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Fuzzy Mathematical Model for a Lot-Sizing Problem In Closed-Loop Supply Chain

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Abstract

The aim of lot sizing problems is to determine the periods where production takes place and the quantities to be produced in order to satisfy the customer demand while minimizing the total cost. Due to its importance on the efficiency of the production and inventory systems, Lot sizing problems are one of the most challenging production planning problems and have been studied for many years with different modeling features. In this paper, we propose a fuzzy mathematical model for the single-item capacitated lot-sizing problem in closed-loop supply chain. The possibility approach is chosen to convert the fuzzy mathematical model to crisp mathematical model. The obtained crisp model is in the form of mixed integer linear programming (MILP), which can be solved by existing solver in crisp environment to find optimal solution. Due to the complexity of the problems harmony search (HS) algorithm and genetic algorithm (GA) have been used to solve the model for fifteen problem. To verify the performance of the algorithm, we computationally compared the results obtained by the algorithms with the results of the branch-and-bound method. Additionally, Taguchi method was used to calibrate the parameters of the meta-heuristic algorithms. The computational results show that, the objective values obtained by HS are better from GA results for large dimensions test problems, also CPU time obtained by HS are better than GA for Large dimensions.

Keywords: Lot-sizing, Harmony search, Returned products

1. Introduction

The lot-sizing problem is to find the right balance between these costs so as to minimize the total costs (Hoesel and Wagelmans, (1991). Wagner and Whitin presented a dynamic programming solution algorithm for single product, multi-period inventory lot-sizing problem. The Wagner-Whitin (1958) proposed algorithm for dynamic lot-sizing has often been misunderstood as requiring inordinate computational time and storage requirements. Montgomery et al. (1973) Presented several single-echelon, single-item, static demand inventory models for situations in which, during the stock out period, a fraction *b* of the demand is backordered and the remaining fraction 1 - *b* is lost forever. Akbalik and Penz (2009) studied a special case of the single-item capacitated lot-sizing problem, where alternative machines are used for the production of a single-item. They proposed an exact pseudo-polynomial dynamic programming algorithm which makes it NP-hard in the ordinary sense. Akbalik and Pochet (2009) presented a new class of valid inequalities for the single-item capacitated lot-sizing problem with step-wise production costs. They proposed a cutting plane algorithm for different classes of valid inequalities introduced. Akbalik and Rapine (2012) considered the single-item uncapacitated lot-sizing problem with batch delivery, focusing on the general case of time-dependent batch sizes. They showed that the problem is polynomial solvable in time $O(T^3)$, where T denotes the number of periods of the horizon. Abad (2001) considered the problem of determining the optimal price and lot-size for a reseller. He assumed that demand can be backlogged and that the selling price is constant within the inventory cycle. Aksen et al. (2003) addressed a profit maximization version of the well-known Wagner–Whitin model for the deterministic single-item uncapacitated lot-sizing problem with lost sales. The authors proposed an $O(T^2)$ forward dynamic programming algorithm to solve the problem.Brahimi et al. (2006) presented four different mathematical programming formulations of the Single-item lot sizing problems. Chu et al. (2013) addressed a real-life production planning problem arising in a manufacturer of luxury goods. This problem can be modeled as a single-item dynamic lot-sizing model with backlogging, outsourcing and inventory

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capacity. They showed that this problem can be solved in *O* ($T^4 logT$) time where T is the number of periods in the planning horizon. Tang (2004) provides a brief presentation of simulated annealing techniques and their application in lot-sizing problems. Sadjadi et al. (2009) proposed an improved algorithm of the Wagner and Whitin method. In so doing, they first assumed that shortage is not permitted and inventory holding and setup costs are fixed, and then assumed the possibility of shortage and variability of setup and holding costs. Golany et al. (2001) studied a production planning problem with remanufacturing. They proved the problem is NP-complete and obtain an *O* (T^3) algorithm for solve the problem. Li et al. (2007) analyzed a version of the capacitated dynamic lot-sizing problem with substitutions and return products. They first applied a genetic algorithm to determine all periods requiring setups for batch manufacturing and batch remanufacturing, and then developed a dynamic programming approach to provide the optimal solution to determine how many new products are manufactured or return products are remanufactured in each of these periods. Pan et al. (2009) addressed the capacitated dynamic lot- sizing problem arising in closed-loop supply chain where returned products are collected from customers. They assumed that the capacities of production, disposal and remanufacturing are limited, and backlogging is not allowed. Moreover, they proposed a pseudo-polynomial algorithm for solving the problem with both capacitated disposal and remanufacturing. Zhang et al. (2012) investigated the capacitated lot-sizing problem in closed-loop supply chain considering setup costs, product returns, and remanufacturing. They formulated the problem as a mixed integer program and propose a Lagrangian relaxation-based solution approach.

