Cost Analysis of Acceptance Sampling Models Using Dynamic Programming and Bayesian Inference Considering Inspection Errors

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Abstract

Acceptance Sampling models have been widely applied in companies for the inspection and testing of the raw materials as well as the final products. A number of lots of the items are produced in a day in the industries so it may be impossible to inspect/test each item in a lot. The acceptance sampling models only provide the guarantee for the producer and consumer confirming that the items in the lots are according to the required specifications so that they can make appropriate decision based on the results obtained by testing the samples. Acceptance sampling plans are practical tools for quality control applications which consider quality contracting on product orders between the vendor and the buyer. Acceptance decision is based on sample information. In this research, dynamic programming and Bayesian inference is applied to decide among decisions of accepting, rejecting, tumbling the lot or continuing to the next decision making stage and more sampling. We employed cost objective functions to determine the optimal policy. First, we used the Bayesian modelling concept to determine the probability distribution of the nonconforming proportion of the lot and then dynamic programming was utilized to determine the optimal decision. Two dynamic programming models have been developed. The first one is for the perfect inspection system and the second one is for imperfect inspection. At the end, a case study is analysed to demonstrate the application the proposed methodology and sensitivity analyses are performed.
Keywords: Acceptance Sampling, Bayesian Inference, Dynamic Programming, Inspection Errors, Quality Cost.

1. Introduction

Acceptance Sampling models have been widely applied in companies for the inspection and testing of the raw materials as well as the final products. A number of lots of the items are produced in a day in the industries, so it may be impossible to inspect/test each item in a lot. The acceptance sampling models only provide the guarantee for the producer and the consumer confirming that the items in the lots are according to the required specifications so that they can make appropriate decisions based on the results obtained by testing the samples.

In this paper, an optimization model is developed for acceptance sampling plan. The proposed approach is based on dynamic programming and Bayesian inference. In deterministic dynamic programming, given a state and a decision, both the immediate payoff and next state are known. If we know either of these only as a probability function, then it is modelled as stochastic dynamic programming. The method of obtaining stages, states, decisions, and recursive formula does not differ. A stochastic dynamic programming has the same approach of a deterministic one, but only the state transition equation differs. The acceptance sampling problem may be modelled as a dynamic programming problem when different sampling stages are available. States of the lot may be defined by the results of the inspection. The lot state is defined as the expected value of nonconforming proportion. The probability distribution function of nonconforming proportion is obtained using the Bayesian inference. Therefore, the lot state is assumed to be known at each stage and the probability density function of nonconforming proportion is determined at the beginning of each stage after sampling the new data.

The state of the lot at the beginning of the next stage depends only on our current decision (tumbling or more sampling). The lot can be tumbled a constant cost. Tumbling will bring the lot to some better state in the next stage and it decreases lot state (nonconforming proportion) with a constant factor. There is also a state-dependent cost of decisions about accepting and rejecting

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the lot at the end of the horizon, reflecting the final decision about the lot. The objective is to minimize the total cost over the horizon of decision making. In this paper, we propose an adaptive optimal policy for lot sentencing problem. This policy is derived based upon a dynamic programming and the Bayesian approach while inspection may be imperfect and first and second type errors existed in the inspection process. Since an inspection process may be imperfect thus inspection process leads to a biased estimation of nonconforming proportion hence the first correct value of nonconforming proportion should be evaluated using conditional probability, then corresponding costs are evaluated based on the correct value of nonconforming proportion. At the end, an optimal framework is developed for the decision-making process at hand.

There are various models for designing an economically optimal sampling system, (Li & Chang, 2005; Aslam & Fallahnezhad, 2013; Fallahnezhad & Niaki, 2013; Fallahnezhad et al., 2014; Fallahnezhad MS, 2012; Fallahnezhad & Nasab, 2012; Fallahnezhad et.al, 2015; Fallahnezhad & Nasab, 2011). Moskowitz and Tang (1992) proposed acceptance sampling plans based on the Bayesian inference. Klassen (2001) introduced an acceptance sampling system based on a new measure named 'credit'. The credit of the producer was defined as the total number of items accepted since the last rejection. Tagaras (1998) proposed a dynamic model for the joint process control and machine maintenance problem of a Markovian deteriorating machine. Kuo (2006) developed an optimal adaptive control policy for joint machine maintenance and product quality control using dynamic programming.

Niaki and Fallahnezhad (2007) utilized Bayesian inference and stochastic dynamic programming to design a decision-making framework in production environment. Also Fallahnezhad et al. (2012), Fallahnezhad (2014) and Fallahnezhad and Niaki (2011) proposed an optimal policy for machine replacement problem in a finite horizon model based on the quality of items produced. They used a stochastic dynamic programming model to design a control threshold policy for the machine replacement problem. In order to determine the optimal policy, the inspection cost in combination with the cost of continuing the production process and cost of rejecting was minimized. Niaki and Fallahnezhad (2009) used Bayesian inferences concept and stochastic dynamic programming to design an acceptance sampling plan. They used a stochastic dynamic programming model to minimize the ratio of the system cost to the system correct choice probability. Wortham and Wilson (1971) proposed an optimal sequential sampling plans using backward recursive inference. They presented a procedure based on dynamic programming for designing optimal acceptance sampling plans for item-by-item inspection. Using a Bayesian procedure, a prior distribution is specified, and a suitable cost model is employed depicting the cost of sampling, accepting or rejecting the lot. Ivy and Nembhard (2005) proposed statistical quality control (SQC) and partially observable Markov decision processes (POMDP) for maintenance decision making of deteriorating systems. Iravani and Duenyas (2002) considered integrated decisions of maintenance and production in a production system with a deteriorating machine. They consider infinite horizon and assumed that the production and repair times follow exponential distribution.

Fallahnezhad and Yousefi (2014) developed a new acceptance sampling plan to decide about the received lot based on cost objective function in the presence of inspection errors. They assumed that the inspection is not perfect and type I and type II errors occur in the inspection process. The problem of acceptance sampling plan in the presence of inspection errors is modelled using decision tree approach. They used from Bayesian inference to update the probability distribution function of nonconforming proportion. Then they analysed the cost at terminal nodes and optimal decisions are determined using a backward recursive approach. Fallahnezhad and Aslam, (2013) proposed a new approach on selecting the different actions for a lot based on Bayesian modeling. They applied Bayesian modeling to determine the probability distribution of the nonconforming proportion of the lot and expected cost of different actions. Fallahnezhad et al., (2012) proposed a decision tree approach for acceptance sampling model. They proposed a new acceptance sampling design to accept or reject a lot based on Bayesian modelling in order to update the probability distribution function of the nonconforming proportion.

