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Research Article

Bi-objective Optimization of a Multi-product Multi-period Fuzzy Possibilistic Capacitated Hub Covering Problem: NSGA-II and NRGA Solutions

Zahra Rajabi [a], Soroush Avakh Darestani [b,*]

[a] *Faculty of Industrial and Mechanical Engineering, Qazvin Branch, Islamic Azad University, Qazvin, Iran*
[b] *Faculty of Industrial and Mechanical Engineering, Qazvin Branch, Islamic Azad University, Qazvin, Iran*
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Abstract

The hub location problem is employed for many real applications, including delivery, airline and telecommunication systems and so on. This work investigates on hierarchical hub network in which a three-level network is developed. The central hubs are considered at the first level, at the second level, hubs are assumed which are allocated to central hubs and the remaining nodes are at the third level. In this research, a novel multi-product multi-objective model for capacitated hierarchical hub location problem with maximal covering under fuzzy condition first is suggested. Cost, time, hub and central hub capacities are considered as fuzzy parameters, whereas many parameters are uncertainty and in deterministic in the real world. To solve the proposed fuzzy possibilistic multi-objective model, first, the model is converted to the equivalent auxiliary crisp model by hybrid method and then is solved by two meta-heuristic algorithms such as Non-Dominated Sorting Genetic Algorithm (NSGA-II) and Non-Dominated Ranked Genetic Algorithm (NRGA) using MATLAB software The statistical results report that there is no significant difference between means of two algorithms exception CPU time criteria. In general, in order to show efficiency of two algorithms, we used Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), the results clearly show that the efficiency of NRGA is better than NSGA-II and finally, figures are achieved by MATLAB software that analyze the conflicting between two objectives.

***Keywords**:* The hierarchical; Hub covering location; Fuzzy possibilistic multi-objective; Multi-product; Meta heuristic algorithms

1. Introduction

Hub location problem is a comprehensive and novel issue in facility location. Hub network is widely used in transportation systems, telecommunication network, logistics, cargo delivery, production-distribution system and etc. Hubs serve as consolidation, connecting and switching points in many-to-many distribution systems. Instead of direct connecting of all origin-destination pairs which is impossible and needs high investment, hub facilities are applied between all demand nodes in order to take advantage of economies of scale.

Hub location problems encounter the movement of traffic, which includes passengers, commodities or information between origin-destination pairs and employed to reduce transportation link between origin-destination points. The purpose of the hub location problem involves finding the location of hubs and allocation of demand nodes to these hubs in order to route traffic between all origin-destination pairs. Generally, there are two hub network structures, single and multiple allocations. In a single allocation; each demand node assigns to exactly one hub whereas in a multiple allocation; each demand node can be assigned to more than one hub.

Most of the researches in the hub location problem literature have been studied the problem in a classical hub network (two level hub and spoke network). In our work, a three-level network namely the hierarchical hub location problem is developed, at the first was presented by Yaman (2009) where the first level (top level) connected central hubs to each other in the complete network, at the second level, the hubs were connected to the central hubs through star networks and finally, at the third level, the demand nodes were connected to the hubs and the central hubs through star networks. In continiuing on Yaman's work, Davari and Fazel-Zarandi (2013) discussed a hierarchical network under uncertainty with fuzzy flows. Karimi et al. (2014) employed capacity constraint and Korani and Sahraeian (2013) suggested hub maximal covering problem in the hierarchical network. Dukkanci and Kara (2017) proposed a hierarchical multimodal hub structure with a service time bound, where as their multimodal network have different types of vehicles in each hierarchical hub network and they used a heuristic solution algorithm based on the subgradient approach to solve the problem. In Yaman's model which was studied in a cargo delivery system, some allocated demand nodes to the hubs and central hubs had longer distance than the

*Corresponding author Email address: avakh@qiau.ac.ir

other allocated demand nodes which may lead to poor service levels to the farther allocated demand nodes and central hubs were scattered on a focused part of Turkey's map therefore, the hubs and central hubs coverage restrictions are enforced to this work.

To the best of our knowledge, hub covering problems have been practiced less on hub location. In fact, Campbell (1994) at the first proposed mixed-integer formulation for both hub center and hub covering problem considering single and multiple allocation and introduced three coverage criteria. Kara and Tansel (2003) suggested nonlinear binary integer programming and the various linearizations for old and new formulation as well. They showed clearly that the hub covering problem is NP-hard. Ernst et al. (2005) proposed a better formulation in hub covering problem with a single assignment and solved their model in less computational time compared with Kara and Tansel's model. They used the cover's radius idea for hub covering problem. Wagner (2008) improved the model formulation for hub covering problem. Introducing three types of coverage by Campbell (1994), most researchers studied the first type of coverage on a hub location problem. Karimi and Bashiri (2011) proved that the first type of coverage is inefficient in many hub location problems as the cost (time or distance) between origin and hub may be too large from other links thus, in this research, the second type of coverage restrictions are applied. In addition, to improve service levels in a rational way, both the capacity and covering constraints are employed which make a competitive advantage for companies.

Fig. 1 shows the hierarchical network where hexagons show central hubs at the first level, squares are candidates for hubs which are assigned to central hubs at the second level and the remaining nodes as drawn circles are candidates for demand nodes which are assigned to hubs and central hubs at the third level. Of course that in our proposed hub network, the demand nodes include potential hubs and central hubs can move multiple products, whereas most researches in the literature devoted only single product (flow or traffic) but in the real world the various types of products (flow or traffic) can be routed in a hub location network. Moreover, Correia et al. (2013) presented the hub classical location problem in a multi-product condition. In our proposed network for each product, demand nodes, hubs and central hubs may be opened in all periods, the complete network between central hubs assumed for each product and each product can be routed in the hierarchical hub covering network.

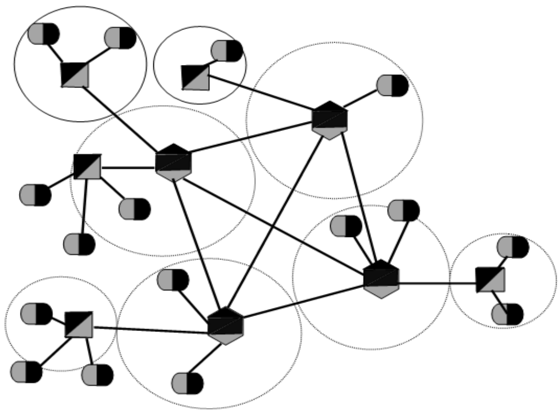

Fig.1. The proposed hierarchical hub covering network for each product

The current investigation tries to take into account two conflicting objectives, the first objective minimizes the total transportation cost and the second one tries to minimize the total transportation time. In fact, another reason to justify the investigation on this model by authors is the importance of minimization of transportation time which can be considered as an objective. For instance, if a faster transportation facility to be used such as air way transportation, simultaneously, time reduces and cost inversely increases.

