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Development of a Novel Lot-sizing Model with Variable Lead Time in Supply Chain Environment

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Abstract

Supply chain management (SCM) addresses the management of materials and information across the entire chain from suppliers to producers, distributors, retailers, and customers. The theory of supply chain management suggests that lead time reduction is the pioneer of using market mediation to reduce transaction uncertainty in the chain, which can be conceptualized as the primary goal of supply chain management. In the past few decades, scholars have had to place attention on the impact of inventory on SCM. This paper is related to the development of a lot-sizing model for a single-component multiple-delivery system with variable demand and lead time of a multinational transformer company. Two models were developed. For the first model, distribution of demand is considered as normal, distribution of procurement lead time is exponential, and the quantity is coming in a single lot. For the second model, distribution of demand is normal, and procurement and administrative delay lead time is exponential, and the quantity is coming in a single lot.
Modification of the first model has been incorporated by taking the effect of multiple deliveries on the models and correcting the Re-order point as obtained from the previous models. The results and analysis by the second model have been done for different parametric conditions. The effect of multiple deliveries is also taken into account. The optimum re-order point and economic ordering quantity with various different inputs have been discussed.

Keywords: Supply chain management, Lot size, Economic order quality, Lead time, Re-order point.

1. Introduction

A lot size is a measure, or quantity addition, acceptable to or specified by a party offering to buy or sell it. It is also used as an alternative term for lot quantity of goods purchased or produced in expectation of the use or sale in the future. Four types of lot-sizing techniques are available: (1) the EOQ, followed by some assumptions for reducing inventory in smaller lots; (2) dynamic lot-sizing, considering the problem of determining production lot sizes when demand is deterministic, but varies with the time; (3) fixed order quantity, a policy is to produce a fixed amount each time by performing a setup; (4) part-period balancing, an idea to balance the inventory-carrying and setup costs.
Traditionally, any manufacturing or service organization performs purchasing, producing, and marketing activities independently, so that it is difficult to make an optimal plan for the supply chain. Research pieces in supply chain management have pre-focused on three major issues: (1) complexity in information flow; (2) mode in management; (3) planning of operation management for processing. In this research, this study puts emphasis on the cost factors, and the effect of the cost factor on supply chain is studied.
It is mentioned here that effective lot size is expected to optimize the supply chain and unwanted cash flow and reduce the possibility of occurrence on inventory shortage caused by variable orders.
A reciprocal cause and effect relationship exists between production planning and control and inventory levels at various stages during processing, and this affects the system.
Eltogral et al. (2007) introduced the complete solution of the problem in an explicit and extended manner. The authors incorporated transportation cost explicitly into the model and developed optimal solution procedures for solving the integrated models. In this paper, they have also developed two new models that integrate the transportation cost explicitly in the single-vendor single-buyer problem. The transportation cost is considered to be in an all-unit-discount format for the first model. The option of over declaring a shipment to exploit the transportation unit cost structure is explored in the second model. The objective in both models is to view the system

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as an integrated the system and determine the production and shipments schedule to minimize the average total cost per unit time.

Moghadam et al. (2008) presented a hybrid intelligent algorithm based on the push SCM, which uses a fuzzy neural network and a genetic algorithm to (i) predict the rate of demand; (ii) determine the material planning; (iii) select the optimal supplier. They also tested the proposed enumerative search algorithm and a heuristic algorithm in a case study conducted in Iran. They claimed that their model is one of the most useful ones for supplier selection in a single-stage category presented in literature.

Inventories make rational system possible for production and distribution. At each stage, in the flow of goods, inventories serve as the vital function of decoupling the various operations in the sequence beginning with raw materials, extending through all the manufacturing operations to finished goods storage, and further to warehouses, distribution points leading to the ultimate consumers, as shown in Fig. 1. The basis of supply chain and inventory management is integration, control, strategic focus, supportive structure, empowerment, physical support, consistency, skill test, and coordination in any organization. Christina et al. (2015) analyzed data from 188 firms and illustrated how the contextual factors pertain to product, market complexity, and factors of SCM. Yaghin et al. (2013) formulated a fuzzy non-linear multi-objective model, where some parameters are not precisely known. In this paper, a hybrid probabilistic-flexible programming approach is proposed to handle imprecise data and soft constraints concurrently. They transformed the original model into an equivalent multi-objective crisp model; it is then converted to a classical mono-objective one by a fuzzy goal programming method. Also, they provided an efficient solution procedure using particle swarm optimization (PSO) to solve the resulting non-linear problem.