In this research, a new dynamic programming model is developed for acceptance sampling problem. Then the developed model is generalized for the imperfect inspection system. Different decisions are employed in objective function. Case studies and sensitivity analysis are performed to elaborate the application of proposed model. For attribute sampling plan, Lindley and Barnett (1965) assumed that the probability for an item to be nonconforming has a beta prior distribution, and provided numerical solution to the sequential sampling plan using Bayesian approach and dynamic programming. Even though several dynamic programming models had been proposed for the problem of acceptance sampling but as the best of author's knowledge, the dynamic modeling of this problem based on a continuous state variable and proposed definition for sate variable is not addressed before. Also solution method of dynamic model is a new heuristics method. On the other hand, in comparison with single sampling plan or double sampling plan, a sequential sampling plan is more attractive in terms of statistical efficiency. The rules for a sequential sampling plan are as follows: at each time after inspection of one lot, we shall accept the batch if all observations up to date are ‘‘close’’ to the specification limit(s), or reject the lot if the observations are ‘‘far’’ from the specification limit(s), or continue sampling one more items from the lot otherwise.

2. The Model

A dynamic programming algorithm will examine all possible methods to solve the problem and will select the optimal solution; therefore, dynamic programming enables us to go through all possible solutions to select the best one. Stochastic dynamic programming is a technique to model a sequential decision making process in a stochastic environment (Ross, 1983). In acceptance sampling plans, we are selecting between decisions of continuing (tumbling the lot or continuing to the next decision making stage and more sampling) or stopping (accepting or rejecting the lot) thus it is a type of optimal stopping problem that can be generalized in order to consider all decisions.

Dynamic programming technique can be employed to design an optimal sequential acceptance sampling plan when the following conditions exist:

i) The cost of accepting a nonconforming item and cost of rejecting a conforming item can be reasonably assessed.

ii) The proportion of nonconforming items is stable and constant or its probability distribution is known.

The most powerful method of acceptance sampling plans is sequential acceptance sampling model. A recessive approach was used to design these models. On the other hands dynamic programing models are solved based on the recessive approach, therefore we use from the dynamic programming to develop a sequential sampling models.

In this paper we impose mandatory fixed sample sizes in each sampling stage. We let the dynamic programming mechanism dictate the optimal policy based on the current state of the system.

However, before doing so, first we need to have some notations and definitions.

2.1. Notations and Definitions

We will use the following notations and definitions in the rest of the section:

p : The proportion of nonconforming items.

Referring to Jeffrey's prior (Nair et. al. (2001)), for the nonconforming proportion *p*, we take a *Beta* prior distribution with parameters 0.5 and 0.5. By use of the Bayesian inference, we can easily show that the posterior probability density function of p is

$$f(p) = \frac{\Gamma(\alpha+\beta+1)}{\Gamma(\alpha+0.5)\Gamma(\beta+0.5)} p^{\alpha-0.5}(1-p)^{\beta-0.5} \qquad (1)$$

Where, α is the number of nonconforming items and β is the number of conforming items in the past stages of the decision- making process.

N : The total number of items a lot.

C_a : The cost of accepting one nonconforming item when the lot is accepted.

C_s : The cost of one nonconforming item which is detected during inspection.

T : The cost of tumbling process.

I : The cost of inspecting one item during decision of inspecting all items in lot.

γ: The coefficient of decreasing detective proportion after improving lot quality (Tumbling).

m : The sample size in each stage of decision making.

λ: The discount factor in stochastic dynamic programming approach.

$V_n(p)$: The cost associated with p when there are n remaining stages to make the decision.

Following assumptions are made to design the proposed sampling plan,

- ✓ The inspections are perfect.
- ✓ A tumbling operation can be performed on lot. The tumbling operation can be expected to eliminate $1-\gamma$ percentage of the nonconforming items.
- ✓ The objective function minimizes the summation of quality costs.
- ✓ Bayesian inference is used to update the proportion of nonconforming items.
- ✓ Dynamic programming is used to find the optimal policy.
- ✓ We can select the optimal policy among decision of accepting, rejecting, tumbling the lot or continuing to the next stage and taking more samples.

2.2. Derivations

We may model an acceptance sampling process as an optimal stopping problem in which in each stage of the decision-making process, we take a sample from a lot and based on the information obtained from the sample we want to decide whether to accept or to reject the lot or continuing to take more samples.

The state variable and stage variable of dynamic programming model is as follows,

- State variable: The expected value of nonconforming proportion. The probability distribution of nonconforming proportion is obtained by sampling.
- Stage variable: The stage of sampling. It is assumed that there are $n_{\max}$ decision making stages.

We mentioned that the probability distribution of the nonconforming proportion (p) could be modelled by the

Bayesian inference as a Beta distribution with parameters $\alpha + 0.5$, $\beta + 0.5$.

If we define n to be the index of the decision-making stage and p to be the state variable, then $NI + C_s NE(p)$ shows the cost when we reject the lot. It is assumed that when we reject the lot, then a 100% inspection plan is done on the lot and all items are inspected thus the total cost of $NI + C_s NE(p)$ is incurred and decision making process ends. $C_a NE(p)$ represents the cost when we accept the lot, and $mI + \lambda(V_{n-1}(E(p)))$ shows the cost when we continue to the next decision making stage therefore sampling continues. It is obvious that new sample data is gathered in this case and then the probability distribution function of the state variable is updated using Bayesian method and dynamic model would be solved again with new sample data. Also a Tumbling operation can be performed on lot. Tumbling operation can be expected to eliminate $1-\gamma$ percentage of the nonconforming items therefore the proportion of nonconforming items in the tumbled lot reduces to $p_1 = \gamma p$ thus $T + \lambda(V_{n-1}(E(p_1 = \gamma p)))$ represent the cost when we tumble the lot. It is obvious that on stage $n-1$ we may choose one of the possible decisions again. It is assumed that if lot continues to the next stage, then the proportion of nonconforming items in the next stage remains p and if tumbling decision is made in one stage then the proportion of nonconforming items in the next stage decreases from p to γp where $\gamma < 1$.

The discount factor λ is used to evaluate the cost of the next stage in the current stage. This factor can be addressed as the rate of decreasing the cost at the next stages which may be resulted from having more information on the state variable in the next stages. Hence the cost associated with p when there are n remaining stages to make the decision is:

$$V_n(E(p)) = Min \begin{Bmatrix} NI + C_s NE(p), C_a NE(p), mI + \\ \lambda\ (V_{n-1}(E(p))), \\ T + \lambda\ (V_{n-1}(E(p_1 = \gamma p))) \end{Bmatrix} \quad (2)$$

Also on stage $n = 0$, we have only two decisions (accepting and rejecting the lot), therefore $V_0(E(p))$ is determined based on the minimization of two terms. Thus, when no decision making stage is available $(n = 0)$, then we should select between decisions of accepting or rejecting the lot that can be easily performed by comparing their corresponding cost.

$$V_0(E(p)) = Min\{NI + C_s NE(p), C_a NE(p)\} \quad (3)$$

It is seen that the value of $V_n(E(p))$ can be easily computed from the values of $V_{n-1}(.)$, thus we may easily compute all cost values recursively based on the values of $V_0(.)$. Thus, when no decision making stage is available $(n = 0)$ then, we should select between decisions of accepting or rejecting the lot, thus following is obtained.

