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Optimizing a Fuzzy Greenp-hub Center Problem using Opposition Biogeography Based Optimization Algorithm

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Abstract

Hub networks have always been acritical issue in locating health facilities. Recently, a study has been investigated by Cocking et al. (2006) in Nouna health district in Burkina Faso, Africa, with a population of approximately 275,000 people living in 290 villages served by 23 health facilities. The travel times of the population to health services become extremely high during the rainy season, since many roads are unusable. In this regard, for many people, travelling to a health facility is a deterrent to seeking proper medical care. Furthermore, in real applications of hub networks, the travel times may vary due to traffic, climate conditions, and land or road type. To handle this challenge this paper considers the travel times are assumed to be characterized by trapezoidal fuzzy variables in order to present a fuzzy green capacitated single allocation *p*-hub center system (FGCSA*p*HCP) with uncertain information. The proposed FGCSA*p*HCP is redefined into its equivalent parametric integer nonlinear programming problem using credibility constraints. The aim is to determine the location of *p*c apacitated hubs and the allocation of center nodes to them in order to minimize the maximum travel time in a hub-and-center network in such uncertain environment. As the FGCSA*p*HCP is NP-hard, a novel algorithm called *opposition* biogeography based optimization is developed to solve that. This algorithm utilizes a binary *opposition* based learning mechanism to generate a diversity mechanism. At the end, both the applicability of the proposed approach and the solution methodologies are demonstrated using GAMS/BARON Software under several kind of problems. Sensitivity analyses on the number of hubs and center nodes are conducted to provide more insights as well.

Keywords: Capacitated *p*-hub center system, Single allocation, Fuzzy travel time, *Opposition* based learning, Biogeography based optimization, Uncertain information.

1. Introduction

Hub networks are utilized in health facilities, emergency services, computer networks, and transportation systems such as airline, railway, and so on. In these networks, instead of generating direct links between origin and destination pairs (OD pairs), the hubs serve as transshipment or switching points for flows between center nodes (non-hub nodes) (Parvaresh et al. 2013). These networks called either hub-and-center or hub-and-spoke provide services via a specified set of hub nodes.Generally,in a hub-and-center network, many origins and destinations can be connected with fewer links, based on which smaller transportation rates and a reduction in total transportation costs can be provided(Kratica et al. 2007). Hub-and-spoke networks play a substantial role in the performance of today's transportation companies. According toUnited Nations conference on trade and developmentUNCTAD (2012), approximately 80% of global trade (by volume) and 70% (by value) is transported through sea and is mainly accomplished by hub ports worldwide. Besides, nearly 40% of the costs are involved in road planning and in transportation operations and management(Haridass et al. 2014), which is often associated with fixed costs (additional costs) along with transportation (shipping) Cost (Kundu et al. 2014). Furthermore, as governments impose more tax regulations promoting so-called environment-friendly policies in transportation activities, investment in such networks became even more important. In these networks, flows departing from an origin node are collected in a hub, transferred between hubs if necessary, and then distributed to a destination node by combining flows (Kratica et al. 2007). The hub facilities consolidate flows in order to utilize the concept of economies of scale in transportation between the hubs.

In real world, hub location is one of the most important issues in hub-and-center network problems. For instance, integration of Taiwanese and Chinese air networks for direct air cargo services was studied by Lin and Chen (2003) in a pure hub-and-spoke network. In addition, Fig. 1 shows an instance of hub airport location of Iranian aviation between 37 cities in which Tehran and Kerman as active airports are hubs. Thisis called as the Iranian aviation dataset (IAD) (Karimi and Bashiri 2011).Hub location problems deal with the location of hubs from a set of candidate hubs and assigning center nodes to the located hubs. If the number of hub nodes is fixed to *p*, we are dealing with *p*-hub location problems.. The prominen t*p*-hub median is to distinguish a set of *p*hubs to serve the

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center nodes so that the total transportation cost of the network is minimized. Nevertheless, the generic *p*-hub center location problem pursues equity among the centers, as opposed to the *p*-hub median, which pursues an economic efficiency. This approach requires minimizing the maximum objective function; so it becomes a min–max problem (Lin et al. 2012).

Fig. 1.The hub-and-center network in Iranian Aviation Dataset

In hub location problems, a center node can usually be assigned either to one or more hubs. Therefore, these problems can be classified into single and multiple assignment types (Bryan and O'Kelly 1999). In the single assignment scheme, each center node is assigned to one and only one hub, while all of the centers can be assigned to several hubs in the multiple assignment schemes (Vidović et al. 2011). These types can also be categorized either uncapacitated or capacitated. Recently, the *p*-hub center problem has received an extensive attention of both researchers and practitioners. In this regard, this paper deals with the capacitated single allocation *p*-hub center problem,which has a widerange of applications in clustering along sidede signing transportation and telecommunication networks (Garey and Johnson 1979). As a result, the literature related to the current research can be presented into the limited works of the *p*-hub center problem.

O'Kelly and Bryan (1998)studied the multi-assignment *p*-hub problem with economies of scale on the trunk lines in an uncapacitated networks. Ebery et al. (2000)and Ebery (2001) proposed formulations and solution approaches for a capacitated multiple allocation hub location problem based on the shortest path. Kara and Tansel (2000) presented several linearization of quadratic programs and provided a NP-completeness proof to single allocation p-hub center problem. Integer programming formulations for both uncapacitated and capacitated *p*-hub center problem was developed by Campbell et al. (2007). A bi-criteria approach in a *p*-hub problem was proposed by Costa et al. (2008). They presented two models, the first minimizes the time to processing flows and the second minimizes the maximum service time at the hubs. Moreover, Camargoa et al. (2008)introduced Benders decomposition algorithms to solve the multiple assignment uncapacitated *p*-hub problem.

A *p*-hub center problem with stochastic time and service-level constraints was modeled by Sim et al. (2009) using mutually independent normal distributions.Ernst et al. (2009)presented two uncapacitated *p*-hub center problems with either single or multi-allocation to minimize the maximum total cost and solved them using a branch-and-bound approach.Yang et al. (2011) studied a *p*-hub center problem with discrete random travel to minimize efficient time point. In addition, Yaman and Elloumi (2012) introduced the star p-hub center problem and the star p-hub median problem, where two mixed integer programming formulations with bounded path lengths were proposed for the first problem in order to minimize the total routing cost. In this matter,Liang (2013) investigated the hardness of the star p-hub center problem. He presented a purely combinatorial 3.5-approximation algorithm for this problem. In addition, two mixed-integer programming for a fuzzy *p*-hub center problem were presented by Yang et al. (2013). They solved the problem by utilizing an improved hybrid particle swarm optimization algorithm by combining genetic operators and local search. However, they did not take into account the capacity restrictions of the hubs. On the other hand, Bashiri et al. (2013) proposed a hybrid approach for a p-hub center problem using both qualitative and quantitative parameters to minimize the longest travel time. They utilized a fuzzy VIKOR to model a hybrid solution and then solved the problem using a genetic algorithm. Interested readers are referred to Farahani et al. (2013)for a more extensive review of the studies on p-hub problems.

1.1. Result of literature review

The majority of existing studies in the capacitated p-hub center problem assume deterministic parameters. This often leads to inadequate and unreliable results in real world, especially when decision makers encounter uncertain information in making their decisions about a hub-and-center network, in particular, in the travel times due to uncertain traffic and climate condition (Farahani et al. (2013). Therefore, it is essential to rely on the decision-makers' subjective judgment or the experts' experience to estimate the value of these uncertain parameters. In this way, the fuzzy set theory is an appropriate representation of the imprecision involved in the travel times. Here, it is worth mentioning that the only work on the fuzzy p-hub center problem is the one proposed by Yang et al. (2013), while capacity restrictions of the hubs were not considered.

Moreover, there is a little discussion about uncertain environments in p-hub center problems especially on the travel time. Note that most of the works on the p-hub center problem only aim to optimize either the fixed cost of the selected hubs or the transportation cost and less attention has been paid to the total travel time in the network with uncertain information; a critical factor that has been overlooked. To understand the importance of the total travel time better, consider a case study investigated by Cocking et al. (2006). In this work, the Nouna health district in Burkina Faso, Africa, with a population of approximately 275,000 people living in 290 villages served by 23 health facilities is involved. The travel times of the population to health services become extremely high important during the rainy season, as many roads are unusable. For many people, In other words, travelling to a health facility is a deterrent to seeking proper medical care.Nonetheless, although there are similar p-hub studies in the literature, to the best of author's knowledge, there is no p-hub model that takes into account both the travel time and most important features of real-world together in hub network transportations such as uncertain travel times. Furthermore, with increasing the global consciousness of environmental protection and the corresponding growth in legislation and regulations, green transportation has become an essential issue. Today's, numerous carriers have begun to perform green transportation and have taken into account environmental concerns and greenhouse gas (GHG) emissions. Nonetheless, unfortunately, the necessary attention has not been paid to the green HLP in depth up to now. Thus, it is critical to study the green transportation approach in HLPs.Additionally, as the HLP belongs to the class of NP-hard problems, exact methods are not appropriate to solve large problems in a reasonable computational time. Consequently, an effective meta-heuristic approach is needed to solve large competitive HLPs.

1.2. Motivation and contribution

Now, this paper tries to overcome this shortcoming by considering the uncertain travel times characterized by trapezoidal fuzzy variables. The objective is to determine the locations of p capacitated hubs and the allocations of center nodes to the located hubs such that the maximum travel time in a fuzzy green capacitated single allocation p-hub center problem (FGCSApHCP) is minimized. As the proposed problem is nonlinear and NP-hard, exact methods are not proper to solve large-scale problems. Therefore, a novel meta-heuristic algorithm called biogeography-based optimization (BBO) is developed. As there is no benchmark available in the literature, the popular genetic algorithm (GA) with a multi-parent crossover is designed in order to validate the results obtained.