Abdesalam and Elassal (2014) considered the joint economic lot-sizing problem (JELP) for multi-layer supply chain with multi-retailers and single manufacturer and supplier. They proposed modifying four computational intelligent algorithms (particle swarm optimization, gravitational search algorithm, cuckoo search, and charged system search) to solve mixed integer problems. The paper, further, tested the effect of adopting a centralized vs. decentralized safety stocks, and the model can be validated with literature favoring a centralized policy for the cost-perspective aspect.

Almender et al. (2015) presented two models: one considering batch production and the other one allowing lot-streaming. They compared their models with traditional models and demonstrated more realistic results. Also, they claimed that generated production plans are always feasible in cost savings as compared to classical models.

Cárdenas-Barrón et al. (2015) proposed a new algorithm based on a reduce-and-optimize approach (ROA), and a new valid inequality is to solve the multi-product multi-period inventory lot-sizing with supplier selection problem. In this paper, the authors have used first-time first-serve to solve the multi-product multi-period inventory lot-sizing with supplier selection problem. They also claimed that this problem is a new variant of the lot-sizing inventory problem. It was observed that the concept of inequality is the set of constrains that has a beneficial impact on both solution and time issues.

Mazdeh et al. (2015) investigated the single-item dynamic lot-sizing problem with supplier selection in their study. The problem is decomposed into two different cases. In the first case, quantity discounts are not taken into account; in the second case, incremental and all-unit quantity discounts are considered. Due to the complexity of the problems, a new heuristic is developed to generate the best ordering policy.

Industries have always focused on the role of inventory in the supply chain due to expanding market and global competition. In order to determine the appropriate ordering quantity in the chain, it is important to find the suitable mechanism for coordinating the inventory processes that are controlled by independent partners as mentioned in Prasertwattana and Chiadamrong (2004a) and Prasertwattana, N. Chiadamrong (2004b).

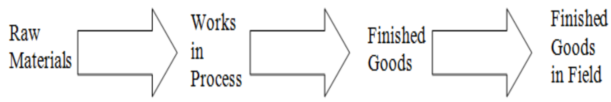

Fig. 1. Inventory Positions in the Supply Chain

The inventory control in supply chain is usually modeled as multi-echelon inventory decision problems. The echelons may consist of two or more of the following characters; supplier(s), manufacturer(s), warehouse(s), and retailer(s). Through inventory control mechanisms, namely centralized, decentralized, and hybrid systems, with different degrees of information sharing are usually used for making inventory-related decisions. In a centralized system, the inventory replenishment decisions are made by a central decision-maker (Abdul-Jalbar et al., 2003). In a decentralized system, inventory replenishment decisions are made by departmental decision-makers. Axsater (2001) and Axsater (2003) derived the exact optimal solutions mathematically on the first two systems under various assumptions. However, some of these models need to be fitted into practical environments.

The two mathematical models of lot-sizing developed by the authors of this paper are based on the application of probability theory. Convolution and marginal distribution techniques are applied in developing the models. The marginal distribution of lead-time demand is established for normal demand and exponential procurement and administrative delay lead time. Forsberg (1996) utilized a similar but simple theoretical model.

It is noted that for any practical situation, sub-lots of materials arrive at an interval, and not the whole lot at a time. Different permissible service levels can be considered, and in each case, the optimum lot-sizing can be calculated with various parameters. Results from the model can be analyzed to investigate the sensitivity of the system in different parametric conditions. Ganeshan (1999) considered low-level inventory to start with and optimized the cost parameter.

In this present paper, a case study has been considered for a power transformer manufacturing unit located in India. The case of a single-item multiple-delivery system with variable demand and lead time has been investigated. In this case, the demand is distributed normally, lead time is distributed exponentially, and the quantity arrives in sub-lots. For manufacturing different ranges of products, the company has to purchase and stock the inputs required for production. Some items may bring into price advantage for larger purchases as suggested by Seo et al. (2002). When the input materials have longer varying lead time, the company has to keep a large amount of inventories as suggested by Tee and Rossetti (2002). In certain cases where input materials have shorter and constant lead time, the company may keep a lower level of inventories. The company has a centralized purchasing procedure for A and some of the B items of its A.B.C. list. Some B items and all C items are purchased independently by the different units of the company.

At the beginning of every month, in this case study, with the production plan in hand, each production unit starts issuing materials and components for A items from central stores and the remaining B and C items from the stores. A summary of issuing materials is recorded according to the types of materials that have been issued. As the consumption date of the items from stores was not available, the monthly production data of the transformers is collected for one year for one of the production unit of the company for analysis. Though the company has two production units producing the power transformers, the case investigated here is limited only for one production unit. Here, it is assumed that the production rate is equal to the demand rate for A items.

The budgeted production schedule of the product (in this case study, the products are power transformers of 50 MvA to 350 MvA) for a year was available and is shown in the Table 1, and the variation is shown in Fig. 2.

