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Research Article

Multiple Items Supplier Selection, Economic Lot-sizing, and Order Allocation Under Quantity Discount: A Genetic Algorithm Approach

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Abstract

The task at hand involves selecting the most suitable supplier(s), determining the optimal lot size, and allocating the total order quantities among the suppliers based on various selection criteria. However, this can become more complex when taking into account quantity discount offers and transportation selection decisions in the selection and order allocation process. To address this challenge, this paper proposes an integrated approach that combines the Analytic Hierarchy Process (AHP) with a multi-objective mixed integer nonlinear program. The approach is designed for a multi-item, capacitated multi-supplier scenario, where the goal is to select suppliers, determine lot sizes, and allocate orders while taking into account unit quantity discounts and intermodal freight costs. The proposed approach aims to minimize costs and the percentage of rejected items, while maximizing the purchasing value. To solve this problem, an efficient genetic algorithm with problem-specific operators is utilized.

Keywords: Multi-criteria supplier selection; Economic Lot-Sizing; Order allocation; AHP; Multi-objective mixed-integer nonlinear programming

1. Introduction

In today's highly competitive market, companies must continually strive to optimize their business processes. One key strategic function within a business is purchasing, which offers opportunities for cost reduction and product quality improvement. Purchasing is more than just obtaining quality products at the lowest price; it also involves acquiring the right quantity of products from the most reliable suppliers, among other objectives (Monczka, Handfield, Giunipero, & Patterson, 2009). In the supplier evaluation and selection process, the best supplier(s) are selected based on a set of both qualitative and quantitative criteria, such as price, quality, delivery reliability, and capacity, among others. This process is crucial, as the selection of the right supplier can significantly impact cost reduction, with raw material and service costs accounting for up to 70% of some companies' expenses (Weber et al., 1991). Selecting the right supplier can also improve overall company competitiveness and customer satisfaction (Wang & Yang, 2009).

The supplier selection problem can be broadly classified as single sourcing or multiple sourcing, depending on the number of suppliers used for procurement. Single sourcing is used when all suppliers can fulfill the supplier's requirements, while multiple sourcing is used when no single supplier can meet the supplier's requirements (Ghodsypour & O'Brien, 1998). The multi-sourcing strategy gives the buyer the opportunity to obtain the item at a reduced cost and minimize the risk of disruption. Furthermore, the multiple sourcing option provides the opportunity for the buyer to use the aggregate demand measure instead of the individual lead time of suppliers (Pan, 1989). Since[1] the longer lead time of a supplier can be compensated by a shorter lead time, the modeler has relative freedom in choosing a supplier with the minimum unit cost offer, which might not satisfy the lead time constraint on its own but satisfies the requirement on the aggregate measure.

The evaluation of suppliers' performance is a multi-criteria decision-making problem that requires the consideration of both qualitative and quantitative criteria, making it necessary to make trade-offs between various conflicting criteria. The problem is complex since it involves the consideration of multiple criteria in the decision-making process, and even more complex for multiple items under the all-unit quantity discount environment. In this article, a multi-objective model is proposed that seeks to minimize cost, minimize percent reject, and maximize the purchasing value to account for the various and conflicting criteria.

Section 2 presents the relevant literature in supplier selection and order allocation, highlighting the various approaches and techniques used in the literature. Section 3 presents the problem description and the multi-objective mathematical model, which includes the formulation of the objective functions and constraints. Section 4 presents

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the solution approach, which involves the use of a genetic algorithm with problem-specific operators to solve the problem. This section also includes a brief discussion of the algorithm's efficiency and effectiveness. Section 5 presents a numerical example to demonstrate the application of the proposed model and solution approach. Finally, section 6 presents the conclusion and recommendations for future work.

2. Literature Review

The supplier selection problem has been a topic of interest for researchers for over five decades (Dickson, 1966). Several studies have been conducted to evaluate and select suppliers, and a comprehensive review of the methods used can be found in Boer, Labro, & Morlacchi (2001), Degraeve, Labro, & Roodhooft (2000), and Weber et al. (1991). Recently, researchers have attempted to integrate the supplier evaluation and selection problem with the order allocation problem. A detailed review of the methods and techniques used in supplier selection and order allocation can be found in Aissaoui, Haouari, & Hassini (2007), Setak, Sharifi, & Alimohammadian (2012), and Wetzstein, Hartmann, Benton, & Hohenstein (2016).

This article focuses on the multiple item supplier selection, lot-sizing, and order allocation problem under quantity discount. The literature review reveals that mathematical programming is the most commonly used method for formulating the multi-criteria supplier selection problem. Gaballa (1977) formulated the procurement of multiple items for the Australian Post Office to multiple competing tenders with quantity discounts as a mixed integer linear model. Pirkul & Aras (1985) solved the multiple items order quantity problem under the presence of the all-unit quantity discount. They considered the sum of the purchasing, inventory, and ordering cost as the objective, while capacity was the only constraint considered. Kasilingam & Lee (1996) proposed a mixed integer model with the objective of minimizing the total cost of purchasing, transportation, establishing vendors, and receiving poor quality items. Chaudhry, Forst, & Zydiak (1993) developed a linear and mixed binary integer programming models for the multi-sourcing vendor selection problem where vendors offer price-breaks depending on the size of order quantities. Sadrian & Yoon (1994) developed a procurement decision support system that considered a business volume discount and saved 15% of the cost. Crama, Pascual, & Torres (2004) developed a non-linear binary integer model for a multi-item vendor selection problem with all-unit quantitsy discounts, and tested it on real-world data obtained from a chemical company.

