Trajectory Planning Using High-Order Polynomials under Acceleration Constraint

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Abstract

The trajectory planning, known as a movement from starting point to ending point by satisfying the constraints along the path, is an essential part of robot motion planning. A common way to create trajectories is to deal with polynomials which have independent coefficients. This paper presents a trajectory formulation as well as a procedure to arrange the suitable trajectories for applications. Created trajectories are aimed to be used for safe and smooth navigation in mobile robots. First, a trajectory problem is formulized by considering a border on the robot's acceleration as the constraint. Also, initial and final conditions for the robot's velocity along the straight path are settled. To investigate the idea that suggested trajectories perform motions with continuous velocity and smooth acceleration, three trajectory problems are formulated using 3rd, 4th, and 5th degrees of polynomials. The high-degree polynomials are used because of providing of smoothness, but there is complexity in the calculation of additional coefficients. To reduce the complexity of finding the high-degree polynomial coefficients, the acceleration constraint is simplified and this process is based on certain scenarios. Afterwards, the coefficients of the used polynomials are found by taking into account the acceleration constraint and velocity conditions. Additionally, to compare the obtained solutions through proposed scenarios, the polynomials` coefficients are solved numerically by Genetic Algorithm (GA). The computer simulation of motions, as well as acceleration constraint, shows that the velocity conditions at the beginning and at the end of motion are fulfilled.

Keywords: Motion planning, Trajectory planning, High-order polynomials, Velocity conditions, Acceleration constraint, Genetic algorithm

1. Introduction

Trajectory planning is related to determining the robot's position, velocity, and acceleration during the motion time. However, when a mobile robot should traverse along a given path, there are infinite possible trajectories that the robot can run. Although only finite numbers of them are appropriate to run in applications. This paper focuses on trajectory planning for point-to-point motion by considering velocity and/or acceleration constraints at the initial and final points, as well as along the path. The trajectory planning taken in hand in here is similar to the vehicle trajectory in optimal control theory. Assume that a car moves through a linear or circular path. The problem here is related to how driver presses accelerator pedal to minimize the total driving time and to maximize the total travelling distance. In such kind of control problems, there exist constraints related to speed and acceleration limits. The trajectory planning taken in hand is similar to the car example, where we used a high-order polynomial as mathematical expression of travelling time and distance in the role of objective function under the velocity and acceleration constraints.

In the robotic studies, trajectory planning is investigated in many research papers. Elnagar et al. (2000) studied smooth piece-wise trajectories considering acceleration constraint. Choi et al. (2001) studied the near-time-optimal trajectory considering battery voltage and obstacle avoidance. Lepetic et al. (2003) studied the time minimizing in the spline curve path. Naguyen et al. (2007) studied polynomial s-curve trajectories. Haddad et al. (2007) focused on trajectories with limited velocities, accelerations, and torques as well as obstacle avoidance. Kardos et al. (2009) considered the trajectories composed of straight-line, circular segments, and continuous-curvature segments. Boryga et al. (2009) planned the trajectory in the form of higher-degree polynomials for serial-link robot manipulators. Korayem et al. (2013) formulated the trajectory based on the dynamic potential function model.
Jond et al. (2014) presented polynomial trajectories from third-order in closed-form solutions. Minh et al. (2014) proposed three trajectory generation techniques including flatness, polynomial, and symmetric polynomial trajectories subject to the vehicle constraints, and

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consequently the third-order symmetric polynomial trajectory was recommended. Zhang et al. (2015) proposed a polynomial trajectory model to be used in the vehicles parking schemes. The model contains multiple constrains related to the parking space parameters as well as the vehicle velocity. Chang et al. (2015) computed a polynomial trajectory considering the continuity of a path using differential values and simple matrix calculation on the curvature. Carbone et al. (2015) considered the trajectory planning with point mass double integrator model. Solving trajectories with respect to constraints as well as conditions is a complicated task. Perhaps, evolutionary methods can be used to solve such problems. Huang et al. (2006) used PSO to search for the global time-optimal trajectories. Barghijand et al. (2011) solved a trajectory optimization problem by the Genetic Algorithm (GA).