$$n = 0 \rightarrow \begin{Bmatrix} NI + C_S NE(p) > C_a NE(p) \Rightarrow Accept \\ NI + C_S NE(p) < C_a NE(p) \Rightarrow Reject \end{Bmatrix} \quad (4)$$

Since p is a stochastic variable, thus we have applied approximation method in equation (5) in order to consider the probability distribution of p in the computations. We characterize properties of the value function at the last stage using a method for approximating the function $V_0(E(.))$ as follows:

$$V_0(E(p)) \cong E(V_0(p)) \quad (5)$$

Since $V_0(p)$ can be obtained for all possible value of p using following equation,

$$V_0(p) = Min\{NI + C_s Np, C_a Np\} \quad (6)$$

Thus following is concluded based on approximation method in equation (5),

$$V_0(E(p)) \cong E(V_0(p)) = \int_{p > \frac{I}{C_a - C_s}} (NI + C_s Np) f(p) dp + \int_{p < \frac{I}{C_a - C_s}} (C_a Np) f(p) dp \quad (7)$$

Also the probability distribution function of the random variable p_1 is needed to evaluate function $V_{n-1}(E(p_1 = \gamma p))$. This probability distribution function is needed in equation (7) for the cases that tumbling action has been selected. This probability distribution is determined using a heuristic approach. Assume that p_1 follows a Beta distribution with parameters α', β' . The approximate values of parameters α', β' can be determined from the following equalities.

$$\begin{Bmatrix} E(p_1) = \dfrac{\alpha'}{\alpha' + \beta'} = E(\gamma p) = \gamma \dfrac{\alpha + 0.5}{\alpha + \beta + 1} \\ \alpha' + \beta' = \alpha + \beta \end{Bmatrix} \quad (8)$$

$$\Rightarrow f(p_1) = Beta(\alpha', \beta')$$

The second equality is defined based on constant sample size in each stage. Therefore the values of parameters α', β' can be determined based on the values of parameters α, β.

Now we can evaluate the value of function $V_n(E(p))$ based on the value of $V_{n-1}(\mathrm{E}(p))$ and $V_{n-1}(\mathrm{E}(p_1 = \gamma p))$ where $V_{n-1}(E(.))$ can be obtained using equation (2) recursively thus it is concluded that the value of $V_n(E(p))$ can be determined based on the values of $V_0(E(.))$ by continuing this recursive method. Also the value of $V_0(E(.))$ can be determined by equation (7). Steps of dynamic programming have been shown in Fig (1).

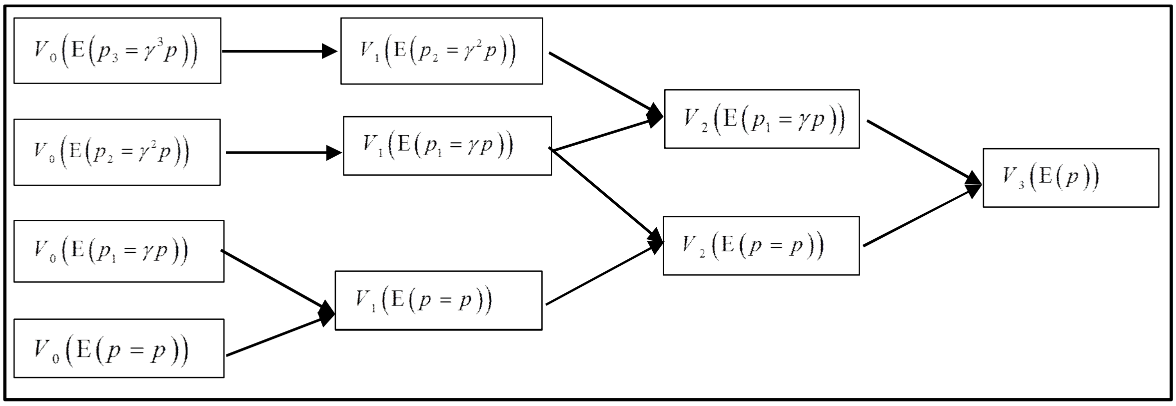

Fig. 1. Diagram of Dynamic Programming's steps

In the next section, a case study is given to illustrate the application of the proposed methodology.

3. Case Study

Assume a juice production industry has produced a lot of $N = 100$ items. The amount of vitamin C in juice is inspected through experimenters. According to the presented approach, first a sample of items is inspected. Also three decision making stages are available for deciding about the lot. The sample size in each stage of decision making is $m = 10$. Assume that stages are available for decision making process and at the start of process, 10 juices are inspected where the number of nonconforming juices is $\alpha = 2$ and the number of conforming juices is $\beta = 8$ in the first sample and the cost of accepting one nonconforming juice is $C_a = 4$, the cost of one nonconforming juice which is detected during inspection is $C_s = 2$, the cost of inspecting one item is $I = 1$, the cost of tumbling process is $T = 150$, the coefficient of decreasing nonconforming proportion in tumbling process is $\gamma = 95\%$ and the discount factor in stochastic dynamic programming approach is $\lambda = 0.9$.

According to dynamic programming approach when three decision making stages is available $(n = 3)$, we have :

$$V_3(E(p)) = Min \begin{Bmatrix} NI + C_s NE(p), C_a NE(p), \\ mI + \lambda V_{3-1}(\mathrm{E}(p)), \\ T + \lambda V_{3-1}(\mathrm{E}(p_1 = \gamma p)) \end{Bmatrix}$$

$V_2(E(p))$ can be calculated using recursive equation when two decision making stages is available, on the other hand we have :

$$V_2(E(p)) = Min \begin{Bmatrix} NI + C_s NE(p), C_a NE(p), \\ mI + \lambda V_{2-1}(E(p)), \\ T + \lambda V_{2-1}(E(p_1 = \gamma p)) \end{Bmatrix}$$

Now we can recursively determine $V_1(E(p))$ as follow:

$$V_1(E(p)) = Min \begin{Bmatrix} NI + C_s NE(p), C_a NE(p), \\ mI + \lambda V_{1-1}(E(p)), \\ T + \lambda V_{1-1}(E(p_1 = \gamma p)) \end{Bmatrix}$$

Also we need to obtain $V_1(E(p_2 = \gamma p_1 = \gamma^2 p))$ for calculating the item $V_2(E(p_1 = \gamma p))$ and then we need to determine $V_0(E(p_2 = \gamma^1 p_1 = \gamma^2 p))$ and the function $V_0(E(p_3 = \gamma p_2 = \gamma^2 p_1 = \gamma^3 p))$ which are obtained by equation (6).

In general, first we must calculate the $V_0(E(p))$, then $V_1(E(p))$, then $V_2(E(p))$ and at the end $V_3(E(p))$ as reported in Tables (1)…(5).

Table 1
Results of calculating E(.)