While the above models included different types of price break offers, they failed to incorporate qualitative criteria that are important and common in the vendor selection decision. Furthermore, most of the models considered the minimization of cost as the only objective and some included other criteria, such as quality and lead time, in their constraints. However, in practice, multi-objective vendor evaluation and selection is common. Ghodsypour & Brien (1998) proposed an integrated analytic hierarchy process and linear programming to incorporate qualitative and quantitative selection criteria and allocate order quantities among suppliers to maximize the overall purchasing value of the buyer. Wang & Yang (2009) introduced an analytical hierarchy process and fuzzy compromise programming to account for the weight of the multiple objectives for a vendor selection and order allocation problem with quantity discount offers. Kokangul & Susuz (2009) developed an integrated AHP and non-linear integer and multi-objective programming to model the vendor selection and order allocation with price break offers. Ebrahim, Razmi, & Haleh (2009) introduced a mathematical model that considers different types of price breaks (all-unit, incremental, and business volume) for a single-item purchasing problem, with suppliers' capacity and demand considered as constraints.

While the above models included both tangible and intangible selection criteria in the supplier selection and order allocation, they did not consider inventory management and transportation issues of the purchased item. Rosenblatt, Herer, & Hefter (1998) developed a model for a single item multiple supplier selection and order allocation, considering the inventory management of the purchased item. Ghodsypour & O'Brien (2001) developed a mixed integer non-linear model to solve the multiple sourcing problems by optimizing the total cost of logistics, including net price, storage, and ordering costs, subject to capacity, demand, and quality requirements. However, both Rosenblatt, Herer, & Hefter (1998) and Ghodsypour & O'Brien (2001) did not explicitly consider inbound transportation in their proposed models. The inclusion of inbound transportation cost, which can account for up to 50% of total logistics cost, is crucial in the supplier selection and order allocation effort (Swenseth, Godfrey, Mendoza, & Ventura, 2002). Mendoza & Ventura (2013) included transportation cost in their mixed integer non-linear program formulation to determine the economic lot-size with demand and capacity of suppliers incorporated as constraints.

In summary, the literature review suggests that the case of multiple items, capacity-constrained multi-sourcing, lot-sizing, and order allocation problem has been studied sufficiently. However, the case with quantity discount and intermodal freight cost has not been adequately addressed in the literature. Moreover, only a few researchers have integrated the qualitative and quantitative aspects of the supplier selection and order allocation problem under a quantity discount environment. Therefore, the objective of this paper is to develop a multi-objective mathematical model that can simultaneously determine the best supplier(s), the order quantity and the economic order quantity allocation, and the selection of the mode of transportation decisions.

The proposed model aims to address the shortcomings of the existing literature by incorporating both qualitative and quantitative criteria in the supplier selection process.

Additionally, the model will consider the inventory management and transportation costs of the purchased item, which are crucial factors in the overall logistics cost. The use of mathematical programming in formulating the multi-criteria supplier selection problem will enable decision-makers to consider a wide range of objectives and constraints while selecting the best suppliers. The proposed model will also consider the quantity discount offers provided by the suppliers, which can significantly impact the overall purchasing value of the buyer.
Furthermore, the model will incorporate transportation costs in the supplier selection and order allocation process, which will enable decision-makers to optimize the logistics costs associated with inbound transportation. The inclusion of transportation costs in the model will also help in selecting the most cost-effective mode of transportation, which can further reduce the overall logistics cost.
In conclusion, the proposed multi-objective mathematical model will provide decision-makers with a comprehensive tool to evaluate and select the best suppliers while optimizing the overall logistics cost of the purchasing process. The model will also consider the quantity discount offers and transportation costs, which are crucial factors in the supplier selection and order allocation decision-making process.

3. Problem Description and Model Formulation

This paper examines the procurement of multiple items from multiple suppliers, which offer quantity discounts, as depicted in Figure 1. The primary objective of the buyer is to allocate its annual demand for the items (D_p) among the suppliers in a manner that minimizes the total cost of logistics, minimizes the total rejected items, and maximizes the total value of the purchase while satisfying capacity, demand, and lead time requirements. To achieve this objective, the buyer must determine the total order quantities (d_{ip}), for each supplier, the economic order quantity (Q_{ip}), and the transportation mode for delivering the items to its warehouse. In the event that foreign suppliers are selected, the items will be shipped via either air (Mode 1) or sea (Mode 2) transport. Once the item arrives at the designated transportation mode's port, it will be transported to the buyer's warehouse via trucks. To effectively select the right suppliers, the buyer must determine the optimal allocation of the total demand among the suppliers while accounting for the quantity discounts offered by each supplier. Additionally, the buyer must select the most cost-effective transportation mode for delivering the items to its warehouse, minimizing the overall logistics cost.