This paper aims to find suitable and possibly near-optimal high-order polynomial trajectories for point-to-point motion with application in mobile or manipulator robots. These trajectories consider velocity and/or acceleration constraints at the initial and final points, as well as along the path. Three trajectory planning problems are proposed containing polynomials from 3rd, 4th, and 5th degrees considering acceleration constraint and velocity conditions. The rest of this paper is organized as follows. The next section describes how the three problems are formulated, and then how the suitable trajectories for the robot are obtained. The simulation results and the discussions are provided in Section 3. The last section includes conclusions.

2. Trajectory Planning Strategy

In this section, three trajectory optimization problems are defined. These problems take into account the boundary conditions for velocity as well as the acceleration constraint. By fulfilling the velocity and acceleration limits, it is expected that generated trajectories satisfy a motion with the below properties; The robot starts to move from origin on and stops gently at the end of time or path, while acceleration/deceleration of the robot is limited to safe navigation.

In accordance with the motion strategy mentioned above, a trajectory optimization problem can be formulated with a high-degree polynomial subject to the velocity boundary condition and the acceleration constraint as follows.

Optimization Problem: Assume that the final time of the motion t_f is given; it is desired to find suitable λ_i's, such that the robot will cover the possible maximum distance when the constraint and boundary conditions are fulfilled. Therefore,

$$max_{\lambda_1..\lambda_{n-1}} q(t) = \sum_{i=1}^{n-1} \lambda_i t^{n-i+1} \quad (1)$$

$$s.t.\ \ \dot{q}(t_f) = 0 \quad (2)$$

$$|\ddot{q}(t)| \le \Phi \quad (3)$$

where λ_i is the coefficient, $t \in [0, t_f]$ is the time, t_f is the trajectory final time, *max q*(*t*) is the objective function that shows traveled distance, Eq. (2) shows the boundary condition which implies zero velocity at the end of the trajectory, and Eq. (3) specifies the acceleration constraint. Also, $n \ge 3$ is the degree of the polynomial and Φ is a positive constant.

To run a smooth and continuous motion, it is needed to use at least a 3rd degree polynomial trajectory. However, high-degree polynomials lead to better smoothness for safe navigation but slower trajectories and vice versa. The defined optimization problem is not often tractable; therefore, the polynomial coefficients cannot be often obtained, particularly when the degree and the terms of the polynomial are large. In the next subsection, maximization problem includes a 3rd degree polynomial objective, and the procedure of finding suitable coefficients is presented.

2.1. 3rd degree Polynomial Trajectory

The optimization problem for 3rd degree polynomial trajectory can be as below.

$$max_{\lambda_1,\lambda_2} q(t) = \lambda_1 t^3 + \lambda_2 t^2 \quad (4)$$

$$s.t.\ \ 3\lambda_1 t_f + 2\lambda_2 = 0 \quad (5)$$

$$|6\lambda_1 t + 2\lambda_2| \le \Phi \quad (6)$$

The trajectory should be optimized and be subject to Eq. (5) and Eq. (6). These equations imply that the initial and final velocities are given as zero, and absolute value of acceleration is limited with a positive boundary. Here, the goal is to determine the polynomial coefficients ($\boldsymbol{\lambda_1}$ and $\boldsymbol{\lambda_2}$) in order to obtain the optimal trajectory. An approach is given below to find the solutions for $\boldsymbol{\lambda_1}$ and $\boldsymbol{\lambda_2}$.