1.1. Fuzzy lot-sizing problem

Fuzzy set theory was first proposed by Zadeh(1965). It is a mathematical tool to describe imprecision in the fuzzy environment. Imprecision refers to the sense of vagueness rather than the lack of knowledge about the value of parameters. The vagueness is due to the unique experiences and judgments of decision makers. Fuzzy mathematical programming or fuzzy optimization, which was proposed by Zimmermann (1996), is one application of fuzzy set theory. There has been a lot of research which deals with vagueness in lot-sizing models as one of the fuzzy mathematical programming models. For example, Yao and Lee (1999) investigated a group of computing schemas for the economic order quantity as fuzzy values of the inventory with/without backorder. Yao and Chiang (2003) fuzzified the total demand and the cost of storing one unit per day to the triangular fuzzy numbers. They defuzzified by the centroid and the signed distance methods. Finally, they compared the results obtained by centroid and signed distance methods in the case of the total cost of inventory without backorders. They expressed the fuzzy order quantity as the normal trapezoid fuzzy number and then solved the aforementioned optimization problem. Pai (2003) applied the fuzzy set theory to solve the capacitated lot-size problem and by using numerical examples. Wong et al. (2012) proposed a stochastic dynamic lot-sizing problem with asymmetric deteriorating commodity, in which the optimal unit cost of material and unit holding cost would be determined. They used artificial neural network (ANN) and modified ant colony optimization (ACO) to solve this stochastic dynamic lot-sizing problem. Guillaume et al. (2003) investigated lot-sizing problem with fuzzy demands. Mandala et al. (2005) investigated multi-item multi-objective inventory model with shortages and demand dependent unit cost has been formulated along with storage space, number of orders and production cost restrictions. They imposed the cost parameters, the objective functions and constraints are in fuzzy environment. They used geometric programming method to solve the model. Chang et al. (2006) presented a fuzzy extension of the economic lot-size scheduling problem (ELSP) for fuzzy demands. They used a genetic algorithm to solve the problem. Chen and Chang (2008) studied fuzzy economic production quantity (FEPQ) model with defective productions which cannot be repaired, fuzzy opportunity cost. They used function principle as arithmetical operations of fuzzy total production inventory cost, and used the graded mean integration representation method to defuzzify the fuzzy total production and inventory cost. Halim et al. (2011) consider a single-unit unreliable production system which produces a single-item. They developed two production planning models on the basis of fuzzy and stochastic demand patterns and defuzzified by using the graded mean integration representation method. Ketsarapong et al. (2011) proposed a single-item lot-sizing problem with fuzzy parameters, which is called the fuzzy single-item lot-sizing problem. They used the possibility approach to convert the fuzzy model to the equivalent crisp single-item lot- sizing problem (EC-SILSP). Sahebjamnia and Torabi (2011) developed a fuzzy stochastic multi-objective linear programming model for a multi-level, multi-item capacitated lot-sizing problem.

In this paper, the single-item capacitated lot-sizing problem is discussed. An integrated model with backlogging, safety stocks and outsourcing with different production methods and limited warehouse space is presented anda fuzzy mathematical model is proposed. Four metaheuristic algorithms named simulated annealing (SA), harmony search (HS), vibration damping optimization (VDO) and

genetic algorithm (GA) have been used to solve the proposed model.
The remaining of this paper is organized as follows: Section 2 describes the single-item capacitated lot-sizing problem with fuzzy parameters and equivalent crisp of the single-item capacitated lot-sizing problem. The solution approaches for solving the proposed model introduced in Section 3. The Taguchi method for tuning the parameters and computational experiments is presented in Section 4. The conclusions and suggestions for future studies are included in Section 5.