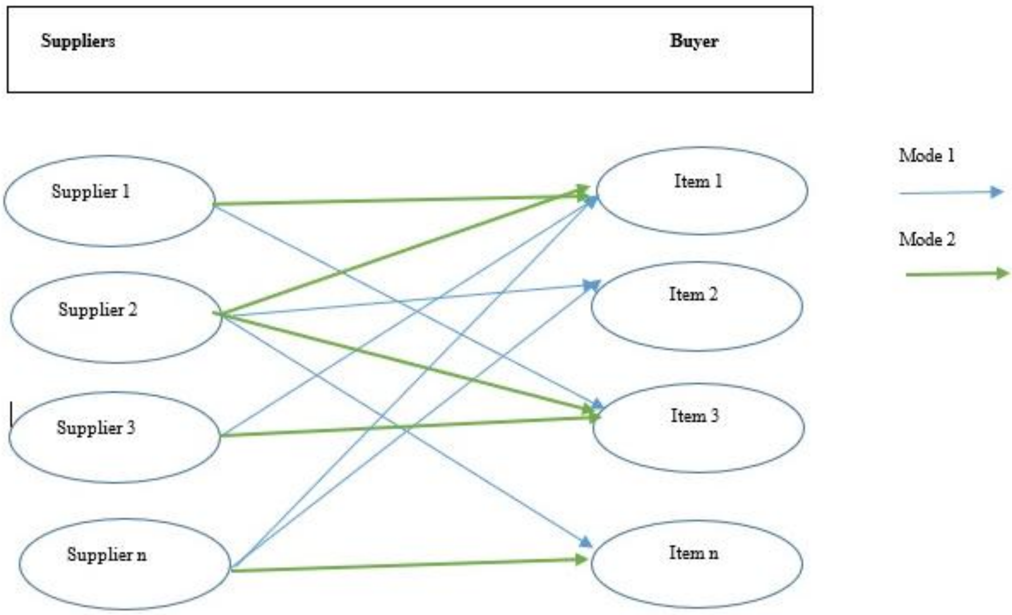

Fig. 1. Multi-item multi-supplier one buyer supply chain

3.1 Notations and assumption

The assumptions made in developing the model of this research include:

- Annual demand for items is deterministic
- No inventory shortage is allowed
- No consolidation or bundling of items for shipment from the different suppliers' is allowed.

The following notation is used in this article:

	Notation	
Sets	i	$the\ set\ of\ suppliers \quad i = 1,2,3,\dots,I$
	p	*the set of products* $\quad l = 1,2,3,\dots,P$
	k	$the\ set\ of\ price\ break\ points \quad k = 1,2,3,\dots,K$
	j	$the\ set\ of\ transportation\ alternative\ modes \quad j = 1,2$
Decision Variables	Q_{ip}	$Economic\ Ordered\ Quantity\ of\ item\ p\ from\ supplier\ i$
	d_{ip}	$the\ total\ amount\ of\ item\ p\ to\ be\ purchased\ from\ supplier\ i$ $d_{ip} \in [q_{i,k-1}, q_{ik})\ \forall p$ where, $q_{i,k}$ o is the k[th] order quantity discount
	x_{ip}	$is\ a\ binary\ variable, its\ value\ set\ equal\ to\ 1\ if\ supplier\ i\ is\ used\ to\ supply\ item\ p\ and\ 0\ otherwise$
	y_{ipk}	$is\ a\ binary\ variable, its\ value\ set\ equal\ to\ 1\ if\ the\ ordered\ quantity\ of\ item\ p\ from\ supplier\ i\ is\ in\ the\ k^{th} interval, \quad and\ 0\ otherwise$
	t_{ipj}	$is\ a\ binary\ variable, its\ value\ set\ equal\ to\ 1\ if\ transportation\ mode\ j\ is\ used\ to\ transport\ item\ p\ from\ supplier\ i, and\ 0\ otherwise$
	n_{ip}	$number\ of\ trucks\ per\ order\ required\ to\ transport\ Q_{ip}, \forall i, p$ $i.e\ \ n_{ip} = \left\lceil \frac{Q_{ip}}{T_p} \right\rceil$
Parameters	C_{ipk}	the unit cost of item p from supplier i at the k^{th} discount interval
	S_{ip}	$the\ ordering\ cost\ of\ item\ P\ from\ supplier\ i$
	u_{ipj}	$Unit\ transportation\ cost\ of\ item\ p\ from\ supplier\ i\ using\ transportation\ mode\ j$
	f_j	$Fixed\ usage\ cost\ per\ truck\ per\ km\ for\ each\ transportation\ mode\ j$
	β_j	$distance\ between\ the\ destination\ point\ of\ transportation\ mode\ j$ to the warehouse of the buyer
	CAP_{ip}	$Capacity\ of\ supplier\ i\ for\ item\ p$
	T_p	$Truck\ load\ capacity\ in\ units\ of\ item\ p$
	l_{ipj}	$the\ lead\ time\ for\ acquiring\ the\ item\ p\ from\ supplier\ i\ using\ transportation\ mode\ j$
	λ_{ip}	$percent\ defective\ of\ supplier\ i\ of\ item\ p$
	ω_{ip}	$Score\ of\ supplier\ i\ with\ respect\ to\ item\ p$
	$\boldsymbol{L_p}$	$the\ maximum\ lead\ time\ the\ buyer\ allow\ for\ item\ p\ to\ be\ delivered$
	h	$the\ annual\ inventory\ holding\ cost\ rate$
	D_p	$Demand\ for\ item\ p$

3.2 Objective Functions

3.2.1 Objective 1: Minimize Cost

The total cost considered include annual inventory holding cost (IHC), ordering cost (OC), transportation cost, and the purchasing cost. The mathematical formulation of each component is as follows:

3.2.1.1 Annual inventory holding cost

The annual inventory holding cost is obtained using eq. (1), where (hC_{ipk}) is the unit annual holding cost, and $\left(\frac{Q_{ip} d_{ip}}{2D_p}\right)$ is the average inventory level during the planning period.

$$\sum_{i=1}^{I} \sum_{p=1}^{P} \sum_{k=1}^{K} \frac{hC_{ipk} Q_{ip}\ d_{ip} x_{ip} y_{ipk}}{2D_p} \tag{1}$$

