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Research Article

An EOQ Model for Defective Items Under Pythagorean Fuzzy Environment

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Abstract

Classical EOQ models can help us decide on the terms of how much to order, to manage an inventory. Any company dealing with physical products needs to manage an inventory to improve and avoid shortages occurring. Many times the lots come with defective items due to which there is a loss in the effectiveness of the model. In the present study, we consider two types of carrying costs for good and defectives items, and also proportionate discount is being considered for defective items. We use the Pythagorean fuzzy environment (PFS) and analyze the score functions with help of (α, β) cuts of the fuzzy parameters. The problem is optimized to get the best solution, utilizing Yager's Ranking. The numerical results obtained from crisp and fuzzy environments are also compared. Lastly graphical and sensitivity illustrations are being used to justify the models.

Keywords: EOQ *inventory system; PFS; Scores function; Yager's ranking function*

1. Introduction:

The EOQ model was first developed in the early nineteenth century, but it is now used by a variety of industries, including finance, agriculture, aeronautical engineering, management science, economics, genetic engineering, and others, to analyze complex real-world processes to improve optimum solutions. Harris presented the classical square root of the EOQ model in 1913, which is covered in every inventory management textbook. There are few surrealistic expectations within the ancient EOQ model. Researchers are fascinated by economic order amount (EOQ) models with a renowned likelihood distribution operate containing a random fraction of defective merchandise and every one unit amount discounts for defective items. Karlin (1934) was a mathematician who studied inventory and development theory. The proportion of faulty products is not constant, according to Rosenblatt and Lee (1986). It may be linear, exponential, or in a multi-state decaying phase. Salameh and Jaber (2000) sold the products at a bargain price after screening all of the items collected and collecting faulty items in a single shipment. Wee et al. (2007) made the same assumption as Salameh and Jaber when it came to shortages due to backorder. Furthermore, Maddah and Jaber (2008) investigated Salameh and Jaber's work and created an EOQ model based on renewal theory to obtain an exact expression for estimated benefit per unit time. Chang (2004) developed an EOQ model for faulty using fuzzy set theory and annual demand. Singh and Singh (2008) developed a finite rate of replenishment fuzzy inventory model using the Signed-distance approach for defuzzification. Chen and Chang (2008), Jaggi (2014), and Jaggi et al. (2015), among others, provide additional extensions to this research field. Ahmadi (2016) established optimum manufacturer-retailer strategies in a supply chain with faulty goods and price dependant demand. Khana et al. (2017) looked at the inventory dilemma of deteriorating items, full backlog, and demand as a result of the sale price. Mohagheghian (2018), with the influence of a two-echelon supply chain, considers multiple retailers with price and promotional effort sensitive non-linear demand. Shekarian et al. (2017) fuzzified demand, storing expense, overall inventory cost, and order quantity using triangular fuzzy numbers and developed crisp and fuzzy models with and without backorders. Because of the increasing complexity of the socioeconomic setting and natural intuitive reasoning, Zadeh initially projected the fuzzy idea in 1965, and Bellman and Zadeh (1970) used it in decision-making scenarios. Since then, this principle of inventory control has been utilized by a variety of scholars. Many researchers like Karmakar et al. (2018), Mao (2018), Patro (2019), De et al. (2019) and (2020), Pal et al.(2020), Maity et al. (2020), Pattnaik & Nayak et al (2021) showed recent decision-making studies on the difficulty of inventory management with the thought of fuzzy distinct fuzzy numbers in fuzzy environments. Mohamadghasem (2020) A note on an alpha cuts-based interval type-2 fuzzy extension of the TOPSIS method. In general, we tend to assume the fundamental flexibilities of a parameter in fuzzy pure mathematics while not encountering any resistance.

The famous Attanassov's intuitionistic fuzzy set opposes the fuzzy set's general summary. Over the past few decades, scholars have focused their efforts on the IFS hypothesis and its uses in a variety of fields, including decision making, medical diagnosis, pattern recognition, business forecasting, facility position selection, and so on. However, it is constrained by its own description. For example, $\boldsymbol{\alpha}$ (membership grade) and $\boldsymbol{\beta}$ (non-

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Table – 1: **The comparison between earlier published work and our present work**