2. Problem Formulation

In this section, we present an MIP formulation of the problem. The single-item capacitated lot-sizing problem with backlogging, safety stocks and limited outsourcing in closed-loop supply chain, is a production planning problem in which there is a time-varying demand for an item over Tperiods. In this section, we present a formulation of the problem. First, the problem assumptions, parameters, and decision variables have thoroughly been introduced and then the proposed model has been defined.

2.1. Assumptions

Before the formulation is considered, the other following assumptions are made on the problem.

I. The shortage, inventory, safety stock deficit costs, variable cost, setup cost, out-sourcing cost and demand are non-deterministic.
II. The amount of the returned products is regarded deterministic over the planning horizon.
III. Shortage is backlogged.
IV. Shortage and inventory costs must be taken into consideration at the end.
V. The quantity of inventory and shortage at the beginning of the planning horizon is zero.
VI. The quantity of inventory and shortage at the end of the planning horizon is zero.

2.2. Parameters

T: Number of periods in the planning horizon, t=1, …, T
J: Number of production manner, j=1, …, J

$\tilde{C}_{jt}$: The production cost of each unit in the period t through the manner j

$\tilde{A}_{jt}$: The setup cost of the production in the period t through the manner j

$\tilde{h}_t^+$: The unit holding cost in the period t

$\tilde{h}_t^-$: Unitary safety stock deficit cost in period t

$\tilde{d}_t$: The demand in the period t

$\tilde{L}_t$: The quantity of the safety stock of product i in the period t

$\tilde{\partial}_t$: Unitary shortage cost in period t

$\tilde{\gamma}_t$: Unit out-sourcing cost at period t

M: A large number

$\tilde{\theta}_t$: The unit holding cost of returned products in period t

$\tilde{C}_t^d$: The maximum number of returned products that could be disposed in period t

$\tilde{C}_t^r$: The maximum number of returned products that could be remanufactured in period t

$\tilde{R}_t$: the number of returned products in period t

2.3. Decision Variables

P_{jt}: Production quantity in the period t through the manner j

P_t^f: The number of returned products that remanufactured in period t

P_t^s: The number of returned products that disposed in period t

y_{jt}: Binary variable, 1 if the produced in the period t through the manner j, otherwise $y_{jt} = 0$

U_t: Out-sourcing level at period t

I_t^-: The quantity of shortage in the period t

S_t^+: The quantity of overstock deficit in the period t

S_t^-: The quantity of safety stock deficit in the period t

2.4. The proposed model

$$MinZ = \sum_{t=1}^{T}\left(\sum_{j=1}^{J}(\tilde{C}_{jt} P_{jt} + \tilde{A}_{jt} y_{jt}) + \tilde{\partial}_t I_t^- + \tilde{h}_t^+ S_t^+ + \tilde{h}_t^- S_t^- + \tilde{\gamma}_t U_t + \tilde{F}_{it} P_{it}^f + \tilde{g}_t P_t^s + \tilde{\theta}_t I_t^r\right) \quad (1)$$

Subject to:

$$S_{t-1}^{+}-S_{t-1}^{-}-I_{t-1}^{-}+I_t^{-}+\sum_{j=1}^{J}P_{jt}+U_t=S_t^{+}-S_t^{-}+\tilde{d}_t+\tilde{L}_t-\tilde{L}_{t-1} \quad \forall t=1,2,....,T \tag{2}$$

$$S_T^{+}=0 \tag{3}$$

$$I_T^{-}=0 \tag{4}$$

$$I_t^{r}=I_{t-1}^{r}-P_t^{f}-P_t^{s}+\tilde{R}_t \tag{5}$$

$$P_{jt}\le My_{jt} \ \forall\ j=1,2,....,J,\ t=1,2,....,T \tag{6}$$

$$I_t^{-}\le \tilde{d}_t \quad \forall \quad t=1,2,....,T-1 \tag{7}$$

$$S_t^{-}\le \tilde{L}_t \quad \forall\ t=1,2,....,T \tag{8}$$

$$0\le U_t\le I_{t-1}^{-}+S_{t-1}^{-}+\tilde{d}_t+\tilde{L}_t \ \forall\ t=1,2,....,T \tag{9}$$

$$P_t^{f}\le \tilde{C}_t^{d} \ \forall\ t=1,2,....,T \tag{10}$$

$$P_t^{s}\le \tilde{C}_t^{r} \ \forall\ t=1,2,....,T \tag{11}$$

$$y_{jt}\in\{0,1\} \quad \forall \quad j=1,2,....,J \quad t=1,2,....,T \tag{12}$$

$$P_{jt},P_t^{f},P_t^{s},I_t^{r},I_t^{-},S_t^{-},S_t^{+}\ge 0 \qquad \forall \quad j=1,2,....,J \quad t=1,2,....T \tag{13}$$