Like as many facilities location problems, uncertainty condition is applied in a hub location problem in fact, two approaches are considered; stochastic and fuzzy approaches, whereas some parameters are influenced by indeterministic and imprecision in data in the real world application and when reliable data need enormous investment or if there is no access to reliable and precise data, the fuzzy possibilistic approach is considered to make it more sensible and reasonable. Taghipourian et al. (2012) presented a fuzzy integer linear programming to deal with imprecise or vague condition when rough weather or emergency situation happens in the air transportation. Ghodratnama et al. (2013) proposed a fuzzy possibilistic bi-objective model in a hub covering problem and in their model some important parameters were considered as fuzzy numbers. Correia and Nickel (2018) suggested a multi-priod stochastic capacitated multiple allocation hub location problem. Uncertainty enforced on the demands, also they assumed capacity between origin-destination pairs of exisiting hubs and the transportation. In this paper, some parameters like cost, time and the capacity of hubs and central hubs for each product are regarded as triangular fuzzy numbers in order to tackle with the uncertainty in a capacitated multi-objective hierarchical hub covering problem with multiple products routed on the proposed network imposing second type of hubs and central hubs coverage.

In addition, majority of problems in the hub location network were solved employing meta-heursitic algorithms. In this context, Calik et al. (2009) solved a hub location problem in an incomplete network using tabu-search method. Moreover, Randall (2008) used ant colony algorithm for the hub location problem considering capacity constraint with a single assignment. Damgacioglu et al. (2015) represented the uncapacitated single allocation plane hub location problem (PHLP). Then they solve their mathematical formulation with genetic algorithm in a reasonable time. Gelareh and Nickel (2015) developed a new mathematical formulation with budget constraints under the multi-period condition. They solved their model with a very efficient metaheuristic algorithm that generates high-quality solutions. Tavakkoli-Moghaddam et al. (2013) discussed multi-objective mathematical model by minimizing the total cost of network and minimizing the maximum travel time and solved the proposed multi-objective model by Multi-objective Imperialist Competitive Algorithm (MOICA) and NSGA-II and the results for both algorithms are compared with each other. Mohammadi et al. (2013) proposed MOICA for multi-objective multi-mode transportation model in a hub covering problem under stochastic condition then reported the results by comparing MOICA with NSGA-II and Pareto Archive Evolution Strategy (PAES). Therefore, due to the complexity of the hierarchical hub covering problem and NP-hardness nature of problem Yaman (2009), NSGA-II, first introduced by Deb et al. (2002) and NRGA, developed by Aljadaan et al. (2008) are proposed. Whereas pervious papers solved hub location problem with other multi-objectives algorithms, our paper solves the hierarchical hub covering problem with NSGA-II and NRGA algorithms because of NP-hardness nature of problem after that we will show the performance of these algorithms by solving model. Therefore, this study will lead to a Pareto optimal solution and the comparison between NSGA-II and NRGA to validate the performance of the proposed model.

Regarding the problem as mentioned before, the main contributions of this work, different from the existing researches in the related literature, are as follows:

- Development of the hierarchical hub network under multi-product situation, remarkably most of the previous researches only considered single product.
- Considering two conflicting objectives in this work.
- Imposing the second type of hubs and central hubs covering restrictions to improve service levels to the demand nodes.
- Proposing a fuzzy possibilistic programming model which deals with uncertainty influencing the hierarchical hub covering network by remarking the imprecision nature of data.
- Converting the proposed fuzzy possibilstic multi-objective model into an equivalent auxiliary crisp model by hybridizing methods that was proposed by Jimenez et al. (2007) and Parra et al.(2005).

The outline of this investigation has been organized as follows:

The proposed fuzzy possibilistic multi-objective mixed integer linear programming is developed in section 2. Section 3 provides the proposed solution methodologies. Computational experiments and the analysis of results are reported in section 4. Finally, conclusion and future research trends are remarked in section 5.

2. Model Formulation

In this section, we propose a fuzzy multi-objective mixed integer linear programming formulation; the problem is an extension of the hierarchical hub location problem that was addressed by Yaman (2009). To increase service levels to demand nodes, the second type of coverage is given. Hence, the main assumptions of this model are discussed as follows:

- The proposed network is considered as multi-product and multi-period.
- One assumption is to establish hubs, central hubs and demand nodes in all periods for each product.
- The number of hubs and central hubs are predetermined.
- Maximum numbers of products that can be moved by demand nodes (the potential central hubs) is predetermined.
- The central hubs network for each product is fully interconnected.
- The single allocation for each product is assumed.
- The coverage radius of hubs and central hubs is considered.
- Some parameters like transportation cost, transportation time and capacities of hubs and central hubs are all assumed to be triangular fuzzy numbers.
- The problem includes two objective functions (minimizing the total transportation cost and total transportation time).

Before introducing the mathematical model, the notations and parameters are defined.

Notations:

N Set of demand nodes

$H \subseteq N$	Set of possible location for hubs
$C \subseteq H$	Set of possible location for central hubs
T	Set of periods
P	Set of products
Parameters:	
w_{irpt}	The amount of flow must be traveled from node $i \in N$ to $r \in N$ at period $t \in T$ for product $p \in P$ so, $sw_{irpt} = \sum_{r \in N} w_{irpt}$ is summation flow of product $p \in P$ destined to node $i \in N$
$\tilde{c}_{ij}$	Transportation cost per unit of product flow from node $i \in N$ to $j \in N$
α_H	Discount factor in routing cost between hubs and central hubs
α_C	Discount factor in routing cost between central hubs as $\alpha_H \geq \alpha_C$
d_{ij}	A unit of distance from node $i \in N$ to node $j \in N$
r_C	Coverage radius for central hubs
r_H	Coverage radius for hubs
P_H	The number of hubs
P_C	The number of central hubs
$\tilde{t}_{ij}$	Transportation time per unit of product flow from node $i \in N$ to $j \in N$
$\overline{\alpha}_H$	Discount factor in routing time between hubs and central hubs
$\overline{\alpha}_C$	Discount factor in routing time between central hubs

If each $i \in N$ is chosen as a hub or central hub, the capacity for hub is denoted by $\widetilde{car}_{jp}$ and for central hub by $\widetilde{car}_{lp}$, also the maximum number of products can be traveled with central hub is discussed by M_l.

Decision variables:

Let u_{ijlpt} be amount of products flow from $i \in N$ as origin or destination node traversing from hub $j \in H$ and central hub $l \in C$ in period $t \in T$ for product $p \in P$. g_{ilkpt} be amount of products flow from $i \in N$ as origin or destination node traversing from two central hub $l \in C$ and $k \in C$ as $l \in C \neq k$ in period $t \in T$ for product $p \in P$ and x_{ijlp} is defined as a binary variable if node $i \in N$ is allocated to hub $j \in H$ and central hub $l \in C$ for product $p \in P$ takes a value 1 and 0 otherwise.