$E(p)$	0.2272727
$E(p_1=\gamma p)=\gamma E(p)$	0.2159091
$E(p_2=\gamma p_1=\gamma^2 p)=\gamma^2 E(p)$	0.20511
$E(p_3=\gamma p_2=\gamma^2 p_1=\gamma^3 p)=\gamma^3 E(p)$	0.19486

Table 2
Results of calculating $V_n(E(p))$

n=0	$V_0(E(p))$	137.799
n=1	$NI+C_sNE(p)$	145.455
	$C_aNE(p)$	90.9091
	$mI+\lambda V_0(E(p))$	128.99
	$T+\lambda V(_0E(p_1=\gamma p))$	263.115
	$V_1(E(p))$	90.9091
n=2	$NI+C_sNE(p)$	145.455
	$C_aNE(p)$	90.9091
	$mI+\lambda V_1(E(p))$	91.8182
	$T+\lambda V_1(\mathrm{E}(p_1=\gamma p))$	227.727
	$V_2(E(p))$	90.9091
n=3	$NI+C_sNE(p)$	145.455
	$C_aNE(p)$	90.9091
	$mI+\lambda V_2(E(p))$	91.8182
	$T+\lambda V_2(E(p_1=\gamma p))$	227.727
	$V_3(E(p))$	90.9091

Considering Tables (1-5), the optimal policy is to accept the lot when three decision making stages is available. In the next section, sensitivity analysis is performed on different parameters.

Table 3
Results of calculating $V_n(E(p_1))$

n=0	$V_0(E(p_1))$	125.684
n=1	$NI+C_sNE(p_1)$	143.182
	$C_aNE(p_1)$	86.3636
	$mI+\lambda V_0(E(p_1))$	123.115
	$T+\lambda V_0(E(p_2))$	257.535
	$V_1(E(p_1))$	86.3636
n=2	$NI+C_sNE(p_1)$	143.182
	$C_aNE(p_1)$	86.3636
	$mI+\lambda V_1(E(p_1))$	87.7273
	$T+\lambda V_1(E(p_2))$	223.841
	$V_2(E(p_1))$	86.3636

Table 4
Results of calculating $V_n(E(p_2))$

n=0	$E(V_0(p_2))$	119.483
n=1	$NI+C_sNE(p_2)$	141.023
	$C_aNE(p_2)$	82.0455
	$mI+\lambda V_0(E(p_2))$	117.535
	$T+\lambda V_0(E(p_3))$	252.233
	$V_1(E(p_2))$	82.0455

Table 5
Results of calculating $V_n(E(p_3))$

n=0	$V_0(E(p_3))$	113.592

4. Sensitivity Analysis

A sensitivity analysis is performed on the parameters of the problem that results are in the Table (6).

Table 6
Sensitivity analysis

cases	$(N, C_a, C_s, T, I, \gamma, m, \lambda, \alpha, \beta)$	Optimal policy	$V_3(E(p))$
1. Case study	$(100, 4, 2, 150, 1, 0.95, 10, 0.9, 2, 8)$	Accept	90.9091
2. Increases N	$(3422, 4, 2, 150, 1, 0.95, 10, 0.9, 2, 8)$	Continue to the next decision making stage	2538.836
3. Increases N	$(3423, 4, 2, 150, 1, 0.95, 10, 0.9, 2, 8)$	Tumble the lot	2539.544
4. Increases C_a	$(100, 5, 2, 150, 1, 0.95, 10, 0.9, 2, 8)$	Continue to the next decision making stage	166.2727
5. Decreases C_s	$(100, 4, 0.53, 150, 1, 0.95, 10, 0.9, 2, 8)$	Continue to the next decision making stage	88.48381
6. Decreases T	$(100, 4, 2, 13, 1, 0.95, 10, 0.9, 2, 8)$	Tumble the lot	90.722773
7. Decreases I	$(100, 4, 2, 150, 0.5, 0.95, 10, 0.9, 2, 8)$	Continue to the next decision making stage	83.13636
8. Decreases γ	$(100, 4, 2, 150, 1, 0.25, 10, 0.9, 2, 8)$	Tumbling	46.54983
9. Decreases λ	$(100, 4, 2, 150, 1, 0.95, 10, 0.5, 2, 8)$	Continue to the next decision making stage	34.02635

A sensitivity analysis is performed on the parameters of the problem that results have been summarized as following:

- By comparing cases one, two and three, it is seen that when the total number of items in a lot (N) is less than 3422 units, then the optimal decision in the proposed method is to continue to the next decision making stage, and when the total number of items in a lot (N) is more than 3423 units, then the optimal decision in the proposed method is to tumble the lot.
- By comparing case one and case six, it is seen that when the cost of tumbling process (T) is less than 13, then the optimal decision in the proposed method is to tumble the lot.
- The result of the proposed model in all cases is reasonable. For example, cost of proposed model increases by increasing cost parameters of the model and it decreases by decreasing different costs of the model and the corresponding optimal decision are made in accordance with the variations.

4.1. Sensitivity analysis of " C_a with C_s ":

Results for Simultaneous variations of cost of accepting one nonconforming item (C_a) and the cost of one detected nonconforming item (C_s) are denoted in

Table 7
Sensitivity analysis of variations of " C_a and C_s "

N.O of Cases	optimal policy
$C_a\downarrow$ $C_s\downarrow$	Accept the lot
$C_a\uparrow$ $C_s\uparrow$	Tumbling
$C_a\uparrow$ $C_s\downarrow$	Continue to the next decision making stage
$C_a\downarrow$ $C_s\uparrow$	Accept the lot

The results of Table 7, confirms the reasonable performance of dynamic programming model in the encountering with the variation of cost parameters.

4.2. Sensitivity analysis of " α : the number of nonconforming items, β : the number of conforming items":

The results of changing α and β are denoted in Table 8. It is seen that when the nonconforming proportion $\left(p = \dfrac{\alpha}{\alpha+\beta}\right)$ is approximately equal or less than 0.31, then optimal policy is to accept the lot, when the nonconforming proportion is more than 0.31, then optimal policy is to continue to the next decision making stage therefore sampling continues. When the nonconforming proportion is more than 0.67 then the optimal decision is to reject. Thus, it is observed that the optimal policy is a type of control threshold policy.