Table 1
Budgeted production plan for transformers

Month	Monthly production (No.)
Jan	2
Feb	12
March	7
April	5
May	5
June	2
July	5
Aug	20
Sept	13
Oct	12
Nov	25
Dec	7
Total	115

Considering the discussion with the Head of Purchasing Department, the authors have taken carrying cost of inventory equal to 30% per year. It is the same with the company's norm.

The Company has ordered 1200 purchase orders for transformer components. The accounting department has taken costs incurred for these 646 orders to be Rs. 16818000/; so, cost per order is Rs. 16818000/1200 = Rs. 14013/- approximately

The ordering cost is divided into the following parts, as shown in the Table 2.

The unit cost of one A item (for this case study, it is "Cold Rolled Grain Oriented" steel) supplied by the vendors in a year is given in the Table 3 followed by Fig. 3.

For the CRGO, the frequency diagram for usage is shown in Fig. 4, and χ^2 test has been carried out in Table 4. χ^2 value from the Table 4 with degrees of freedom 3 and at 5% level is 7.815. So, the fit is not very good, because the materials are supplied according to the received order of the transformers.

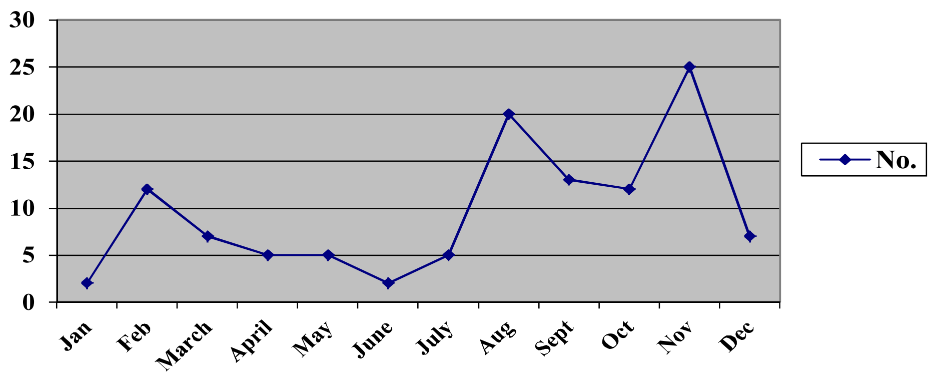

Fig. 2. Monthly production plan for transformers

Table 2
Division of cost per order

SI. NO.	Description of the cost component	% of cost per order
1.	Salary of staffs and overheads	45.66
2.	Stationary and postage charges	5.95
3.	Receiving, unpacking, and counting in stores	12.71
4.	Inward inspection	23.78
5.	Invoice processing for payment in account section	4.76
6.	Handling and temporary storage	5.35
7.	E-mails and temporary calls	1.78
	Total	100

Table 3
Unit cost of CRGO

Vendor	Unit cost (Rs/T)	CRGO (T)
V1	185000	15
V2	268000	432
V3	231000	500
V4	255000	600
V5	201000	450
V6	271000	1800
V7	285000	150
Total		3947