3.2.1.2. Ordering cost

Eq. (2) calculates the total ordering cost for the selected suppliers where S_{ip} is the ordering cost per order of item p from supplier$i$, and $\left\lceil \frac{d_{ip}}{Q_{ip}} \right\rceil$ is the smallest integer greater than or equal to $\frac{d_{ip}}{Q_{ip}}$, which is the number of orders to be made over the planning period.

$$\sum_{i=1}^{I}\sum_{p=1}^{P} S_{ip}\, x_{ip} \left\lceil \frac{d_{ip}}{Q_{ip}} \right\rceil \quad (2)$$

3.2.1.3 Transportation cost

The transportation cost is the sum of the fixed and variable transportation costs and is obtained by eq. (3). The fixed transportation cost, which is the first term in eq. (3), is the cost of using trucks to transport items from the port of the selected mode of transportation to the warehouse of the buyer. While the variable transportation cost is the transportation cost of using either air or sea to transport the items. Where u_{ipj} and $\left(d_{ip}x_{ip}t_{ipj}\right)$ are the unit variable transportation cost and the amount of item p transported from supplier i using transportation mode j respectively.

$$\sum_{i=1}^{I}\sum_{p=1}^{P}\sum_{j=1}^{J}\left(n_{ip}\beta_j\, f_j \left\lceil \frac{d_{ip}}{Q_{ip}} \right\rceil + u_{ipj}d_{ip}\right)x_{ip}t_{ipj} \quad (3)$$

3.2.1.4 Purchase cost

The total purchasing cost is the sum of the product of the unit cost of the item and the total quantity supplied from each selected supplier(s).

$$\sum_{i=1}^{I}\sum_{p=1}^{P}\sum_{k=1}^{K} C_{ipk}d_{ip}x_{ip}y_{ipk} \quad (4)$$

$$Z_1 = Annual\ inventory\ holding\ cost + Ordering\ cost + Transportaion\ cost + Purchase\ cost$$

$$Z_1 = \sum_{i=1}^{I}\sum_{p=1}^{P}\sum_{k=1}^{K}\frac{hC_{ipk}Q_{ip}\ d_{ip}x_{ip}y_{ipk}}{2D_p} + \sum_{i=1}^{I}\sum_{p=1}^{P} S_{ip}\, x_{ip}\left\lceil \frac{d_{ip}}{Q_{ip}} \right\rceil + \sum_{i=1}^{I}\sum_{p=1}^{P}\sum_{j=1}^{J}\left(n_{ip}\beta_j\, f_j \left\lceil \frac{d_{ip}}{Q_{ip}} \right\rceil + u_{ipj}d_{ip}\right)x_{ip}t_{ipj} + \sum_{i=1}^{I}\sum_{p=1}^{P}\sum_{k=1}^{K} C_{ipk}d_{ip}x_{ip}y_{ipk} \quad (5)$$

3.2.2 Objective 2: Maximize the purchasing value

As in Ghodsypour & Brien (1998), the total value of purchasing is calculated using eq. (6) where (ω_{ip}) and (d_{ip}) are the score of a supplier and the quantity of supply respectively. Maximizing the total purchasing value objective function prioritizes the allocation of supply quantities to suppliers with the highest supplier score (ω_{ip}).

$$Z_2 = \sum_{i=1}^{I}\sum_{p=1}^{P}\omega_{ip}d_{ip} \quad (6)$$

3.2.3. Objective 3: Minimize percent of rejected items

Our third objective is the minimization of the total amount of non-conforming item, which is the sum of the product of the percent reject of product $\lambda_{ip}$by the total quantity of supply from each selected supplier d_{ip}.

$$Z_3 = \sum_{i=1}^{I}\sum_{p=1}^{P}\lambda_{ip}d_{ip} \quad (7)$$

3.3. Model constraints

The constraints of the model include the demand of the buyer, supplier's capacity, deliver lead-time requirement of the buyer, and transportation mode and truck load capacity. Eq. (8) ensures that the total supplied quantity from each selected supplier should at least exceed the demand of the buyer. Eq (9) states that the total order quantity assigned to a selected supplier must not exceed the supplier's capacity.

$$\sum_{i=1}^{I} d_{ip}x_{ip} \geq D_p, \forall p \quad (8)$$

$$d_{ip} \leq CAP_{ip}, \forall i, p \quad (9)$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J} l_{ipj}d_{ip}t_{ipj} \leq L_pD_p, \forall p \quad (10)$$

$$\sum_{j=1}^{J} t_{ipj} - x_{ip} = 0, \forall i, p \quad (11)$$

$$Q_{ip} - n_{ip} * T_p \leq 0, \forall i, p \quad (12)$$

$$q_{ip(k-1)}y_{ipk} \leq d_{ip} \quad (13)$$

$$d_{ip} \leq q_{ipk}y_{ipk} \quad (14)$$

$$x_{ip}, y_{ipk}, and\ t_{ij} \in (0,1) \quad (15)$$

$$n_{ip}, Q_{ip}\ \ and\ d_{ip} \in Integer \quad (16)$$

In Eq. (10), the aggregate lead-time, which is the cumulative lead-time of supplied quantity from selected suppliers, must not exceed the lead-time demand of the buyer. The lead-time is a function of the transportation mode and thus the transportation mode, which is a binary decision variable, is used in the equation. Eq. (11) restricts the selection of one of the transportation modes. Eq. (12) ensure the truck capacity requirement. Eq. (13) and (14) enforces that the ordered quantity falls in the valid quantity discount interval. In Eq. (15), the supplier selection, quantity discount offer interval, and the mode of transportation selection variables are binary. Eq. (16), states that the number of trucks, the EOQ, the allocated order quantity, and the number of orders made to each supplier are all integer values. The next section presents a numerical example to show the applicability of the proposed model.