For 3rd degree polynomial, the acceleration is linear, then it reaches the maximum or minimum values at beginning or ending of the time interval $[0, t_f]$. Therefore, when the acceleration constraint is fulfilled at *t=0* and *t=t_f*, it would be satisfied for all time of the interval. In other words, the acceleration constraint can be reordered at *t=0* and *t=t_f* as below:

$$|2\lambda_2| \le \Phi\ ,\ |6\lambda_1 t_f + 2\lambda_2| \le \Phi \quad (7)$$

The boundary values for λ_2 are obtained as below:

$$\lambda_2 \le \pm\frac{\Phi}{2} \quad (8)$$

Choosing the positive boundary value for λ_2 , the maximum value of the trajectory function is achieved. In this case, the polynomial coefficients are as follows:

$$\lambda_2 = \frac{\Phi}{2}\ ,\ \lambda_1 = -\frac{\Phi}{3t_f} \quad (9)$$

Also, the negative value for λ_2 could be chosen in the Eq. (8). However, it leads to obtaining negative values for traveled distance, and it is an impossible state for trajectories. The discussed 3rd degree polynomial trajectory is obtained as below.

$$q(t) = -\frac{\Phi}{3t_f}t^3 + \frac{\Phi}{2}t^2 \; , t \in [0 \quad t_f] \quad (10)$$

As it is seen, the 3rd degree polynomial solution is obtained. However, dealing with the higher-degree polynomials is difficult, because the number of possible scenarios increases exponentially.

2.2. 4th -degree Polynomial Trajectory

The optimization problem for 4th degree polynomial can be defined as follows:

$$max_{\lambda_1..\lambda_3} q(t) = \lambda_1 t^4 + \lambda_2 t^3 + \lambda_3 t^2 \quad (11)$$

$$s.t. \;\; 4\lambda_1 t_f^2 + 3\lambda_2 t_f + 2\lambda_3 = 0 \quad (12)$$

$$|12\lambda_1 t^2 + 6\lambda_2 t + 2\lambda_3| \le \Phi \quad (13)$$

Here, the problem is complicated and the solution cannot be found by classical methods. To handle the problem, we propose an approach. The main idea is to simplify the acceleration constraint in Eq. (13). By this approach, we would only expect to find the near-optimal solutions.

Firstly, the nonlinear Eq. (13) can be reduced to two linear inequalities. In other words, Eq. (13) can be reordered at t=0 and t=t_f as below.

$$|2\lambda_3| \le \Phi \; , \left|12\lambda_1 t_f^2 + 6\lambda_2 t_f + 2\lambda_3\right| \le \Phi \quad (14)$$

Afterwards, we consider Eq. (14) instead of Eq. (13). Here, we can choose a number of values for the right-hand side of Eq. (14) in order to convert the inequalities to equalities. Each chosen value establishes a scenario, and each scenario leads to finding a solution. However, the number of values which can be chosen for the right-hand side of Eq. (14) is infinite; they can be considered among the fractional values within the interval$[-\Phi, +\Phi]$. Table 1 shows the problem solutions for some chosen values. Each solution gives a trajectory. Based on the maximum traveled distance generated by each trajectory during a given time, the suitable trajectories and also the suitable scenarios can be distinguished.

It is seen from Table 1 that the scenario (profile) number 2 is more suitable among the other scenarios. The scenario number 1 leads to losing one order of the polynomial. Among the rest of scenarios, the one that generates the maximum traveled distance can be a suitable choice for 4th degree polynomial trajectory. By the scenario number 2, the acceleration must be arranged at times t=0 and t=t_f to 0.5Φ and –Φ, respectively. Afterwards, the acceleration nonlinear constraint can be replaced with two linear inequalities that are given as follows:

$$\ddot{q}(0) \le \frac{\Phi}{2} \;\; \Rightarrow \;\; 2\lambda_3 \le \frac{\Phi}{2} \quad (15)$$

$$\ddot{q}(t_f) \le -\Phi \;\; \Rightarrow \;\; 12\lambda_1 t_f^2 + 6\lambda_2 t_f + 2\lambda_3 \le -\Phi \quad (16)$$

