Research Article

Designing Tolerance of Assembled Components Using Weibull Distribution

Mohammad Mehdi Movahedi [a,*], Seyedreza Seyedghasemi [b]

[a] *Department of Industrial Management, Firoozkooh Branch, Islamic Azad University, Firoozkooh, Iran.*

[b] *Department of Industrial Management, Science and Technology Branch, Islamic Azad University, Tehran, Iran.*

Abstract

Tolerancing is one of the most important tools for planning, controlling, and improving quality in the industry. In order to meet the customer needs and enhance product and service quality, the design engineers use handbooks to determine the tolerance. Although the use of the statistical methods to determine the tolerance is not a new concept, the engineers for this purpose typically use the known statistical distributions such as the normal distribution. However, if the statistical distribution of the variable is unknown, a new statistical method is used. Therefore, we want to offer a flexible and proper statistical method to determine the tolerance of components of a product to enhance its performance. In this regard, Weibull distribution is proposed. To illustrate the proposed method first technical characteristics of production components were selected randomly, and thenmanufacturing parameters were determined using maximum likelihood method. Finally, the Goodness of Fit test was used to ensure the accuracy of the obtained results.

Keywords: Tolerance; Weibull distribution; Statistical quality control.

1. Introduction

Determining the technical specifications of components for achieving quality and process capabilityis of utmost importance. Six Sigmaprograms play a significant role to improve the quality of a process according to the tolerance of quality characteristics.So, to use the Six Sigma and the analysis of the process capability including the C_{pk}, the tolerancing plays an important role in this area. Tolerancing refers to the selection of specifications for individual components using formal optimization. Specifications might relate to the acceptable length of a shaft, for example, or the acceptable resistance of a specific resistor in a printed circuit board.

Tolerance can be considered as physical or chemical characteristics (such as size, weight, tensile strength, specifications of the assembled components) or the geometric specifications (such as shape, dimensions, and surface roughness). Since it is not actually possible to manufacture many components as much accurate as the nominal or target value, the deviation from the nominal value is something inevitable. Therefore, for any product and service, it is necessary to consider the tolerance in addition to the nominal value.

Sometimes the engineer determines the limits of the product specifications more tightly than the condition in which the product can yield positive performance or being used by the customer. On the other hand, if therange of the product specifications limits is open, it will reduce the product performance. In such condition, the design specifications named as the tolerancing in order to meet the customer satisfaction and enhance product performance. (Devor et al. 2007).

Therefore, in practice:

- Engineers try to produceas many components aspossible at the acceptable range of variationsofquality characteristics.
- Managers rely on inspection plans to select the defective pieces and send the rest to the market.
- Customers suffer from poor quality products.

Some common methods for determining tolerance found in technical books and handbooks of engineering are as follows:

1. Determine the maximum tolerance that may still meet the practical needs.
2. Allocate tolerance so that the allocated tolerance is equal to or greater than the process capability.

Tolerance is determined based on engineering handbooks. As the tolerance of the process is determined sometimes less and other times more, many components either are produced as waste or have greater precision than needed. In any case, the total costs increase.

Statistical tolerancing is the study of the properties of an ensemble of components using assumed properties of the individual components. Stochastic optimization from can also be used to select the optimal combination of tolerances to achieve a variety of possible objectives.

When two or more components are assembled together, at present, the random selection of components for

*Corresponding author Email address: mmmovahedi@gmail.com

assembling and the property of the central limit theorem are not considered, and the total tolerance of several assembled components is considered equal to the total tolerance of each one. Considering the process capability to determine the tolerance of the components can also help to rationalize it. Now, if the probability relationships between components are used to determine the tolerance, process capabilities and product performance requirements are appropriately considered, and it is possible to consider the tolerance of each assembly component wider than the current value, so that the production of components becomes easier in practice, while the function of the assembled assembly is not reduced.

The use of statistical methods in the determination of component tolerance is not a new method (Chandra,2001). But only a few probability distributions have been used for this purpose. Therefore, when different conditions arise, users that most of them are mechanical or another related engineers, are less familiar with how to use different statistical distributions. In addition, design engineers, especially in Iran, rarely used this method. So the development of some statistical methods that designers can use to convince them and can cover a variety of situations is a necessity. Therefore, the first goal of this research is to introduce the statistical methods and its effective properties to the industry sector; and the second goal is to develop a new statistical method for situations where the use of a flexible distribution is needed. However, we have proposed Weibull distribution, which can be considered along with other statistical distributions, where other probability distributions are not suitable.