4. Genetic Algorithm Approach

Genetic Algorithm (GA) is a popular search heuristic that has gained widespread usage among researchers for solving complex optimization problems. GA is modeled after the natural selection process in nature, where it searches for near-optimal solutions to optimization problems by operating on potential solutions known as chromosomes. Each chromosome is assigned a fitness value that represents its performance in solving a specific problem.

The GA process begins by generating a population of potential solutions or artificial chromosomes. Then, a fitness-based selection process, crossover, and mutation operators are applied to produce a new population of offspring. These offspring will then undergo the same process of selection, crossover, and mutation until a termination condition is met, such as reaching a maximum number of generations or achieving a satisfactory level of fitness.

In the case of supplier selection, GA can be utilized to find the best supplier and the corresponding optimal values of the decision variable. The GA process involves generating a population of potential supplier solutions, where each supplier is represented by a chromosome with a fitness value that describes its ability to satisfy the buyer's requirements. The fitness function is determined by the buyer's objective, which is to minimize the total cost of logistics, minimize total rejected items, and maximize the total value of the purchase while meeting capacity, demand, and lead time requirements.

The GA process then applies selection, crossover, and mutation operators to produce a new population of potential suppliers that are better suited to the buyer's requirements. The selection operator chooses the best chromosomes based on their fitness values, and the crossover operator combines two chromosomes to create a new one. The mutation operator introduces random changes to the chromosomes to increase the diversity of the population.

By repeating the GA process, the population of potential suppliers evolves to better satisfy the buyer's requirements, leading to a near-optimal solution.

Step 1: Chromosome representation

The chromosome representation (Table 1.) for the problem in this paper is a n-dimensional matrix where the column corresponds to the suppliers, the rows correspond to the decision variables, and n corresponds to the number of items. The economic order quantity, the order quantity, and the number of trucks are represented as positive integer value while the selection of supplier(s), mode of transportation are represented as binary integers. As shown in the mathematical model, the quantity of supply and the number of trucks are an integer multiple of the economic order quantity, and hence are functions of the EOQ. Therefore, the EOQ and the transportation mode selection variables are the sole decision variables required to be represented in the GA. The first step in utilizing GA is determining the chromosome representation for the problem at hand. The decision variables in the chromosome representation include the economic order quantity (EOQ), the order quantity, the number of trucks required for transportation, the selection of supplier(s), and the mode of transportation.

Table 1.
Genetic representation of sample solution for a single item

		Suppliers				
		A	**B**	**C**	**D**	**E**
Decision Variable	EOQ	30	45	70	20	120
	Supplier selection	0	0	1	0	1
	Transportation Mode 1	0	1	0	0	0
	Transportation Mode 2	1	0	1	0	1

Step 2: Initial population generation

The second step in utilizing GA for supplier selection and procurement optimization is the generation of the initial population. The initial population is generated by creating a set of randomly generated chromosomes, where each chromosome represents a potential solution to the procurement problem.

To generate the initial population, the maximum capacity of the supplier [UB] and the minimum amount [LB] that a supplier is willing to supply are taken into account. The economic order quantity is then generated randomly within the range [LB, UB]. This ensures that the initial solutions are feasible and within the supplier's capacity constraints.

In addition to the economic order quantity, the chromosome also includes genes for supplier selection and transportation mode selection. The genes for supplier selection and transportation mode selection are binary values, where 0 corresponds to not selecting the option, and 1 corresponds to selecting the option.

The initial population is generated by creating a set of chromosomes with randomly generated values for the economic order quantity, supplier selection, and transportation mode selection. These chromosomes are evaluated using the fitness function, which measures their ability to satisfy the buyer's requirements, and the chromosomes with the highest fitness values are selected for the next generation.

Step 3: fitness value

Once the initial population is generated, the next step in utilizing GA for supplier selection and procurement optimization is to evaluate the fitness of each chromosome in the population. The fitness function measures how well a chromosome satisfies the buyer's requirements and constraints, and it is used to select the best chromosomes for the next generation.

For the model proposed in this paper, the fitness function is the minimum of the sum of the objective function and the constraint penalty function. The objective function is to minimize the total cost of logistics, minimize total rejected items, and maximize the total value of the purchase while meeting capacity, demand, and lead time requirements.

The constraint penalty function is used to handle the inequality constraints, which include the lead time of delivery of items. If a chromosome violates any constraint, a penalty is added to the fitness function to reflect the degree of violation. The fitness function for is expressed as follows:

$$fitness = objfun + leadtime\_penalty$$

$$= objfun + \begin{cases} 2^{num_{gen}} * \left(\sum_{i=1}^{I} \sum_{j=1}^{J} l_{ipj} d_{ip} t_{ipj} - L_p D_p \right), \\ \text{if } \sum_{i=1}^{I} \sum_{j=1}^{J} l_{ipj} d_{ip} t_{ipj} > L_p D_p \\ 0, otherwise \end{cases}$$

Where num_gen is the number of generations. The penalty will only be applied to late deliveries and hence the above inequality condition included.

Step 4: Crossover

After evaluating the fitness of each chromosome in the population, the next step is crossover. Crossover is a genetic operator that combines the genetic material of two parent chromosomes to create offspring chromosomes with improved fitness.

In this paper, a standard vertical one-cut-point operator is used for the n-dimensional matrix representation of the chromosome. This operator selects a random column index and exchanges the genetic material of the two parent chromosomes at that column index to create two offspring chromosomes (Fig. 2.).