Moreover, to enhance the performance of the proposed BBO, this paper intends to extend the opposition-based learning (OBL) algorithm proposed as an *opposition* BBO in a binary solution space. To do so, instead of a pure random generation, the objective of OBL implementation is to consider two approaches of randomness and oppositions in generating the solutions simultaneously. In other words, after generating the population of solutions, a second chance is given to the population by checking the opposite solutions, represented as opposite population. Therefore, a novel algorithm namely *opposition*BBO (OBBO) is presented. Afterward, to tune the parameters of the algorithms, the Taguchi method is performed. Besides, sensitivity analyses on the number of hubs and center nodes are conducted to provide more insights as well. This study is motivated by the need of considering GHG emissions and their consequences in making the decisions related to HLP in a real environment. In short, the highlights of the differences of this study with the mentioned studies are as follow:

- Developing a fuzzy HLP with uncertain transportation time;
- Redefining the problem into its equivalent parametric integer nonlinear programming; and
- Proposing an *opposition* binary BBO;

The application of this study is to generate additional opportunities and cost effective alternatives for companies that operate in the hub transportation networks under such uncertain environment.The remainder of the paper is organized as follows. Section 2 describes the problem definition while Section 3 contains the solving methodologies. Opposition-based learning will be introduced in Section 4. In order to demonstrate the application of the proposed model, several problems are investigated in Section 5. Moreover, a sensitivity analysis is performed in this section. At the end, conclusion and future researches are provided in Section 6.

2. Problem Definition

The proposed problem is useful especially for delivery of perishable or time sensitive items such as the ones in express mail services, emergency services, and healthcare's, where aminimum service (travel) time is desirable for all customers. Here, consider areal transportation network in city logistics including multiple center nodes and candidate hubs. For instance, a network for two hubs and four center nodes is illustratedin Fig. 2.

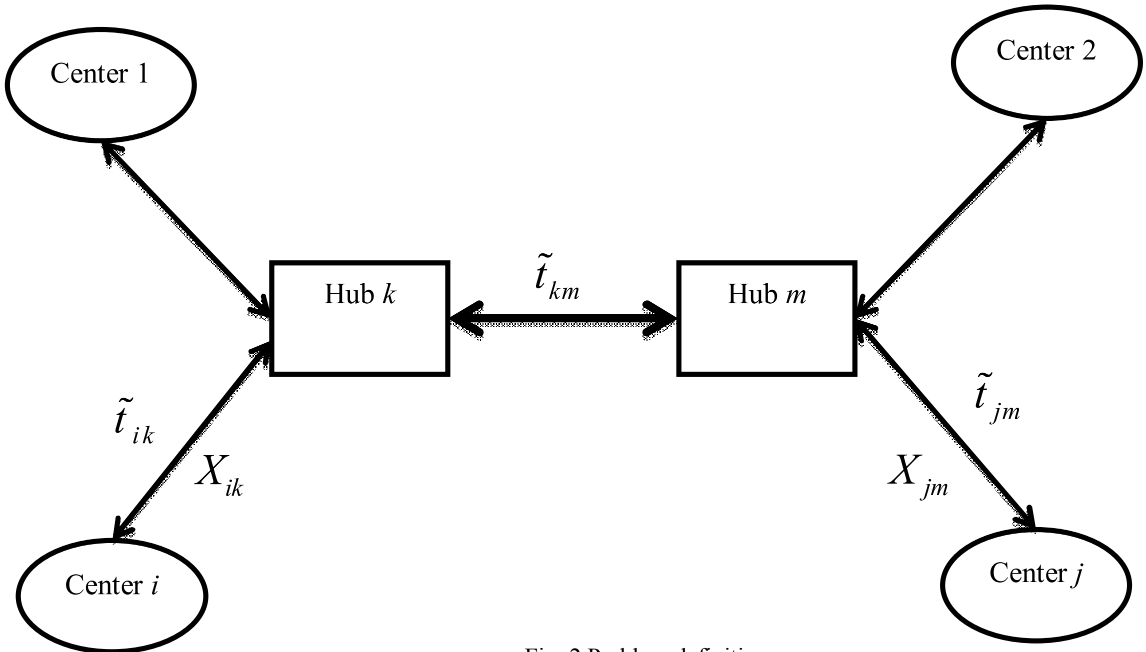


Fig. 2.Problem definition

Let H be the set of candidate hubs $h, m \in H$ in the design network with a provided capacity for handling to the center node $i, j \in N$. Collectively, the design network contains a set $O = \{N \cup H\}$. This network contains a set of directed links that connect from all of the origin center nodes to all of the hub nodes; all of the hub nodes to all of the other hub nodes; and all of the hub nodes to all of the destination center nodes with no intermediate nodes. A center pickup node i with a center delivery node j forms an origin–destination pair (OD).A discount factor is assumed on the hub-to-hub trunk transportation links to provide reduced time on arcs between hubs for reflecting economies of scale. In addition, a capacity limitation is considered on the volume of traffic entering a hub. In real applications of hub-and-center networks, the travel times may vary because of traffic or climate conditions, speed ambulances, land or road type. To handle this uncertainty, an appropriate representation of the imprecision can be described by using fuzzy variables. There by, due to the uncertainty of relevant data, the uncertain travel times are associated with fuzzy features characterized by trapezoidal fuzzy variables.

The aim is to locate p hubs from the set H and allocate the center nodes from set N to them, in order to minimize the maximum travel time with fuzzy travel times determined by experts' experience in practice. To do so, an integer nonlinear programming model is developed based on the following assumptions:

1- The number of hubs to be located is predetermined (p).

2- Each center node is only assigned to a single hub.

3- All hubs are connected to one another.

4- Direct transportation between center nodes is not allowed.

5- The installation cost of the hub nodes is not considered.

The mathematical notations usedin the proposed formulation are:

Indices

i, j Index for center nodes $i, j = 1,...,N$

k, m Index for hub $k, m = 1,...,H$

Input parameters

W_{ij} Flow between center node *i* and center node *j* (unit)

$\tilde{t}_{ik}$ Fuzzy transfer time for commodity to travel from center node*i* to hub *k* (second)

$\tilde{t}_{km}$ Fuzzy transfer time for commodity to travel from hub *k* to hub *m* (second)

$\tilde{t}_{jm}$ Fuzzy transfer time for commodity to travel from hub *m*to center node *j* (second)

β Discount factor for trips between two hub nodes

O_i Total commodity to be transferred from center node*i* (unit)

C_k Capacity of hub *k* (unit)

Decision variables

X_{ik} A binary variable, set equal to 1 if center node*i* is allocated to hub k, 0 otherwise

Y_k A binary variable, set equal to 1 if a hub is established at node *k*, 0 otherwise

Then, the proposed FCSApHCP without considering green approach is formulated as follows:

$$Min \quad Z = \underset{i,j,k,m}{Max}\left\{W_{ij}\left(\tilde{t}_{ik}X_{ik} + \beta\tilde{t}_{km}X_{ik}X_{jm} + \tilde{t}_{jm}X_{jm}\right)\right\}; \quad (1)$$

Subject to:

$$\sum_{k\in H} X_{ik} = 1, \quad \forall i \in N; \quad (2)$$

$$\sum_{i\in N} X_{ik} \geq Y_k, \quad \forall k \in H; \quad (3)$$

$$Y_k \geq X_{ik}, \quad \forall i \in N, \ k \in H; \quad (4)$$

$$\sum_{k\in H} Y_k = p, \quad (5)$$

$$\sum_{i\in N} O_i X_{ik} \leq C_k, \quad \forall k \in H; \quad (6)$$

$$X_{ik}, Y_K \in \{0,1\}, \quad \forall i \in N, \ k \in H; \quad (7)$$

The objective function(Z) presented in Eq. (1)deals with minimizing the maximum of travel time including the travel time between origin- hub, hub-hub, and hub-destination associated with each flow, respectively. Constraint (2) shows that every center node is assigned to one and only one hub. Constraints (3) and (4) ensure that the flow is sent only via open hubs. They prevent direct transmission between center nodes.Constraint (5)ensures that exactly *p* hubs are chosen. Constraint (6) guarantees that the total flow into hub *k* via collection cannot exceed its maximum capacity.Finally, Constraint (7) ensures binary values for decision variables where$\tilde{t}_{ik}$, $\tilde{t}_{km}$, and $\tilde{t}_{jm}$ are fuzzy numbers. Note that the capacitated p-hub center problem is a well-known nonlinear model in the literature(Farahani et al. (2013)).Moreover, it is clear that the above formulation is a fuzzy integer nonlinear programming (FINLP). In the next section, this model is then converted to a parametric integer nonlinear formulation.

2.1.Equivalent parametric integer nonlinear programming

As mentioned before, fuzzy travel times are characterized by trapezoidal fuzzy variables. In order to solve the proposed problem, the FCSApHCP is then redefined into its equivalent parametric integer nonlinear programming problem using credibility constraints (*Cr*) where they are transformed to their equivalent forms. In the following, the framework of credibility constraints is briefly described (Liu and Liu 2002; Yang et al. 2013). The fuzzy set theory introduced by Zadeh (1965) is increasingly applied to a wide variety of practical problems. In fuzzy decision making environments, possibility and necessity measures are prominent fuzzy measures (Davari et al. 2013).

Definition 2.1 Let X be a universe of discourse, A is a fuzzy subset of X if for all $x \in X$ there is a number $\mu_A(x) \in [0,1]$ assigned to represent the membership of x to A. $\mu_A(x)$ is called the membership function of A.

Definition 2.2 The (crisp) set of elements that belongs to the fuzzy set A for which the degree of its membership function exceeds the level α is called α-cut and denoted by $A_\alpha = [x \in X \mid \mu_A(x) \geq \alpha]$.

Definition 2.3 (Zimmermann 1978). A fuzzy decision is defined in an analogy to non-fuzzy environments as the selection of activities that simultaneously satisfies objective functions and constraints. In fuzzy set theory, the intersection of sets normally corresponds to the logical ''and''. In addition, the ''decision'' in a fuzzy environment can be viewed as the intersection of fuzzy constraints and fuzzy objective functions.

Definition 2.4 Let ξ be a fuzzy variable along with membership function μ of $\Re \rightarrow [0,1]$. A fuzzy variable ξ is said to be normal if there exists a real number r such that $\mu(r) = 1$.