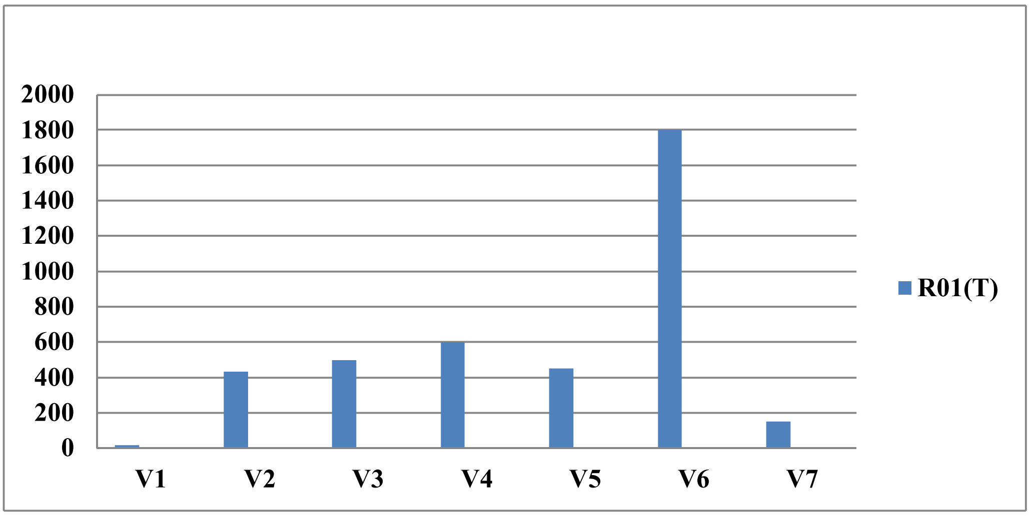

Fig. 3. CRGO supplied by vendors

Table 4
Quantity distribution at plant for CRGO

S.No.	Particulars	Quantity (T) (a_i)	Observed frequency (f_o)	Expected frequency (f_e)	$(fo - fe)^2 / fe$
	Mean of quantity $a = \frac{\Sigma f_i a_i}{\Sigma f_i} = \frac{4315}{15} = 287.67$				
1		15	1	17.22	15.28
2		100	7	8.56	0.28
3	Standard deviation	150	2	5.45	2.18
4		450	2	40.65	36.75
5	$\sigma = \sqrt{\frac{\Sigma f_i (a - a_i)^2}{\Sigma f_i}} = \sqrt{\frac{1438943.33}{100}} = \sqrt{95929.56}$	600	2	13.65	9.94
6		1200	1	0.13	5.82
	$= 309.72$				
Total			15		70.26

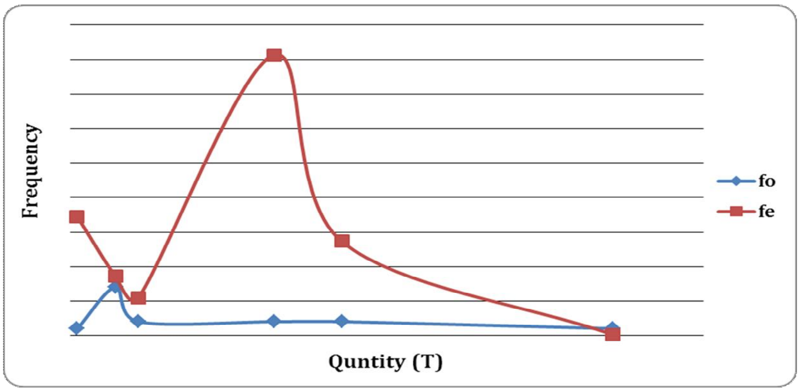

Fig. 4. Frequency distribution of quantity (T) CRGO

Received and used Quantities are shown in Table 5 followed by Fig. 5.

Table 5
Quantity of CRGO received.

Month	Quantity received (T)	Balanced Quantity (T)	Quantity used (T)	No.
Jan	69	69	8	1
		61	8	1
Feb	291	344	63	1
		281	64	1
		217	200	10
March	137	154	100	5
March		54	40	2
April	100	114	100	5
May	150	164	100	5
June	100	164	156	2
July	100	108	90	5
August	600	618	100	4
		518	286	13
		232	28	2
		204	19	1
Sep	450	635	455	7
		180	108	6
Oct	600	672	218	4
		454	157	2
		297	88	3
		209	159	3
Nov	1200	1250	265	5
		985	122	6
		863	93	2
		770	33	2
		737	554	10
Dec	150	333	46	1
		287	57	1
		230	123	3
		107	51	1
		56	56	1
Total	3947		3947	115

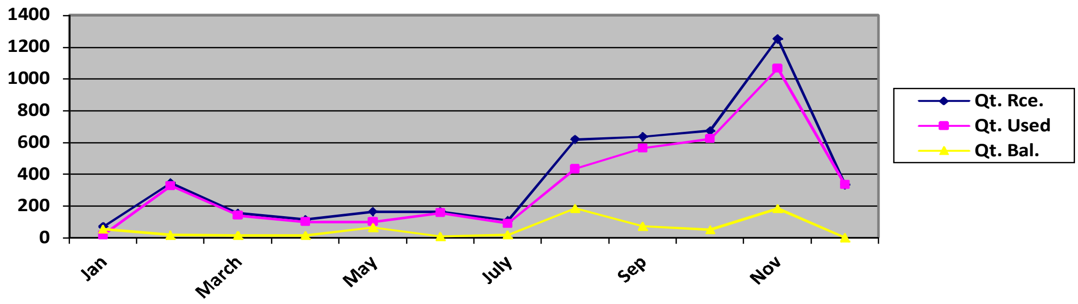

Fig. 5. Chart between quantities received, used, and balanced

2. Model Development

In this case study, the authors have collected the relevant problem data that include detailed data of all materials listed in ABC list. Once the procedure is developed and established, the results can be readily extended to similar types of A item.

Pujawan (2004) compared two popular lot-sizing techniques: the Silver Meal (SM) and the Least Unit Cost (LUC) and observed several properties of two traditional lot-sizing rules on the variability of orders created by a supply chain channel receiving demand with variability. Although the two rules appear to be very similar, they exhibit interestingly different behaviours; the SM rule is shown to produce a series of orders with more stable interval between orders, but with more variable order quantities. Conversely, the LUC rule results in more stable order quantities, but more variable order intervals. The study also reveals that the addition of an appropriate amount of extra quantity to an order could significantly reduce order variability. An economic lot size model has been developed with four-echelon inventory as suggested by Rahman and Sarker (2006). Raw-material assembly lines, process raw material, ready raw material, and finished product inventory are the types in the model as suggested by Rahman and Sarker. An integrated inventory model between supplier, manufacturer, and buyer can be found in (Lee, 2005; Lee and Wu, 2006). Both of the works explained integrated inventory, but did not consider remanufacturing as an option for recovery of the product.