The paper consists of the following sections: literature review, tolerance introduction, discussion of Weibull distribution, research methodology, data analysis, results, Model Validation, and conclusion.

2. Literature Review

In terms of different states of tolerancing, we consider only the assembled components, in other words additive relationship.In condition where the two-way technical specification is considered, the tolerance can be defined as the difference between the upper and lower limits. Also when two or more components are assembled in which the components have a linear relation with each other, if we consider the tolerance of the component X_i as T_i, $i=1,2,\ldots,k$, then the tolerance of the X assembled component can be assumed as T. In such condition, we have:

If

$$X = X_1 \pm X_2 \pm X_3 \pm \ldots \pm X_k \tag{1}$$

Then

$$T = T_1 + T_2 + \ldots + T_k \tag{2}$$

This relationship which is an additive relationship can be used for the design of tolerance for each component of the set $T_1,\ldots,T_k$.

Through the consideration of relation between the technical specifications of the components with the assembled set, we consider the following assumptions to determine the components tolerance (Chandra 2001):

1. Components are independent of each other.
2. The selection and assembling of components are performed randomly.
3. The technical specification of each component with the mean μ and variance σ_i^2 follows the normal distribution.
4. The production process of each component is carried out under the statistical control and the mean technical specification of X_i is equal to the target or nominal value, that is,

$$\mu_i = \frac{(U_i - L_i)}{2} \tag{3}$$

5. The standard deviation of each componentthrough this process is performed in such a way that 99.73 % of any componentspecifications falls within its tolerancing range specifications.

$$U_i - L_i = T_i = 6\sigma_i \tag{4}$$

Let μ and σ^2 be the mean and variance of X respectively.As$X = X_1 \pm X_2 \pm X_3 \pm \ldots \pm X_k$, then

$$\mu = \mu_1 \pm \mu_2 \pm \mu_3 \pm \ldots \pm \mu_k \tag{5}$$

and as the X_i's are mutually independent,

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \ldots + \sigma_k^2 \tag{6}$$

As the components are randomly selected and assembled, according to central limit theorem the technical specification of the assembled set follows the normal distribution. If 99.73 % of each component technical specifications falls within its tolerance range of technical specifications that is L and U, then the equation (7) can be reached as following.

$$U - L = T = 6\sigma \tag{7}$$

Combining Eqs. (5), (6), and (7) yields:

$$T = \sqrt{T_1^2 + T_2^2 + \ldots + T_k^2} \tag{8}$$

The equation (8) is a probable relation, which provides the method of tolerancing the component of an assembled

set. Below different approaches and views related to the tolerance are discussed.

Fattahi et al (2005) suggest tolerance as a means to adjust control limits.In fact, if the components tolerance increases, the production costs will decrease and vice versa.

Devor et al. (2007) used the concept of loss function to determine the tolerance. By quantifying the loss function, they showed that the tolerance could be determined in such a way to minimize the loss function. They have taken into account the reduction of quality costs (including residual components, re-working and other costs). In addition, if necessary they have used different statistical distributions to determine the tolerance.

Sampath Kumar et al. (2009) attempted to determine the optimal tolerance in such a way to minimize the quality, manufacturing and assembly costs. They implemented a new global nonlinear optimization technique named as the Pattern search algorithm to allocate the optimal tolerance and reduce costs in order to overcome the existing tolerancing problems.

Macko et al. (2012) studied the negative effects of the trigger technical specifications tolerance. The trigger specifications are defined as the dependence between the trigger force and the force angle applied to it. Their study shows an example of a trigger mechanism, which is a type of Glock mechanism. The designers proposed the MW software as the solution to design the trigger technical specifications in small armies.

Kaisarlis, (2012) considered reverse engineering to determine the dimensions of tolerance. Since, tolerance is widely used in reverse engineering;the most research is done in this area.They argued that their proposed method is only an initial attempt to solve such RE problems, whichcan be directly implemented within the CAD environment.

Ginsberg(1981) presented an optical design and flowchart method putting emphases on tolerancing the components on the early stage of the design process. The authors have introduced the analysis tables development process whose application is to determine the tolerancing budget.