Definition 2.5 (Liu and Liu 2002). Let u and r be two real numbers. Then, the possibility degree of occurrence of a fuzzy event, characterized by $\xi \leq r$, is defined by:

$$Pos\{\xi \leq r\} = \sup_{u\leq r} \mu(u)$$

While the necessity degree of occurrence of this fuzzy event is introduced using:

$$Nec\{\xi \leq r\} = 1 - Pos\{\xi > r\} = 1 - \sup_{u>r} \mu(u)$$

Therefore, the *credibility measure* (Cr) is then defined as an average of possibility and necessity measures as follows:

$$Cr\{\xi \leq r\} = \frac{1}{2}(Pos\{\xi \leq r\} + Nec\{\xi \leq r\})$$

It should be noted that a fuzzy event might fail even though its possibility achieves 1, and holds even though its necessity is 0. However, the fuzzy event must hold if its credibility is 1 and fails if its credibility is 0.
Let us consideraga in the sample network shown in Fig. 2. The travel time from nodes i to j must route through only two installed hubs. Consider the travel times $\tilde{t}_{ik} = (r_{ik}^1, r_{ik}^2, r_{ik}^3, r_{ik}^4)$, $\tilde{t}_{km} = (r_{km}^1, r_{km}^2, r_{km}^3, r_{km}^4)$, and $\tilde{t}_{jm} = (r_{jm}^1, r_{jm}^2, r_{jm}^3, r_{jm}^4)$ .Then, the total travel time on this path of $i \to k \to m \to j$ is $\tilde{t}_{ik} + \beta\tilde{t}_{km} + \tilde{t}_{jm}$ where the discount factor $0 \le \beta \le 1$ represents the scale economies on the inter-hub linkage.

Theorem. Let the travel times $\tilde{t}_{ik}$, $\tilde{t}_{km}$, and $\tilde{t}_{jm}$ be mutually independent trapezoidal fuzzy variables. Then, the proposed FCSApHCPis equivalent to the following parametric integer nonlinear programming problem:

$$\begin{aligned} &\text{Minimize} \quad Z \\ &\text{Subject to:} \quad f(X_{ik}, X_{jm}) \le Z, \qquad \forall i, j \in N \;\; k, m \in H \\ &\qquad (2)\text{-}(7); \end{aligned} \tag{8}$$

Where the term of $f(X_{ik}, X_{jm})$ is the following piecewise function as:

$$f(X_{ik}, X_{jm}) = \begin{cases} W_{ij}((1-2\alpha)(r_{ik}^1 X_{ik} + \beta r_{km}^1 X_{ik} X_{jm} + r_{jm}^1 X_{jm}) + 2\alpha(r_{ik}^2 X_{ik} + \beta r_{km}^2 X_{ik} X_{jm} + r_{jm}^2 X_{jm})), & \alpha \le \dfrac{1}{2} \\ W_{ij}((2-2\alpha)(r_{ik}^3 X_{ik} + \beta r_{km}^3 X_{ik} X_{jm} + r_{jm}^3 X_{jm}) + (2\alpha - 1)(r_{ik}^4 X_{ik} + \beta r_{km}^4 X_{ik} X_{jm} + r_{jm}^4 X_{jm})), & \alpha > \dfrac{1}{2} \end{cases}$$

Proof. Asthe travel times $\tilde{t}_{ik}$, $\tilde{t}_{km}$, and $\tilde{t}_{jm}$ are mutually in dependent by the properties of trapezoidal fuzzy variables, $\tilde{t}_{ik} + \beta\tilde{t}_{km} + \tilde{t}_{jm}$ is a trapezoidal fuzzy variable and can be denoted by

$(r_{ik}^1 + \beta r_{km}^1 + r_{jm}^1, r_{ik}^2 + \beta r_{km}^2 + r_{jm}^2, r_{ik}^3 + \beta r_{km}^3 + r_{jm}^3, r_{ik}^4 + \beta r_{km}^4 + r_{jm}^4)$ .Denote $\xi_{ikmj} = \tilde{t}_{ik} + \beta\tilde{t}_{km} + \tilde{t}_{jm}$ for simplicity of presentation. Therefore, the credibility constraints (*Cr*) can be obtained as follows:

$$Cr\{\xi_{ikmj} \le Z\} \ge \alpha, \qquad \forall i, k, m, j.$$

where $0 < \alpha < 1$ is apre scribed credibility level. It should be noted that if $0 < \alpha \le 1/2$, then

$$Cr\{\xi_{ikmj} \le Z\} \ge \alpha \iff Z \ge (\xi_{ikmj})_{2\alpha}^L$$

in which $(\xi_{ikmj})_{2\alpha}^L$ is the left extreme point of the term of $2\alpha - \text{cut}$ of ξ_{ikmj}. Hence, $Cr\{\xi_{ikmj} \le Z\} \ge \alpha$ is equivalent to

$$(1-2\alpha)(r_{ik}^1 + \beta r_{km}^1 + r_{jm}^1) + 2\alpha(r_{ik}^2 + \beta r_{km}^2 + r_{jm}^2) \le Z.$$

It follows that the credibility constraint $Cr\{W_{ij}(\tilde{t}_{ik} X_{ik} + \beta\tilde{t}_{km} X_{ik} X_{jm} + \tilde{t}_{jm} X_{jm}) \le Z\} \ge \alpha$ can be expressed equivalently as follows

$$W_{ij}((1-2\alpha)(r_{ik}^1 X_{ik} + \beta r_{km}^1 X_{ik} X_{jm} + r_{jm}^1 X_{jm}) + 2\alpha(r_{ik}^2 X_{ik} + \beta r_{km}^2 X_{ik} X_{jm} + r_{jm}^2 X_{jm})) \le Z.$$

On the other hand, if the credibility level $1/2 < \alpha < 1$, then one has

$$Cr\{\xi_{ikmj} \le Z\} \ge \alpha \iff Z \ge (\xi_{ikmj})_{2-2\alpha}^R ,$$

where $(\xi_{ikmj})_{2-2\alpha}^R$ is the right extreme point of the $(2-2\alpha) - \text{cut}$ of ξ_{ikmj}. Finally, the credibility constraint $Cr\{W_{ij}(\tilde{t}_{ik} X_{ik} + \beta\tilde{t}_{km} X_{ik} X_{jm} + \tilde{t}_{jm} X_{jm}) \le Z\} \ge \alpha$ is then equivalent to

$$W_{ij}((2-2\alpha)(r_{ik}^3 X_{ik} + \beta r_{km}^3 X_{ik} X_{jm} + r_{jm}^3 X_{jm}) + (2\alpha - 1)(r_{ik}^4 X_{ik} + \beta r_{km}^4 X_{ik} X_{jm} + r_{jm}^4 X_{jm})) \le Z.$$

This concludes the proof.

2.2. Green approach

Green transportation is a critical issue in particular in road freight transportation that is a major contributor to emit carbon dioxide (CO_2) equivalent emissions. Many carriers have started taking into account the negative externalities of their activities such as pollution, accidents, noise, resource consumption, climate change risk, and land use deterioration to reduce carbon dioxide emissions that are harmful for both human health and environment, (Demir et al. 2014b). Meanwhile, most trucks use diesel engines for running, which are major sources of emissions such as nitrogen oxides (N_2O), particulate matter (PM) and CO_2. In this regard, N_2O-based smog and PM have been related to a wide range of human health problems. Particularly, greenhouse gases (GHGs) significantly contribute to global warming at the global level. According to a review of recent research on the green transportation approach studied by Demir et al. (2014b):

"*The selection of the right vehicles from an available set is a promising area to minimize CO_2 emissions*".

Now that the selection of the appropriate vehicles from an available is an area in need of more effort for comprehensive investigations, unfortunately, this fact has not been investigated much and there exist only a few studies carried out about it. To fill this gap, the carriers not only should select right vehicle so that the total penalty due to the delivery schedule violations is to be minimized, but also they should take into account the green transportation approach in their planning, especially in road freight transportation connected to the hubs. Thus, in the proposed problem, vehicle selection and its speed level for transportation is investigated. Obviously, decreasing these emissions in transportation planning requires an understanding of vehicle emission models. So, the vehicle emission model proposed in this paper is based on a *comprehensive modal emission model* (CMEM)[1] developed by Scora and Barth (2006), Barth et al. (2005), and Barth and Boriboonsomsin (2008), which is an instantaneous model estimating fuel consumption for heavy-goods vehicles. According to the model described by Demir et al. (2012), the fuel rate is calculated using:

$$FR = \xi(kNV + P/\eta)/\kappa, \tag{9}$$

where ξ is fuel-to-air mass ratio, k is the engine friction factor, $N$is the engine speed, V is the engine displacement, and η and κ are constants. Besides, P denotes the second-by-second engine power output and can be obtained as follows:

$$P = P_{tract}/\eta_{tf} \quad P_{acc}, \tag{10}$$

where η_{tf} denotes the vehicle drive train efficiency, and P_{acc} is the engine power demand associated with the operation of vehicle accessories such as air conditioning and running losses of the engine. Usually, this parameter is assumed to be zero. Moreover, P_{tract} is the total *tractive* power requirement on the wheels of vehicle:

$$P_{tract} = (M\tau + Mg\ \sin\theta + 0.5C_d\rho A\upsilon^2 + MgC_r\ \cos\theta)v/1000, \tag{11}$$

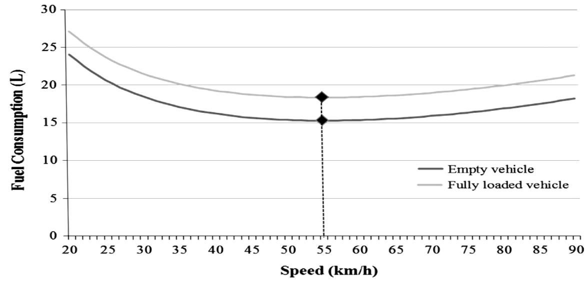


Fig. 3. Behaviour of Fuel consumption versus speed (Demir et al. 2014a)