Authors	Different Holding cost	Deterioration	Defective items	discount	Types of fuzzy
M.K.Salameh, M.Y.Jaber (2000)	No	no	yes	no	no
M.Y.Jaber, M.I.M Wahab (2010)	Yes	no	yes	no	no
R.R.Yager(2013)	no	no	no	no	Yes(PF)
C.KJaggi(2014)	no	yes	no	no	no
C.K.Jaggi, A.Sharma, S.Tiwari(2015)	no	yes	no	no	no
S.K.De,S.S.Sana (2015)	no	no	no	no	Yes(IFS)
E.Shekarian, E.U.Olugu, S.H.Abdul-Rashid, N.Kazemi (2016)	yes	no	yes	no	yes
Z.M.Teksan, L.Geunes(2016)	no	no	no	no	no
S,C,Chen, J.Min, J.T.Teng, F.Li (2016)	no	no	no	no	no
A.Khana, P.Goutam, C.K.Jaggi (2017)	no	yes	yes	no	no
S.Karmakar, S.K.De (2018)	no	no	no	no	Yes(Dense fuzzy)
X.B.Mao,S.S.Hu (2018)	no	no	no	no	Yes(Interval valued fuzzy)
S.K.De, G.C.Mahato (2019)	no	no	no	no	Yes(Fuzzy Monsoon demand)
R.Patro. M.M.Nayak, M. Acharya (2019)	no	no	yes	yes	Yes(Triangular)
S.Pal, A.Chakraborty (2020)	no	yes	no	no	Yes(Triangular neutrosophic)
S.Maity, A.Chakraborty (2020)	no	no	no	no	Yes(Dense fuzzy)
S.K.De(2021)	no	no	yes	no	Yes(Lock fuzzy)
This research	yes	no	yes	yes	Yes(PFS)

$\boldsymbol{\alpha+\beta\leq 1}, \forall \boldsymbol{\alpha,\beta} \in [\mathbf{0},\mathbf{1}]$. Suppose $\boldsymbol{\alpha=0.8,\beta=0.6}$ satisfying $\alpha \geq \beta$, but contradicting $\boldsymbol{\alpha+\beta\leq 1}$. This condition can occur in a decision-making dilemma where all of the knowledge is unknown and the catchment region of exact customers is not finite. As a result, we employ the PFS to transform the real-world problem into a mathematical model. The sum of the support degree (membership degree) and the against degree (non-membership degree) to which an option meets a decision maker's criterion can be greater than one, but their square sum must be equal to or less than one. Thus the membership and non-membership function in the Pythagorean fuzzy set satisfies $\boldsymbol{\mu_p^2+\vartheta_p^2\leq 1}$ whether $\boldsymbol{\mu_p+\vartheta_p \leq or \geq 1}$. Peng and Yang (2015) introduce the score function in PFS as $\boldsymbol{S(x)=\mu_p^2-\vartheta_p^2}$ and the new operation of interval-valued PFS also discusses Peng (2019). According to the results of the study, researchers have created various inventory models for imperfect quality goods, assuming that the keeping expense for both good and faulty items is the same. Based on Wahab and Jaber (2010) and Shekarian, Olugu, Abdul-Rasid, Kazemi (2016), the optimum lot sizes for goods of imperfect content, where separate keeping costs for decent and faulty products are considered.In the actual production world, though, the good and defective goods are handled differently and held separately as two lots with varying keeping costs. For a unit of good and faulty goods per year, we allocate holding costs C_{h_g} *and* C_{h_d}. For example, in many industries such as textiles, cosmetics, and semiconductors, faulty products are isolated from good items and sold at a discounted price following a 100 percent screening procedure. In the inventory models, this paper contains the proportionate discount per percentage of defectives using two distinct forms of holding prices. We maximize the overall benefit function in crisp environments and look at two unknown parameters, the rate of defectives and demand, in PFS to see how efficient and influential fuzzy environments are, analyses of the score functions are performed using the member and non-membership functions, as well as alpha cuts of the fuzzy parameters. We refine the dilemma and find the right answer using Yager's ranking index. To validate the model, a comparative analysis of numerical outcomes is conducted and graphical illustrations are produced. We can see from the sensitivity analysis that the parameters are very sensitive when we adjust the value from -25 to +25 percent of the original input data.Table-1 shows the comparison between our present work and the already published work of few different authors.

2. Basic Concepts

Definition 2.1 (K.T.Atanassov 1986) Let $\boldsymbol{X} = \{\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots\ldots\ldots, \boldsymbol{x_n}\}$ be a finite universal set. Then IFS *I* in X is given by $I = \{< \boldsymbol{x_i}, \boldsymbol{\mu_I}(\boldsymbol{x_i}), \boldsymbol{\vartheta_I}(\boldsymbol{x_i}) > |\boldsymbol{x_i} \in \boldsymbol{X}\}$, $\boldsymbol{\mu_I} : \boldsymbol{X} \to [\boldsymbol{0}, \boldsymbol{1}]$ denotes the membership degree and $\boldsymbol{\vartheta_I} : \boldsymbol{X} \to [\boldsymbol{0}, \boldsymbol{1}]$ denotes the non-membership degree of the element $\boldsymbol{x} \in \boldsymbol{X}$ to the set *I* respectively with condition that $\boldsymbol{0} \le \boldsymbol{\mu_I}(\boldsymbol{x}) + \boldsymbol{\vartheta_I}(\boldsymbol{x}) \le \boldsymbol{1}$. The degree of indeterminacy $\boldsymbol{\pi_I}(\boldsymbol{x}) = \boldsymbol{1} - \boldsymbol{\mu_I}(\boldsymbol{x}) - \boldsymbol{\vartheta_I}(\boldsymbol{x})$. For convenience, Xu and Yager (2006) called ($\boldsymbol{\mu_I}(\boldsymbol{x}), \boldsymbol{\vartheta_I}(\boldsymbol{x})$) an **intuitionistic fuzzy set (IFS).**