Hasenauer and Shannon (2013) used an optical technique instead of the mechanical methods to determine the tolerance. The optical technique requires an extensive cost, whichis usually applied in a condition where there are variations in components dimension. It determines the permissible changes by using the effects of the whole set of waves passing through the lens.

Zhang and Huq (1992) studied the status of the theory and practice of how the process of components manufacturing and assembly is shaped in order to reduce the costs related to the production process through determining their desired tolerance.

Wheeler et al. (1999) have developed a probability method to select an optimal subset of technology-based processes required to design the tolerance together with the under the control conventional strategy process. They presented an implicit enumeration approach to select an optimum subset of processes based on its required technology. They also presented a probabilistic approach

for the problem solution and used the first-order second moment method (FOSMM) to estimate the system performance related to the functional requirements.

Chandra (2001) has used the multiple probability distributions to determine the tolerance. He has also utilized the linear, nonlinear and dynamic planning while studying the variety of statistical tolerance determination techniques.

Mansuy et al (2013) has described a general method of tolerance analysis for the assembled mechanical systems. The innovation in this paper refers to the use of the mathematical tools including the generic algorithm to analyze the tolerance.

Movahedi et al (2016) proposed a five-parameter generalized lambda distribution to design tolerance.

Beaucaire et al (2013) showed that tolerance analysis could be used to assess product quality. Jianxin Yang.et al (2013) proposed a tolerancing model based on the variability of the assembly goals.

Yan, et al. (2015) used a Monte Carlo simulation for statistical analysis of a nonlinear tolerance. Geometric tolerancing with the nominal value of zero wasdesigned and Monte Carlo simulation was used to analyze the tolerance.

China, Zhanga and Lub (2016) introduced qualitative statistical method oftolerancing. New geometrical specifications of production and statistical process control (SPC) urged to develop a new statistical method of tolerancing to deal with the challenges of the modern manufacturing.

Alain Van Hoecke (2016) has pointed out the risks of using the statistical tolerancing tool to optimize the risks which threaten the customers and suppliers. He also showed that through the risk control we could increase the benefits of the statistical tolerancing.

Yan,et al (2016) presented a new method of tolerancing. At first produced *n* samples based on the distribution of each dimension, then, determined functional requirements according to the limits of their assembly for each sample, further established a homogeneous transformation matrix to describe the performance, and finallyused the Newton-Raphson method to solve the derived repetitive formula.

In spite of the abundant use of statistical methods, the normal distribution is the most important and most widely used statistical tool in tolerance design. However,to determine tolerance, we use Weibulldistribution that gives us more flexibility.

The main features of Weibull distribution include better tolerance design, realistic assessment, speed in obtaining tolerance, long-term tolerance analysis, and betterquality control.

Huang et al (2004) used Monte Carlo (MC) method prevails in statistical simulation approaches for multi-dimensional cases with general (i.e., non-Gaussian) distributions and/or complex response functions. A method is proposed based on number theory (NT-net) to reduce computing effort and the variability of MC's results in tolerance design and circuit performance simulation. NoorulHaq et al (2005) used genetic algorithm for tolerance design of machine elements. A probabilistic

approach has been used by Rout and Mittal (2007) to model the effects of noise factors and an experimental design technique has been adopted by them to select optimal tolerance of kinematic and dynamic parameters for minimal performance variations. Lee et al (2011) show that a higher resistance to cytotoxic drug (Ara-C) can be developed in the subpopulation of promyelocytic leukemia cells that survived radiation treatment. Armillotta (2016) has studied the role of the planar datum parameters on the tolerance analysis problems. In order to evaluate the translational and rotational errors related to the form tolerances usually neglected in the tolerance analysis, the relation between the datum planes has been simulated by a stochastic model, where two surface profiles are randomly generated and then are registered to reproduce a mating status. A simulation plan has provided a condition to predict the amount of the contact errors as a performance function of size, tolerance and process-related assumptions on two features. A practical example

$$F(x) = p(X \le x) = 1 - e^{-(\lambda x)^{\beta}} \qquad \text{For } x > 0$$

The corresponding probability density is:

$$f(x) = \frac{d}{dt} F(x) = \beta \lambda^{\beta} x^{\beta-1} e^{-(\lambda x)^{\beta}} \quad \text{For } x > 0$$

Where λ is a scale parameter and β is referred to as the shapeparameter. Note that these two parameters are the continuous parameters and their acceptable limits are $0 < \beta < \infty$ and $0 < \lambda < \infty$. By increasing β the distributiongets closer to λ .