In Eq. (11), M is the total vehicle weight (kilogram), v is the vehicle speed (meter/second), τ is the acceleration (meter/second 2), θ is the road angle, g is the gravitational constant, and C_d and C_r are the coefficient of the aerodynamic drag and rolling resistance, respectively. Besides, ρ is the air density and A is the frontal surface area of the vehicle. It is supposed that for a given route (i, j) connected to hub k and $m$withthe length d, v be the speed of a vehicle speed traversing. Now, if all variables in FR except for the vehicle speed v remain constant,the fuel consumption (in liter) on this route can be calculated as follows:

$$F(v) = kNV\lambda d/v + P\lambda\gamma d/v, \tag{12}$$

by considering $\lambda = \xi/\kappa/\psi$ and $\gamma = 1/1000\eta_{tf}\eta$ where ψ is the conversionfactor of fuel from gram/second to liter/second. If $\alpha = \tau + g\sin\theta + gC_r\cos\theta$ be a vehicle-arc specific constant and $\beta = 0.5C_d\rho A$ be avehicle specific constant, Then, $F(v)$ given in Eq. (12) can be rewritten as

$$F(v) = \lambda(kNV + w\gamma\alpha v + \gamma\alpha f v + \beta\gamma v^3)d/v. \tag{13}$$

where w is the curb weight (i.e., the weight of an empty vehicle) and f is the vehicle load. The value of these parameters are obtained from the MEET reported table by Demir et al. (2012). Thus, the vehicle speed v is a variable in the proposed emission model, which is important tool in a wide-area emission assessment. It is worthwhile to mention that the CO_2emissions are directly related to fuel consumption and therefore can be easily calculated if the amount of fuel consumption is known (Demir et al. 2014b). Fig. 3 presents how fuel consumption (liter/100 kilometer) changes with respect to speed (kilometer/hour) for an empty and fully loaded medium-duty vehicle using the Eq. (13). It can be seen that the vehicle speed that minimizes fuel consumption is around 55 kilometer/hour using the assumed values. Note that as the vehicle speed v is a variable in the proposed emission model, the model works with a discrete speed function defined by R non-decreasing speed levels $\bar{v}^r = (r = 1,...,R)$.

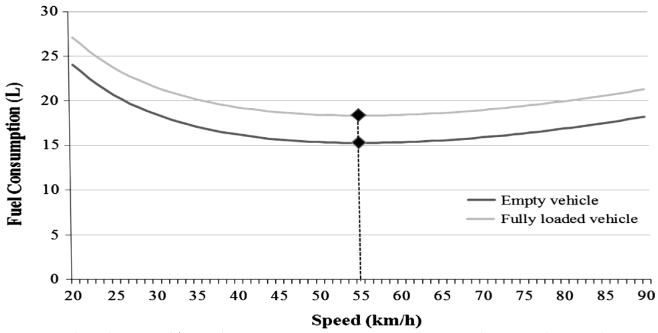


Fig. 3. Behaviour of Fuel consumption versus speed (Demir et al. 2014a)

To do so, a binary decision variable U_{ijt} is defined to show if route (*i*, *j*) is visited by vehicle *t* to serve the request where $t \in T$. Secondly, it is not a necessity that all requests must be served by the carriers. In this matter, a binary variable $S_{ijt}^r \in \{0,1\}$ indicates whether or not route (*i,j*) connected to the hubs is traversed by vehicle *t* at a speed level *r* with respect the discrete speed function defined by R. Thereby, the CO_2 emissions of the vehicles, which are directly related to fuel consumption, can be easily calculated based on the value of $F(\bar{v}^r)$ if the speed level is known. Therefore, the fuel consumption is a function of the speed level and as a result, the carriers should choose a good speed level for their vehicles. The following constraint ensures that one and only one speed level should be selected for each vehicle *t* in each route (*i,j*), and $S_{ijt}^r = 1$ only if that vehicle has already been selected for that route where $U_{ijt} = 1$:

$$\sum_{r=1}^{R} S_{ijt}^r = U_{ijt}, \qquad \forall i,j,t \in T \tag{14}$$

In this regard, the following constraints can be considered:

$$U_{ijt} \le \sum_k \sum_m X_{ik} X_{jm}, \qquad \forall i,j = 1,...,N, j \ne i, t \in T \tag{15}$$

$$\sum_{t \in T} U_{ijt} = \sum_{t \in T} U_{jit}, \qquad \forall i,j = 1,...,N, j \ne i,\ t \in T \tag{16}$$

Constraint (15)expresses a route (*i,j*) can be allocated to a vehicle if the center nodes *i* and *j* are assigned to the hubs *k* and *m*, respectively, i.e., if there exists a route (*i,j*) connected to the hubs *k* and *m* where both X_{ik} and X_{jm} would take a value of 1. In addition, it ensures that each route is visited at most once by vehicles of carriers. Finally, Constraint (16)ensures that if vehicle *t*is selected by its owner (carrier) to serve the route (*i,j*), conversely, that vehicle should be chosen to serve the route (*j,i*). The proposed objective functions will be presented later.

Finally, based on what was assumed and derived above, the total fuel cost in the green approach can be modeled by:

$$\sum_{i=1}^{N} \sum_{j=1, j\ne i}^{N} \sum_{t \in V_l} \sum_{r=1}^{R} f_{CO_2} F\left(\bar{v}^r\right) S_{ijt}^r$$

Consequently, the objective of the proposed FGCSA*p*HCP can be redefined as:

$$\text{Minimize} \quad Z + \sum_{i=1}^{N} \sum_{j=1, j\ne i}^{N} \sum_{t \in V_l} \sum_{r=1}^{R} f_{CO_2} F\left(\bar{v}^r\right) S_{ijt}^r \tag{17}$$

$$\text{Subject to:} \quad Cr\left\{W_{ij}(\tilde{t}_{ik} X_{ik} + \beta \tilde{t}_{km} X_{ik} X_{jm} + \tilde{t}_{jm} X_{jm}) \le Z\right\} \ge \alpha, \qquad \forall i,j \in N\ k,m \in H, (2)\text{ - }(7); \tag{18}$$

where Equation (18)together with objective function (17) is to minimize the α-efficient time point of the travel time.

3. Solving Methodologies

As the proposed problem is an INLP and belongs to the class of NP-hard problems, a novel algorithm, called *opposition* biogeography-based optimization (OBBO) is presented in this section.

3.1. The BBO

In this section, a brief description of the BBO algorithm is first given. Afterward, BBO will be extended to OBBO. BBO is a type of evolutionary algorithm that was firstly introduced bySimon (2008). As its name implies, biogeography is the study of the migration, speciation, and extinction of species. Biogeography has often been considered as a process that enforces equilibrium in the number of species in habitats. Moreover, it has been inspired by the study of the distribution of animals and plants over time and space. BBO has demonstrated good performance when used in various unconstrained and constrained benchmark functions(Ergezer et al. 2009; Simon 2008). In this algorithm, the habitat suitability index (HSI) is a measure of the goodness of the solution that is represented by the habitat, which is also called fitness. Therefore, a high value of HSI means large number of species and low value of HSI means less number in habitat. In BBO, each habitat contains several features called suitability index variables (SIVs) that are similar to genes of chromosomes in genetic algorithm. These SIVs emigrate from high-HSI habitats to low-HSIs during several iterations under emigration process with an emigration rate(μ_i) .On the other hand, low-HSI habitats accept new SIVs from high-HSI habitats through an immigration process with an immigration rate(λ_i). Hence, BBO utilizes a migration operator that includes two sub-operators of the emigration and immigration. Fig. 4 illustrates the linear BBO immigration and emigration curves.

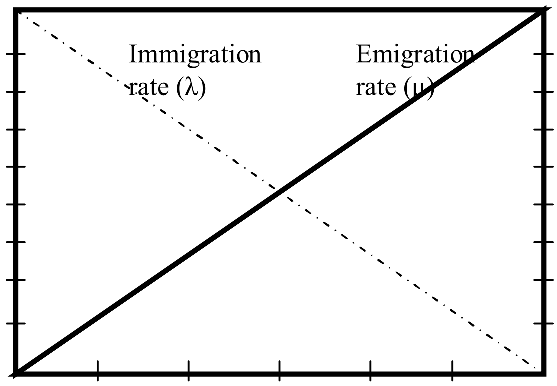


Fig .4. The immigration and emigration curves in BBO

In Fig. 4, the worst solution has the highest immigration rate; hence, it has a very high chance of borrowing features from other solutions to improve the next generation. Meanwhile, the best solution has a very low immigration rate, indicating that it is less likely to be altered by other solutions(Simon 2008). As the number of species increases, more species are able to leave the habitat and the emigration rate increases. In addition to the migration operator, the BBO utilizes another operator called mutation, similar to GA. Now that each habitat has been characterized by its rank, in BBO, the ranks are utilized to implement the migration operator in which the emigration rate (μ_i) and the immigration rate (λ_i) for habitat *i*are calculated as follows:

$$\lambda_i = 1 \times (R_i / nPop) \quad (19)$$

$$\mu_i = 1 \times (1 - R_i / nPop) \quad (20)$$

where R_i represents the rank of the habitat *i* (or its front position) after sorting all habitats according to sorting strategy and *nPop*represents the size of the population.

3.1.1. The habitat representation

In this research, thehabitatsare generated in a way that they satisfy all constraints all times; hence, all of their corresponding solutions are feasible. The proposed habitat is structured as a $N \times (H + 2 \times (N \times V_l))$ matrix provided in four sections to satisfy all constraints all the times and hence to avoid generating infeasible solutions. The fourth three sections of a habitat are:

Section 1: A $N \times H$ matrix of the X_{ik} ;

Section 2: A $N \times (N \times V_l)$ matrix of the U_{ijt} ;

Section 3: A $N \times (N \times V_l)$ matrix of the S_{ijt} ;

Hereby, each section of the habitat is related to a decision variable. In Section 1 as an example,each row and column of the matrix represents a center node and a hub, respectively. Therefore, each cell represents a bitthatrefers to an arc between a hub and a center node. It takes 1 ifcenternode*i*is allocated to hub *k*,and 0 otherwise. By applyingConstraint (3), all bits except one in each row are zero. Fig. 5 illustrates the general form of the proposed habitat for five geographically dispersed center nodes $(N=5)$ and three candidate hubs $(H=3)$, where *p* is equal to two. In this figure, it can be seen that hub 1 and 3 are located, center nodes 1-3 are allocated to hub 3, and center nodes4-5are allocated to hub 1. In addition, hub 2 is not located due to the value of *p*.