Definition 2.2 (K.T.Atanassov 1986) $(\boldsymbol{\alpha}, \boldsymbol{\beta})$level interval or $(\boldsymbol{\alpha}, \boldsymbol{\beta})$-cuts

If we consider, A to be an IFS of a universal set X. Then $(\boldsymbol{\alpha}, \boldsymbol{\beta})$-cut of A is a crisp subset $\boldsymbol{A_{\alpha,\beta}}$ of the IFS A is given by $\boldsymbol{A_{\alpha,\beta}} = \{\boldsymbol{x} \in \boldsymbol{X}: \boldsymbol{\mu_A}(\boldsymbol{x}) \ge \boldsymbol{\alpha}, \boldsymbol{\vartheta_A}(\boldsymbol{x}) \le \boldsymbol{\beta}\}$,where$\boldsymbol{\alpha}, \boldsymbol{\beta} \in [\boldsymbol{0}, \boldsymbol{1}]$ with $\boldsymbol{\alpha} + \boldsymbol{\beta} \le \boldsymbol{1}$.

Definition 2.3 (Peng and Yang 2015) Let set X be a universe of discourse and a PFS is an object having the form $\boldsymbol{P} = \{< \boldsymbol{x}.\boldsymbol{P}(\boldsymbol{\mu_P}(\boldsymbol{x}), \boldsymbol{\vartheta_P}(\boldsymbol{x})) > |\boldsymbol{x} \in \boldsymbol{X}\}$ where $\boldsymbol{\mu_P} : \boldsymbol{X} \to [\boldsymbol{0}, \boldsymbol{1}]$ is the membership degree and $\boldsymbol{\vartheta_P} : \boldsymbol{X} \to [\boldsymbol{0}, \boldsymbol{1}]$ is the non-membership degree of the element $\boldsymbol{x} \in \boldsymbol{X}$ to the set *P*, respectively. For every $\boldsymbol{x} \in \boldsymbol{X}$ and *P* satisfies $\boldsymbol{\mu_P^2} + \boldsymbol{\vartheta_P^2} \le \boldsymbol{1}$.We call $\boldsymbol{\pi_P}(\boldsymbol{x}) = \sqrt{\boldsymbol{1} - (\boldsymbol{\mu_P}(\boldsymbol{x}))^2 - (\boldsymbol{\vartheta_P}(\boldsymbol{x}))^2}$, the indeterminacy degree of x to P and the score of the PFS P is given by $\boldsymbol{S}(\boldsymbol{x}) = \boldsymbol{\mu_P^2}(\boldsymbol{x}) - \boldsymbol{\vartheta_P^2}$. The feasible region of PFS and IFS are shown in Fig-1.

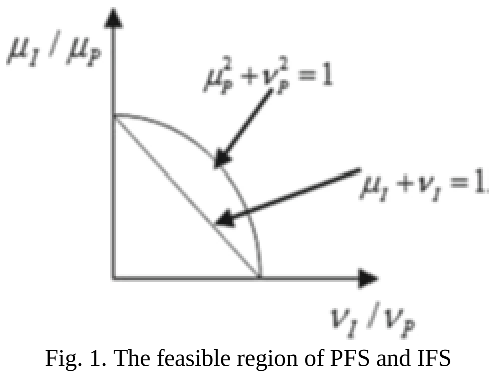


Fig. 1. The feasible region of PFS and IFS

In the above Fig-1, the intuitionistic (non) membership grades are all points under the line $\mu_I(x) + \vartheta_I(x) = 1$ and all points with $\mu_P^2 + \vartheta_P^2 \le 1$ are Pythagorean (non)membership grades. So, a PFS represents a larger body of non-standard membership grades than intuitionistic membership grades.

3. Symbols and Mathematical Model

The following are the representations and hypotheses used to build the suggested inventory models.

z	Order quantity
C_p	The unit cost of set-up
C_k	The cost associated with an order
C_h	Cost associated with holding
p	% of defective items in z
S_g	Unit wise fixed selling price for good items
w	Rate of screening
C_s	One items screening cost
T	Cycle time
TR	Total revenue cycle wise
TP	Total profit cycle wise
TPU	Unit time total profit

3.1. Assumptions

1. Demand rate per cycle is constant.
2. Instantaneous delivery and no shortage.
3. After 100% screening of the items, defective ones are sold at a proportionate discounted price.
4. There is no gap between placing and receiving an order.
5. The holding cost varies for both good and defective items.