Table 8
Sensitivity analysis of "α and β"

m=9	α	1								
	β	8								
	p	0.11								
	optimal policy	Accept								
m=10	α	1	a	2	α	3	α	4		
	β	9	b	8	β	7	β	6		
	p	0.10	p	0.20	p	0.3	p	0.40		
	optimal policy	Accept	optimal policy	Accept	optimal policy	Continue	optimal policy	Continue		
m=11	α	1	α	2	a	3	α	4		
	β	10	β	9	b	8	β	7		
	p	0.09	p	0.18	p	0.27	p	0.36		
	optimal policy	Accept	optimal policy	Accept	optimal policy	Continue	optimal policy	Continue		
m=12	α	1	α	2	α	3	α	4		
	β	11	β	10	β	9	β	8		
	p	0.08	p	0.17	p	0.25	p	0.33		
	optimal policy	Accept	optimal policy	Accept	optimal policy	Accept	optimal policy	Continue		
m=13	α	2	α	3	α	4	α	5		
	β	11	β	10	β	9	β	8		
	p	0.15	p	0.23	p	0.31	p	0.38		
	optimal policy	Accept	optimal policy	Accept	optimal policy	Accept	optimal policy	Continue		
m=14	α	2	α	3	α	4	α	5		
	β	12	β	11	β	10	β	9		
	p	0.14	p	0.21	p	0.29	p	0.36		
	optimal policy	Accept	optimal policy	Accept	optimal policy	Accept	optimal policy	Continue		
m=15	α	2	α	3	a	4	α	5	α	10
	β	13	β	12	b	11	β	10	β	5
	p	0.13	p	0.2	p	0.27	p	0.33	p	0.67
	optimal policy	Accept	optimal policy	Accept	optimal policy	Accept	optimal policy	Continue	optimal policy	Reject

Consequently following decision making strategy is resulted,

$$\begin{cases} p \in [0.00, 0.31] \rightarrow accept\ the\ lot \\ p \in (0.31, 0.67) \rightarrow continue\ to\ the\ next\ decision\ making\ stage \\ p \in [0.67, 1.00) \rightarrow reject\ the\ lot \end{cases}$$

5. The Presence of Inspection Error

In this section, a dynamic programming model is developed for acceptance sampling problem in the presence of inspection errors. There are two types of inspection errors: 1. The first type of error 2. the second type of error.

We may model an acceptance sampling process as an optimal stopping problem. First, we take a sample from a lot in each stage of the decision-making process and the objective is to decide whether to accept, to reject and inspect all items, to tumble the lot, or to continue sampling based on the information obtained from the sample.

We mentioned that the probability distribution of the nonconforming proportion (p) could be modelled by the Bayesian inference as a Beta distribution with parameters $(\alpha+0.5\ ,\beta+0.5)$. But (p) denotes the nonconforming proportion obtained by imperfect inspection, thus its value is different from the true nonconforming proportion. Following notations are used in the rest of this paper;

p : The apparent nonconforming proportion (its value is obtained by imperfect inspection).

p^* : True proportion of nonconforming.

p_{T} : The apparent proportion of nonconforming items in the tumbled lot.

p^*_{T} : True proportion of nonconforming items in the tumbled lot.

C_r : The cost of rejecting one conforming item.

S : The cost of inspecting one item during sampling.

I_1 : The probability of first type error in inspecting one item.

I_2 : The probability of second type error in inspecting one item.

If we define n to be the index of the decision-making stage and $E(p^*)$ to be the state variable, then the cost functions of different decisions can be obtained as follows:

1) for accepting the lot

$$a_1 = C_a N$$

2) for rejecting the lot and inspecting all items in the lot

$$a_2 = NI + NI_1C_r(1-p^*) + NI_2C_ap^* + C_sN(1-I_2)p^* \Rightarrow$$

$$a_2 = NI + NI_1C_r + (NI_2C_a - NI_1C_r + C_sN(1-I_2))p^*$$

3) for tumbling the lot

$$a_3 = T + \lambda V_{n-1}(p^*_{\mathrm{T}})$$

4) for continuing to the next decision making stage and more sampling

$$a_4 = ms + \lambda V_{n-1}(p^*) \tag{9}$$

It is assumed that when the lot is rejected then all items are inspected and three types of cost are incurred. These costs are as follows,

1. $NI_2C_ap^*$: The cost of accepting one nonconforming item multiplied by second type error probability, I_2 (Probability of accepting one nonconforming item).
2. $C_sN(1-I_2)p^*$: The cost of one detected nonconforming item during inspection multiplied by probability of detecting a nonconforming item, $1-I_2$ (Probability of rejecting one nonconforming item).
3. $NI_1C_r(1-p^*)$: The cost of rejecting one conforming item multiplied by first type error probability, I_1 (Probability of rejecting one conforming item).

Np^* is the number of nonconforming items in the lot. True value of nonconforming proportion (p^*) is determined using conditional probability as follows,

$$\begin{aligned} p = {} & pr\left\{\left(\begin{matrix}\text{item is categorized}\\ \text{as nonconforming}\end{matrix}\right)\middle|\left(\begin{matrix}\text{item is}\\ \text{conforming}\end{matrix}\right)\right\} pr\left\{\begin{matrix}\text{item is}\\ \text{conforming}\end{matrix}\right\} \\ & + pr\left\{\left(\begin{matrix}\text{item is categorized}\\ \text{as nonconforming}\end{matrix}\right)\middle|\left(\begin{matrix}\text{item is}\\ \text{nonconforming}\end{matrix}\right)\right\} pr\left\{\begin{matrix}\text{item is}\\ \text{nonconforming}\end{matrix}\right\} \end{aligned} \tag{10}$$

Since,

$$pr\left\{\begin{pmatrix}\text{item is categorized}\\ \text{as nonconforming}\end{pmatrix}\middle|\begin{pmatrix}\text{item is}\\ \text{conforming}\end{pmatrix}\right\}=I_1$$
$$pr\{\text{item is conforming}\}=1-p^*$$
$$pr\left\{\begin{pmatrix}\text{item is categorized}\\ \text{as nonconforming}\end{pmatrix}\middle|\begin{pmatrix}\text{item is}\\ \text{nonconforming}\end{pmatrix}\right\}=1-I_2 \tag{11}$$
$$pr\{\text{item is nonconforming}\}=p^*$$

Thus,

$$p=I_1(1-p^*)+(1-\mathrm{I}_2)p^* \quad \rightarrow p^*=\frac{p-I_1}{1-I_1-I_2} \tag{12}$$

Since $E(p)=\dfrac{\alpha+0.5}{\alpha+\beta+1}$, Thus following is obtained,

$$E(p^*)=\frac{E(p)-I_1}{1-I_1-I_2}=\frac{\frac{\alpha+0.5}{\alpha+\beta+1}-I_1}{1-I_1-I_2} \tag{13}$$

Tumbling operation eliminates $(1-\gamma)$ percent of the nonconforming items therefore the proportion of nonconforming items in the tumbled lot reduces to $p_{\mathrm{T}}^*=\gamma p^*$. So the true value of nonconforming proportion after the tumbling process (p_{T}^*) is determined as follows,

$$p_{\mathrm{T}}^*=\gamma p^* \Rightarrow E(p_{\mathrm{T}}^*)=E(\gamma p^*)=\gamma E(p^*)=\gamma\frac{E(p)-I_1}{1-I_1-I_2}=\gamma\frac{\frac{\alpha+0.5}{\alpha+\beta+1}-I_1}{1-I_1-I_2} \tag{14}$$