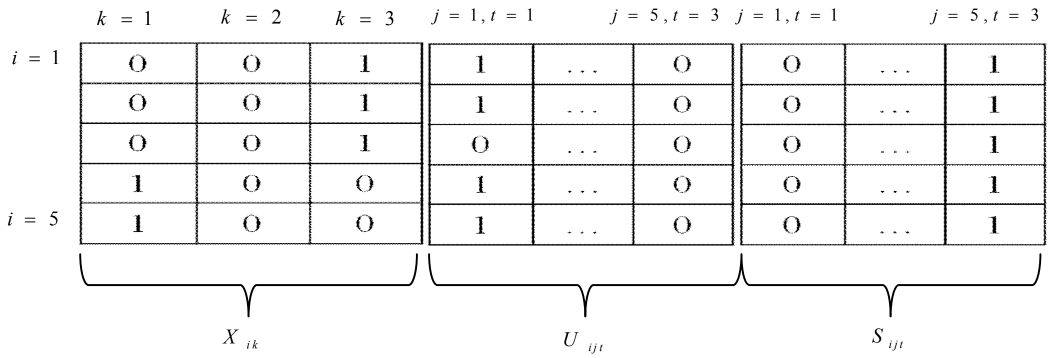
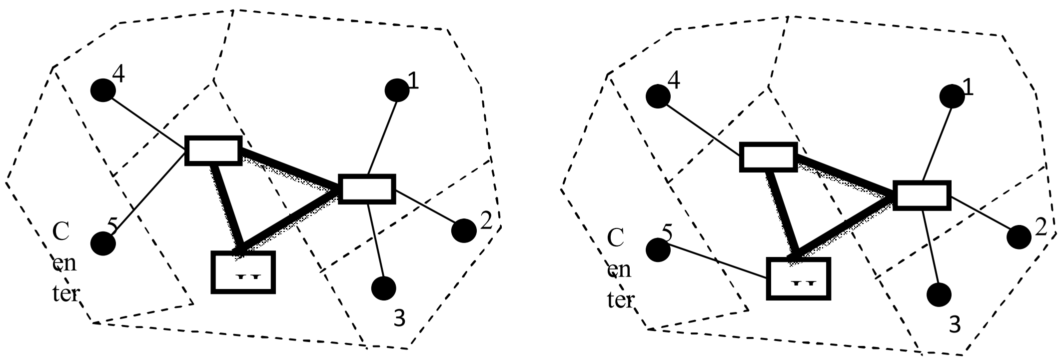


Fig. 5. The habitat representation

3.1.2. The migration and mutation operators

The mutation operator makes a mutated habitat randomly by modifying its SIVs. This operator helps to generate a reasonable level of diversity in the habitats. It also serves the search by jumping out of local optimal solutions. For this operator, a predetermined parameter P_m that represents the probability of mutation operator is applied. This operator randomly selects a habitat and then randomly selects one of its center node and reassigns it to another hub selected at random (an exchange mutation.) It is worthwhile to mention that the exchange mutation had a good performance in binary-coded solution(Ramezanian et al. 2012).More precisely, as shown in Fig. 6 for the initial habitat, the operator generates a mutated habitat such that it switches the assignment of a single node (center node5) from one hub to another.



Initial habitat

Mutated habitat

Fig. 6. Illustration of the exchange mutation

In the migration operator, some characteristics swap directly within the emigrating habitats that share their information to immigrating habitats that accepts shared characteristics due to binary space. Thus, habitat with appropriate suitability index variables (SIVs) are scattered throughout the population.

4. Opposition-Based Learning

Opposition-based learning (OBL)was originally introduced by Tizhoosh (2005)and was first proposed as machine intelligence scheme for reinforcement learning. Afterward, it has been employed to improve soft computing methods such as fuzzy systems (Tizhoosh 2009)and artificial neural networks(Shokri et al. 2007; Ventresca and Tizhoosh 2006). In addition, Rahnamayan et al. (2008)illustrated the capabilities of OBL by combining it with differential evolution to solve continuous optimization problems. OBL has been employed in a wide range of evolutionary algorithms such as biogeography-based optimization(Ergezer et al. 2009), particle swarm optimization(Wang et al. 2011), ant colony optimization (Malisia 2008),and simulated annealing (Ventresca and Tizhoosh 2007).

Most of the evolutionary algorithms start with an initial random population without any preliminary knowledge about the solution space. Additionally, the computation time is directly related to the quality and distance of the solutions in the initial population from the optimal solution. Here, two questions arise as follows: how to enrich the initial population and the population generated in each it eration? what advantage is between simultaneous consideration of randomness and oppositions versus pure randomness? To answer these questions, this paper tries to explore the simultaneous implementation of two approaches (randomness and oppositions) in generating the solutions instead of pure randomness. After generating the population of solutions, a second chance is given to this population by checking the opposite solutions (opposite population)and select the best solutions (fittest) among them to start the algorithm. The aim of the OBL algorithm as a diversity mechanism is to enhance the performance of the proposed meta-heuristic algorithms and to enrich the Pareto-fronts. However, as the solution space of this paper is binary, a new version called the binary opposition-based scheme is proposed. To do this, the concept of OBL in continuous spaces is first presented. Then, it will be modified to be used in a binary solution space.

4.1. Opposition in continuous space

The continuous space encompasses the variants of opposition, quasi-opposition, and quasi-reflection defined below.

Definition 1. The opposite of any real number $x \in [a, b]$, denoted by $\breve{x}$, is generated using $\breve{x} = a \quad b \quad x$. This definition can easily be extended to higher dimensions.

Definition 2. The quasi-opposite of any real number $x \in [a, b]$, denoted by $\breve{x}_{qo}$, is defined as $\breve{x}_{qo} = rand(c, \breve{x})$.

Definition 3. The quasi-reflected point, $\breve{x}_{qr}$, of any real number in $[a, b]$ is calculated using $\breve{x}_{qr} = rand(x, c)$.

In other words, quasi-opposition reflects a variable to a random point between the center of the range (c) and its opposite point, whereas quasi-reflection shifts the variable x to a random point between the center of the domain and x, as illustrated in Fig. 7(Ergezer et al. 2009).

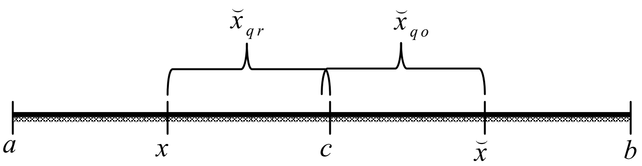


Fig.7. Opposite points in continuous space

Among of the above opposite points, it has been mathematically proved that the quasi-reflection with an expected probability of $11/16$ has a higher chance of being closer to the solution than the quasi-opposite with an expected probability of $9/16$. This means the quasi-reflection point yields the highest probabilityof being closer to the solution of an optimization problem(Ergezer et al. 2009).

4.2. Opposition in binary space

To extend the use of opposite points in a binary space, this paper utilizes a binary OBL as follows:

Definition 4. The opposite point of $X(x_1, x_2, ..., x_d)$ in a d-dimensional binary space with $x_i \in \{0,1\}, \quad i = 1,2,...,d$ is obtained by $\breve{X}(\breve{x}_1, \breve{x}_2, ..., \breve{x}_d)$, where $\breve{x}_i = 1 - x_i, \quad i = 1,2,...,d.$

(Seif and Ahmadi 2015)

Definition 5. The Hamming distance between t wo$d$-dimensional binary vectors $x, y \in \{0,1\}$ is calculated by $HD(x, y) = \sum_{i=1}^{d} x(i) \oplus y(i)$. Another alternative to obtain the Hamming distance is the use of the absolute value of the arithmetic subtraction as $|x(i) - y(i)|$.

Proposition 1. There is a unique opposite point for $X(x_1, x_2, ..., x_d)$ denoted by $\breve{X}(\breve{x}_1, \breve{x}_2, ..., \breve{x}_d)$.

Proof. If there are two opposite points of $\breve{X}(\breve{x}_1, \breve{x}_2, ..., \breve{x}_d)$ and $X'(x'_1, x'_2, ..., x'_d)$, then based on **Definition 4**, $\breve{x}_i = 1 - x_i$ and $x'_i = 1 - x_i$ for $i = 1,2,...,d$. Therefore, $\breve{X} - X'$. □

Proposition 2. Reconsider $X(x_1, x_2, ..., x_d)$ and its opposite point $\breve{X}(\breve{x}_1, \breve{x}_2, ..., \breve{x}_d)$. Then, for each $Y \in \{0,1\}^d$ we have $HD(X, Y) = d - HD(\breve{X}, Y)$.

Proposition 2 that has been mathematically proved by Seif and Ahmadi (2015), represents that the distance between X and Y, generated at random, is equal to the difference between the size of dimension space and the Hamming distance between the opposite point X and YwhereHD denotes the Hamming distance presented in Definition 5. Thus, after generating a binary solution X, its opposite point can be created and evaluated in order to give another chance to explore a candidate solution closer to the global optimum. Recently, it has been claimed that an opposite point is more effective and beneficial than an independent random point by the following proposition (Seif and Ahmadi 2015):

Proposition 3. Consider $X(x_1, x_2, ..., x_d)$ as a binary solution for objective function $y = f(:)$ where $d > 1$. Suppose $X_1(x_{11}, x_{12}, ..., x_{1d})$ and

$X_2(x_{21},x_{22},...,x_{2d})$ are two random solutions in the solution space. Then

$$Pr(\|\breve{X},X\| \le \min\{\|X_1,X\|,\|X_2,X\|\}) > Pr(\|X_2,X\| \le \min\{\|X_1,X\|,\|\breve{X},X\|\})$$

This proposition states that the probability the distance between X and $\breve{X}$ to be less than or equal to the distance between $\{X_1,X_2\}$ with X is more than the probability the distance between X_2 and X is less than or equal to the distance between $\{X_1,\breve{X}\}$ and X. Thus, $\breve{X}$ is more probable than X_2 to be the closest to X among $\{X_1,X_2,\breve{X}\}$. That is why an opposite solution is more effective than an independent random solution. Although Proposition 3 is true for an arbitrary objective function, this paper seeks to exhibit its beneficial application in multi-objective optimization problems where more than one criterion is considered and therefore the non-dominated solution concept is displayed. For instance, consider a multi-objective optimization problem involving P, $(P>1)$, objective functions to be optimized simultaneously. In this problem, $f_i(x)$; $i=1,2,...,P$, is the *i*- th objective function and x is a feasible solution. Then, a typical multi-objective maximization problem is defined as:

Maximize $f_1(x), f_2(x),..., f_P(x)$ (21)

As some objective functions in (21) may conflict each other, there is not a unique solution that maximizes all the objectives, simultaneously. Hence, the non-dominated concept is used, in which solution x is said to dominate y, $(x \succ y)$, if and only if (Pasandideh et al. 2014):

$$f_i(x) \ge f_i(y), \quad (\forall i=1,2,\dots,P) \text{ and}$$
$$f_i(x) > f_i(y), \quad (\exists i \in \{1,2, \quad ,P\}) \tag{22}$$

This provides a set of optimal solutions called Pareto-optimal. These Pareto-optimal solutions create the Pareto front(Kubotani and Yoshimura 2003). Under these considerations, this paper tries to show that the binary OBL works well in multi-objective optimization schemes.