3.3. Mathematical Model Formulation

The proposed inventory model is built using the assumptions mentioned above. A lot of *z* products are used at the start of the cycle with buying c_p and ordering c_k prices. For the period 0 to t_1, the screening process is carried out at a rate of w units per unit time with a screening cost that is greater than the demand rate *D*. Each lot includes *p* percentage of defectives and *(1-p)z* high-quality products after 100 percent screening. Since both values are handled differently in the real world scenario, we conclude that their keeping costs are not the same. The faulty products *pz* were delivered in a single shipment at a proportionate discount price at period t_1. Shortages are avoided by assuming, the number of good items $(1-p)z$ is at least equal to the demand during the screening time t, (Salameh and Jaber 2000) that is

$$(1-p)z \ge Dt \qquad (3.3.1)$$

and

$$p \le 1 - \frac{D}{w} \qquad (3.3.2)$$

Now the cycle-wise total cost ($TC(z)$) includes the procurement, screening, and holding (both good and defective items) cost per cycle and total revenue consist of total sales concerning good and imperfect quality items. The holding cost of defective ($C_{h_d}(z)$) and good items ($C_{h_g}(z)$) are as follows:

$$C_{h_d}(z) = C_{h_d}\left(\frac{pz^2}{2w}\right) \tag{3.3.3}$$

and

$$C_{h_g}(z) = C_{h_g}\left(\frac{pz^2}{2w} + \frac{z(1-p)T}{2}\right) \tag{3.3.4}$$

The cycle wise total cost and total revenue are calculated as follows:

$$TC(z) = C_k + C_p z + C_s z + C_{h_g}\left(\frac{pz^2}{2w} + \frac{z(1-p)T}{2}\right) + C_{h_d}\left(\frac{pz^2}{2w}\right) \tag{3.3.5}$$

$$TR(z) = \frac{2S_g z^2 + \left(C_k + C_p z + C_s z + C_{h_g}\left(\frac{pz^2}{2w} + \frac{z(1-p)T}{2}\right) + C_{h_d}\left(\frac{pz^2}{2w}\right)\right)(zp+1)}{2z + (zp+1)} \tag{3.3.6}$$

The cycle wise total profit is calculated as:

$$TP(z) = TR(z) - TC(z)$$

$$TP(z) = \left\{\frac{2S_g z^2 - 2z\left\{C_k + C_p z + C_s z + C_{h_g}\left(\frac{pz^2}{2w} + \frac{z(1-p)T}{2}\right) + C_{h_d}\left(\frac{pz^2}{2w}\right)\right\}}{2z + (zp+1)}\right\} \tag{3.3.7}$$

Finally, the unit time-wise total profit $TPU(z)$ is given as follows:

$$TPU(z) = TP(z)/T$$

Where $T = \frac{z(1-p)}{D}$

$$TPU(z) = \frac{2D(S_g z - C_k - C_p z - C_s z)}{2z + zp + 1}\left(\frac{1}{1-p}\right) - \frac{z^2}{2z + zp + 1}(C_{h_d} p + C_{h_g}) \tag{3.3.8}$$

The optimality criteria for the model is checked by finding derivatives of the total profit concerning the lot size

$$TPU'(z) = \left(\frac{1}{(2z + zE[p] + 1)^2}\right)$$

$$\left[\begin{array}{l}\left(2DS_g - 2DC_p - 2DC_s + 4DC_k + 2DC_k p\right)\left(\frac{1}{1-p}\right) \\ -2C_{h_d} z^2 p - C_{h_d} z^2 (p)^2 - 2C_{h_g} z^2 - C_{h_g} z^2 p - 2C_{h_g} z - 2C_{h_d} zp\end{array}\right] \tag{3.3.9}$$

$$TPU''(z) = -\left(\frac{2}{(2z + zp + 1)^3}\right)$$

$$\left[(2+p)\left(2DS_g - 2DC_p - 2DC_s + 4DC_k + 2DC_k p\right)\left(\frac{1}{1-p}\right) + C_{h_g} + C_{h_d} p\right] < 0 \tag{3.3.10}$$

The optimal solution of the model is given by the first and second derivative $TPU(z)$s. For all values of z, the second derivative $TPU(z)$ is negative which shown that the maximizes/ concave nature of $TPU(z)$. When the first derivative equals 0, the ideal lot size is as follows:

$$z_{\max} = \sqrt{\frac{\left(2DS_g - 2DC_p - 2DC_s + 4DC_k + 2DC_k p\right)\left(\frac{1}{1-p}\right)}{2C_{h_d} p + C_{h_d}(p)^2 + 2C_{h_g} + C_{h_g} p + \frac{2C_{h_d} p}{z} + \frac{2C_{h_g}}{z}}} \tag{3.3.12}$$

For a large value of z, $\frac{1}{z} \to 0$

$$z_{\max} = \sqrt{\frac{\left(2DS_g - 2DC_p - 2DC_s + 4DC_k + 2DC_k E[p]\right)E\left(\frac{1}{1-p}\right)}{2C_{h_d} E[p] + C_{h_d}(E[p])^2 + 2C_{h_g} + C_{h_g} E[p]}} \tag{3.3.13}$$