The mean and the variance of the distribution can be calculated based on the following formulas (Rousand and Hoyland, 2004 p37-41):

$$E(X) = \mu = \frac{1}{\lambda} \Gamma(\frac{1}{\beta} + 1) \qquad (11)$$

And

$$\text{var}(X) = \sigma^2 = \frac{1}{\lambda^2} (\Gamma(\frac{2}{\beta} + 1) - \Gamma^2(\frac{1}{\beta} + 1)) \qquad (12)$$

Some important features of Weibull distribution are as follows:

1. Weibull distribution acts like an exponential distribution, when $\beta = 1$ and like a normal distribution, when $\beta = 2.5$

2. The distribution of a random variable that is defined as the minimum of several random variables, each have a different Weibull distribution

3. Weibull distribution is skewed to the right.

To obtain the mean and variance of the distribution first parameters must be estimated using parametric inference methods such as moments and maximum likelihood

shows that the effects of the shape errors may actually be related to the specific cases. They include the tolerance chains with contacts between the small-sized datum planes gathered through the production processes with a limited accuracy as well as variety of possibilities for matching those features.

3. Weibull Distribution

Weibull distribution is one of the most widely used distributions in reliability analysis. The distribution is named after the Swedish professor Waloddi Weibull (1887-1979) who developed the distribution for modeling the strength of materials. Weibull distribution is very flexible, and can, through an appropriate choice of parameters, model many types of failure rate behaviors.

The formula for the cumulative distribution function of2-parameter Weibull distribution is:

$$(9)$$

methods. In this study, the likelihood method is considered.

In all observations, method of maximum likelihood (ML) and statistical inference based on the ML plays the main role. If we use likelihood function based on observed samples $x_1, x_2, ..., x_n$, distribution parameters can be estimated through 13 and 14:

$$\lambda = (\frac{1}{n} \sum_{i=1}^{n} x_i^{\beta})^{-\frac{1}{\beta}} \qquad (13)$$

And

$$\hat{\beta}_{n+1} = \left(\frac{\sum_{i=1}^{n} x_i^{\hat{\beta}} \ln(x_i)}{\sum_{i=1}^{n} x_i^{\hat{\beta}n}} - \frac{1}{n} \sum_{i=1}^{n} \ln x_i \right) \qquad (14)$$

This equation can be solved by Newton-Raphson method. For example, if $\beta_\circ = 1$ the outcome of MLE for λ will be $\hat{\lambda}$. After calculating $\hat{\lambda}$ and $\hat{\beta}$, the mean and standard deviation can be calculated.

4. Research Methodology

The population of the study consists of specific industrial products used in the process of manufacturing and assembly. The sample of the study consists of the randomly selected components of the same product.

First,100 data randomly was generated. Then, assuming that the obtained data is distributed according to Weibull distribution, equations 13 and 14 were used to estimate distribution parameters by MATLAB software.Further, a goodness of fit testwas applied. Afterwards, the mean and variance of the distribution was determined, with the help of equations 11 and 12, that was used to determine tolerance. Finally, the results were assessed using a

Weibull Plot.and compared to the results of the previous studies.

5. Data Analysis and Results

To determine the mean and standard deviation of randomly generated data with Weibull distribution first β and then λ was estimated.

Table 1
Goodness of fit test for the first component (C1)

Category	Mean	Absolute frequency	The cumulative frequency (Y_i)	The relative cumulative frequency observed (O_i)	The expected relative cumulative frequency(E_i)	$(O_i - E_i)^2$	$\dfrac{(O_i - E_i)^2}{E_i}$
13.9501-13.9698	13.9599	22	22	0.22	0.1107	0.011946	0.1079
13.9698-13.9895	13.9796	14	36	0.36	0.2276	0.017529	0.0770
13.9895-14.0091	13.9993	25	61	0.61	0.4333	0.031222	0.0720
14.0091-14.0288	14.0189	18	79	0.79	0.7115	0.006162	0.0086
14.0288-14.0484	14.0386	21	100	1.00	0.9347	0.004264	0.0045
						total	0.2700

As a result parameters of the first component are $\beta = 559.664$ and $\lambda = 0.0713$. Chi-square test was applied to assess the results. Then, to calculate the expected relative cumulative frequency ((E_i)) the equation (9)was considered. Taking into account that $\chi^2(0.2700) < \chi^2_{(0.95,4)}(0.71)$ it can be concluded that the results in the Table 1 are valid.