The crossover probability used in this paper is 0.85, which means that there is an 85% chance that crossover will occur for each pair of parent chromosomes. This probability is chosen to balance the exploration of new solutions with the exploitation of existing solutions.

By applying crossover, the process can combine the genetic material of the best performing chromosomes to create new, potentially better performing offspring chromosomes. The GA process will then use selection, crossover, and mutation operators to generate new chromosomes and improve the fitness of the population over successive generations.

a. parent 1 solution

		Suppliers				
		A	B	C	D	E
Decision Variable	EOQ	30	45	70	20	120
	Supplier selection	0	0	1	1	0
	Mode 1	0	1	0	0	0
	Mode 2	0	0	1	0	1

b. parent 2 solution

		Suppliers				
		A	B	C	D	E
Decision Variable	EOQ	80	60	60	40	90
	Supplier	1	0	1	0	1
	Mode 1	1	0	0	0	0
	Mode 2	0	0	1	0	1

c. offspring 1 solution

		Suppliers				
		A	B	C	D	E
Decision Variable	EOQ	30	45	70	40	90
	Supplier selection	0	0	1	0	1
	Mode 1	0	1	0	0	0
	Mode 2	0	0	1	0	1

d. offspring 2 solution

		Suppliers				
		A	B	C	D	E
Decision Variable	EOQ	80	60	60	20	120
	Supplier selection	1	0	1	1	0
	Mode 1	1	0	0	0	0
	Mode 2	0	0	1	0	1

Fig. 2. Example of the vertical one-cut-point crossover

Step 5: Mutation

Mutation is a genetic operator that introduces random changes to the genetic material of a chromosome, creating new potential solutions that were not present in the initial population. In this paper, the mutation operator is applied to the offspring chromosomes generated by the crossover operator. The mutation operator alters the value of a randomly selected gene in the offspring chromosome (Table 2.). The mutation probability used in this paper is 0.01, which means that there is a 1% chance that a gene will be mutated for each offspring chromosome.

To perform the mutation operation, a random number is first generated from a uniform distribution with a range of [0,1]. If the generated number is less than or equal to the mutation probability, a randomly selected gene in the offspring chromosome is mutated. The mutation process is performed while ensuring that the capacity constraint of the supplier is not violated.

Table 2
Gene selected for mutation

		Suppliers				
		A	B	C	D	E
Decision Variables	EOQ	30	45	70	20	120
	Mode 1	0	1	0	0	0
	Mode 2	1	0	1	0	1

Step 6: Problem Specific Operators

After applying crossover and mutation operators, the next step is to include problem-specific operators. These operators are tailored to the specific problem at hand and can speed up the GA process while ensuring that the constraints of the problem are met.

In this paper, similar to Liao & Rittscher (2007), problem-specific operators are included to speed up the GA and include the supply-demand equality constraint. However, unlike Liao & Rittscher (2007), our proposed problem-specific operator does not add or subtract the difference between demand and supply to one or more randomly selected suppliers. Instead, our proposed operator uses an elitist selection approach to add or subtract the difference between demand and supply.

The problem-specific operator is designed to guarantee the demand constraint and speed up the search for near-optimal solutions. It works as follows:

1. Calculate the difference between the total supply and the demand for each item$(\mathrm{supply.difference})$. And let $smallest_i$ *and* $largest_i$ represent the i^{th} smallest and largest supplied quantities respectively.
2. If $\mathrm{supply.difference} > 0$:
   If $smallest_i \geq \mathrm{supply.difference}$:
   $$smallest_i = smallest_i - \mathrm{supply.difference}$$
   Else:
   $$smallest_i = 0$$
3. If $\mathrm{supply.difference} < 0$:
   If $\mathrm{supply.difference} + largest_i > Capacity_i$:
   $$largest_i = \mathrm{supply.difference} + largest_i$$
   Else:
   $$largest_i = Capacity_i$$
4. Repeat step 1-3 until the difference between the supplied quantity and the demand for each item is zero.

By adding or subtracting the difference between demand and supply using an elitist selection approach, our proposed problem-specific operator ensures that the solutions generated by the GA process are feasible and practical in the real world. It also speeds up the search for near-optimal solutions by directing the search towards solutions that meet the supply-demand equality constraint. In the results and discussion section, a comparison of the solution obtained using both random selection and elitist selection approaches is provided. This comparison shows that our proposed problem-specific operator produces better results than the random selection approach, which is consistent with the literature.

Step 7: Termination

The final step in utilizing GA for supplier selection and procurement optimization is termination. The GA process is repeated multiple times, with each iteration involving the evaluation of the fitness of the population, selection, crossover, mutation, and problem-specific operators. This cycle continues until a defined stopping criterion is met.

In this paper, the stopping criterion is set to a maximum number of population generations, which is set to 200. After generating 200 populations, the GA process is terminated, and the best solution obtained from all the populations is selected as the final solution.

The choice of the stopping criterion is critical in determining the quality of the solutions obtained from the GA process. If the stopping criterion is too small, the GA process may not have sufficient time to converge to a globally optimal solution. On the other hand, if the stopping criterion is too large, the GA process may continue to generate new populations without producing any significant improvements in the solutions obtained.

It is worth noting that the stopping criterion used in this paper is problem-specific and may vary depending on the complexity of the procurement problem at hand. Therefore, decision-makers should carefully choose the stopping criterion based on their understanding of the problem and the performance of the GA process.