Nurshanti and Suparno (2010) developed the Pull System Inventory Model (Fig. 6) in which finished goods are used to fulfill demand due to serviceable inventory. Manufacturing process is run with constant rate, where the production rate (p) is higher than customer demand rate (λ); n and x are the number of manufacturing delivery lot sizes for every production setup and number of manufacturing lot sizes needs for production of every raw material order, respectively. Remanufacturing process is run in the return product rate (γ) with assumption that all of return product can be remanufactured. Both demand and return rates have assumptions following a Poisson process. In this system, remanufactured and manufactured products are delivered to the warehouse as serviceable inventory when they reach optimal quantity of remanufactured products (Qr) and manufactured product will deliver with Qm.

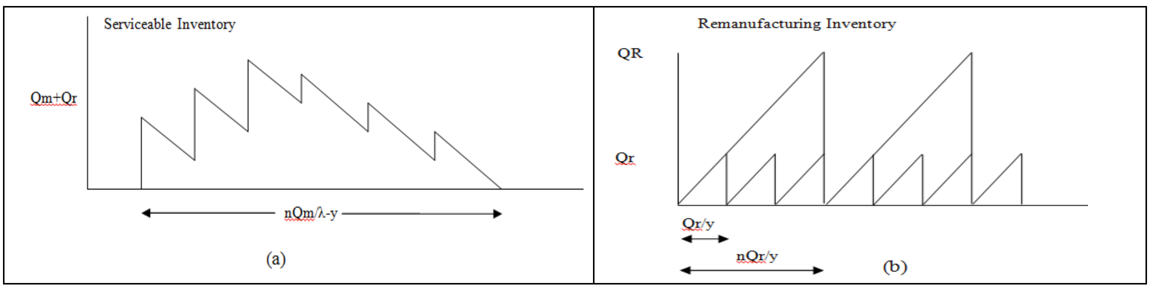

Fig 6. (a,b). Pull system inventory model

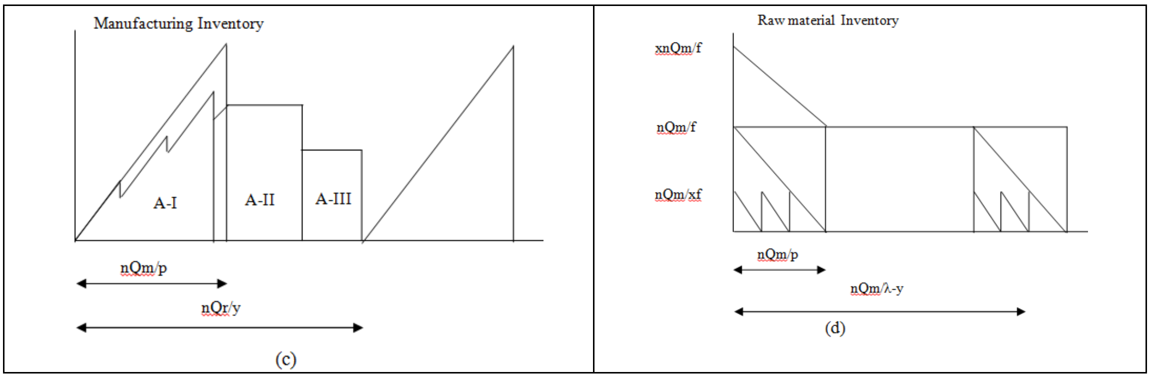

Fig 6. (c,d). Pull system inventory model

3. Model Formulation

The two models are based on the application of probability theory. Convolution and joint distribution techniques have been used to derive the distribution of lead time and marginal distribution of demand in lead time.

In the first model, the delay in placing the order has been neglected, while in second model, this delay has been considered and is taken to be distributed exponentially.

The concepts of convolution and marginal distribution have been widely discussed in (Hadley and Whitin, 1963; Cramer, 1946). The marginal distribution concept is now widely used in analyzing the inventory system, where any one or both of lead time and demand follow probability distribution pattern.