Solving the system containing Eq. (15) and Eq. (16) in equality condition, we can obtain:

$$\lambda_1 = -\frac{\Phi}{8t_f^2} \; , \lambda_2 = 0, \lambda_3 = \frac{\Phi}{4} \quad (17)$$

Therefore, a suitable 4th degree polynomial trajectory is obtained as follows:

$$(t) = -\frac{\Phi}{8t_f^2}t^4 + \frac{\Phi}{4}t^2 \quad , t \in [0 \quad t_f] \quad (18)$$

Table 1

Some possible scenarios with corresponding solutions for the 4th degree polynomial trajectory

Sce. No.	$\ddot{q}(0)$	$\ddot{q}(t_f)$	λ_1	λ_2	λ_3	max $q(t_f)$
1	Φ	- Φ	zero	-0.333 (Φ/t_f)	0.5Φ	0.166 Φt_f^2
2	0.5Φ	- Φ	-0.125 (Φ/t_f^2)	zero	0.25Φ	0.125 Φt_f^2
3	zero	- Φ	-0.25(Φ/t_f^2)	0.333 (Φ/t_f)	zero	0.083 Φt_f^2
4	-0.5Φ	- Φ	-0.375(Φ/t_f^2)	0.666 (Φ/t_f)	-0.25Φ	0.041 Φt_f^2
5	- Φ	- Φ	-0.5(Φ/t_f^2)	Φ/t_f	-0.5Φ	zero

2.3. 5th -degree Polynomial Trajectory

The optimization problem for 5th degree polynomial is formulated as follows:

$$max_{\lambda_1..\lambda_4} q(t) = \lambda_1 t^5 + \lambda_2 t^4 + \lambda_3 t^3 + \lambda_4 t^2 \quad (19)$$

$$s.t. \;\; 5\lambda_1 t_f^3 + 4\lambda_2 t_f^2 + 3\lambda_3 t_f + 2\lambda_4 = 0 \quad (20)$$

$$|20\lambda_1 t^3 + 12\lambda_2 t^2 + 6\lambda_3 t + 2\lambda_4| \le \Phi \quad (21)$$

Reordering the constraint given in Eq. (24) at t=0, t=t_c, and t=t_f

$$|2\lambda_4| \le \Phi \; , |20\lambda_1 t_c^3 + 12\lambda_2 t_c^2 + 6\lambda_3 t_c + 2\lambda_4| \le \Phi, \left|20\lambda_1 t_f^3 + 12\lambda_2 t_f^2 + 6\lambda_3 t_f + 2\lambda_4\right| \le \Phi \quad (22)$$

where t_c is the critical point of the acceleration function. It is well-known that a function reaches its maximums or minimums at the critical points. Therefore, when the acceleration constraint inequality is held at the critical point(s) as well as at t=0 and t=t_f, then it will be held in all instances of interval [0, t_f]. Table 2 shows the problem solutions for 15 chosen states for the mentioned times (t=0, t=t_c, and t=t_f). It is seen from the table that the scenario number 6 is a suitable case. Using this scenario, the acceleration must be arranged at times t=0 and t=t_f to 0.5Φ and –Φ, respectively. Also, t_c is arranged to $\frac{3t_f}{4}$. Therefore, Eq. (21) can be replaced with two linear inequalities as well as a linear equality as below.

$$\ddot{q}(0) \le \frac{\Phi}{2} \;\; \Rightarrow \;\; 2\lambda_4 \le \frac{\Phi}{2} \quad (23)$$

$\ddot{q}(t_c) = \ddot{q}\left(\frac{3t_f}{4}\right) = 0 \quad \Rightarrow \quad \frac{135}{16}\lambda_1 t_f^3 + \frac{27}{4}\lambda_2 t_f^2 + \frac{9}{2}\lambda_3 t_f + 2\lambda_4 = 0$ (24)

$\ddot{q}(t_f) \le -\Phi \quad \Rightarrow \quad 20\lambda_1 t_f^3 + 12\lambda_2 t_f^2 + 6\lambda_3 t_f + 2\lambda_4 \le -\Phi$ (25)