In terms of mentioned above, the mathematical formulation for the present model is put forward as follows:

$$\min \sum_{i\in N}\sum_{r\in N}\sum_{p\in P}\sum_{t\in T}(w_{irpt}+w_{ript})\sum_{j\in H}\tilde{c}_{ij}\sum_{l\in C}x_{ijlp} + \sum_{i\in N}\sum_{j\in H}\sum_{l\in C,l\neq j}\sum_{p\in P}\sum_{t\in T}\alpha_H\tilde{c}_{jl}u_{ijlpt} + \sum_{i\in N}\sum_{l\in C}\sum_{k\in C,k\neq L}\sum_{p\in P}\sum_{t\in T}\alpha_C\tilde{c}_{lk}g_{ilkpt} \quad (1)$$

$$\min \sum_{i\in N}\sum_{r\in N}\sum_{j\in H}\sum_{l\in C}\sum_{p\in P}\sum_{t\in T} w_{irpt}(\tilde{t}_{ij}+\overline{\alpha}_H\,\tilde{t}_{jl})\,x_{ijlp} + \sum_{l\in C}\sum_{k\in C,k\neq l}\sum_{p\in P}\overline{\alpha}_C\,\tilde{t}_{lk}\,x_{kkkp} + \sum_{i\in N}\sum_{j\in H}\sum_{l\in C,l\neq j}\sum_{p\in P}\sum_{t\in T}\overline{\alpha}_H\,\tilde{t}_{jl}\,u_{ijlpt} + \sum_{i\in N}\sum_{l\in C}\sum_{k\in C,k\neq l}\sum_{p\in P}\sum_{t\in T}\overline{\alpha}_C\,\tilde{t}_{lk}\,g_{ilkpt} \quad (2)$$

s.t.

$$\sum_{j\in H}\sum_{l\in C}x_{ijlp} = 1 \qquad \forall\, i \in N, p \in P, \quad (3)$$

$$x_{ijlp} \leq x_{jjlp} \qquad \forall\, i \in N, j \in H, j \neq i, l \in C, p \in P, \quad (4)$$

$$\sum_{r\in N}x_{jrlp} \leq x_{lllp} \qquad \forall\, j \in H, l \in C, l \neq j, p \in P, \quad (5)$$

$$\sum_{j\in H}\sum_{l\in C}x_{jjlp} \leq P_H \qquad \forall\, p \in P, \quad (6)$$

$$\sum_{l\in C}x_{lllp} \leq P_C \qquad \forall\, p \in P, \quad (7)$$

$$\sum_{p\in P} x_{lllp} \le M_l \qquad \forall\, l \in C, \tag{8}$$

$$\sum_{k\in C, k\neq l} g_{ilkpt} - \sum_{k\in C, k\neq l} g_{iklpt} = \sum_{r\in N} w_{irpt} \sum_{j\in H} (x_{ijlp} - x_{rjlp}) \qquad \forall\, i \in N, l \in C, p \in P, t \in T, \tag{9}$$

$$\sum_{r\in N, r\neq j} (w_{irpt} + w_{ript})(x_{ijlp} - x_{rjlp}) \le u_{ijlpt} \qquad \forall\, i \in N, j \in H, l \in C, l \neq j, p \in P, t \in T, \tag{10}$$

$$d_{il} x_{illp} \le r_C \qquad \forall\, i \in N, l \in C, l \neq i, p \in P, \tag{11}$$

$$d_{ij} x_{ijlp} \le r_H \qquad \forall\, i \in N, j \in H, l \in C, l \neq j, p \in P, \tag{12}$$

$$\alpha_H d_{jl} x_{jjlp} \le r_C \qquad \forall\, j \in H, l \in C, l \neq j, p \in P, \tag{13}$$

$$\sum_{t\in T} \sum_{i\in N} \sum_{l\in C} sw_{ipt} x_{ijlp} \le \widetilde{car}_{jp} \qquad \forall\, j \in H, p \in P, \tag{14}$$

$$\sum_{t\in T} \sum_{i\in N} \sum_{j\in H} sw_{ipt}\, x_{ijlp} \le \widetilde{car}_{lp} \qquad \forall\, l \in C, p \in P, \tag{15}$$

$$x_{ljlp} = 0 \qquad \forall\, j \in H, l \in C, l \neq j, p \in P, \tag{16}$$

$$x_{ijlp} \in \{0,1\} \qquad \forall\, i \in N, j \in H, l \in C, p \in P, \tag{17}$$

$$g_{ilkpt} \ge 0 \qquad \forall\, i \in N, l \in C, l \neq k, p \in P, t \in T, \tag{18}$$

$$u_{ijlpt} \ge 0 \qquad \forall\, i \in N, j \in H, l \in C, p \in P, t \in T, \tag{19}$$

The first objective function minimizes the total cost of routing in the network as:

$\sum_{i\in N}\sum_{r\in N}\sum_{p\in P}\sum_{t\in T}(w_{irtp} + w_{ript})\sum_{j\in H}\tilde{c}_{ij}\sum_{l\in C} x_{ijlp}$, The first term minimizes the transportation cost of products flow node i to other node that traverses from hub j.

$\sum_{i\in N}\sum_{j\in H}\sum_{l\in C, l\neq j}\sum_{p\in P}\sum_{t\in T}\alpha_H \tilde{c}_{jl} u_{ijlpt}$, the second term minimizes the transportation cost of products flow from i as an origin or destination to other nodes which passes the connection routes of hub j and central hub l.

$\sum_{i\in N}\sum_{l\in C}\sum_{k\in C, k\neq L}\sum_{p\in P}\sum_{t\in T}\alpha_C \tilde{c}_{lk} g_{ilkpt}$, the third term minimizes the transportation cost of products flow from i as an origin or destination to other nodes which passes the connection routes of two central hub l and k as k≠l.

The second objective function with formula (2) interprets the transportation time of products as:

$\sum_{i\in N}\sum_{r\in N}\sum_{j\in H}\sum_{l\in C}\sum_{p\in P}\sum_{t\in T} w_{irpt}(\tilde{t}_{ij} + \overline{\alpha}_H\, t_{jl})\, x_{ijlp} + \sum_{l\in C}\sum_{k\in C, k\neq l}\sum_{p\in P}\overline{\alpha}_C\ \tilde{t}_{lk}\, x_{kkkp}$, this term calculates to minimize the transportation time of the possible longest route in the network.

$\sum_{i\in N}\sum_{j\in H}\sum_{l\in C, l\neq j}\sum_{p\in P}\sum_{t\in T}\overline{\alpha}_H \tilde{t}_{jl} u_{ijlpt}$, tries to minimize the transportation time of product flow from i as an origin or destination to other nodes which passes the connection routes of hub j and central hub l.

$\sum_{i\in N}\sum_{l\in C}\sum_{k\in C, k\neq l}\sum_{p\in P}\sum_{t\in T}\overline{\alpha}_C \tilde{t}_{lk} g_{ilkpt}$, this term minimizes the transportation time of product flow from i as an origin or destination to other nodes which passes the connection routes of two central hub l , k as k≠l.

Since the model considers the single allocation, so constraints (3) and (17) represent single allocation, constraint (3) states every node is allocated to exactly one hub and one central hub. In this context, constraint (4) indicates that a node i is allocated to hub j and central hub l, so node j should be a hub in the network and assigned to a central hub.

Constraint (5) ensures that hub j cannot be assigned to another node unless that node must be a central hub this shows that node j is assigned to central hub l. Due to constraints (6) and (7), maximum number of hubs and central hubs are determined. Constraint (8) guarantees that each central hub l can handle at most M_l products. Constraint (9) describes that if node i is not assigned to central hub l, then the product flow from node i to the nodes that are assigned to central hub l enters node l therefore, Constraints (9) and (10) are the product flow balance constraints. To improve high service levels to demand nodes covering the most of demand nodes, r_H and r_C state hubs and central hubs coverage radius as stated in constraints (11) to (13) respectively. Constraint (14) is the capacity constraint for hubs and constraint (15) limits capacity for central hubs in each product in order to provide reasonable services to demand nodes. Constraint (16) is redundant, but it strengthens the model. Constraint (17), limits x_{ijlp} variable to take binary values and constraints (18) and (19) enforce u_{ijlpt} and g_{ilkpt} decision variables to take non-negativity values.