5. Computational Results and Discussion

The computational results and discussions section of this paper presents the implementation details and performance of the GA with the problem-specific operator. The GA was coded in Python 3.6.5 and executed on a personal computer with a Core™ i5-6200U @ 2.3 GHz, with 3.48 GB of RAM.

For the numerical experiment, data obtained from the Ethiopian Pharmaceutical Fund and Supply Agency (PFSA) was used. The problem involved selecting the best supplier, allocating order quantities, determining economic order quantities, and transportation mode in such a way that it minimizes total logistics cost, maximizes purchase value, and minimizes the total percentage of non-conforming items while obtaining the items within the delivery time and desired quantity. The problem had three items and six suppliers, of which only one supplier was local. All suppliers offered price-breaks of the all-quantity discount type for two items and no discount offer for the third item. To solve the problem, the Analytic Hierarchy Process (AHP) was first used to evaluate and calculate a supplier's score, which was used in the purchasing value function. Seven purchasing experts from PFSA participated in the pairwise comparison of predetermined criteria used by the agency. A sample summary pairwise comparison matrix and the resulting weight of criteria are shown in Table 3., with a group consensus value of 97%. The consistency ratio value was 0.022, which is less than 0.1, indicating that the decision-makers' evaluation of the criteria provided in the pairwise comparison matrix is consistent. The score of a supplier was then calculated by summing the product of the weight of a criterion and the corresponding value of the supplier as evaluated by the buyer.

The GA with the problem-specific operator was then applied to the problem, and the results were compared with the results obtained using random selection. The comparison showed that the proposed problem-specific operator produced better results than the random selection approach. The proposed operator reduced the total logistics cost by 17.5% and increased the purchase value by 15.2% compared to the random selection approach. Additionally, the proposed operator reduced the total percentage of non-conforming items by 10.4%.

The results demonstrate the effectiveness of the proposed GA approach with the problem-specific operator in supplier selection and procurement optimization. The approach can help decision-makers obtain near-optimal solutions that meet the supply-demand equality constraint and are feasible and practical in the real world. The results also highlight the importance of using problem-specific operators to speed up the GA process and ensure that the solutions obtained are of high quality.

Table 3
Pairwise comparison matrix (consistency ratio=0.022<0.1)

	Unit Cost	Percent reject	Delivery	Shelf Life	Logistics	Weight
Unit Cost	1	4	2	5.285714	2.714286	0.4163
Percent reject	0.25	1	0.333333	2	0.4	0.0933
Delivery	0.5	3	1	3.555556	2	0.2572
Shelf Life	0.19	0.5	0.28125	1	0.333333	0.0606
Logistics	0.37	2.5	0.5	3	1	0.1726

The weights obtained from the pairwise comparison matrix in Table 3 were used in conjunction with the supplier score table to obtain the weighted supplier score (Table 4). The weighted supplier score value was used in the purchasing value function that the buyer was interested in maximizing.

To determine the total logistics cost, the estimated order processing or management cost per order was set to 1000 Birr for local suppliers and 5000 Birr for foreign suppliers. The estimated fixed transportation cost per truck per kilometer was set to 40 Birr for all distances greater than 100km and 100 Birr otherwise. The distance between the seaports and the buyer's warehouse was 884km, while the distance from the airport to the warehouse was 50km.

The annual demand for items and other input parameters of the model were provided in Tables 4, 5, and 6. These input parameters were used to determine the optimal solution that minimizes total logistics cost, maximizes purchase value, and minimizes the total percentage of non-conforming items while meeting the supply-demand equality constraint.

In summary, the weighted supplier score obtained from Table 3 was used in the purchasing value function, and the estimated costs of order processing, transportation, and distances were used to calculate the total logistics cost. The annual demand for items and other input parameters provided in Tables 4, 5, and 6 were used in the model to obtain the optimal solution that meets the supply-demand equality constraint and minimizes total logistics cost, maximizes purchase value, and minimizes the total percentage of non-conforming items.

Table 4
Discount plan, transportation cost, and lead time values of suppliers for item 1.

Suppliers	Purchase quantity	Unit cost	Transportation cost		Lead Time (months)		%Defective	Supplier Score
			Mode 1	Mode 2	Mode 1	Mode 2		
A	0 up to 4999 5000 or more	344.9 343.9	0	0	1	1	0.012	5.6
B	0 up to 1999 2000 or more	387.4 385.4	407	2.7	2	3	0.008	6.5
C	0 up to 1999 2000 or more	586.7 585.7	17.4	1.8	2	3	0.002	8.5
D	0 up to 1499 1500 or more	257.3 255	51.9	5.7	2	3	0.004	6.9
E	0 up to 2500 25001 or more	459.4 459	57.4	14.5	2	3	0.006	5.8
F	0 up to 4000 4001 or more	462.7 462	50.2	8.3	1	2	0.004	6.2

Table 5
Discount plan, transportation cost, and lead time values of suppliers for item 2.

Suppliers	Purchase quantity	Unit cost	Transportation cost		Lead Time (months)	
			Mode 1	Mode 2	Mode 1	Mode 2
A	0 up to 1499 1500 or more	344.9 343.9	0	0	2	2
B	0 up to 1999 2000 or more	308.9 306.9	26.2	3.1	2	3
C	0 up to 1999 2000 or more	457.8 455.8	13.3	1.4	3	4
D	0 up to 999 1001 or more	306.3 306	16.5	10.2	3	4
E	0 up to 3500 3501 or more	220.5 220	51.9	3.8	2	3
F	>0	378.6	26.5	4.8	2	3

Table 6
Unit cost, transportation cost, and lead time values of suppliers for item 3.