Consequently, solving a system containing Eq. (23), Eq. (24), and Eq. (25) all in equality condition, the problem solutions will be obtained as follows:

$\lambda_1 == -\frac{11\Phi}{30t_f^3}, \lambda_2 = \frac{19\Phi}{24t_f^2}, \lambda_3 = -\frac{11\Phi}{18t_f}, \lambda_4 = \frac{\Phi}{4}$ (26)

Finally, a suitable 5th degree polynomial trajectory is obtained as in Eq. (27).

$q(t) = -\frac{11\Phi}{30t_f^3}t^5 + \frac{19\Phi}{24t_f^2}t^4 - \frac{11\Phi}{18t_f}t^3 + \frac{\Phi}{4}t^2 \quad , \quad t \in [0 \quad t_f]$ (27)

3. Results and Discussions

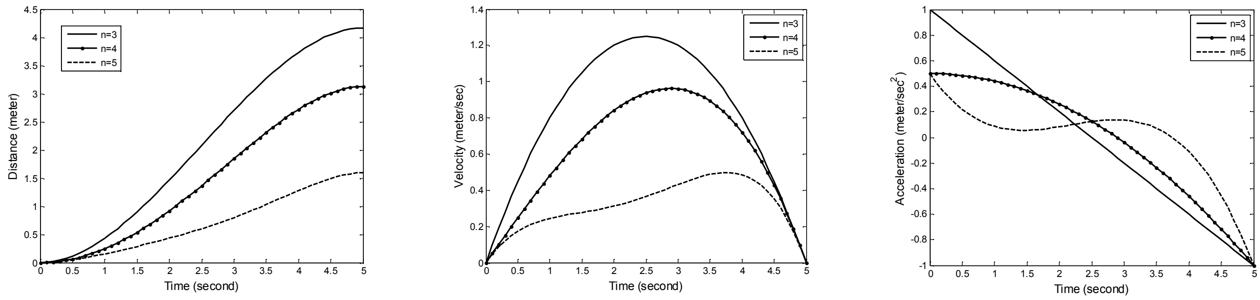
This paper discusses planning high-degree polynomial trajectories with application for robots. Safe navigation imposes limitation on the acceleration of the robot, and it is shown as constraints inside the trajectory problems. Three trajectory problems were formulated with polynomials of 3rd, 4th, and 5th degrees. These polynomials' coefficients were founded through a number of scenarios presented.

Here, the computer simulation should be carried out to investigate if the obtained (suitable) trajectories satisfy the velocity conditions as well as the acceleration constraint during a motion. The profiles that the obtained trajectories generate for the robot's position, velocity, and acceleration are shown in Fig. 1. In these computer simulations, the motion parameters are assumed as $\Phi = 1m/s^2$ and t_f=5*sec*. The motion simulations show that the velocity conditions, as well as acceleration constraint, at the beginning and at the end of motion are fulfilled. As seen in Fig. 1a, 1b, and 1c, the travelled distance decreases as the degrees of the polynomials increase; the velocity boundary conditions are satisfied in the initial (t=0) and the final times ($t = t_f$), and the acceleration constraint is satisfied during the whole motion time.

Additionally, the mentioned three trajectory problems are solved through Genetic Algorithm Toolbox of MATLAB in order to look for optimal solution for the polynomials' coefficients. The GA results are shown in Table 3. When both results are compared in this Table, it is explicit that the scenario-based solutions are an approach to the GA outputs. For the problem of 5th degree polynomial, GA is unable to reach a solution due to local minima.