The objective function (1) shows total cost. Constraints (2) are the inventory flow conservation equations through the planning horizon. Constraints (3) and (4) define respectively, the demand shortage and the safety stock deficit for item at end period is zero. Constraints (5) are the inventory flow conservation equations for returned products. constraints (6) impose that the quantity produced must not exceed a maximum production level M_t. M_t is the total demand requirement for product on section [t, T] of the horizon, M_t is then equal Eq. (14) to:

$$M_t=\left(\sum_{t=1}^{T}d_t\right) \tag{14}$$

Constraints (7) and (8) define upper bounds on, respectively, the demand shortage and the safety stock deficit in period t. Constraints (9) ensure that outsourcing level U_t at period t is nonnegative and cannot exceed the sum of the demand, safety stock of period t and the quantity backlogged, safety stock deficit from previous periods. Constraints (10) and (11) are the capacity constraints of disposal, remanufacturing.Constraints (12) and (13) characterize $y_{jt}$is a binary variable and the variable's domains: $P_{jt},P_t^{f},P_t^{s},I_t^{r},I_t^{-},S_t^{-},S_t^{+}$ are non-negative for $j\in$ J and $t\in$ T.

2.5. Equivalent crispsingle-item capacitated lot-sizing problem

The possibility approach in the context of fuzzy set theory was introduced by Zadeh (1978) to deal with non-stochastic imprecision and vagueness.

According to the Dubois and Prade (1988) and Dubois (2006), the possibility approach appropriately was used to model various kinds of information, such as linguistic information and uncertain formulate, in logical settings. In this section, the possibility approach will be used to convert the fuzzy model to the equivalent crisp model.

In this paper, Chance-Constrained Programming (CCP) by Charnes and Cooper (1959), which is normally used to confront stochastic linear programming (SLP), is adopted as a way to convert the fuzzy single-item capacitated lot-sizing problem to the Equivalent crisp single-item capacitated lot-sizing problem. The concept of CCP guarantees that the probability of stochastic constraints is greater than or equal to a pre-specified minimum probability. Lertworasirikul et al. (2003) proved and proposed the below Lemma:

Let $\tilde{a}_i$ for i = 1,..., n be fuzzy variables with normal and convex membership functions and b be a crisp variable. The lower and upper bounds of the α-level set of $\tilde{a}_i$ are denoted by $(\tilde{a}_i)_\alpha^L$ and $(\tilde{a}_i)_\alpha^U$, respectively. Then, for any given possibility levels α_1, α_2 and α_3 with $0 \leq \alpha_1, \alpha_2, \alpha_3 \leq 1$, (Ketsarapong et al, 2011):

(i) $\pi(\tilde{a}_1 + \cdots + \tilde{a}_n \leq b) \geq \alpha_1$ iff $(\tilde{a}_1)_{\alpha_1}^L + \cdots + (\tilde{a}_n)_{\alpha_1}^L \leq b$,

(ii) $\pi(\tilde{a}_1 + \cdots + \tilde{a}_n \geq b) \geq \alpha_2$ if $(\tilde{a}_1)_{\alpha_2}^U + \cdots + (\tilde{a}_n)_{\alpha_2}^U \geq b$,

(iii) $\pi(\tilde{a}_1 + \cdots + \tilde{a}_n = b) \geq \alpha_3$ if $(\tilde{a}_1)_{\alpha_3}^L + \cdots + (\tilde{a}_n)_{\alpha_3}^L \leq b$ and $(\tilde{a}_1)_{\alpha_3}^U + \cdots + (\tilde{a}_n)_{\alpha_3}^U \geq b$.

