorithm. Nature inspired cooperative strategies for optimization,studies in computational intelligence, J. R.Gonzalez, ed., Vol. 284, Springer, Berlin, 65–74.

Zhu JH, He F, Liu T, FH, Zhang WH, Liu Q, Yang C. (2018).Structural topology optimization under harmonic base acceleration excitations. Struct Multidiscip Optim, 57: 1061-1078.