Table 2
Fifteen possible scenarios with corresponding solutions for the 5th degree polynomial trajectory

Sce. No.	$\ddot{q}(0)$	$\ddot{q}(t_c)=0$	$\ddot{q}(t_f)$	λ_1	λ_2	λ_3	λ_4	max $q(t_f)$
1	Φ	t_c=0.25t_f	-Φ	-0.533 (Φ/t_f^3)	1.333 (Φ/t_f^2)	-1.222 (Φ/t_f)	0.5Φ	0.077 Φt_f^2
2	Φ	t_c=0. 5t_f	-Φ	zero	zero	-0.333 (Φ/t_f)	0.5Φ	0.166 Φt_f^2
3	Φ	t_c=0.75t_f	-Φ	-0.533 (Φ/t_f^3)	1.333 (Φ/t_f^2)	-1.222 (Φ/t_f)	0.5Φ	0.077 Φt_f^2
4	0.5Φ	t_c=0.25t_f	- Φ	-0.433 (Φ/t_f^3)	0.958 (Φ/t_f^2)	-0.722 (Φ/t_f)	0.25Φ	0.052 Φt_f^2
5	0.5Φ	t_c=0. 5t_f	-Φ	zero	zero	-0.333 (Φ/t_f)	0.5Φ	0.166 Φt_f^2
6	0.5Φ	t_c=0.75t_f	-Φ	-0.366 (Φ/t_f^3)	0.791 (Φ/t_f^2)	-0.611 (Φ/t_f)	0.25Φ	0.063 Φt_f^2
7	zero	t_c=0.25t_f	-Φ	-0.333 (Φ/t_f^3)	0.583 (Φ/t_f^2)	-0.222 (Φ/t_f)	zero	0.027 Φt_f^2
8	zero	t_c=0. 5t_f	-Φ	zero	zero	-0.333 (Φ/t_f)	zero	-0.333 Φt_f^2
9	zero	t_c=0.75t_f	-Φ	-0.2 (Φ/t_f^3)	0. 25 (Φ/t_f^2)	zero	zero	0.050 Φt_f^2
10	-0.5Φ	t_c=0.25t_f	-Φ	-0.233 (Φ/t_f^3)	0.208 (Φ/t_f^2)	0.277 (Φ/t_f)	-0.25Φ	0.002 Φt_f^2
11	-0.5Φ	t_c=0. 5t_f	-Φ	zero	zero	-0.333 (Φ/t_f)	-0.25Φ	-0.583 Φt_f^2
12	-0.5Φ	t_c=0.75t_f	-Φ	-0.033 (Φ/t_f^3)	-0.291 (Φ/t_f^2)	0.611 (Φ/t_f)	-0.25Φ	0.036 Φt_f^2
13	-Φ	t_c=0.25t_f	-Φ	-0.133 (Φ/t_f^3)	-0.166 (Φ/t_f^2)	0.777 (Φ/t_f)	-0. 5Φ	-0.022 Φt_f^2
14	-Φ	t_c=0. 5t_f	-Φ	zero	zero	-0.333 (Φ/t_f)	-0. 5Φ	-0.833 Φt_f^2
15	-Φ	t_c=0.75t_f	-Φ	0.066 (Φ/t_f^3)	-0.833 (Φ/t_f^2)	1.222 (Φ/t_f)	-0. 5Φ	-0.044 Φt_f^2



(a) (b) ©

Fig. 1. Plots of the trajectories for, (a) position, (b) velocity and (c) acceleration

Table 3
Scenario-based solutions versus GA optimal solutions

n	Scenario Based Solutions					GA Solutions				
	λ_1	λ_2	λ_3	λ_4	max $q(t_f)$	λ_1	λ_2	λ_3	λ_4	max $q(t_f)$
3	-0.0666	0.5000	-	-	4.1666	-0.0667	0.5005	-	-	4.1792
4	-0.0050	zero	0.2500	-	3.1250	-0.0042	-0.0104	0.2893	-	3.2969
5	-0.0029	0.0316	-0.1222	0.2500	1.5972	No feasible solution found				