It is obvious that when $n-1$ stages are available, then we can select one of the possible decisions again. Also a discount factor λ is needed to evaluate the cost of the next stage in the current stage (according to the approach of stochastic dynamic programming). Hence the cost associated with $E(p^*)$ when there are n remaining stages to make the decision is:

$$V_n(E(p^*))=Min\begin{cases}a_1=C_aNE(p^*)\\ a_2=NI+NI_1C_r+(NI_2C_a-NI_1C_r+C_sN(1-I_2))E(p^*)\\ a_3=T+\lambda V_{n-1}(E(p_{\mathrm{T}}^*))\\ a_4=ms+\lambda V_{n-1}(E(p^*))\end{cases} \tag{15}$$

Since in stage $n=0$, we have only two decisions of accepting and rejecting the lot therefore $V_0(E(p^*))$ is determined based on the minimization of two terms. When no decision making stage is available $(n=0)$, then we must select between two decisions of accepting or rejecting the lot that can be easily performed by comparing their corresponding cost.

$$V_0(E(p^*))=Min\begin{cases}a_1=C_aNE(p^*),\ a_2=NI+NI_1C_r+\\ (NI_2C_a-NI_1C_r+C_sN(1-I_2))E(p^*)\end{cases} \tag{16}$$

It is assumed that if decision making continues to next stage, then the nonconforming proportion in the next stage remains p^*, and if tumbling decision is made in one stage then the nonconforming proportion in the next stage decreases form p^* to γp^* where $0<\gamma<1$.

We characterize properties of the value function using a method for approximating the function $E\left(V_{n\text{-}1}(.)\right)$ in order to consider the probability distribution function of p in computations thus following approximation is applied,

$$V_0\left(E\left(p^*\right)\right) \cong E\left(V_0\left(p^*\right)\right) \tag{17}$$

Since function $V_0(p^*)$ is obtained using equation (18) thus $E\left(V_0\left(p^*\right)\right)$ can be obtained using equation (19),

$$V_0(p^*) = Min\begin{Bmatrix} a_1 = C_a N p^*, \\ a_2 = NI + NI_1C_r + \\ (NI_2C_a - NI_1C_r + C_sN(1-I_2))p^* \end{Bmatrix} \tag{18}$$

hence $V_0(E(p^*))$ can be obtained as follows:

$$V_0\left(E(p^*)\right) \cong E\left(V_0\left(p^*\right)\right) = \begin{bmatrix} \displaystyle\int_{p^* > \frac{I+I_1C_r}{C_a-(I_2C_a-I_1C_r+C_s(1-I_2))}} N\left(I+I_1C_r+(I_2C_a-I_1C_r+C_s(1-I_2))p^*\right) f(p)\,dp \\ + \\ \displaystyle\int_{p^* < \frac{I+I_1C_r}{C_a-(I_2C_a-I_1C_r+C_s(1-I_2))}} \left(C_aNp^*\right) f(p)\,dp \end{bmatrix} \tag{19}$$

Since,

$$p^* = \frac{p-I_1}{1-I_1-I_2}, \tag{20}$$

thus $V_0\left(E(p)\right)$ is obtained as follows:

$$V_0\left(E(p^*)\right) = \begin{bmatrix} \displaystyle\int_{p > \left(\frac{I+I_1C_r}{C_a-(I_2C_a-I_1C_r+C_s(1-I_2))}(1-I_1-I_2)+I_2\right)} N\left(I+I_1C_r+(I_2C_a-I_1C_r+C_s(1-I_2))\frac{p-I_1}{1-I_1-I_2}\right) f(p)\,dp \\ + \\ \displaystyle\int_{p < \left(\frac{I+I_1C_r}{C_a-(I_2C_a-I_1C_r+C_s(1-I_2))}(1-I_1-I_2)+I_2\right)} \left(C_aN\frac{p-I_1}{1-I_1-I_2}\right) f(p)\,dp \end{bmatrix} \tag{21}$$

When $(n=0)$ stages are available then we must select between decisions of accepting or rejecting the lot, thus following strategy is obtained for single stage model:

$$n=0 \Rightarrow \begin{Bmatrix} N\left(I+I_1C_r+(I_2C_a-I_1C_r+C_s(1-I_2))E(p^*)\right) > C_aNE(p^*) \Rightarrow \text{Accept} \Rightarrow V_0 = a_1 \\ N\left(I+I_1C_r+(I_2C_a-I_1C_r+C_s(1-I_2))E(p^*)\right) < C_aNE(p^*) \Rightarrow \text{Reject} \Rightarrow V_0 = a_2 \end{Bmatrix} \tag{22}$$

It is necessary to determine $V_{n-1}\left(E(.)\right)$ recursively to solve dynamic model in equation (14). To evaluate equation (21), the probability distribution function of random variable p_T is needed. This function is determined using a heuristic approach. Since p_T is the apparent nonconforming proportion of the lot in the imperfect inspection process thus we assume that p_T follows a Beta distribution with parameters α'', β''. The approximate values of parameters α'', β'' can be determined using the equation (27). To explain this heuristic method, first we have,

$E\left(p_T\right)$ = Mean of Beta distribution

$$\text{with parameters } \alpha'', \beta'' = \frac{\alpha''}{\alpha''+\beta''}. \tag{23}$$

Since tumbling operation eliminates $(1-\gamma)$ percentage of the nonconforming items thus the proportion of nonconforming items in the tumbled lot reduces to $p_{\mathrm{T}}^{*}=\gamma p^{*}$ thus

$$E\left(p_{\mathrm{T}}^{*}\right)=\gamma E\left(p^{*}\right)=\gamma E\left(\frac{p-I_1}{1-I_1-I_2}\right)=$$

$$\gamma\left(\frac{\frac{\alpha+0.5}{\alpha+\beta+1}-I_1}{1-I_1-I_2}\right) \tag{24}$$

Also since p_{T} is apparent nonconforming proportion of the lot after tumbling obtained by imperfect inspection and p_{T}^{*} is the true proportion of nonconforming items after tumbling with considering inspection errors thus using equation (12) following is obtained

$$E\left(p_{\mathrm{T}}^{*}\right)=E\left(\frac{p_{\mathrm{T}}-I_1}{1-I_1-I_2}\right)=\frac{1}{1-I_1-I_2}\frac{\alpha''}{\alpha''+\beta''}-\frac{1}{1-I_1-I_2}I_1 \tag{25}$$

Thus using two equalities (24) and (25), following is obtained,

$$\gamma\left(\frac{\frac{\alpha+0.5}{\alpha+\beta+1}-I_1}{1-I_1-I_2}\right)=\frac{1}{1-I_1-I_2}\frac{\alpha''}{\alpha''+\beta''}-\frac{1}{1-I_1-I_2}I_1 \tag{26}$$

Assuming equal number of sample size for both cases, we have, $\alpha''+\beta''=\alpha+\beta$. Thus following equalities are resulted to obtain the values of α'',β'',

$$\begin{cases}\gamma\left(\frac{\frac{\alpha+0.5}{\alpha+\beta+1}-I_1}{1-I_1-I_2}\right)=\frac{1}{1-I_1-I_2}\frac{\alpha''}{\alpha''+\beta''}-\frac{1}{1-I_1-I_2}I_1 \\ \alpha'+\beta'=\alpha+\beta\end{cases} \tag{27}$$

Following results are obtained by solving equations(27)

$$\begin{cases}\alpha''=(\alpha+\beta)\left[\gamma\frac{\alpha+0.5}{\alpha+\beta+1}+(1-\gamma)I_1\right] \\ \beta''=(\alpha+\beta)\left[1-\left[\gamma\frac{\alpha+0.5}{\alpha+\beta+1}+(1-\gamma)I_1\right]\right]\end{cases} \tag{28}$$

Thus the values of parameters α'',β'' can be determined using the values of parameters α, β.