The single-item capacitated lot-sizing problem with fuzzy parameters is transformed into the equivalent crispsingle-item capacitated lot-sizing problem by the above Lemma, as follows:

$$MinZ = \sum_{t=1}^{T} \begin{array}{l} (\sum_{j=1}^{J}((\alpha(\tilde{C}_{jt})_1^L + (1-\alpha)(\tilde{C}_{jt})_0^L)P_{jt} + (\alpha(\tilde{A}_{jt})_1^L + (1-\alpha)(\tilde{A}_{jt})_0^L)y_{jt}) + (\tilde{\partial}_t)_\alpha^L I_t^- + \\ (\alpha(\tilde{h}_t^+)_1^L + (1-\alpha)(\tilde{h}_t^+)_0^L)S_t^+ + (\alpha(\tilde{h}_t^-)_1^L + (1-\alpha)(\tilde{h}_t^-)_0^L)S_t^- + (\alpha(\tilde{\gamma}_t)_1^L + (1-\alpha)(\tilde{\gamma}_t)_0^L)U_t + \\ (\alpha(\tilde{F}_{it})_1^L + (1-\alpha)(\tilde{F}_{it})_0^L)P_{it}^f + (\alpha(\tilde{g}_t)_1^L + (1-\alpha)(\tilde{g}_t)_0^L)P_t^s + (\alpha(\tilde{\theta}_t)_1^L + (1-\alpha)(\tilde{\theta}_t)_0^L)I_t^r) \end{array} \tag{15}$$

Subject to:

$$S_{t-1}^+ - S_{t-1}^- - I_{t-1}^- + I_t^- + \sum_{j=1}^{J} P_{jt} + U_t \leq S_t^+ - S_t^- + (\tilde{d}_t)_\alpha^U + (\tilde{L}_t)_\alpha^U - (\tilde{L}_{t-1})_\alpha^U \quad \forall t = 1,2,....,T \tag{16}$$

$$S_{t-1}^+ - S_{t-1}^- - I_{t-1}^- + I_t^- + \sum_{j=1}^{J} P_{jt} + U_t \geq S_t^+ - S_t^- + (\tilde{d}_t)_\alpha^L + (\tilde{L}_t)_\alpha^L - (\tilde{L}_{t-1})_\alpha^L \quad \forall t = 1,2,....,T \tag{17}$$

$$S_T^+ = 0 \tag{18}$$

$$I_T^- = 0 \tag{19}$$

$$I_t^r = I_{t-1}^r - P_t^f - P_t^s + \tilde{R}_t \tag{20}$$

$$P_{jt} \leq M y_{jt} \quad \forall j = 1,2,....,J, \; t = 1,2,....,T \tag{21}$$

$$I_t^- \leq (\tilde{d}_t)_\alpha^U \quad \forall \quad t = 1,2,....,T-1 \tag{22}$$

$$S_t^- \leq (\tilde{L}_t)_\alpha^U \quad \forall t = 1,2,....,T \tag{23}$$

$$0 \leq U_t \leq I_{t-1}^- + S_{t-1}^- + (\tilde{d}_t)_\alpha^U + (\tilde{L}_t)_\alpha^U \quad \forall t = 1,2,....,T \tag{24}$$

$$P_t^f \leq (\tilde{C}_t^d)_\alpha^U \quad \forall t = 1,2,....,T \tag{25}$$

$$P_t^s \leq (\tilde{C}_t^r)_\alpha^U \quad \forall t = 1,2,....,T \tag{26}$$

$$y_{jt} \in \{0,1\} \quad \forall \quad j = 1,2,....,J \quad t = 1,2,....,T \tag{27}$$

$$P_{jt}, P_t^f, P_t^s, I_t^r, I_t^-, S_t^-, S_t^+ \geq 0 \quad \forall \quad j = 1,2,....,J \quad t = 1,2,...T \tag{28}$$

3. Solution Approaches

3.1. Harmony search algorithm

Harmony search (HS) is a new heuristic method that mimics the improvisation of music players. HS was proposed by Geemet al. (2001). Inspiration was drawn from musical performance processes that occur when a musician searches for a better state of harmony, improvising the instrument pitches towards a better aesthetic outcome. The HS algorithm imposes fewer mathematical requirements and does not require specific initial value settings of the decision variables. Because the HS algorithm is based on stochastic random searches, the derivative information is also not necessary. In the HS algorithm, musicians search for a perfect state of the harmony which determined by aesthetic estimation, as the optimization algorithms search for a best state (i.e., global optimum) determined by an objective function.