The following notation will be used to develop the model:

A $\sqrt{\text{square of the mean of demand rate (montihly in) million) + 2 x square of standard deviation of demand rate x reciprocal of mean of administrative delay lead time.}}$ $= \sqrt{\Delta^2 + 2\sigma^2 K}$

a Sub lot quantity in million

a_n Net accumulation quantity in million

B_{ST} Extra buffer stock required to eliminate stock out between inter-arrival time of sub lots

i index of time period

B $\sqrt{\ }$ square of the mean of demand rate + 2x square of standard deviation of demand rate x reciprocal of mean of procurement lead time = $\sqrt{\Delta^2 + 2\sigma^2\lambda}$

c_u Unit cost of time

c_c Cost of carrying inventory in percentage per year

c_s Ordering cost in Rupees per order

c(t) Cumulated generating function of lead time demand

D Demand in lead time

Do Optimum Re-order point

d Demand in any time period

d_i Number of stock units demanded in time period i

f(D) Distribution of demand in lead time

F(D) Cumulative distribution of demand in lead time

g^(x) Probability density function of administrative delay lead time

g(L) Probability density function of procurement lead time

h Number of standard deviation

h(T) Probability density function of the total lead time.

H(T) Cumulative Probability density function of h (T)

K Reciprocal of mean of administrative delay lead time.

L Procurement lead time

M Minimum stock in any time period

M(t) Moment-generating function of lead time demand

N(t) Moment-generating function of demand

n Number of order per year

P(s) Probability of shortage inter-arrival time of sub lots

Q Order quantity

Q_o Economic order quantity

R.O.P Re-order point

T Total lead time

T_i Inter-arrival time between sub lots in time period i

T_C Total variable cost of carrying inventory

x Administrative delay lead time

U Loss per unit inventory if there is no demand

V Salvage value of one unit of inventory

α Probability of stock out in a cycle (order)

β Permissible number of stock outs per year

λ Reciprocal of mean of procurement lead time

Δ Mean of demand rate of the item

Δ_a Mean of arrival quantity of the item in T_i

Δ_n Net accumulation of inventory in T_i

σ Standard deviation of demand rate

σa Standard deviation of arrival quantity

σ_n Standard deviation of net accumulation of Inventory μ Mean of lead time demand

3.1. Model I (Demand normal and procurement lead time exponential)

This model was formulated by considering procurement lead times to be distributed exponentially and demand pattern distributed normally. In this model, the administrative delay lead time is neglected and is taken zero. When stock reaches the R.O.P., the order can be sent to the supplier for replenishment.
The probability density function of demand in lead time was found by the method of joint probability distribution. In this model, the optimal ordering quantity and the re-order level have been derived and are given in equations (8) and (13), respectively.

3.1.1. Probability density function of demand in procurement lead time

The density function of procurement lead time is

$$g(L) = \lambda \exp(-\lambda L), \qquad \text{where } L > 0 \tag{1}$$

and $1/\lambda$ is the mean of the lead time.

The probability density function of demand is:

$$N(d, \Delta, \sigma) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(\frac{-(d-\Delta)^2}{2\sigma^2}\right) \tag{2}$$

The marginal distribution of demand in procurement lead time (L) is:

$$f(D) = \int_0^\infty N(D, \Delta L, \sigma\sqrt{L})\,\lambda e^{-\lambda L}\, dL$$

$$= \frac{\lambda}{\sqrt{2\pi}} \int_0^\infty L^{-1/2} \exp\left(-\frac{D^2}{2\sigma^2} \times \frac{L}{L} - \frac{\Delta^2 + 2\sigma^2\lambda}{2\sigma^2} L\right) dL$$

$$= \frac{\lambda \exp(\Delta D/\sigma^2)}{\sqrt{2\pi}} 2\left(\frac{D^2}{2\sigma^2} \times \frac{2\sigma^2}{\Delta^2 + 2\sigma^2\lambda}\right)$$

$$\times K_{1/2}\left(2\sqrt{\frac{D^2}{2\sigma^2} \times \frac{\Delta^2 + 2\sigma^2\lambda}{2\sigma^2}}\right) \tag{3}$$

Here, $\int_0^\infty L^{v-1/2} e^{-\eta/x-\gamma x}\, dx = 2(\eta/\gamma)^{\upsilon/2} K_\upsilon(2\sqrt{\eta\gamma})$

(Half basis function)

and $K_{1/2}^{(z)} = \sqrt{\frac{\pi}{2Z}}\, e^{-z}$

So, when D > 0

$$= \frac{\lambda \exp\left(-D/\sigma^2\left(\sqrt{\Delta^2 + 2\sigma^2\lambda} - \Delta\right)\right)}{\sqrt{\Delta^2 + 2\sigma^2\lambda}} \tag{4}$$

when D < 0, let us put D = -z

$$f(D) = \frac{\lambda \exp\left(D/\sigma^2\left(\sqrt{\Delta^2 + 2\sigma^2\lambda} + \Delta\right)\right)}{\sqrt{\Delta^2 + 2\sigma^2\lambda}} \tag{5}$$