Now we can evaluate the value of function $V_n\left(E(p^{*})\right)$ based on the value of $V_{n-1}\left(\mathrm{E}\left(p^{*}\right)\right)$ and $V_{n-1}\left(\mathrm{E}\left(p_{\mathrm{T}}^{*}\right)\right)$ therefore it is concluded that we can determine the value of $V_n\left(E(p^{*})\right)$ based on the value of $V_0\left(E(.)\right)$ recursively.

In the next section, a case study is given to illustrate the application of the extended model methodology.

6. Case Study of Extended Model

In addition to the parameter values of the case study in the third section, assume that the cost of rejecting one conforming juice is $C_r=5$ and the cost of inspecting one item during sampling is $S=2$ and the probability of first type error in inspecting one juice is $I_1=0.05$, and the probability of second type error in inspecting one juice is $I_2=0.1$.

According to stochastic dynamic programming approach where three stages $(n=3)$ are remained, we have,

$$V_3\left(E(p^{*})\right)=Min\begin{Bmatrix}a_1=C_aNE(p^{*}) \\ a_2=NI+NI_1C_r+ \\ (NI_2C_a-NI_1C_r+ \\ C_sN(1-I_2))E(p^{*}) \\ a_3=T+\lambda V_2\left(E(p_{\mathrm{T}}^{*})\right) \\ a_4=ms+\lambda V_2\left(E(p^{*})\right)\end{Bmatrix}$$

$V_2\left(E(p^{*})\right)$ can be obtained recursively. Thus when two stages are remained $(n=2)$, following is obtained:

$$V_2\left(E(p^{*})\right)=Min\begin{Bmatrix}a_1=C_aNE(p^{*}) \\ a_2=NI+NI_1C_r+ \\ (NI_2C_a-NI_1C_r+ \\ C_sN(1-I_2))E(p^{*}) \\ a_3=T+\lambda V_1\left(E(p_{\mathrm{T}}^{*})\right) \\ a_4=ms+\lambda V_1\left(E(p^{*})\right)\end{Bmatrix}$$

Also $V_1\left(E(p^{*})\right)$ can be obtained recursively:

Table 9

The results of $E(.)$

$E(p)$	0.22727
$E(p^*)$	0.20855
$E(p_T^*)=\gamma E(p^*)$	0.19812
$E(p_{T2}^*)=E(\gamma p_T^*)=\gamma^2 E(p^*)$	0.18822
$E(p_{T3}^*)=E(\gamma p_{T2}^*)=E(\gamma^2 p_T^*)\Rightarrow$ $E(p_{T3}^*)=\gamma^3 E(p^*)$	0.17881

Table 10

Results of $V_n(E(p^*))$

n=0	$V_0(E(p^*))$	103.522
n=1	$C_aNE(p^*)$	125.133
	$NI+NI_1C_r+(NI_2C_a-NI_1C_r+C_sN(1-I_2))E(p^*)$	119.839
	$T+\lambda V_0(E(p_T^*))$	190.140
	$ms+\lambda V_0(E(p^*))$	103.169
	$V_1(E(p^*))$	103.169
n=2	$C_aNE(p^*)$	125.133
	$NI+NI_1C_r+(NI_2C_a-NI_1C_r+C_sN(1-I_2))E(p^*)$	119.839
	$T+\lambda V_1(E(p_T^*))$	190.126
	$ms+\lambda V_1(E(p^*))$	102.852
	$V_2(E(p^*))$	102.852
n=3	$C_aNE(p^*)$	125.133
	$NI+NI_1C_r+(NI_2C_a-NI_1C_r+C_sN(1-I_2))E(p^*)$	119.839
	$T+\lambda V_2(E(p_T^*))$	190.114
	$ms+\lambda V_2(E(p^*))$	106.738
	$V_3(E(p^*))$	106.738
	$T+\lambda V_0(E(p_{T2}^*))$	187.26

$$V_1(E(p^*))=Min\left\{\begin{array}{l} a_1=C_aNE(p^*) \\ a_2=NI+NI_1C_r+ \\ (NI_2C_a-NI_1C_r+ \\ C_sN(1-I_2))E(p^*) \\ a_3=T+\lambda V_0(E(p_T^*)) \\ a_4=ms+\lambda V_0(E(p^*)) \end{array}\right\}$$

The calculations are reported in Tables (9-10). Considering Tables (9-10), the optimal policy is to continue to the next decision making stage and more sampling when three stages are available. In the next section, sensitivity analysis is performed on different parameters.

7. Sensitivity analysis of extended Model

Results of sensitivity of analysis are shown in Table (11). A sensitivity analysis is performed on the parameters of the problem that results have been explained in following:

- All results coincide with the type of variations. For example, increasing the cost of one decision leads to not selecting this decision as optimal.
- It is seen that when the total number of items in a lot (N) decreases, then the optimal policy in the proposed method is to reject the lot, and when the total number of items in a lot (N) becomes more than 3800 units, then the optimal decision in the proposed method is to tumble the lot.
- It is seen that when the probability of first type error in inspecting one item (I_1) increases, then the optimal decision in the proposed method is to accept the lot. This action is logical because when the probability of first type error in inspecting one item increases, then it's better to accept the lot because first type error is the probability of incorrect rejection of an acceptable item.

7.1. Sensitivity analysis of changing cost parameters

Simultaneous variations of the cost of accepting one nonconforming items (C_a), the cost of one detected nonconforming items (C_s) and the cost of rejecting one conforming items (C_r) are investigated and the results are denoted in Table 12. The results show the valid and logical performance of the proposed method.