Harmony search

Objective function $f(x_i)$, i=1 to N
Define HS parameters: HMS, HMCR, PAR, and BW
Generate initial harmonics (for i=1 to HMS)
Evaluate $f(x_i)$
While (until terminating condition)
Create a new harmony: x_i^{new}, i=1 to N
If($U(0,1) \geq HMCR$),
$x_i^{new}=x_j^{old}$, where x_j^{old} is a random from {1,…,HMS}
Else if ($U(0, 1) \leq PAR$),
$x_i^{new}=x_L(i)+ U(0, 1)\times[x_U(i) - x_L(i)]$
Else
$x_i^{new}=x_j^{old} + BW[(2\times U(0,1))-1]$, where x_j^{old} is a random from {1,…,HMS}
end if
Evaluation $f(x_i^{new})$
Accept the new harmonics (solutions) if better
End while

Fine the current best estimates

3.2. Genetic algorithm

The genetic algorithm (GA) has been proven to be powerful method for combinatorial optimization problems. It was first proposed by Holland (1975) to encode the features of a problem by chromosomes, in which each gene represents a feature of the problem. In general, the GA consists of the following steps:

Step 1: Initialize a population of chromosomes.

Step 2: Evaluate the fitness of each chromosome.

Step 3: Create new chromosomes by applying genetic operators (e.g., crossover and mutation) to the current selected chromosomes.

Step 4: Evaluate the fitness of the new population of chromosomes.

Step 5: If the termination condition is satisfied, stop and return the best chromosome; otherwise, go to Step 3.

3.2.1. Representation schema

In this paper, each chromosome is a production plan and each chromosome formed as an integer vector with T genes as shown in Fig 1, where T is the number of periods.

3.2.2. Selection

The selection provides the opportunity to deliver the gene of a good solution to next generation. There are various selection operators available that can be used to select the parents. In this study, the tournament selection is employed.

3.2.3. Crossover

Crossover is a process, in which chromosomes exchange genes through the breakage and reunion of two chromosomes to generate a number of children. Crossover's offspring should represent solutions that combine substructures of their parents. In this study, crossover generates an offspring by combining two selective parents as shown in Fig. 2.

3.2.4. Mutation

A mutation scheme that swaps the value of the (number of periods ×strongly mutation) random selected genes of the current solution with each other. Fig. 3 illustrates this operation on the*T*=8 and *Sm*=0.5.

In our implemented GA, crossover and mutation operators are used with the given probabilities.

3.2.5. Fitness function

The fitness function is the same as the objective function defined in Section 2.

3.2.6. Termination condition

The search process stops if the stopping criterion (i.e., maximum number of improvisations) is satisfied, then computation is terminated.

4. Experimental Results

We try to test the performance of the GA and HS in finding good quality solutions in reasonable time for the problem. For this purpose, 15 problems with different sizes are generated for each algorithms. The production manners and periods has the most impact on problem hardness. The proposed model coded with Lingo (ver.8) software using for solving the instances, GA and HS is implemented to solve each instance in five times to obtain more reliable data.
The genetic algorithm and harmony search are coded in MATLAB R2011a and all tests are conducted on a not

book at Intel Core i5 Processor 1.7 GHz and 6 GB of RAM.

In this paper, the Taguchi method applied to calibrate the parameters of the proposed algorithms. The Taguchi method was developed by Taguchi (2000). This method is based on maximizing the performance measures, called signal-to-noise (S/N) ratio, in order to find the optimized levels of the effective factors in the experiments. The S/N ratio refers to the mean-square deviation of the objective function that minimizes the mean and variance of quality characteristics to make them closer to the expected values. For the factors that have significant impact on the S/N ratio, the highest S/N ratio provides the optimum level for that factor. Table 1 lists different levels of the factors for HS and GA. In this paper, according to the levels and the number of the factors, respectively the Taguchi method L_{25} is used for the adjustment of the parameters for the HS and GA. Figs 4 and 5 shown S/N ratios. Best Level of the factor for each algorithm is shown in table 2.