3. Solution Methodology

3.1. The equivalent auxiliary crisp model

To change the possibilistic mixed integer linear programming model into an equivalent auxiliary crisp model, in this subsection one efficient way is applied by hybridizing method of Jimenez et al. (2007) and Parra et al. (2005). Thus, the main work of the proposed methods is based on the method of Jimenez et al. (2007). This hybrid approach involves several advantages that are defined as follows:

- Applying this approach is an efficient way to solve fuzzy linear programming and also the number of objective functions and constraints do not increase thus, this method is used for the proposed fuzzy multi-objective model.
- The method is used for the variety of membership functions like triangular, trapezoidal and non-linear cases in both symmetric and asymmetric forms.
- The approach relies on the mathematical concepts like: expected interval and expected value of fuzzy numbers.

As mentioned above, this hybrid method is employed to defuzzy the present model. Additional information can be found in Jimenez (1996), Parra et al. (2005), Jimenez et al. (2007) and Pishvaee and Torabi (2010). Therefore, the fuzzy multi-objective mixed integer linear programming in the hierarchical hub covering problem is converted into the equivalent auxiliary crisp model as follows:

$$\min \sum_{i\in N}\sum_{r\in N}\sum_{p\in P}\sum_{t\in T}(w_{irpt}+w_{ript})\sum_{j\in H}\left(\frac{c_{ij}^{o}+2c_{ij}^{m}+c_{ij}^{p}}{4}\right)\sum_{l\in C}x_{ijlp} + \sum_{i\in N}\sum_{j\in H}\sum_{l\in C, l\neq j}\sum_{p\in P}\sum_{t\in T}\alpha_H\left(\frac{c_{jl}^{o}+2c_{jl}^{m}+c_{jl}^{p}}{4}\right)u_{ijlpt} + \sum_{i\in N}\sum_{l\in C}\sum_{k\in C, k\neq L}\sum_{p\in P}\sum_{t\in T}\alpha_C\left(\frac{c_{lk}^{o}+2c_{lk}^{m}+c_{lk}^{p}}{4}\right)g_{iklpt} \quad (20)$$

$$\min \sum_{i\in N}\sum_{r\in N}\sum_{j\in H}\sum_{l\in C}\sum_{p\in P}\sum_{t\in T} w_{irpt}\left(\left(\frac{t_{ij}^{o}+2\,t_{ij}^{m}+t_{ij}^{p}}{4}\right)+\bar{\alpha}_H\left(\frac{t_{jl}^{o}+2\,t_{jl}^{m}+t_{jl}^{p}}{4}\right)\right)x_{ijlp} + \sum_{l\in C}\sum_{k\in C, k\neq l}\sum_{p\in P}\bar{\alpha}_C\left(\frac{t_{lk}^{o}+2\,t_{lk}^{m}+t_{lk}^{p}}{4}\right)x_{kkkp} + \sum_{i\in N}\sum_{j\in H}\sum_{l\in C, l\neq j}\sum_{p\in P}\sum_{t\in T}\bar{\alpha}_H\left(\frac{t_{jl}^{o}+2t_{jl}^{m}+t_{jl}^{p}}{4}\right)u_{ijlpt} + \sum_{i\in N}\sum_{l\in C}\sum_{k\in C, k\neq l}\sum_{p\in P}\sum_{t\in T}\bar{\alpha}_C\left(\frac{t_{lk}^{o}+2\,t_{lk}^{m}+t_{lk}^{p}}{4}\right)g_{ilkpt} \quad (21)$$

$$\sum_{t\in T}\sum_{i\in N}\sum_{l\in C} sw_{ipt}x_{jjlp} \leq \beta\left(\frac{car_{jp}^{p}+car_{jp}^{m}}{2}\right)+(1-\beta)\left(\frac{car_{jp}^{m}+car_{jp}^{o}}{2}\right) \quad \forall\ j\in H, p\in P \quad (22)$$

$$\sum_{t\in T}\sum_{i\in N}\sum_{j\in H} sw_{ipt}\,x_{ijlp} \leq \beta\left(\frac{car_{lp}^{p}+car_{lp}^{m}}{2}\right)+(1-\beta)\left(\frac{car_{lp}^{m}+car_{lp}^{o}}{2}\right) \quad \forall\ l\in C, p\in P \quad (23)$$

and constraints (3)-(13) and (16)-(19).
β is the feasibility degree of constraint between 0 and 1, $(1-\beta)$ is the maximum infeasibility degree of constraint. Constraints (22) and (23) convert fuzzy capacity to defuzzy capacity by hybrid approach as mentioned above.

3.2. The proposed non-dominated sorting genetic algorithm (NSGA-II)

Non-dominated sorting genetic algorithm (NSGA-II) inspired multi-objective computational algorithm which was introduced by Deb et al. [17]. NSGA-II algorithm is based on Pareto approach which is one of the complicated problem due to showing the set of Pareto solution in multiple solution space. In this proposed algorithm, ranking the population was performed using both the concept "fast non-dominated sorting (FNDS)" and "crowding distance (CD)". The fast non-dominated sorting finds the non-dominated frontiers that the individual of the frontier set are not dominated by any other individual and when the individuals of the population have the same rank, ranking the individual of new population is performed by crowding distance which was calculated for each individual.

3.2.1. Non-dominated set

The fast Non-dominate sorting procedures are described as follows:

- For each member of the population like p
- For each member of the population like q
- If p dominated q (p<q) then add q to s_p, i.e., s_p=$s_p \cup \{q\}$
- Else if q dominated p (q<p) then add to n_p, i.e., $n_p = n_p + 1$
- If $n_p = 0$, no solution dominates p then p is a member of first front which is created $F_1 = F_1 \cup \{p\}$
- Initialize front counter K=1
- If the i th front is not empty $f_i \neq 0$, the following implemented
- For each member of the population like p in F_K
- For each member of the population like q in the set of s_p
- Decrease n_q by one
- If $n_q = 0$, then q is member of Q (Q=Q∪ $\{q\}$);
- Increment K (the front counter) by one
- Current front is shaped with members of Q.

3.2.2. Crowding distance

After ranking the population by fast non-dominated sorting, the density of solution around a certain solution in the population is estimated by crowding distance as the average distance of two points on the either side of this certain point along each objective is calculated as shown in Fig. 2.

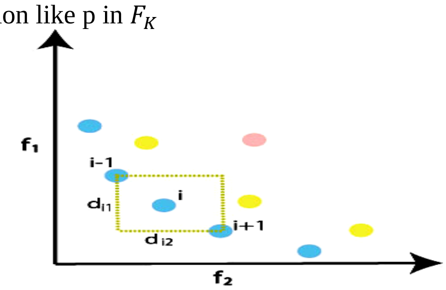

Fig. 2. Crowding distance calculation Deb et al. (2002)

3.2.3. Solution representation

The solution of the proposed model should represent the network by encoding three chromosome strings.

i) The first discrete chromosome string, Hub matrix a ($P \times p_H$) matrix, where P is the number of products and p_H is the number of hubs that represents the indices of open hubs in the network which includes p_C, the number of central hubs, randomly.

ii) The second discrete chromosome string, assign matrix a ($2 \times I \times P$) matrix that shows the allocation of demand nodes to hubs and central hubs for each product which includes two rows, the first row shows the indices of open hubs and the second row is the indices of open central hubs and each demand node is assigned to them for each product.

iii) Third chromosome string G is continuous matrix a ($p_C \times p_C \times P$) matrix is filled with random values with continues uniform distribution limited between 0 and 1 and the sum of each column and row should be one. G matrix expresses what percentage of each product flow is routed from one central hub to another one in the complete network.