and when $p = 0$, $C_s + C_p = S_g$ and $C_{h_d} = C_{h_g} = C_h$

Eq. (3.3.13) reduces to the traditional EOQ formulae,

$$z_{\max} = \sqrt{\frac{2C_k D}{C_h}}$$

3.3 Implication of PFS environment in Inventory Management

Due to the increasing complexity of the socioeconomic environment and natural intuitive thinking, various constraints such as incomplete knowledge, information fission, information fusion, and incorrect data sets may arise in decision-making situations. Therefore, it is very much important to use the best membership function and their de-fuzzification method to convert the real-world problems as an appropriated mathematical model. In the general fuzzy set, we usually consider the simple flexibility of a parameter without getting any opposition. The IFS contradicts the general overview of the fuzzy sets, but it is limited in its definition. For example, α (membership grade) and β (non-membership grade) are related such that $\alpha \geq \beta$ and $\alpha + \beta \leq 1, \forall \alpha, \beta \in [0,1]$. Suppose $\alpha = 0.8, \beta = 0.6$ satisfying $\propto \geq \beta$, but contradicting $\alpha + \beta \leq 1$. In a decision-making problem, this situation may arise where all the information are not known and the catchment zone of exact customers is not finite. This concept is more realistic than existing A-IFS and the best fitted fuzzy relaxation might be done through PFS.

3.4 Pythagorean Fuzzy Mathematical Model

The demand level and defective are adaptive in the proposed model. Assuming that the demand and defect of consumers are all fuzzy, the total profit becomes fuzzy, and the problem is given by

$$Maximize\ \widetilde{TPU}(z) = \frac{2\tilde{D}(S_g z - C_k - C_p z - C_s z)}{2z + z\tilde{p} + 1}\left(\frac{1}{1-\tilde{p}}\right) - \frac{z^2}{2z + z\tilde{p} + 1}\left(C_{h_d}\tilde{p} + C_{h_g}\right) \quad (3.4.1)$$

subject to

$$z = \frac{T\tilde{D}}{1-\tilde{P}} \quad (3.4.2)$$

Let the membership and non-membership function of the uncertain parameters are defined as follows

$$\mu(k_i) = \begin{cases} \sqrt{\frac{k_i - k_{i1}}{k_{i2} - k_{i1}}}, & k_{i1} \le k_i \le k_{i2} \\ \sqrt{\frac{k_{i3} - k_i}{k_{i3} - k_{i2}}}, & k_{i2} \le k_i \le k_{i3} \\ 0 & otherwise \end{cases} \quad (3.4.3)$$

and

$$\vartheta(k_i) = \begin{cases} \sqrt{\frac{k_{i2} - k_i}{k_{i2} - k'_{i1}}}, & k'_{i1} \le k_i \le k_{i2} \\ \sqrt{\frac{d_i - d_{i2}}{k'_{i3} - k_{i2}}}, & k_{i2} \le k_i \le k'_{i3} \\ 0 & otherwise \end{cases} \quad (3.4.4)$$

where $\widetilde{k_i} = (k'_{i1}, k_{i1}, k_{i2}, k_{i3}, k'_{i3})$ and we take $k_1 = D$ and $k_2 = p$.

Peng and Yager's (2015) score function of the uncertain parameters k_i are given by

$$\omega(k_i) = \begin{cases} \frac{k_i - k_{i1}}{k_{i2} - k_{i1}} - \frac{k_{i2} - k_i}{k_{i2} - k'_{i1}}, & \beta_{1i} \le k_i \le k_{i2} \\ \frac{k_{i3} - k_i}{k_{i3} - k_{i2}} - \frac{k_i - k_{i2}}{k'_{i3} - k_{i2}}, & k_{i2} \le k_i \le \beta_{3i} \\ 0 & otherwise \end{cases} \quad (3.4.5)$$

Rewriting the above score function (3.4.5). we obtain

$$\omega(k_i) = \begin{cases} \gamma_{1i}(k_i - \beta_{1i}), & \beta_{1i} \le k_i \le k_{i2} \\ \gamma_{2i}(\beta_{3i} - k_i), & k_{i2} \le k_i \le \beta_{3i} \\ 0 & otherwise \end{cases} \quad (3.4.6)$$

where $\gamma_{1i} = \frac{2k_{i2} - k_{i1} - k'_{i1}}{(k_{i2} - k_{i1})(k_{i2} - k'_{i1})}$, $\gamma_{2i} = \frac{k_{i3} + k'_{i3} - 2k_{i2}}{(k_{i3} - k_{i2})(k'_{i3} - k_{i2})}$, $\beta_{1i} = \frac{k_{i2}^2 - k_{i1}k'_{i1}}{2k_{i2} - k_{i1} - k'_{i1}}$, $\beta_{3i} = \frac{k_{i3}k'_{i3} - k_{i2}^2}{k_{i3} + k'_{i3} - 2k_{i2}}$