Computational experiments are conducted to validate and verify the behavior and the performance of the meta-heuristic algorithms employed to solve the considered single-item capacitated lot-sizing problem. We try to test the performance of the proposed GA and HS finding good quality solutions in a reasonable time for the problem. For this purpose, 15 problems with different sizes and $\alpha = 0.4$ are generated. For these problems, we used the trapezoidal fuzzy setup cost and unit price which would be classified as three levels cheap, normal and expensive. It is respectively denoted by:

$\tilde{A}_C = (17000, 18000, 19000, 20000)$, $\tilde{A}_N = (19000, 20000, 21000, 22000)$, $\tilde{A}_E = (21000, 22000, 23000, 24000)$, $\tilde{C}_C = (60, 65, 70, 75)$, $\tilde{C}_N = (70, 75, 80, 85)$ and $\tilde{C}_E = (70, 75, 80, 85)$ The trapezoidal fuzzy demand and safety stock are classified as three levels, low, medium and high. It is respectively denoted by: $\tilde{d}_L = (8, 9, 10, 11)$, $\tilde{d}_M = (10, 11, 12, 13)$, $\tilde{d}_H = (13, 14, 15, 16)$, $\tilde{L}_L = (1, 2, 3, 4)$, $\tilde{L}_M = (3, 4, 5, 6)$ and $\tilde{L}_H = (5, 6, 7, 8)$.

The triangular fuzzy inventory holding cost, shortage cost, safety stock deficit cost, out-sourcing cost, repair cost, dispose cost and inventory holding cost for return products are classified as three levels cheap, normal and expensive.

$\tilde{h}_C^+ = (6, 7, 8)$, $\tilde{h}_N^+ = (7, 8, 9)$, $\tilde{h}_E^+ = (8, 9, 10)$, $\tilde{\delta}_C = (16, 17, 18)$, $\tilde{\delta}_N = (17, 18, 19)$, $\tilde{\delta}_E = (18, 19, 20)$, $\tilde{h}_C^- = (11, 12, 13)$, $\tilde{h}_N^- = (12, 13, 14)$, $\tilde{h}_E^- = (13, 14, 15)$, $\tilde{u}_C = (30000, 35000, 40000)$, $\tilde{u}_N = (35000, 40000, 45000)$, $\tilde{u}_E = (40000, 45000, 50000)$, $\tilde{g}_{Low} = (55, 60, 65)$, $\tilde{g}_{Medium} = (60, 65, 70)$, $\tilde{g}_{High} = (65, 70, 75)$, $\tilde{F}_L = (15, 20, 25)$, $\tilde{F}_{Medium} = (20, 25, 30)$, $\tilde{F}_{High} = (25, 30, 35)$, $\tilde{\theta}_{Low} = (4, 5, 6)$, $\tilde{\theta}_{Medium} = (5, 6, 7)$, $\tilde{\theta}_{High} = (6, 7, 8)$.

The number of manners and periods has the most impact on the problem hardness. The proposed model coded with Lingo (Ver.8) software using for solving the instances. The best results are recorded as a measure for the related problem. Table 3 shows details of computational results obtained by solution methods for all test problems. The results of running the proposed GA and HS are compared with the optimal solution of the instances, obtained from Lingo software, Fig 6 depict comparison between solution quality of the Lingo, GA and HS of the instances, and Fig 7 depict comparison between solution quality of GA and HS.

- ✓ The GA and HS algorithms can solve all the test problems for both models.
- ✓ The GA and HS can find good quality solutions for small dimensions problems.
- ✓ The objective values obtained by HS are better from GA results for Large dimensions test problems.
- ✓ Fig 8 shows the CPU time of the HS is less than GA.

5. Conclusion

In this paper, we propose a mathematical formulation of a new single-item capacitated lot-sizing problem with backlogging, safety stocks and limited outsourcing in closed-loop supply chain. This formulation takes into account several industrial constraints such as shortage costs, safety stock deficit costs and limited outsourcing. Due to the complexity of the problem, GA and HS algorithms is used to solve problem instances. Several problems with different sizes generated and solved by HS and Lingo software. The results show that the HS algorithm able to find good quality solutions in reasonable time. One straightforward opportunity for future research is extending the assumption of the proposed model for including real conditions of production systems such as limited inventory and etc. Also, developing a new heuristic or meta-heuristic to construct better feasible solutions.

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