3.2.4. Selection and crossover

The fast non-dominated sorting (FNDS) and crowding distance (CD) were applied to the individual in the population. Selection of parents randomly is carried out based on tournament selection and then the crossover operator is used to create offspring by combining two or more parents and the inherited property of selected parents transferring information between them to obtain better quality results. In this problem, uniform crossover for discrete and continues parts of solution representation is applied. Feasibility of generated new offspring by crossover operator should be checked.

3.2.5. Mutation

In our approach, we use uniform mutation for both discrete and continue parts of solution representation.

4. Experimental Results

In this section, for a better understanding of the proposed solution method, the effectiveness and performance of the proposed NSGA-II in comparison with NRGA is surveyed. Next subsection describes NRGA algorithm.

4.1. Non-dominated ranked genetic algorithm (NRGA)

Aljadaan et al. (2008) developed a population based multi-objective evolutionary algorithm called non-dominated ranked genetic algorithm (NRGA). This algorithm is a two-stage ranking based on roulette wheel selection operator which selects new generation from parents generation, randomly. It may differ with NSGA-II in selection strategy and sorting of the population.

4.2. Parameters setting

Since the results of an algorithm are influenced by its parameters, an appropriate setting of their parameters for both algorithms NSGA-II and NRGA is carried out using the Taguchi design method in design of experiments (DOE). First of all, influential parameters on the algorithms results are identified in Tables 1 and 2. Three-level is considered for each parameter. Tables 3 and 4 display nine experiments whereas, for each scenario MID/D measured at least three iterations. Tuned parameters of two algorithms have been analyzed as Fig. 3 and 4 depict output of analysis, for NSGA-II algorithm, nPop, p_c parameters were set on 70 and 0.7 for p_m parameter on 0.3 and for NRGA algorithm, nPop, p_c parameters were set on 75 and 0.7 for p_m parameter on 0.3.

Table 1
Considered levels for NSGA-II parameters

Parameters	Level 1	Level 2	Level 3
nPop	50	70	80
P_c	0.6	0.7	0.75
P_m	0.1	0.2	0.3

Table 2
Considered levels for NRGA parameters

Parameters	Level 1	Level 2	Level 3
nPop	50	75	85
P_c	0.6	0.7	0.8
P_m	0.1	0.2	0.3

Table 3
Design of experiment with Taguchi method for NSGA-II parameters

Experiment	Experiment levels			First iteration	Second iteration	Third iteration	average
	nPop	P_c	P_m	MID1/D1	MID2/D2	MID3/D3	MID/D
1	1	1	1	0.692595	0.709	0.668	0.690
2	1	2	2	0.716566	0.700	0.729	0.715
3	1	3	3	0.660618	0.649	0.675	0.662
4	2	1	2	0.724753	0.700	0.734	0.720
5	2	2	3	0.573947	0.572	0.606	0.584
6	2	3	1	0.688357	0.684	0.687	0.686
7	3	1	3	0.711354	0.673	0.749	0.711
8	3	2	1	0.724630	0.695	0.757	0.725
9	3	3	2	0.688652	0.703	0.667	0.686

Table 4
Design of experiment with Taguchi method for NRGA parameters

Experiment	Experiment levels			First iteration	Second iteration	Third iteration	average
	nPop	P_c	P_m	MID1/D1	MID2/D2	MID3/D3	MID/D
1	1	1	1	0.678	0.680	0.648	0.669
2	1	2	2	0.723	0.728	0.718	0.723
3	1	3	3	0.735	0.730	0.723	0.729
4	2	1	2	0.773	0.722	0.724	0.737
5	2	2	3	0.610	0.627	0.627	0.621
6	2	3	1	0.715	0.646	0.676	0.679
7	3	1	3	0.653	0.684	0.654	0.664
8	3	2	1	0.729	0.726	0.708	0.721
9	3	3	2	0.738	0.727	0.758	0.741

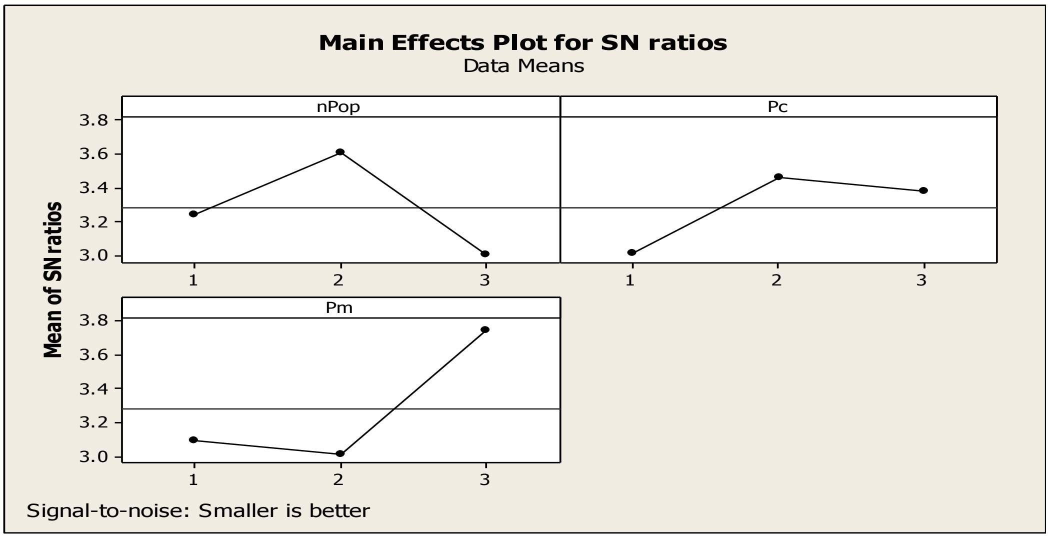

Fig. 3. The analysis of NSGA-II parameters by Taguchi method in design of experiment

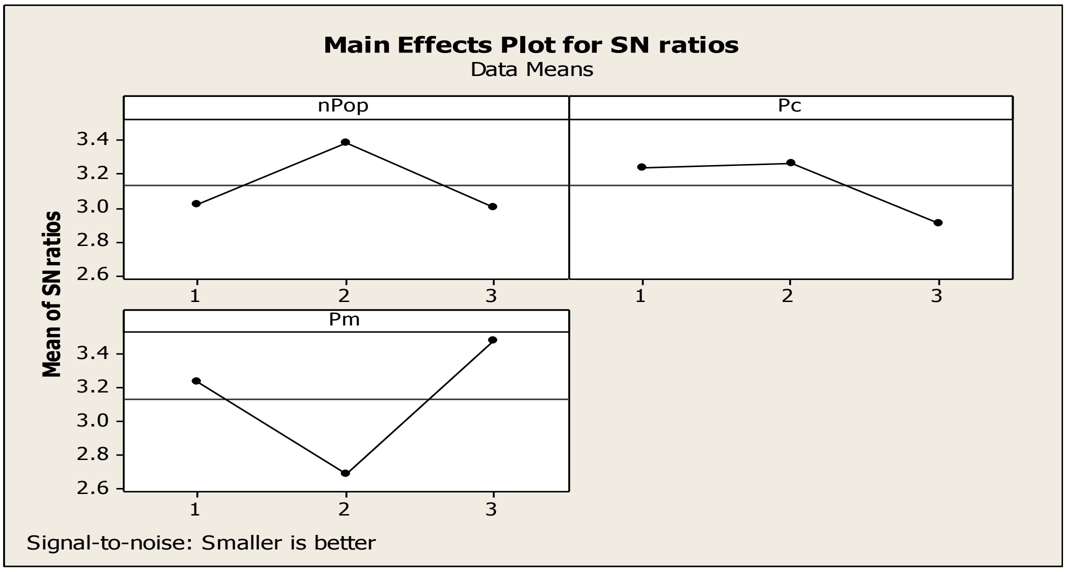

Fig. 4. The analysis of NRGA parameters by Taguchi method in design of experiment