Assuming that the nominal value of the second component (C_2) is 19 mm, 100"X"datahave beenrandomly generated between 18.95 to 19.05. These data are shown in the table (2). The parameters of the second component are $\beta = 776.141$ and $\lambda = 0.0525$. Chi-square test has been applied to test the results and as $\chi^2(0.1046) < \chi^2_{0(.95,4)}(0.71)$ the results are accurate. The expected relative cumulative frequency ((E_i)) was calculated according to the equation (9).

Table 2
Goodness of fit test for the second component (C_2)

Category	Mean	Absolute frequency	The cumulative frequency (Y_i)	The relative cumulative frequency observed (O_i)	The expected relative cumulative frequency (E_i)	$(O_i - E_i)^2$	$\dfrac{(O_i - E_i)^2}{E_i}$
18.9510-	18.9608	19	19	0.19	0.1186	0.00509	0.0429
18.9706-	18.9804	16	35	0.35	0.2455	0.01092	0.0444
18.9901-	18.9999	20	55	0.55	0.4661	0.00703	0.0150
19.0097-	19.0195	20	75	0.75	0.7512	0.00001	0.0001
19.0293-	19.0360	25	100	1.00	0.9541	0.00210	0.0022
						total	0.1046

Consequently, technical specifications of the components can be calculate based on the obtained results. Thus, having $\beta = 559.664$ and $\lambda = 0.0713$ for the first component the mean and variance are calculated according to the formulas (11) and (12) as indicated below:

$$\mu_1 = \frac{1}{\lambda}\Gamma(1+\frac{1}{\beta})$$

$$= \frac{1}{0.0713}\Gamma(1+\frac{1}{559.664})$$

$$= 13.998$$

And

$$var(X_1) = \frac{1}{\lambda^2}(\Gamma(\frac{2}{\beta}+1)-\Gamma^2(\frac{1}{\beta}+1))$$

$$= \frac{1}{(0.0713)^2}(\Gamma(\frac{2}{559.664}+1)-\Gamma^2(\frac{1}{559.664}+1))$$

$$= 0.0103$$

The result will be $\sigma_1 = 0.032$. Figure 1 presents Weibull plot of the first component obtained by the MINITAB software.
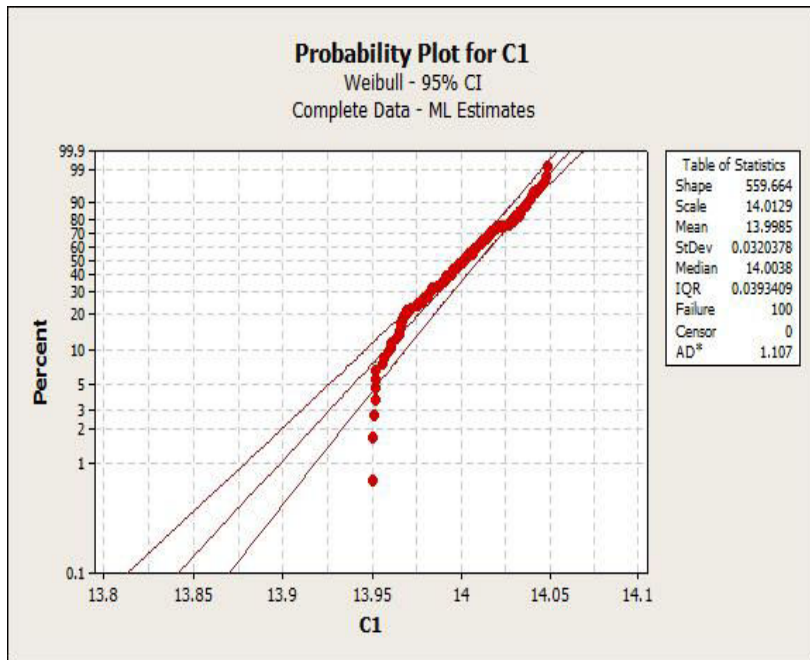