4.3. Opposition BBO(OBBO)

Now, the OBL algorithm is added to BBO as a diversity mechanism to enhance the performance and to enrich the convergence rate. In OBBO, the opposite population is created based on the **Definition 4** of the binary OBL proposed in Section 3.Here,it is concluded that the proposed OBBO first generates an opposite population (OP_0) of the initial population (P_0) with the same size (*n Pop*). In other words, an opposite habitat is made for each habitat in the initial population. Afterward, both P_0 and OP_0 are merged and then habitats with the best fitness are preserved with respect to *n Pop*. Note that this procedure is also implemented during each iteration of OBBO. Nevertheless, to save the computational time, the opposite population of the current population has a chance of being generated $OJR\%$ (usually 30%) of the time during each iteration. This probability is determined by the *Opposition Jumping Rate* ∈[0, 1] parameter as specified in Fig. 8. At the end, the generated population in iteration *t*, (P_t), the new population (Q_t), and the opposite population of the new population (OQ_t) are merged and truncated according to. Afterward, the sorting procedure is done to recognize the habitats, similar to BBO steps.

Step 1: Set the parameters
Step 2: Create population P_0 of size with randomly created habitats and set t=0.
Step 3: Calculate the habitats *HIS*
Step 4: Calculate λ_i and μ_i based on the habitats
Step 5: If the stopping criterion is met, stop and return P_t.
Step 6: In population P_t, for each habitat *i*:
a. If Rand∈[0,1] <=λ_i
b. Select a source habitat using the binary tournament selection as habitat *j*
c. Do the migration operator and transfer the SIVs from habitat *j* to habitat *i*
d. If Rand∈[0,1] <=P_m
e. Perform the mutation operator
Step 7: Calculate the generated habitats *HIS*
Step 8: Put the generated habitats in $Q_t$as a new population
Step 9:SetR_t= $P_t \cup Q_t$
Step 10:Usesortingprocedure
Step 11:Set P_{t+1}=∅ and *i*=1.
Step 12:Until $|P_{t+1}|+|R_t|$< nPop:
a. Add all solutions in F_i to P_{t+1}.
b. Set *i*=*i*+1.
Step 13:Sortsolutionsin F_i according to the sorting strategy.
Step 14:Addfirst N-$|P_{t+1}|$solutions of $F_i$to P_{t+1}.
Step 15: Calculate λ_i and μ_i based on the habitats' rank
Step 16:Set t=t+1, and go to Step 5.

Fig .8. The pseudo-code of the OBBO

4.4. Parameter tuning

To obtain better solutions, the parameters of the proposed algorithms are calibrated using the Taguchi method. This method is a fractional factorial experiment that is known as an efficient alternative for full factorial experiments (Peace 1993). It uses a special set of arrays called orthogonal arrays. These arrays stipulate the way of conducting the minimal number of experiments that can give the full information of all the factors that affect the performance parameter. There are two groups of factors including signal and noise, where maximizing the signal to noise (S/N) ratio is the aim. As a "smaller-is-better "objective is appropriate, the S/N ratio is defined as:

$$\frac{S}{N} = -10 \times \log\left(\frac{S(Y^2)}{n}\right) \quad (23)$$

where Y represents the response and n denotes the number of orthogonal arrays. To employ the Taguchi method, the multi-objective coefficient of variation (*MOCV*) measure (Rahmati et al. 2013)is utilized as the response. In this response, *MID* and *MS* are used in Eq. (24).

$$MOCV = \frac{MID}{MS} \quad (24)$$

Where *MID* is a representation of the first goal (convergence) and *MS* is a representation of the second goal (diversity). Hence, two mentioned goals of multi-objective optimization are considered simultaneously. In order to run the Taguchi method, the levels of the factors (algorithms parameters) such as the maximum generation (*MaxG*), population (*Pop*), and the opposition jumping rate(*OJR*) are determined in Table 1. The values given in Table 1 are chosen based on some trial experiments and literature.

Table 1
The levels of algorithms parameters

Algorithm	Parameter	Range	Low (1)	Medium (2)	High (3)
GA	MaxG	100-400	100	**250**	400
	Pop	50-250	50	150	**250**
	P_c	0.6-			
	P_m	0.1-		0.15	
OBBO	Max	100-400	100	**250**	400
		50-250	50	150	**250**
	P_m	0.1-0.25		0.15	0.25
	OJR	0.1-			

To set the parameters of the model, 15 test problems with random parameters are generated. Then, the L29 design is used for ONSGA-II while L9 is designed for OMOBBO using the Minitab Software. Figs. 9 and 10 illustrate the S/N ratio resulted by OMOBBO and ONSGA-II, respectively. In these figures, the best level of each parameter is selected to be the one with the highest S/N ratio. As a result, the proper values of the parameters are shown in bold in Table 1.

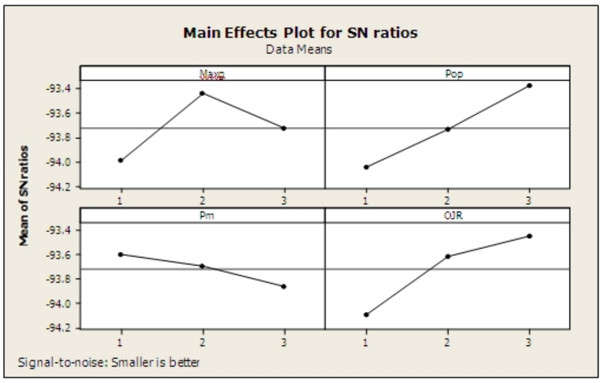


Fig. 9. The S/N ratio plot for parameters of OBBO

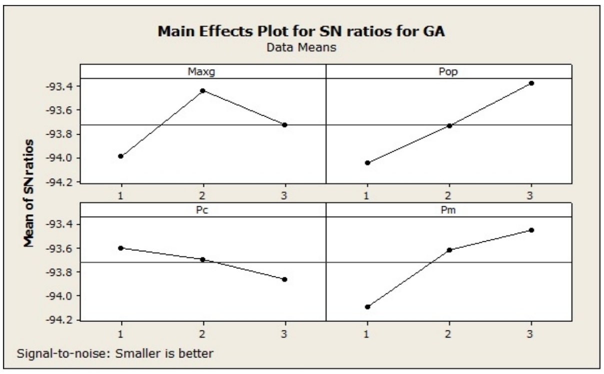


Fig. 10. The S/N ratio plot for parameters of GA

5. Computational Results

To assess the applicability of the proposed methodology, several problems with different numbers of hubs, center nodes, and p are designed. These problems are randomly generated according to the given information of a case study inBashiri et al. (2013), which are classified into four classes. The values of the parameters of each problem are obtained using the last column of Table 2. In order to validate the results obtained using the proposed algorithms, these problems are solved using GAMS/BARON 23.5 software on an Intel (R), core (TM) i7, 3.23 GHz lap top with 512 Mb RAM. Note that the branch-and-reduce optimization navigator (BARON) presented by Tawarmalani and Sahinidis (2005) is algorithms for the global solutions of NLP and MINLP. They also showed this algorithm reduces root-node relaxation gaps by up to 100% and expedites the solution process, often by several orders of magnitude. BARON implements the branch-and-bound approach enhanced with a variety of a constraint propagation and duality techniques in order to reduce the ranges of variables in the course of the algorithm (Sahinidis 2013).The GA and OBBO are first implemented in 20independent runs to solve each problem with the same input parameters. Afterward, the best fitness value of the problem and its corresponding CPU time (seconds)are considered for comparison. The solutions of 30 different problems of various sizes obtained using GA, OBBO, and BARON are reported in Table 3.The main aims of the results reported in Table 3 are:

(i) Validating the results obtained using OBBO and GA,
(ii) Comparing their performance together; and
(iii) Performing sensitivity analyses on the impact of the number of hubs, center nodes, and p.

Note that the second column in Table 3 represents the size of the problems. For example, 5*6*6 means there are five potential hubs vendors, six center nodes, and p=6.

Table 2
The sources of random generation

Parameter	Value
W_{ik}	~U (5,15)
$\tilde{t}_{ik}$	(~U (1,3), ~U (4,5), ~U (6,8))
$\tilde{t}_{km}$	(~U (0.5,2), ~U (3,4.5), ~U (5,6.5
$\tilde{t}_{jm}$	(~U (0.5,3), ~U (4,6), ~U (7,8
O_i	~U (2,4)
C_k	~U (25, 50)
f_{CO2}	~U (100, 200)

5.1. Validation and sensitivity analyses

In order to validate the results obtained using OBBO and GA, they are compared to the ones obtained by BARON. To make the comparisons, a quality measure namely the percent deviation of the objective function is defined for each solution as follows:

$$\%Deviation_{Obj.} = \frac{z_{algorithm} - z_{BARON}}{z_{BARON}} \times 100 \qquad (25)$$

A lower value of this measure implies a good performance of the proposed algorithm. The first class contains Problems 1-8, where the number of hubs and p both are equal to two. The aim of the problems of this class is to analyze the influence of center nodes variety on the objective function and computation time where the number of the hubs and p are fixed. The solutions in this class reveal that when the number of center nodes is 1, the percentage deviations of the objective function for both OBBO and GA are zero, as they both find the lowest cost obtained using BARON ($884,645). However, GA requires more CPU time as compared to OBBO. Furthermore, as the number of center nodes increases from one to two, the minimum total cost of the firm increases up to $901,984 for all algorithms. Considering the percent deviations in the above two problems are zero, it can be interpret that the results obtained by both GA and OBBO are optimal. This ascending trend can also be seen in the other problems of this class and that the percentage deviations of the solutions obtained by OBBO and GA are slightly greater than zero. Besides, as the number of center nodes increases, the required CPU times of all algorithms increases as well. In other words, increasing the number of center nodes can lead to increase in the objective function and the required computation time simultaneously. Meanwhile, while the percent deviations of the objective function in OBBO are relatively good, they are better than those obtained by GA.