Now, the α-cut of the score function $\omega(\widetilde{k_i})$ is given by

$$\left[L_{\widetilde{K_i}}^{-1}(\alpha) R_{\widetilde{K_i}}^{-1}(\alpha)\right] = \left[\frac{\alpha}{\gamma_{1i}} + \beta_{1i}, \beta_{3i} - \frac{\alpha}{\gamma_{2i}}\right] \quad , i = 1, 2. \quad (3.4.7)$$

where i =1, 2 are the rate of defective and demand, whose alpha-cuts are

$$\left[L_{\tilde{p}}^{-1}(\alpha) R_{\tilde{p}}^{-1}(\alpha)\right] = \left[\frac{\alpha}{\gamma_{11}} + \beta_{11}, \beta_{31} - \frac{\alpha}{\gamma_{21}}\right] \quad (3.4.8)$$

and

$$\left[L_{\tilde{D}}^{-1}(\alpha) R_{\tilde{D}}^{-1}(\alpha)\right] = \left[\frac{\alpha}{\gamma_{12}} + \beta_{12}, \beta_{32} - \frac{\alpha}{\gamma_{22}}\right] \quad (3.4.9)$$

Using the above two alpha-cut we determined the left and right alpha-cut of unit wise total profit as

$$L_{\widetilde{TPU}}^{-1}(\alpha) = \frac{2\left(\frac{\alpha}{\gamma_{12}} + \beta_{12}\right)(S_g z - C_k - C_p z - C_s z)}{2z + z\left(\frac{\alpha}{\gamma_{11}} + \beta_{11}\right) + 1}\left(\frac{1}{1 - \left(\frac{\alpha}{\gamma_{11}} + \beta_{11}\right)}\right) - \frac{z^2}{2z + z\left(\frac{\alpha}{\gamma_{11}} + \beta_{11}\right) + 1}\left(C_{h_d}\left(\frac{\alpha}{\gamma_{11}} + \beta_{11}\right) + C_{h_g}\right) \quad (3.4.10)$$

$$R_{\widetilde{TPU}}^{-1}(\alpha) = \frac{2\left(\beta_{32} - \frac{\alpha}{\gamma_{22}}\right)(S_g z - C_k - C_p z - C_s z)}{2z + z\left(\beta_{31} - \frac{\alpha}{\gamma_{21}}\right) + 1}\left(\frac{1}{1 - \left(\beta_{31} - \frac{\alpha}{\gamma_{21}}\right)}\right) - \frac{z^2}{2z + z\left(\beta_{31} - \frac{\alpha}{\gamma_{21}}\right) + 1}\left(C_{h_d}\left(\beta_{31} - \frac{\alpha}{\gamma_{21}}\right) + C_{h_g}\right) \quad (3.4.11)$$

We have calculated the Index value of the unit wise total profit function as

$$I(\widetilde{TPU}) = \frac{1}{2}\int_0^1 \left(L_{\widetilde{TPU}}^{-1}(\alpha) + R_{\widetilde{TPU}}^{-1}(\alpha)\right) d\alpha$$

$$= \frac{1}{2}\int_0^1 \left(\frac{2\left(\frac{\alpha}{\gamma_{12}} + \beta_{12}\right)(S_g z - C_k - C_p z - C_s z)}{2z + z\left(\frac{\alpha}{\gamma_{11}} + \beta_{11}\right) + 1}\left(\frac{1}{1 - \left(\frac{\alpha}{\gamma_{11}} + \beta_{11}\right)}\right) - \frac{z^2}{2z + z\left(\frac{\alpha}{\gamma_{11}} + \beta_{11}\right) + 1}\left(C_{h_d}\left(\frac{\alpha}{\gamma_{11}} + \beta_{11}\right) + C_{h_g}\right) + \frac{2\left(\beta_{32} - \frac{\alpha}{\gamma_{22}}\right)(S_g z - C_k - C_p z - C_s z)}{2z + z\left(\beta_{31} - \frac{\alpha}{\gamma_{21}}\right) + 1}\left(\frac{1}{1 - \left(\beta_{31} - \frac{\alpha}{\gamma_{21}}\right)}\right) - \frac{z^2}{2z + z\left(\beta_{31} - \frac{\alpha}{\gamma_{21}}\right) + 1}\left(C_{h_d}\left(\beta_{31} - \frac{\alpha}{\gamma_{21}}\right) + C_{h_g}\right) \right) d\alpha$$

$$= (S_g z - C_k - C_p z - C_s z)\int_0^1 \left(\frac{\left(\frac{\alpha}{\gamma_{12}} + \beta_{12}\right)}{\left(2z + z\left(\frac{\alpha}{\gamma_{11}} + \beta_{11}\right) + 1\right)\left(1 - \left(\frac{\alpha}{\gamma_{11}} + \beta_{11}\right)\right)} + \frac{\left(\beta_{32} - \frac{\alpha}{\gamma_{22}}\right)}{\left(2z + z\left(\beta_{31} - \frac{\alpha}{\gamma_{21}}\right) + 1\right)\left(1 - \left(\beta_{31} - \frac{\alpha}{\gamma_{21}}\right)\right)} \right) d\alpha - \frac{z^2}{2}\int_0^1 \left(\frac{C_{h_d}\left(\frac{\alpha}{\gamma_{11}} + \beta_{11}\right) + C_{h_g}}{2z + z\left(\frac{\alpha}{\gamma_{11}} + \beta_{11}\right) + 1} + \frac{C_{h_d}\left(\beta_{31} - \frac{\alpha}{\gamma_{21}}\right) + C_{h_g}}{2z + z\left(\beta_{31} - \frac{\alpha}{\gamma_{21}}\right) + 1} \right) d\alpha \quad (3.4.12)$$