4.3. Comparison metric

To compare the performance of two proposed algorithms, five comparison metrics have been taken into account as follows:

1- Diversity (D): This metric measures the spread of non-dominated set of solution and is computed by Eq. (24). A higher value of diversity reports a better performance of the algorithm.

$$D = \sqrt{\sum_{j=1}^{m}(\max_i f_i^j - \min_i f_i^j)^2} \quad (24)$$

2- Spacing (S): This metric specifies the uniform distribution of the Pareto solution and is measured by Eq. (25) and (26).

$$S = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(d_i - d^-)^2} \quad (25)$$

$$d_i = \min_{k\in n \cap k\neq i} \sum_{j=1}^{m}\left|f_j^i - f_j^k\right| \quad (26)$$

d^- is average of distance between consecutive solution in non-dominated set of solutions. A lower value of spacing shows a better performance of the algorithm.

3- Number of Pareto solution (NOS): This metric reports the number of obtained optimal Pareto solutions.

4- Mean ideal distance (MID): This metric computes the distance between solution fronts and ideal point by Eq. (27) so the algorithm with a lower value of MID defines more closeness between Pareto solution and ideal point.

$$MID = \frac{1}{NOS}\sum_{i=1}^{NOS} c_i \quad (27)$$

c_i is defined as the distance between each member of the population with ideal point.

5- CPU time: This metric is main criteria to express the efficiency of each meta-heuristic algorithm.

4.4. Computational results

After tuning parameters, in this section, the efficiency and the significant difference between two proposed algorithms are executed. For this reason, the proposed model is tested over a number of problem instances. The input data are generated with uniform distribution. Fuzzy parameters are considered with 20 percent tolerance for mean point of membership function as illustrated in Table 5. Tables 6 and 7 elucidate 32 test problems were run on MATLAB 7.8.0 (R2009a) executed on a computer with the characteristic of 2 GHz, Intel® Core TM i5-2430M CPU, equipped with 4.00 GB of RAM. Also these tables illustrate the results of the comparison with deferent values of parameters. The first column is the comparison between two algorithms based on diversity. The second column is the comparison based on spacing. The third column is the comparison based on Number of Pareto solution (NOS) parameter. The forth column is comparison based on Mean ideal distance (MID) and the fifth column shows the comparison based on the running time (CPU time).

Table 5
The interval of generated input parameter with uniform random distribution

Problem parameters	Corresponding random distribution
c^m	~ uniform (10000,20000)
W	~ uniform (100,250)
car^m	~ uniform ($6*10^8$,$9*10^8$)
D	~ uniform (550,1300)
t^m	~ uniform (300,1000)
β	~ uniform (0,1)

Table 6
Comparison of computational result of NSGA-II and NRGA for $(\alpha_H, \alpha_C, \bar{\alpha}_H, \bar{\alpha}_C) = (0.2, 0.1, 0.3, 0.2)$

				Diversity *(10^8)		Spacing *(10^6)		NOS		MID *(10^8)		CPU time	
N	$(r_H,r_C)*10^2$	p_H	p_C	NSGA-II	NRGA	NSGA-II	NRGA	NSGA-II	NRGA	NSGA-II	NRGA	NSGA-II	NRGA
5	11 ,13												
		3	1	2.07	2.52	0.304	0	70	75	1.42	1.61	258.6	265.2
		3	2	3.19	3.57	1.64	2.01	55	75	2.20	2.09	252.1	269.4
		3	3	3.06	3.26	2.46	4.15	56	13	2.09	2.39	271.2	298.8
	7.5 ,11												
		3	1	2.20	2.22	0	0	70	75	1.57	1.60	239.7	254.1
		3	2	3.59	3.34	0.622	0	70	75	2.30	2.36	250.9	268.2
		3	3	4.14	4.70	3.87	2.38	7	13	3.48	3.48	279.3	315.8
10	11 ,13												
		5	1	12.9	14.3	2.12	3.30	69	75	9.56	10.2	1136	1193.8
		5	2	16.8	16.6	1.65	0	60	75	11.9	12.6	1235	1236.2
		5	3	22.8	22	5.92	6.83	70	56	16.1	13.8	1273	1300.9
		5	4	27.2	20.3	19.1	11.5	21	20	19.7	14.3	1326.3	1340.5
		5	5	23.9	21.0	9.17	72.9	23	5	17.5	17.5	1425.7	1407.3
	7.5 , 11												
		5	1	17.9	18.1	9.89	0	3	75	16.9	13.6	1187.4	1191.0
		5	2	26.1	20.3	4.22	0	44	75	17.1	14.9	1236.0	1236.4
		5	3	20.3	22.7	15.9	5.11	10	75	14.5	16.7	1218.4	1304.6
		5	4	28.7	25.6	97.4	20.2	8	69	19.4	17.3	1278.4	1356.1
		5	5	23.3	29.3	27.6	15.3	11	17	17.9	21.3	1344.5	1411.5

Table 7
Comparison of computational result of NSGA-II and NRGA for $(\boldsymbol{\alpha_H}, \boldsymbol{\alpha_C}, \bar{\boldsymbol{\alpha}}_{\mathbf{H}}, \bar{\boldsymbol{\alpha}}_{\mathbf{C}}) = (\mathbf{0.8}, \mathbf{0.7}, \mathbf{0.8}, \mathbf{0.7})$

				Diversity *(10^8)		Spacing *(10^6)		NOS		MID *(10^8)		CPU time	
N	$(r_H,r_C)*10^2$	p_H	p_C	NSGA-II	NRGA	NSGA-II	NRGA	NSGA-II	NRGA	NSGA-II	NRGA	NSGA-II	NRGA
5	11 ,13												
		3	1	2.20	2.99	0.043	1.85	70	65	1.60	2.02	242.5	256.6
		3	2	5.07	4.84	0.968	2.51	70	75	3.19	3.07	250.0	273.1
		3	3	5.50	3.52	0	0	70	72	3.29	2.61	274.9	326.3
	7.5 , 11												
		3	1	3.30	3.19	0	0	70	75	2.23	1.98	231.7	254.0
		3	2	6.92	6.56	8.24	10.5	70	75	3.26	2.90	253.8	267.4
		3	3	8.04	7.46	10.9	8.27	17	75	4.94	5.43	270.0	301.5
10	11 ,13												
		5	1	28.1	13.3	11.4	0	70	75	14.3	10.1	1121.5	1182.1
		5	2	32.4	17.1	0	5.93	70	15	18.7	13.6	1185.8	1203.9
		5	3	35.2	66.6	32.1	69.1	70	75	17.0	22.4	1249.9	1275.2
		5	4	47.2	40.6	691	363.1	6	10	24.7	21.9	1356.3	1342.3
		5	5	37.3	37.4	65.8	38.8	20	20	22.1	20.9	1364.1	1450.0
	7.5 , 11												
		5	1	15.1	20.1	0.43	0	68	75	11.6	14.9	1114.5	1194
		5	2	44.8	29.3	0	0	70	75	27.2	22.0	1176.9	1230.2
		5	3	31.3	29.3	227	133	6	3	20.2	27.2	1238.6	1341.3
		5	4	51.6	64.8	12.1	243	70	17	32.4	36.9	1287.9	1339.8
		5	5	44.2	67.4	6.01	55.0	63	16	32.5	45.3	1330.3	1430.0