Fig. 1.WeibullPlot of the first component (C_1)

Then, having $\beta = 776.141$ and $\lambda = 0.0525$ for the second component the mean and variance are calculated according to the formulas (11) and (12) as follows:

$$\mu_1 = \frac{1}{\lambda}\Gamma(1+\frac{1}{\beta})$$

$$= \frac{1}{0.0525}\Gamma(1+\frac{1}{776.141})$$

$$= 19.025$$

$$var(X_1) = \frac{1}{\lambda^2}(\Gamma(\frac{2}{\beta}+1)-\Gamma^2(\frac{1}{\beta}+1))$$

$$= \frac{1}{(0.0525)^2}(\Gamma(\frac{2}{776.141}+1)-\Gamma^2(\frac{1}{776.141}+1))$$

$$= 0.000984$$

The result will be $\sigma_1 = 0.031$. Figure 2 presents Weibull plot of the second component obtained by the MINITAB software.
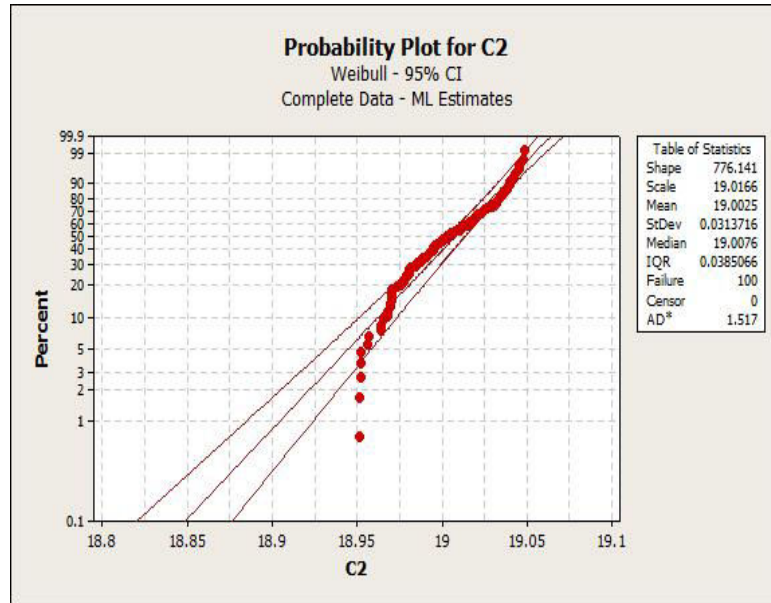
Fig. 2. Weibull Plotof the second component (C_2)

Assuming that the nominal valueis 14 and 18 mm for the first and second components respectively, the tolerance of the two components will be calculated according to the equation (7):

First component (C_1): $T_1 = 6 \times 0.032 = 0.192$

Second component (C_2): $T_2 = 6 \times 0.031 = 0.186$

Consequently, technical specifications for the components can be determined as follows:

First component (C_1): $C1 = 14 \pm 0.096$

Second component (C_2): $C2 = 19 \pm 0.083$

If the two components are assembled based on Central Limit Theorem, the mean is equal to the sum of the averages:

$$\mu = \mu_1 + \mu_2$$

$$= 14 + 19 = 33$$

Based on Central Limit Theorem and formula (8) tolerance of the assembled components is calculated as follows:

$$T = \sqrt{T_1^2 + T_2^2}$$

$$= \sqrt{(0.192)^2 + (0.186)^2} = 0.267$$

If we use the additive relationship and assume that the tolerance of the components is equal, we can calculate tolerance of each component as follows:

$$0.267 \div 2 = 0.133$$

Thus, toleranceis 0.192 and 0.186 for C_1 and C_2 respectively; that is both are more than 0.133. Accordingly, more tolerance can be attributed to $C_1$and C_2, which will lead to production process improvement and will dramatically reducewaste and costs.

6. Model Validation

Where the use of real data is impossible or difficult, simulation is a powerful method in this study. So, simulation has been used to validate the proposed method. To this end, five more random data sets have been generated. Each set consists of two randomly assigned 100 series data sets. It has been assumed that these data are quality characteristic of the components manufactured. The proposed method is then applied to them, and the results are summarized in table 3 with the results of the previous random data sets.. The results show that in all cases the total tolerance is higher than the traditional methods for tolerancing.