3.1.2. Cumulative probability density function of lead time demand

$$F(D) = \int_{-\infty}^{D} f(D)\, dD = \int_{-\infty}^{0} f(D)\, dD + \int_{0}^{D} f(D)\, dD$$

$$\lim_{D \to \infty} F(D) = 1$$

$$\frac{dF(D)}{dD} = \frac{\lambda\sigma^2 \exp\left(-\frac{D}{\sigma^2}\sqrt{\Delta^2 + 2\sigma^2\lambda} - \Delta\right)}{\sqrt{\Delta^2 + 2\sigma^2\lambda}} = f(D) \tag{6}$$

When D > 0.

3.1.3. Determination of Re-order Point and Economic Order Quantity

3.1.3.1. Determination of R.O.P.

If α be the probability of stock out per cycle, then

$$1 - \alpha = 1 - \frac{\lambda\sigma^2 \exp\left(-\frac{D}{\sigma^2}\sqrt{\Delta^2 + 2\sigma^2\lambda} - \Delta\right)}{\left(\sqrt{\Delta^2 + 2\sigma^2\lambda}\right) - \Delta\sqrt{\Delta^2 + 2\sigma^2\lambda}} \tag{7}$$

or Re-order Point = 2.3 σ^2

$$\frac{\log\left(\alpha/\lambda\sigma^2\left(\Delta^2 + 2\sigma^2\lambda - \Delta\sqrt{\Delta^2 + 2\sigma^2\lambda}\right)\right)}{\Delta - \sqrt{\Delta^2 + 2\sigma^2\lambda}} \tag{8}$$

3.1.3.2. Determination of E.O.Q.

$$\text{Total Cost (T.C.)} = \frac{12\Delta C_s}{Q} + (D - \Delta/K)\, C_u C_c + \frac{1}{2} Q C_u C_c \tag{9}$$

If we take β = permissible stock out per year, then

$$\beta = \alpha \times \frac{12\Delta}{Q} = (M \exp(-LD)) \times \frac{12\Delta}{Q} \tag{10}$$

$$\text{where} \quad L = \frac{\left(\sqrt{\Delta^2 + 2\sigma^2\lambda} - \Delta\right)}{\sigma^2} \tag{10.1}$$

$$M = \frac{\lambda\sigma^2}{\left(\Delta^2 + 2\sigma^2\lambda\right) - \Delta\sqrt{\Delta^2 + 2\sigma^2\lambda}} \tag{10.2}$$

So, Total cost,

$$T.C. = \frac{\beta C_S}{N \exp(-LD)} + C_u C_c (D - \Delta/K)$$

$$+\frac{1}{2}\frac{C_uC_c \times 12\ \Delta M \exp(-LD)}{\beta} \quad (11)$$

Differentiating with respect to D and equating it to zero

$$\frac{L\beta C_s}{M\exp(-LD)} + C_uC_c - 6\ \frac{C_uC_c\Delta\ ML\exp(-LD)}{\beta} = 0 \quad (12)$$

$$\text{or } Q_o = \frac{1}{2}\left(\frac{2}{L} + \sqrt{\frac{4}{L^2} + \frac{8C_s}{C_uC_c} x12\Delta}\right)$$

$$= \frac{1}{L} + \sqrt{\frac{1}{L^2} + \frac{24C_s\Delta}{C_uC_c}} = \frac{1}{L}\left[1 + \sqrt{1 + \frac{24C_s\Delta L^2}{C_uC_c}}\right] \quad (13)$$

So, for any particular permissible stock out per year, one can get the Economic Order Quantity directly by the equation (13).

3.2. Model II: Demand normal, procurement lead time and administrative delay lead time exponential

This model was formulated by considering administrative delay and procurement lead times to be distributed exponentially and demand pattern distributed as a normal distribution. Resultant distribution of total lead time was first derived by applying the method of convolutions. For any order level D, to obtain the desired service level of β (permissible probability of stock outs per year) by equation (40), it was found that the order quantity Q and total cost TC are dependent on D by equation (38).

As the object is to minimize the total cost associated with holding up the order level D, a trial solution was obtained from the equation (4) by increasing D in steps and computing the corresponding Q and T.C.