The Index value of the lot size is

$$I(z) = \frac{1}{2}\int_0^1 \left(L_z^{-1} + R_z^{-1}\right) d\alpha$$

$$= \frac{T}{2}\int_0^1\left(\frac{\left(\frac{\alpha}{\gamma_{12}}+\beta_{12}\right)}{1-\left(\frac{\alpha}{\gamma_{11}}+\beta_{11}\right)}+\frac{\left(\beta_{32}-\frac{\alpha}{\gamma_{22}}\right)}{1-\left(\beta_{31}-\frac{\alpha}{\gamma_{21}}\right)}\right)d\propto \quad (3.4.13)$$

We take the help of MATLAB software for numerical computation of the equations (3.4.12) and (3.4.13)

4. Numerical Example

We have considered the numerical example taken in case of Salameh and Jaber (2000) to have the numerical clarification of the proposed inventory model, D =70,000 /year, C_k =150/cycle, C_{h_g} = \$30/year, C_{h_d} = \$25/year, w =1/min, C_s =0.6, C_p =35, S_g = \$65. The optimal values of decision variables of the models under crisp and Pythagorean fuzzy environments are provided in following Table-2.

Optimal solution	Crisp	Fuzzy
Lot size	877.921units	871.793units
Screening time	0.00501	0.00497
Cycle length	0.0122year	0.0122year
Total profit	2052711.358/year	2053171.670/year

4.1 Graphical Analysis of models

- Fig.2 indicated that order quantity vs. profit function. When range from 600 to 1300 then is concave in z.
- Fig. 3, 4, 5, and 6 all illustrate the % of defective versus the total profit in four varying ranges of defective items. We can conclude that as the percentage of defective items increase in the range of 0 and 0.8, the profit of the model increases, and in the range of 0.8 to 1.2, there is a gap when defective items percent is almost 1, indicating that % of defective items cannot be taken exactly 1, after which there is a decrease in the profit of the items.

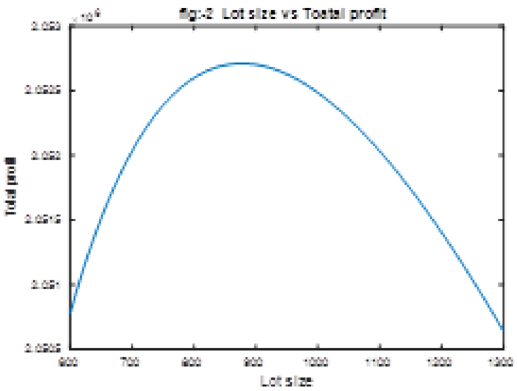


Fig. 2. Lot size vs total profit

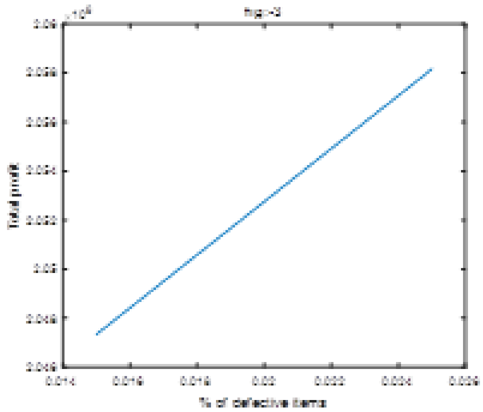


Fig. 3.

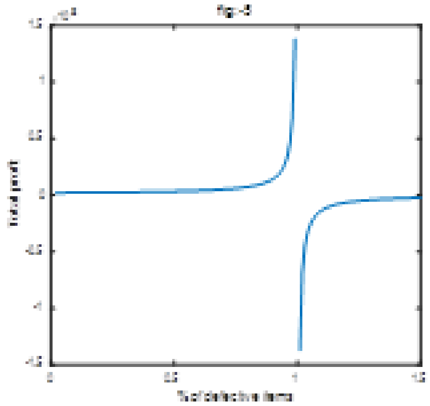


Fig. 4

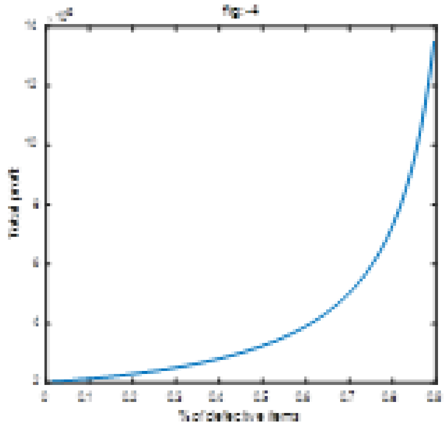


Fig. 5

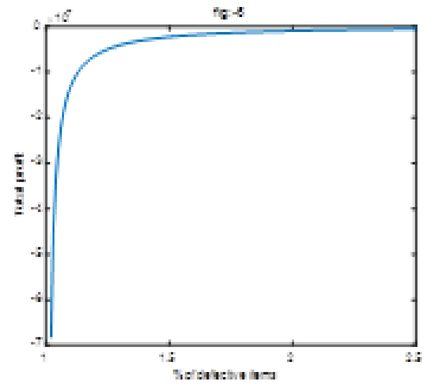


Fig.6. (Fig. 3, 4, 5, and 6 all illustrate the % of defective versus the total profit in four varying ranges of defective items)