To compare and achieve the performance of each algorithm with respect to the 32 problem instances, the significant differences between NSGA-II and NRGA are carried out by one-way ANOVA using Minitab software version 16 according to Figs. 5-9. Results report that in four criteria namely, Diversity, Spacing, NOS and MID, zero hypothesis is accepted with 95% confidence level (p-value>0.05) thus, there is no significant difference between the two algorithms. But the statistical results regarding CPU time metric revealed that the zero hypothesis is not accepted (p-value<0.05) thus, there is actually a significant difference between NSGA-II and NRGA in which the mean CPU time of NSGA-II algorithm is less than the mean CPU time of NRGA algorithm.

```
Source  DF       SS       MS     F      P
Factor   1  0.00180  0.00180  0.37  0.544
Error   62  0.29978  0.00484
Total   63  0.30158

S = 0.06954   R-Sq = 0.60%   R-Sq(adj) = 0.00%


                                    Individual 95% CIs For Mean Based on
                                    Pooled StDev
Level         N     Mean    StDev   -------+---------+---------+---------+--
NSGA-II-D-n  32  0.50530  0.06954          (----------------*---------------)
NRGA-D-n     32  0.49470  0.06954   (---------------*---------------)
                                    -------+---------+---------+---------+--
                                         0.480     0.495     0.510     0.525

Pooled StDev = 0.06954
```

Fig. 5. Output of one-way ANOVA for Diversity

```
Source  DF     SS     MS     F      P
Factor   1  0.108  0.108  0.82  0.368
Error   62  8.125  0.131
Total   63  8.233

S = 0.3620   R-Sq = 1.31%   R-Sq(adj) = 0.00%


                                   Individual 95% CIs For Mean Based on
                                   Pooled StDev
Level         N    Mean   StDev   -+---------+---------+---------+--------
NSGA-II-S-n  32  0.6036  0.3546            (-----------*------------)
NRGA-S-n     32  0.5214  0.3692   (------------*------------)
                                  -+---------+---------+---------+--------
                                 0.40      0.50      0.60      0.70

Pooled StDev = 0.3620
```

Fig. 6. Output of one-way ANOVA for Spacing

```
Source  DF      SS      MS     F      P
Factor   1  0.0213  0.0213  0.55  0.462
Error   62  2.4041  0.0388
Total   63  2.4253

S = 0.1969   R-Sq = 0.88%   R-Sq(adj) = 0.00%


                                   Individual 95% CIs For Mean Based on
                                   Pooled StDev
Level         N    Mean   StDev   --------+---------+---------+---------+-
NSGA-II-N-n  32  0.4818  0.1969   (-------------*-------------)
NRGA-N-n     32  0.5182  0.1969           (-------------*-------------)
                                  --------+---------+---------+---------+-
                                        0.450     0.500     0.550     0.600

Pooled StDev = 0.1969
```

Fig. 7. Output of one-way ANOVA for NOS

```
Source  DF       SS       MS     F      P
Factor   1  0.00009  0.00009  0.04  0.836
Error   62  0.13423  0.00217
Total   63  0.13432

S = 0.04653   R-Sq = 0.07%   R-Sq(adj) = 0.00%

                                     Individual 95% CIs For Mean Based on
                                     Pooled StDev
Level         N     Mean    StDev   ---------+---------+---------+---------+-
NSGA-II-M-n  32  0.50121  0.04653     (---------------*----------------)
NRGA-M-n     32  0.49879  0.04653   (----------------*---------------)
                                    ---------+---------+---------+---------+-
                                         0.490     0.500     0.510     0.520

Pooled StDev = 0.04653
```

Fig. 8. Output of one-way ANOVA for MID

```
Source  DF        SS        MS       F      P
Factor   1  0.011065  0.011065  106.66  0.000
Error   62  0.006432  0.000104
Total   63  0.017497

S = 0.01019   R-Sq = 63.24%   R-Sq(adj) = 62.65%

                                     Individual 95% CIs For Mean Based on
                                     Pooled StDev
Level         N     Mean    StDev   -------+---------+---------+---------+--
NSGA-II-T-n  32  0.48685  0.01019   (---*--)
NRGA-T-n     32  0.51315  0.01019                             (--*---)
                                    -------+---------+---------+---------+--
                                         0.490     0.500     0.510     0.520

Pooled StDev = 0.01019
```

Fig. 9. Output of one-way ANOVA for CPU time

In a special case to obtain the general performance of algorithms, we rank two algorithms considering five metrics (criteria) by TOPSIS method. Firstly, criteria weighting was obtained by Antropy technique and after that equivalent criteria weighting is considered. Table 8 illustrates decision making matrix putting obtained mean of each criterion with one-way ANOVA for each algorithm then two proposed algorithms are ranked by TOPSIS method.

Table 8
Decision making matrix

	Diversity	Spacing	NOS	MID	CPU time
NSGA-II	0.5053	0.5363	0.52	0.50121	0.48685
NRGA	0.4947	0.52	0.5572	0.4987	0.51315

Obtained criteria weighting by Antropy technique are as follows:

W_1= 0.1 W_2=0.3 W_3=0.4 W_4=0.1 W_5= 0.1

Relative alternative distance:

Cl_{1+}=0.91701 NRGA>NSGA-II
Cl_{2+}=0.08299

Then equivalent weighting for five criteria:

W_1= 0.2 W_2=0.2 W_3=0.2 W_4=0.2 W_5= 0.2

Relative alternative distance:

Cl_{1+}=0.778065 NRGA>NSGA-II
Cl_{2+}=0.221935

The findings reported that the performance of NRGA algorithm is better than NSGA-II algorithm, in general also, in relation to the selected problem, in Figs. 10-12 for both algorithms, the achieved results showed that increasing the number of central hubs, the obtained Pareto solutions would have decreasing trend, whereas CPU time would increase. Also Figs. 10-12 show the conflicting results between two objectives in the proposed model.