The second class contains Problems 9-14, where the aim is to analyze the influence of center nodes variety on the total cost and CPU when the number of hubs and p are increased from 2 to 3. When the number of center nodes is one in this class, OBBO is able to find the optimal solution, while GA is not. Note that the minimum cost obtained for this problem is lower than the one obtained for the first problem of Class 1. Besides, similar to Class 1, as the number of center nodes increases from one to two, the total cost of the firm obtained by GA and OBBO increases from $823,989 to $1,027,541 and from $818,257 to $1,026,000, respectively. It should be noted that the amount of this increase is higher than the one in Class 1. Moreover, the maximum percent deviation of the objective function for GA and OBBO are1.469 and 3.50, respectively. Similarly, increasing the number of center nodes can lead to an increase in the total cost of the firm and the required computation time of the proposed algorithms.

Table 3
The results obtained using the algorithms

Class	Problem	Size	BARON z ($)	BARON CPU time(s)	GA Best z($)	GA CPU time(s)	GA % Dev of Obj. function	OBBO Best z($)	OBBO CPU time(s)	OBBO % Dev of Obj. function
1	1	2*2*1	884,645	0.180	884,645	19	0.000	884,645	25	0.000
	2	2*2*2	901,984	1	901,984	46	0.000	901,984	29	0.000
	3	2*2*3	1,046,498	4	1,055,113	51	0.823	1,050,925	44	0.423
	4	2*2*4	1,417,948	35	1,420,300	64	0.166	1,420,300	75	0.166
	5	2*2*5	2,955,391	74	2,965,004	62	0.325	2,957,000	66	0.054
	6	2*2*7	3,263,983	69	3,289,487	77	0.781	3,265,000	82	0.031
	7	2*2*8	3,628,335	71	3,645,005	89	0.459	3,631,000	93	0.073
	8	2 2 10	7,482,371	73	7,526,355	107	0.588	7,485,355	115	0.040
2	9	3*3*1	818,257	5	823,989	22	0.701	818,257	34	0.000
	10	3*3*2	1,024,060	49	1,027,541	67	0.340	1,026,000	72	0.189
	11	3*3*3	1,364,750	125	1,382,227	222	1.281	1,368,112	189	0.246
	12	3*3*4	2,787,128	209	2,828,058	776	1.469	2,792,310	556	0.186
	13	3*3*5	3,106,475	240	3,119,255	900	0.411	3,215,200	803	3.500
	14	3*3*6	3,789,984	338	3,804,000	1,005	0.370	3,801,000	1,102	0.291
3	15	4*1*2	269,874	3	275,444	25	2.064	269,874	37	0.000
	16	4*2*2	572,010	9	578,689	58	1.168	572,010	66	0.000
	17	4*3*2	800,267	68	815,005	145	1.842	803,600	130	0.416
	18	4*4*2	1,336,156	603	1,352,565	1,127	1.186	1,338,110	905	0.146
	19	4*5*2	1,756,363	632	1,769,797	1,200	0.765	1,761,000	1,011	0.264
	20	4*6*2	2,000,458	647	2,030,800	1,408	1.517	2,009,451	1,334	0.450
	21	4 10* 2	2,900,054	640	2,905,330	1,882	0.182	2,900,700	1,665	0.022
	22	4 12* 2	3,000,444	709	3,001,000	2,056	0.019	3,000,550	2,001	0.004
	23	4 15* 2	3,405,900	716	3,411,200	2,794	0.156	3,405,900	2,455	0.000
	24	4 18* 2	Infeasible	-	4,380,000	3,211	-	4,100,220	3,007	-
	25	4 22* 2	Infeasible	-	5,298,610	4,420	-	5,009,120	3,905	-
4	26	1*8*5	4,390,181	1	4,405,477	24	0.348	4,395,000	15	0.110
	27	3*8*5	338,993	2	341,511	36	0.743	338,993	44	0.000
	28	4*8*5	271,886	2	274,050	43	0.796	271,886	55	0.000
	29	5*8*5	231,746	3	231,856	43	0.048	231,746	49	0.000
	30	6*8*5	219,057	10	220,117	45	0.484	219,900	40	0.385

The third class contains Problems 15-25. Unlike the previous two classes, the number of center nodes and *p* in this class are fixed at 4 and 2, respectively while the number of hubs varies between 1 and 22. Note that as the number of hubs in Problem 23 increases from 12 to 15 stores, the total cost of the firm obtained using GA increases from $3,001,000 to $3,411,200, while the cost varies from $3,000,550 to $3,405,900 in OBBO. In addition, the solutions of the problems in this class show that OBBO has a relatively good performance in terms of the cost, because the maximum percent deviation of the objective function is 4.861 (lower than the one of GA). This result is valid for the other problems of this class that can better be confirmed by Fig. 11. It is worth noting that while BARON was not able to find a feasible solution for Problems 24 and 25 with the number of stores of 18 and 22, respectively, the proposed algorithms were able to find solutions. This is the advantage of employing the two meta-heuristics over BARON, as they are able to find solutions of larger problems in reasonable computational times.

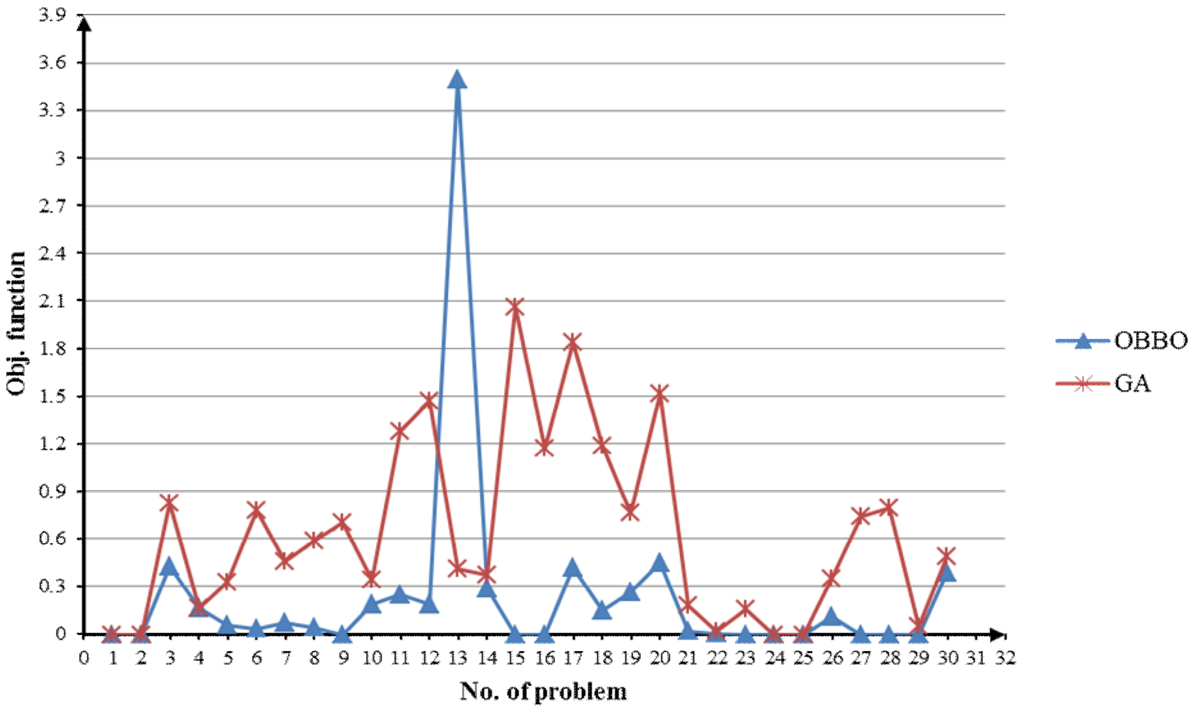


Fig. 11. Percentage of deviations for the algorithms

Finally, the number of the hubs and center nodes in the fourth class are fixed at 8 and 5, respectively. The aim of this class is critical because the impact of the *p*-hub center problem can be demonstrated. Note that the total cost for this class of problems is higher than the ones in the previous classes. Moreover, while the average CPU times of GA and OBBO to solve the problems of this class are almost equal, the percentage deviations of the objective function are lower with OBBO compared to the ones obtained by GA. That is, they could verify the optimality of solutions. On the other hand, as the number of *p* increases from one to three in Problems 26 and 27, the total cost of the firm obtained by GA and OBBO decreases from $4,405,477 to $341,511 and from $4,395,000 to $338,993, respectively. This descending trend is noticeable and continues by increasing the number of *p*in the next three problems. This is a good implication of the effect of *p*-center strategy that decreases the total cost of the firm. In other words, the solutions of the problems in fourth class with multiple *p* indicate that the proposed fuzzy *p*-hub center model can reduce the total transportation cost. At the end, regarding the percent dev. % columns, it can be seen that that the results obtained by GA and OBBO are valid and near optimal.

5.2. Comparison

Table 4 summarizes the worst, the mean, and the best fitness function and computation time obtained by implementations of both OBBO and GA on 11 large-size problems (Problems 31-41), each independently replicated 20 times. The number of *p* in all of these problems is 50. As BARON is not able to solve any of these problems in reasonable computation time, the proposed algorithms are compared together based on the same input parameter of each problem. In the second column of Table 4, 70*60 means there are 70 potential hubs and 60 center nodes.

The results in Table 4not only show that required computation times of OBBO is less compared to the ones in its counterpart, but also OBBO is the better algorithm in terms of the best, the mean, and the worst total costs obtained. In addition, a comparison based on the fitness values and CPU times reveals that increasing the number of potential vendors increases the required computation times, as expected.