4.2 Sensitivity Analysis

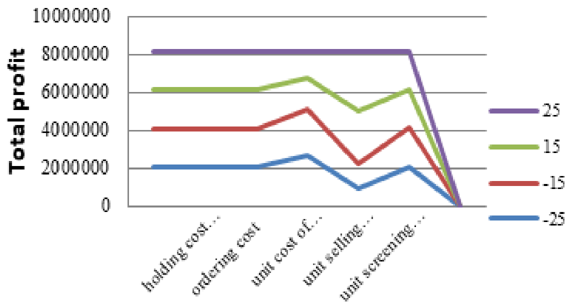
We have considered the effects of the lot size and profit function under the changes from -25% to +25% of the parameters $\left(D, \quad Sg, Ck, CP, C_s, C_{h_g}, C_{h_d}, p\right)$ the result as summarized in table -3.

Table-3
Sensitivity Analysis

Parameter	% change	$z_{\max}$	$ETPU(z)$	% change in $ETPU(z)$
D	+25	981.546	2569384.081	25.17
	+15	941.466	2362673.631	15.1
	-15	809.403	1742899.275	-15.09
	-25	760.302	1536461.245	-25.14
S_g	+25	899.125	3201293.124	55.95
	+15	890.704	2741858.588	33.57
	-15	864.950	1363569.833	-33.57
	-25	856.193	904145.445	-55.95
C_k	+25	972.826	2049845.436	-0.14
	+15	936.019	2050956.914	-0.08
	-15	815.696	2054590.448	0.09
	-25	771.429	2055927.229	0.16
C_p	+25	866.289	1434250.745	-30.12
	+15	870.961	1681634.427	-18.07
	-15	884.827	2423789.943	18.07
	-25	889.401	2671176.566	30.12
C_s	+25	877.723	2042109.137	-0.52
	+15	877.803	2046350.026	-0.31
	-15	878.040	2059072.692	0.31
	-25	878.119	2063313.581	0.52
C_{h_g}	+25	786.527	2049634.404	-0.15
	+15	819.543	2050825.058	-0.09
	-15	950.865	2054742.930	0.1
	-25	1010.978	2056196.907	0.17
C_{h_d}	+25	876.128	2052657.148	-0.0002
	+15	876.844	2052678.819	-0.0001
	-15	879.003	2052743.938	0.0001
	-25	879.726	2052765.680	0.0002
p	+25	878.276	2058160.138	0.27
	+15	878.132	2055970.863	0.15
	-15	877.718	2049480.944	-0.16
	-25	877.587	2047343.383	-0.26

Fig.7 explores the behavior of total profit, for each of the cost parameters, the sensitivity analysis concludes that the behavior of the lot size and profit changes over the percentage of changes of the demand, cost parameters, and defective on and from (-25% to +25%) exclusively. We can also observe that as the percentage of demand increases then there is an increase in lot size as well as total profit and vice versa. Change of holding cost doesn't affect the total profit but the profit function varies in such a way that it neither never decreases nor never increases due to change of selling price and ordering cost within the specific range of parameters. Moreover, the profit value of the model gives almost the same due to variation of variable and screening cost.

Fig-7 Profit vs Cost



5. Managerial Implications

- In the real-world production setting, good and defective products are managed differently. For example, in many semiconductor factories, a separate warehouse is typically set aside for faulty products to better separate them from the good ones. This allows for independent cost and quantity monitoring of faulty products. As a result, the annual interest rates for good and defective goods are different, which is important for the smooth functioning of the company.

- The decision-maker has no direct knowledge of the defect rate of the item or the rate of demand of the consumers who will be buying it. Uncertainty of this type can be defined as a non-standard intuitionistic fuzzy system. To solve the difficulties of decision-making challenges, PFS carefully considers those cases.

6. Conclusion and Scope

A comparative analysis of EOQ models for defectives products using various keeping costs under crisp and PFS conditions have been considered in this paper. Allowable proportionate discount is taken into account in this formula based on the number of faulty goods found in each lot, which results in an improvement in the net benefit and lot size. This model is only limited to defective items and only items where a proportionate discount is considered. The implementation of the EOQ order requires that demand remain constant throughout the year, which is not feasible. Since it includes transportation costs, the ordering cost per order cannot be constant. To explain the model, a numerical analysis of the crisp, as well as fuzzy model, is done. PFS may be a new resolution to decision-making in any inventory management scenario. Yager's rating index fuzzified the best results of the fuzzy model in terms of lot size and overall profit. This model may be generalized to incorporate varying demand and variable costs. This model ought to be extended to require into consideration backlogs and shortages. This model can also be extended to a multi-objective programming model.

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