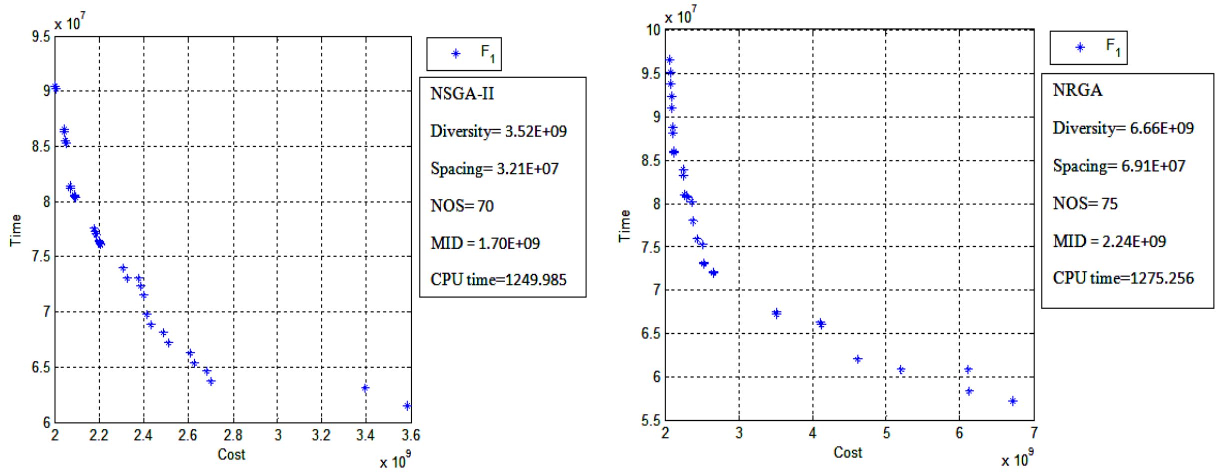

Fig. 10. The comparison of optimal Pareto front using proposed algorithms for $(\alpha_H, \alpha_C, \bar{\alpha}_H, \bar{\alpha}_C) = (0.8, 0.7, 0.8, 0.7)$, n=10, (r_H, r_C)=(1100,1300) and P_C=3

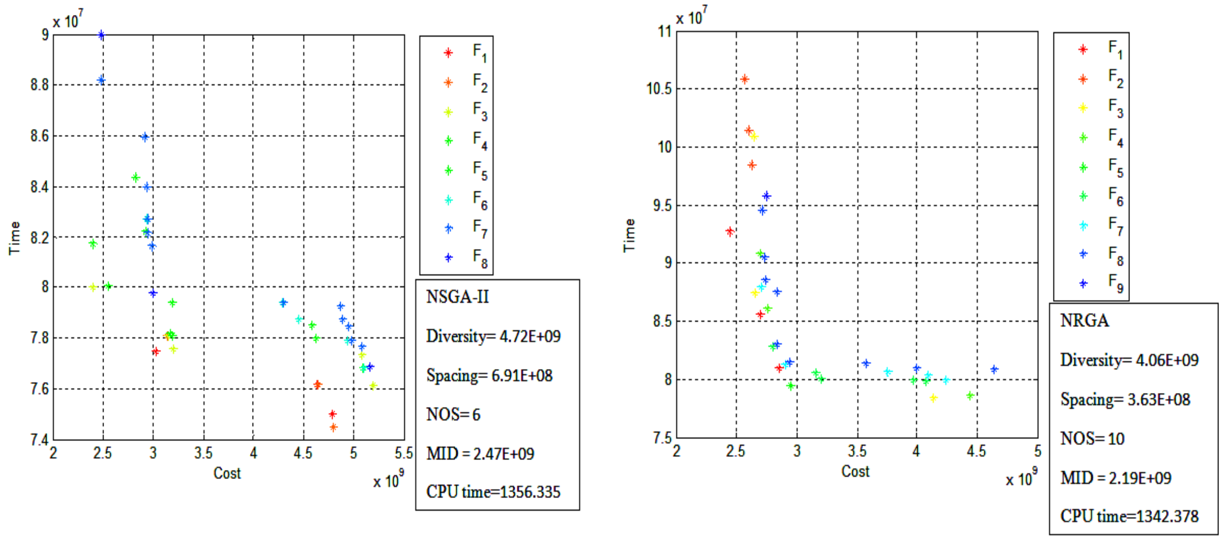

Fig. 11. The comparison of optimal Pareto front using proposed algorithms for $(\alpha_H, \alpha_C, \bar{\alpha}_H, \bar{\alpha}_C) = (0.8, 0.7, 0.8, 0.7)$, n=10, (r_H, r_C)=(1100,1300) and P_C=4

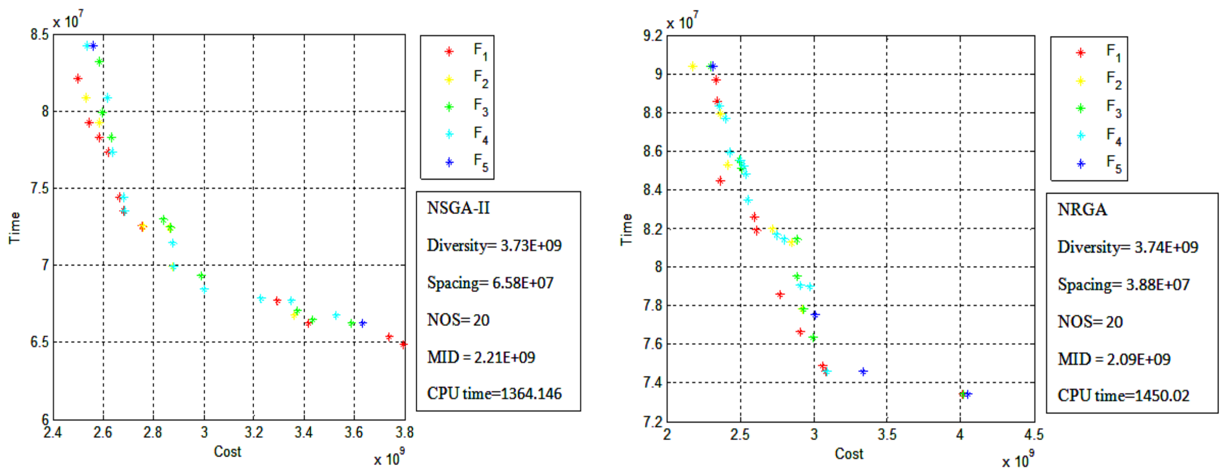

Fig. 12. The comparison of optimal Pareto front using proposed algorithms for $(\alpha_H, \alpha_C, \bar{\alpha}_H, \bar{\alpha}_C) = (0.8, 0.7, 0.8, 0.7)$, n=10, (r_H, r_C)=(1100,1300) and P_C=5

5. Conclusion

This research is an extension of the hierarchical hub location problem in which each demand node, including potential hubs and potential central hubs can move several products in their routes over the hierarchical network, whereas our multi-objective multi-product hub covering problem was first investigated in the hierarchical hub network under fuzzy condition. The usual application of the presented model is satisfying the demand from nodes by enforcing hubs and central hubs covering radius restrictions, two conflicting objective functions were taken into account. The first objective function attempted to minimize the total transportation cost while the second objective function tried to minimize the total transportation time, respectively. In this study, due to available to vague and uncertainty data, some parameters were considered fuzzy ones. The hierarchical hub problem is type of NP-hard problem therefore, to solve the fuzzy multi-objective mixed integer linear programming in a small-sized and middle-sized, two meta-heuristic algorithms (NSGA-II and NRGA) were proposed and their efficiencies were compared to each other over 32 problems. The purpose of solving this problem with two algorithms was specifying the conflicting between two objective functions that was first studied in the hierarchical hub network and then obtaining the value of algorithms metrics and comparing means of two algorithms metrics by statistical method, finally showing the efficiency of algorithms. The results showed that on all comparison metrics, there is no significant difference between means of two algorithms metrics exception of CPU time metric and in general, to show the efficiency of two algorithms, TOPSIS method was used and results reported that the NRGA algorithm outperformed the NSGA-II algorithm. The results also revealed the conflicting between two objective functions moreover, by increasing the numbers of central hubs, the obtained Pareto solutions were decreased in most cases and CPU time was increased.

For further research, a new model considering uncertainty under stochastic approach may be proposed. Moreover, the multiple allocations assuming multi-product and considering the competitive environment to this model may be worthy for investigation.

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