Suppliers	Purchase quantity	Unit cost	Transportation cost		Lead Time (months)	
			Mode 1	Mode 2	Mode 1	Mode 2
A	>=0	33.7	0	0	2	2
B	>=0	34.8	4.6	0.8	3	4
C	>=0	37.6	3.8	1.4	2	3
D	>=0	33.7	3.9	0.9	2	3
E	>=0	85.6	6.1	4.1	3	4
F	>=0	75.2	5.3	2.9	2	3

In the context of supplier selection and procurement optimization, the weighted supplier score obtained in Table 3 was used as an input in the purchasing value function, which is a critical component in the objective function of the optimization problem. The purchasing value function aims to maximize the value of the purchase while considering the total logistics cost and the percentage of non-conforming items.

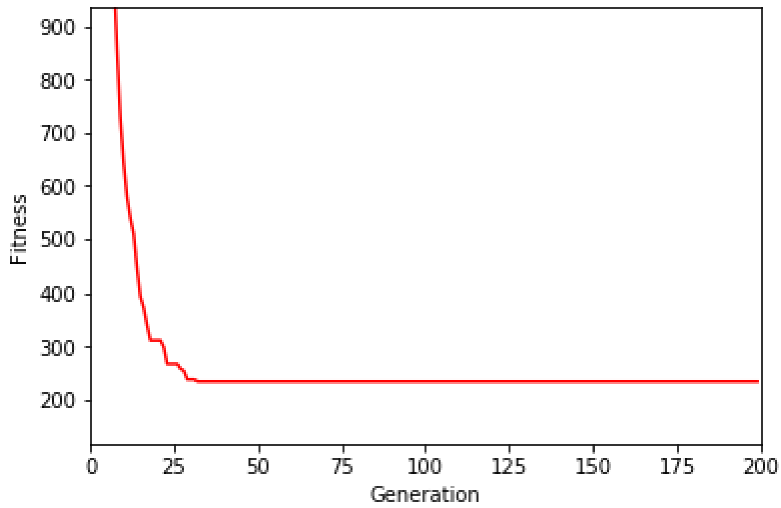

Fig. 3. Convergence of the GA approach

The proposed GA approach was evaluated using three objective functions: total logistics cost, purchase value, and the total percentage of non-conforming items. As indicated in Fig 3 the iterative process for finding a solution converges around the 20th generation, resulting in the attainment of the optimal solution. The results obtained from the GA approach were compared with the results obtained from randomly selecting a supplier and adding or subtracting the difference between the demand and supply. The values of the three objective functions obtained from the GA approach were 400,714, 5,798, and 0.0272, respectively. In contrast, the values obtained from randomly selecting a supplier and adjusting the order quantities were 441,485, 5877, and 0.025. The results showed that the GA approach outperformed the random selection approach in terms of the first objective function, which is the total logistics cost, as it was able to reduce it by 10%. Although there was no significant improvement in the values of the second and third objective functions, the GA approach was still able to provide a better overall solution.

Furthermore, the importance of the first objective function was emphasized by considering it twice as important as the other two objective functions. In this case, the value of the first objective function decreased to 380,168, and supplier F was also selected in addition to suppliers C and D. The table (Table 7.) also provided tradeoffs between the objective functions, giving decision-makers alternative solutions to choose from based on their priorities.

In summary, the results showed that the GA approach proposed in step 6 of the optimization process was more effective in obtaining an optimal solution than randomly selecting a supplier and adjusting the order quantities. The tradeoffs between the different objective functions provided alternative solutions for decision-makers to choose from based on their priorities.

Table 7
Summary of results

Case		1	2	3	4
Weights	W_1	0.001	0.0005	0.00025	0.00025
	W_2	0.1	0.025	0.05	0.025
	W_3	1	0.25	0.25	0.5
Objectives	Z_1	400,714	380,168	391,777	410,508
	Z_2	5,798	5,729	5,607	5,918
	Z_3	0.0272	0.0276	0.03	0.0258
Total order	Item 1	$(69_C, 191_D)$	$(200_D, 60_F)$	$(200_D, 60_E)$	$(62_C, 198_D)$
	Item 2	$(48_C, 182_D)$	$(70_C, 160_D)$	$(39_C, 191_D)$	$(130_C, 100_D)$
	Item 3	$(100_C, 200_D)$	$(130_C, 170_D)$	$(100_C, 200_D)$	$(100_C, 200_D)$

5. Conclusion

In conclusion, this paper addresses the complex decision-making problem of multi-criteria supplier selection, order allocation, and economic order quantity and transportation mode selection under quantity discount. The inclusion of various qualitative selection criteria, price break offers, supplier capacity limitations, and demand and lead time requirements of the buyer increases the complexity of the problem. To tackle this problem, an integrated AHP and multi-objective nonlinear mixed integer model is proposed to minimize the total cost, minimize the total percentage of rejected items, and maximize the total purchasing value simultaneously.

Furthermore, a GA approach with problem-specific operators is developed and used to solve the proposed model. The results show that the GA approach is more effective in obtaining an optimal solution than randomly selecting a supplier and adjusting the order quantities. The proposed approach can be extended to include different types of quantity discount schemes and bundling of procured items for transportation.

Overall, this paper provides a comprehensive approach for decision-makers to optimize supplier selection, order allocation, and economic order quantity and transportation mode selection under quantity discount. By considering various qualitative and quantitative factors, the proposed approach can help decision-makers make informed decisions while minimizing costs, maximizing value, and meeting other constraints and requirements.

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