Table 3
Summarized simulation results

No. of assemble	Part No.	Target value	β	λ	μ	σ^2	Statist. Tolerance	Statist. Tolerance
1	First	14	559.664	0.0713	13.988	0.032	0.192	0.133
	Second	19	776.141	0.0525	19.025	0.031	0.186	0.133
2	First	20	744.5	0.0499	20.005	0.031	0.186	0.161
	Second	13	317.3	0.0769	12.997	0.044	0.264	0.161
3	First	16	408.3	0.0625	15.997	0.042	0.252	0.158
	Second	24	837.7	0.0417	24.000	0.032	0.192	0.158
4	First	17	481.3	0.0589	16.999	0.040	0.240	0.152
	Second	18	618.9	0.0555	17.999	0.031	0.186	0.152
5	First	26	643.5	0.0384	25.997	0.045	0.270	0.141
	Second	33	2600	0.0303	33.000	0.014	0.084	0.141
6	First	11	287.5	0.0909	11.001	0.041	0.246	0.144
	Second	25	1137	0.0400	24.999	0.025	0.150	0.144

7. Conclusion

The literature review of the topic shows that in traditional methods for statistical tolerancing, a few statistical distributions have been proposed and implemented; however, when we faced with specific situations, a flexible statistical distribution can help us in statistical tolerancing. In addition, design engineers are less familiar with the statistical tolerancing.This study was carried out to introduce this method to designer engineers, and to introduce a flexible statistical distribution for statistical tolerancing. So, we offers Weibull statistical distribution as an alternative to the traditional methods for statistical tolerancing. We demonstrated that tolerance could be designed much faster by this method.

Further,simulation method, andMinitab software were used for data processing and the Minitab software was used to draw the graph.However, it could not be achieved because of the lack of manufacturing data.

In this paper, we propose the use of Weibul Distribution as an alternative method to consider the process capability and central limit theorem. In addition, the introduction of components tolerancing with statistical methods was another major goal of this article that we have achieved. The proposed method could be used to determine the tolerance of the components in a way that is more open than conventional methods. In addition, when the process of assembly of two or more components is considered, this method can be considered as more tolerance for each piece, while the product has the same function.

The research limitation includes the use of random data and assumptions.However, simulation results with six pairs of components show the performance of the proposed method. In addition, one of the goals of this research was to offer a method for cost, losses, reworking, and time reduction.

To conclude this paper we have proposed flowchart for the tolerance design using Weibull distribution. The chart can befound in the Appendix 1.

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Appendix1. Proposed flowchart for tolerance design with Weibull distribution

```
                    ┌─────────┐
                    │  Start  │
                    └─────────┘
                         │
          ┌──────────────────────────────┐
          │ Make sure the process is       │
          │ under statistical control      │
          └──────────────────────────────┘
```

Start

Make sure the process is under statistical control

Take a sufficient number of random samples from the production process

Calculate β and λ with following formula and MATLAB software

$$\textbf{And } \lambda = \left(\frac{1}{n}\sum_{i=1}^{n} x_i^{\beta}\right)^{-\frac{1}{\beta}} \quad \hat{\beta}_{n+1} = \left(\frac{\sum_{i=1}^{n} x_i^{\hat{\beta}} \ln(x_i)}{\sum_{i=1}^{n} x_i^{\hat{\beta}n}} - \frac{1}{n}\sum_{i=1}^{n} \ln x_i\right)$$

Run Chi-square test to ensure the accuracy of the results (β and λ)

Calculate the components' mean and variance by the following equations:

$$\textbf{and } \mathrm{var}(X) = \frac{1}{\lambda^2}\left(\Gamma\left(\frac{2}{\beta}+1\right) - \Gamma^2\left(\frac{1}{\beta}+1\right)\right) \quad \mu = \frac{1}{\lambda}\Gamma\left(\frac{1}{\beta}+1\right)$$

Consider the target value as technical specification

No

The process mean is equal to the specified value

Consider the technical specification as the process mean

Calculate the tolerance by $T = 6\sigma$

No

Use handbooks

Determined tolerance is equal to the proposed tolerance or tolerance of handbooks.

Consider calculated tolerance as the basis for technical characteristics

Periodically examine your tolerance

188

Finish