Table 11
Sensitivity analysis

N.O of cases	$(N, C_a, C_s, T, I, \gamma, m, \lambda, \alpha, \beta, C_r, \mathrm{S}, \mathrm{I}_1, \mathrm{I}_2)$	Optimal policy	$V_3(E(p))$
Case study	$(100,4,2,150,1,0.95,10,0.9,2,8,5,2,0.05,0.1)$	Continue to the next decision making stage	106.738
Increases N	$(3880,4,2,150,1,0.95,10,0.9,2,8,5,2,0.05,0.1)$	Tumble the lot	2927.59072
Decreases N	$(70,4,2,150,1,0.95,10,0.9,2,8,5,2,0.05,0.1)$	Reject the lot	83.88770053
Increases C_s	$(100,4,5,150,1,0.95,10,0.9,2,8,5,2,0.05,0.1)$	Accept the lot	125.1336898
Decreases T	$(100,4,2,16,1,0.95,10,0.9,2,8,5,2,0.05,0.1)$	Tumble the lot	106.1140686
Decreases I	$(100,4,2,150,0.25,0.95,10,0.9,2,8,5,2,0.05,0.1)$	Reject the lot	94.83957219
Decreases γ	$(100,4,2,150,1,0.1,10,0.9,2,8,5,2,0.05,0.1)$	Tumble the lot	121.2727273
Increases λ	$(100,4,2,150,1,0.95,10,0.95,2,8,5,2,0.05,0.1)$	Reject the lot	98.52092074
Decreases C_r	$(100,4,2,150,1,0.95,10,0.9,2,8,0.1,2,0.05,0.1)$	Reject the lot	100.4491979
Increases S	$(100,4,2,150,1,0.95,10,0.9,2,8,5,3,0.05,0.1)$	Reject the lot	119.8395722
Increases I_1	$(100,4,2,150,1,0.95,10,0.9,2,8,5,2,0.1,0.1)$	Accept the lot	95.45454545
Increases I_2	$(100,4,2,150,1,0.95,10,0.9,2,8,5,2,0.05,0.5)$	Tumble the lot	141.3544837

Table 12
Sensitivity analysis of changing two parameters

N.O of cases	Optimal policy
$C_a \downarrow\ C_s \downarrow$	Accept the lot
$C_a \uparrow\ C_s \uparrow$	Continue to the next decision making stage
$C_a \uparrow\ C_s \downarrow$	Continue to the next decision making stage
$C_a \downarrow\ C_s \uparrow$	Accept the lot
$C_a \downarrow\ C_r \downarrow$	Accept the lot
$C_a \uparrow\ C_r \uparrow$	Continue to the next decision making stage
$C_a \uparrow\ C_r \downarrow$	Reject the lot
$C_a \downarrow\ C_r \uparrow$	Accept the lot
$C_r \downarrow\ C_s \downarrow$	Reject the lot
$C_r \uparrow\ C_s \uparrow$	Accept the lot
$C_r \uparrow\ C_s \downarrow$	Continue to the next decision making stage
$C_r \downarrow\ C_s \uparrow$	Accept the lot

7.2. Sensitivity analysis of "α: the number of nonconforming items, β: the number of nonconforming items"

The results of changing α and β are denoted in Table 13. It is seen that when the nonconforming proportion $\left(p=\frac{\alpha}{\alpha+\beta}\right)$ is equal or less than 0.18, then optimal policy is to accept the lot, when the nonconforming proportion is more than 0.18, then optimal policy is to continue to the next decision making stage therefore sampling continues. When the nonconforming proportion is more than 0.43, then optimal policy is to reject. Thus, it is found that the optimal policy is a type of control threshold policy.

$$\begin{cases} p \in [0.00, 0.18] \rightarrow \textit{accept the lot} \\ p \in (0.18, 0.43) \rightarrow \textit{continue to the} \\ \textit{next decision making stage} \\ p \in [0.43, 1.00) \rightarrow \textit{reject the lot} \end{cases}$$

Table 13
Sensitivity analysis of "α and β"

m=9	α	1								
	β	8								
	p	0.11								
	optimal policy	Accept								
m=10	α	1	a	2	α	3	α	4		
	β	9	b	8	β	7	β	6		
	p	0.10	p	0.20	p	0.3	p	0.40		
	optimal policy	Accept	optimal policy	Continue	optimal policy	Continue	optimal policy	Continue		
m=11	α	1	α	2	a	3	α	4		
	β	10	β	9	b	8	β	7		
	p	0.09	p	0.18	p	0.27	p	0.36		
	optimal policy	Accept	optimal policy	Accept	optimal policy	Continue	optimal policy	Continue		
m=12	α	1	α	2	α	3	α	4		
	β	11	β	10	β	9	β	8		
	p	0.08	p	0.17	p	0.25	p	0.33		
	optimal policy	Accept	optimal policy	Accept	optimal policy	Continue	optimal policy	Continue		
m=13	α	2	α	3	α	4	α	5		
	β	11	β	10	β	9	β	8		
	p	0.15	p	0.23	p	0.31	p	0.38		
	optimal policy	Accept	optimal policy	Continue	optimal policy	Continue	optimal policy	Continue		
m=14	α	2	α	3	α	4	α	5		
	β	12	β	11	β	10	β	9		
	p	0.14	p	0.21	p	0.29	p	0.36		
	optimal policy	Accept	optimal policy	Continue	optimal policy	Continue	optimal policy	Continue		
m=15	α	2	α	3	a	4	α	5	α	6
	β	13	β	12	b	11	β	10	β	9
	p	0.13	p	0.2	p	0.27	p	0.33	p	0.40
	optimal policy	Accept	optimal policy	Continue	optimal policy	Reject	optimal policy	Continue	optimal policy	Continue
m=16	α	3	α	4	a	5	α	6	α	7
	β	13	β	12	b	11	β	10	β	9
	p	0.18	p	0.25	p	0.31	p	0.37	p	0.43
	optimal policy	Accept	optimal policy	Continue	optimal policy	Continue	optimal policy	Continue	optimal policy	Reject

8. Conclusions

In this paper, we developed two optimization models for acceptance sampling plan. The first model is written for the cases that inspection is perfect and the second one considers inspection errors in the model. It is observed that the obtained dynamic model can be solved recursively using a heuristic method. To achieve this goal, we used a dynamic programming model and

Bayesian inference to design a sampling plan in quality environments considering quality costs. In order to determine the optimal policy, we considered a cost function and tried to optimize the function. first the probability distribution of nonconforming proportion is obtained by Bayesian inference then the correct value of nonconforming proportion is obtained by considering inspection errors then, the dynamic programming approach is utilized to make the optimal decision among decisions of accepting, rejecting and inspection all items, tumbling the lot or continuing and more sampling in the presence of inspection errors. Using sensitivity analysis, it is observed that the optimal policy is a type of control threshold policy. It is observed that when the lot size is less than a threshold, then the optimal decision is to continue to the next decision making stage, and when the lot size is more than this threshold, then the optimal decision is to tumble the lot. Also when the cost of tumbling process is less than a threshold, then the optimal decision is to tumble the lot. Furthermore, in the second model, when the lot size decreases, then the optimal policy tends to reject the lot, and when the probability of first type error increases, then the optimal decision tends to accept the lot.

By comparing the first model and the second model, it is observed that, although the cost of the second model (considering inspection errors) is more than the cost of the first model (perfect inspection), when the inspections are not perfect, then the optimal policy is completely different in some cases.

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