Table 4
The optimization results in large size problems

Problem	Size	GA				OBBO			
		Best	Mean	Worst	CPU time(s)	Best	Mean	Worst	CPU time(s)
31	75*60	1,435,356	1,438,465	1,438,736	1,324	1,433,000	1,434,154	1,434,420	1,220
32	100*80	2,014,000	2,014,598	2,014,911	1,708	2,007,000	2,008,804	2,010,137	1,528
33	140*120	2,651,652	2,657,139	2,657,203	2,111	2,640,101	2,642,322	2,644,006	2,000
34	145*130	2,854,554	2,858,424	2,863,000	2,249	2,842,000	2,848,426	2,900,229	2,355
35	150*150	3,066,455	3,060,322	3,067,988	2,805	3,057,562	2,904,684	3,067,652	2,669
36	170*160	3,411,188	3,413,543	3,414,940	4,546	3,395,884	3,397,736	3,401,622	3,497
37	185*180	3,770,900	3,778,341	3,778,900	4,993	3,462,910	3,469,125	3,476,000	3,799
38	195*190	4,208,841	4,209,471	4,213,262	5,844	4,184,851	4,197,405	4,222,514	4,431
39	210*200	4,680,554	4,682,708	4,689,924	6,202	4,600,036	4,606,006	4,610,110	4,929
40	250*240	5,719,702	5,721,112	5,722,578	7,112	5,378,912	5,379,000	5,379,222	5,524
41	300*280	9,005,012	9,008,270	9,012,738	8,804	6,900,679	6,905,736	6,907,325	6,630

The convergence curves of both algorithms on Problems 31 and 32 are drawn in Fig. 12andFig. 13, respectively. As illustrated in these figures, it is obvious that OBBO searches out the better solutions and converges faster than GA, where it can provide better global searching ability and can prevent trapping in the local optima.

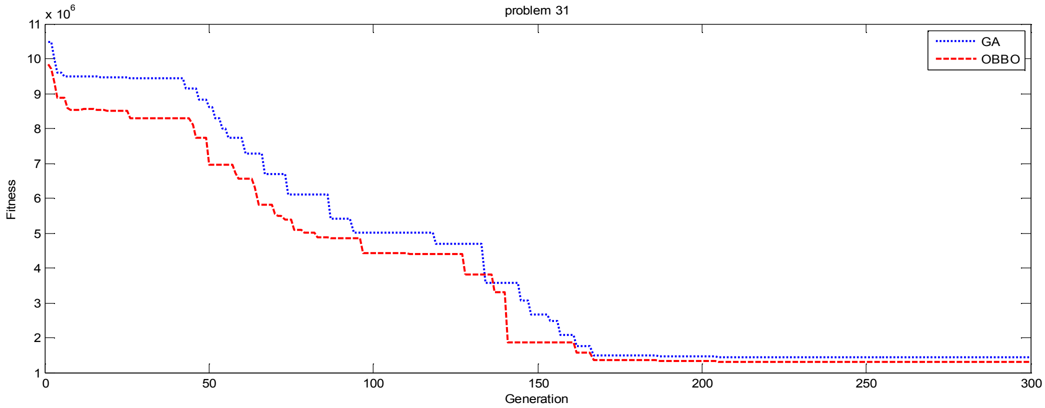


Fig 12. Convergence curvesof GA and OBBOon Problem 31

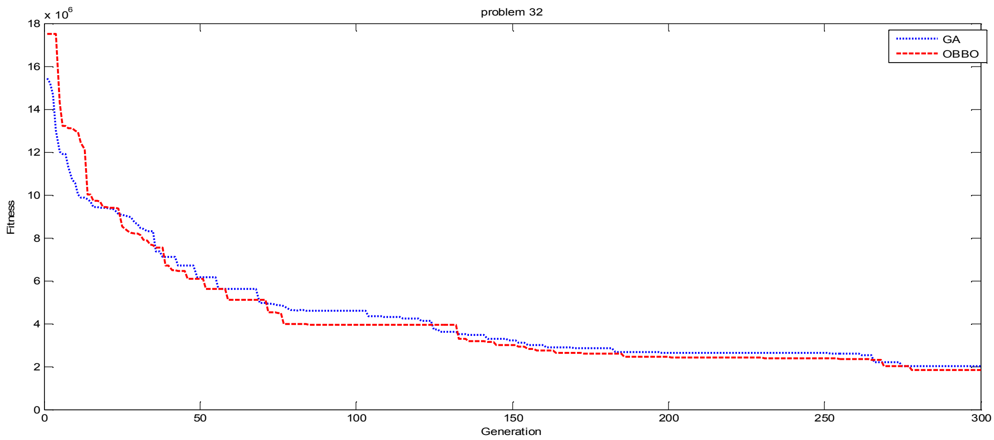


Fig. 13. Convergence curves of GA and Obion Problem 32

In order to assess the performance of OBBO and compare it to GA statistically, the results obtained in Table 3 are analyzed using paired samples *t*-tests to test the equality of the means of the two algorithms in terms of total cost, computation time, and percentage deviation of the objective function at 95% confidence level. The null hypotheses for the equality of the means of these metrics are:

1- The mean of the total cost of the firm obtained by GA is equal to one of OBBO.
2- The mean of computation times in GA is equal to one of OBBO.
3- The mean of the percent deviation of the objective function obtained by GA is equal to one of OBBO.

The alternatives in all the above tests refer to OBBO to be a better algorithm. The summary results of the tests are presented inTable5, based on which the above null hypotheses are rejected in favor of OBBO. In other words, OBBO is the better algorithm and can provide less cost in less CPU time on average. Fig. 14 shows this effectiveness, better.

Table 5.
The *P*-values of paired sample *t*-tests

Metric	*T*-value	*P*-value	Test results	Chosen algorithm
Best fitness	2.13	0.042	Null hypothesis is rejected	OBBO
Computation time	2.82	0.009	Null hypothesis is rejected	OBBO
Percent deviation	5.32	0.001	Null hypothesis is rejected	OBBO

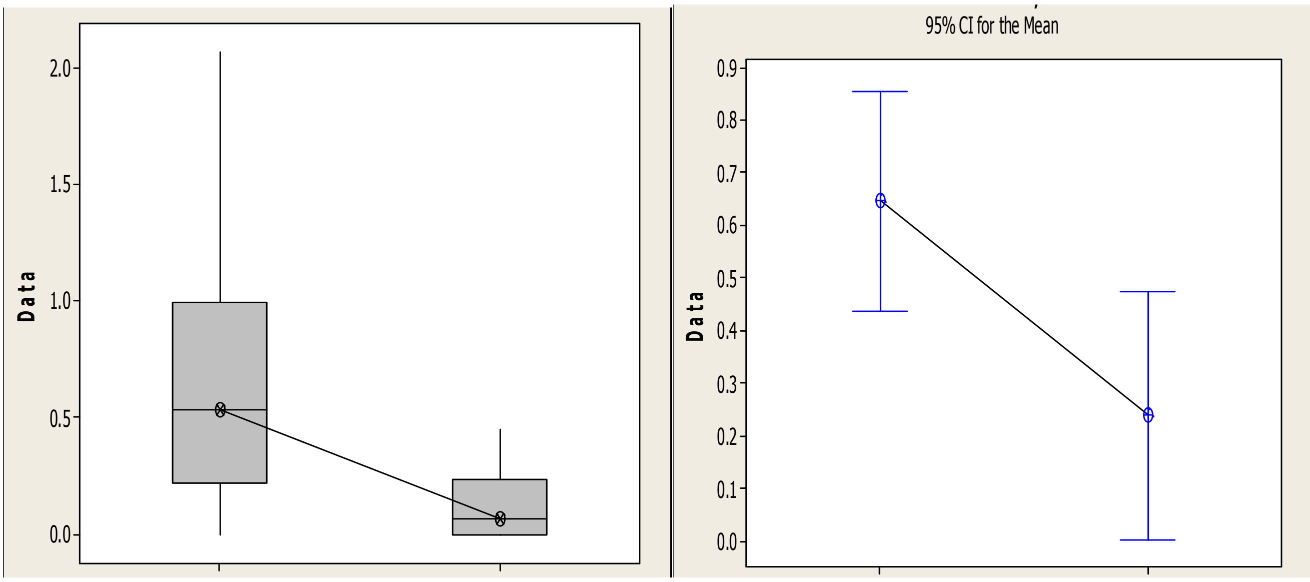


Fig 14. Box and interval plots of the percentage of deviationsfor OBBO and GA

6. Conclusion and Future Research

Hub networks are utilized in telecommunications, transportation systems, postal delivery systems, computer networks, etc. In these networks, instead of generating direct links between origin and destination pairs, the hubs serve as switching or transshipment pointsf or flows between center nodes. The hub facilities consolidate flows in order to utilize the concept of economies of scale in transportation between hubs. In a hub-and-center network, many origins and destinations can be connected with fewer links. In this paper, a fuzzy green capacitated single allocation *p*-hub center problem was presented in order to select the hubs and to assign the center nodes to located hubs. The aim of this paper was to minimize the maximum travel time in a hub-and-center transportation network under green consideration. In order to solve the problem, the proposed FGCSA*p*HCP was re defined in to its equivalent parametric integer nonlinear programming problem usingcredibilityconstraints.Due to the NP-harness of the problem,an opposition BBOwas proposed to improve the performance of BBO in obtaining better solutions. Then,theTaguchimethodwas applied to tune the parameters of both GA and OBBO, based on which their results were compared together using several randomly generated problems. In addition, the performanceof these proposed algorithmswere statistically analyzed.

Computational results showed that OBBO had desirable performance in terms of the maximum travel time and percent deviation, while GA had better performance to solve the test problems in terms of required computation times at 95% confidence level. In addition, the solutions obtained by OBBO provided better maximum travel time than those obtained by the GA. Therefore, it can be concluded that OBBO outperforms GA. Finally, the results confirm the applicability of the proposed model and the solution methodologies taken to solve the problem. For future work extensions, the use of OBBO to solve redundancy allocation and clustering problems is recommended. Besides, multiple allocation schemes in *p*-hub center problem can be considered and that the hub location model can be extended to include competition environments.

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[1]**Note:**This model is based upon second-by-second tailpipe emissions data collected from 343 light-duty vehicles(LDVs) tested using a variety of laboratory driving cycles (Demir et al. 2014b).

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