4. Conclusion

This study proposes a practical formulation for trajectory planning problem denoted by 3rd, 4th, and 5th degrees of polynomials. The formulation ensures that during the generated trajectories, the robot moves according to a bounded acceleration in order to have a safe navigation. The formulation also is taken into consideration of the velocity conditions. In this research, an approach based on scenarios is presented to find the coefficients of these polynomials. By simulating the trajectories by means of the obtained solutions, the consistency of the proposed scenarios is also investigated. The graphs of velocity and acceleration (shown in Fig. 1) of the generated trajectories show that resulting polynomials satisfy considered constraints and conditions. Additionally, suitability of the obtained solutions against the numerical optimal solutions of GA is clearly observed in this work (in Table 3).

References

Barghijand, H., Akbarimajd, A. and Keighobadi, J. (2011). Quasi-Static Object Manipulation by Mobile Robot: Optimal Motion Planning Using GA. Proceedings of 11th International Conference on Intelligent Systems Design and Applications, IEEE Press, 202–207.

Boryga, M. and Grabos, A. (2009). Planning of manipulator motion trajectory with higher-degree polynomials use. Mechanism and Machine Theory, 44, 1400–1419.

Carbone, G. and Gomez-Bravo, F. (2015). Off-line and on-line trajectory planning, Springer International Publishing Switzerland. Motion and Operation Planning of Robotic Systems, Mechanisms and Machine Science, 29, 29-62.

Chang, S. and Huh, U. (2015). G2 Continuity Smooth Path Planning using Cubic Polynomial Interpolation with Membership Function. *Journal of Electrical Engineering & Technology*, 10, 676-687.

Choi, J.S. and Kim, B.K. (2001). Near-Time-Optimal Trajectory Planning for Wheeled Mobile Robots with Translational and Rotational Sections. IEEE Transactions on Robotics and Automation, 17, 85-90.

Elnagar, A. and Hussein, A. (2000). On optimal constrained trajectory planning in 3D environments. *Robotics and Autonomous Systems*, 33, 195–206.

Haddad, M., Chettibi, T., Hanchi, S. and Lehtihet, H.E. (2007). A random-profile approach for trajectory planning of wheeled mobile robots. European Journal of Mechanics A/Solids, 26, 519–540.

Huang, P. and Xu, Y. (2006). PSO-Based Time-Optimal Trajectory Planning for Space Robot with Dynamic Constraints. Proceedings of the 2006 IEEE International Conference on Robotics and Biomimetics, IEEE Press, 1402-1407.

Jond, H.B., Akbarimajd, A. and Ozmen, N.G. (2014). Time-Distance Optimal Trajectory Planning For Mobile Robots On Straight And Circular Paths. *Journal of Advances in Computer Research*, 5, 23-36.

Kardos,E.S. and Kiss, B. (2009). Continuous-curvature paths for mobile robots. *Periodica polytechnic Electrical Engineering*, 53, 63-72.

Korayem, M.H., Nazemizadeh, M. and Rahimi, H.N. (2013). Trajectory optimization of nonholonomic mobile manipulators departing to a moving target amidst moving obstacles. *Acta Mechanica*, 224, 995-1008.

Lepetic, M., Klancar, G., Skrjanc, I., Matko, D., et. al. (2003). Time optimal path planning considering acceleration limits, Robotics and Autonomous Systems , 45, 199–210.

Minh, V.T. and Pumwa, J. (2014). Feasible Path Planning for Autonomous Vehicles. Mathematical Problems in Engineering, 2014,1-12.

Naguyen, K.D., Chen, I.M. and Ng, T.Ch. (2007). Planning algorithms for s-curve trajectories. Proceedings of the IEEE/ASME International conf. on Advanced Intelligent Mechatronics, IEEE Press, 1-6.

Zhang, D., Li, S., Yang, Q. and Liu, L. (2015). Optimization Based Trajectory Planning of Parallel Parking with Multiple Constraints. *SAE International Journal of Passenger Cars - Electronic and Electrical Systems*, 8, 413-418.

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