3.2.1. Distribution of total lead time:

If L =Procurement lead time with mean 1/λ and exponentially distributed;

x =Administrative lead time with mean 1/K and exponentially distributed;

Then, T = Total lead time = L + x (14)

The probability density function (p.d.f.) of 1 is

$g(L) = \lambda \exp(-\lambda L) \qquad L > 0$ (15)

The p.d.f. of x is

$g\hat{}(x) = K \exp(-Kx) \qquad x > 0$ (16)

The p.d.f. of T is obtained by the law of convolution as

$$h(T) = \int_{\propto}^{+\propto} g(T-x)\,\hat{g}(x)\,dx \quad (17)$$

$$= \frac{K\lambda}{\lambda - K}(\exp(-KT) - \exp(-\lambda T)) \quad (18)$$

$$\text{or } h(T) = 0 \qquad -\propto < T < 0$$

$$= \frac{K\lambda}{\lambda - K}(\exp(-KT) - \exp(-\lambda T)) \quad 0 \le T < \propto \quad (19)$$

Cumulative distribution function of T is

$$H(T) = \frac{K\lambda}{\lambda - K}\int_o^T (\exp(-KT) - \exp(-\lambda T))\,dT$$

$$= \frac{K\lambda}{\lambda - K}\left(\frac{1}{K}(1 - \exp(-KT)) - \frac{1}{\lambda}((1 - \exp(-\lambda T)))\right) \quad T > 0 \quad (20)$$

when T = 0, H(T) = 0

when T = ∝ H(T) = 1

3.2.2. Distribution of demand in lead time

If f(D) represents distribution of demand in lead time

$$f(D) = \int_o^{\propto} N(D, \Delta T, \sigma\sqrt{T})\, x\, h(T)\,dT$$

and $N(D, \Delta T, \sigma\sqrt{T})$ represents lead time demand distribution,

h(T) represents distribution of total lead time.

$$F(D) = \int_o^{\propto} N(D, \Delta T, \sigma\sqrt{T})\frac{K\lambda}{\lambda - K}(\exp(-KT) - \exp(-\lambda T))\,dT$$

$$= \frac{K\lambda}{\lambda - K}\int_o^{\propto} N(D, \Delta T, \sigma\sqrt{T})\exp(-KT)dT$$

$$- \frac{K\lambda}{\lambda - K}\int_o^{\propto} N(D, \Delta T, \sigma\sqrt{T})\exp(-\lambda T)dT$$

$$= I_1 - I_2 \quad (21)$$

Now,

$$I_1 = \frac{K\lambda}{(\lambda - K)\sqrt{2\pi\sigma}}\int_o^{\propto} T^{-1/2}\exp\left(-\frac{(D-\Delta T)^2}{2\sigma^2 T}\right)\exp(-KT)dT$$

$$= \frac{K\lambda\exp(\Delta D/\sigma^2)}{(\lambda - K)\sqrt{2\pi}\,\sigma}\int_o^{\propto} T^{-1/2}\exp\begin{pmatrix} -\left(D^2/2\sigma^2\right)^{1/T} \\ -\left(\frac{\Delta^2}{2\sigma^2} + K\right)T \end{pmatrix} dT$$

Now, we know that

$$\int_o^{\propto} x^{\nu - 1/2}\exp(-\beta/x - \gamma x)\,dx =$$

$$2(\beta/\gamma)^{\nu/2} K_\nu\left(2\sqrt{\beta\gamma}\right) \quad \text{where } \beta > 0, \quad \gamma > 0$$

$$\text{and } K_{\pm 1/2}(Z) = \sqrt{\frac{\pi}{2z}}\ e^{-z}$$

The integrates have been given by (Gradshteyn and Ryzhik, 1965):

$$\text{So, } I_1 = \frac{K\lambda\exp(\Delta D/\sigma^2)}{(\lambda - K)\sqrt{2\pi\sigma}}\exp\left(-\frac{D}{\sigma^2}\sqrt{\left(\Delta^2 + 2\sigma^2 K\right)}\right) \quad (22)$$

when D ≥ 0

when D ≤ 0

putting D = Z when Z > 0

from equation (22)

$$I_1 = \frac{K\lambda \exp\left(-D/\sigma^2\left(\sqrt{\Delta^2+2\sigma^2 K}+\Delta\right)\right)}{(\lambda-K)\sqrt{\Delta^2+2\sigma^2 K}}$$

(23)when D ≤ 0.

$$\text{So, } I_1 = \frac{K\lambda\, p\left(-D/\sigma^2\left(\sqrt{\Delta^2+2\sigma^2 K}-\Delta\right)\right)}{(\lambda-K)\sqrt{\Delta^2+2\sigma^2 K}}, \quad D \geq 0. \quad (23')$$

$$= \frac{K\lambda \exp\left(D/\sigma^2\left(\sqrt{\Delta^2+2\sigma^2 K}+\Delta\right)\right)}{(\lambda-K)\sqrt{\Delta^2+2\sigma^2 K}}, \quad D \leq 0 \quad (23'')$$

Similarly,

$$I_2 = \frac{K\lambda \exp\left(-D/\sigma^2\left(\sqrt{\Delta^2+2\sigma^2\lambda}-\Delta\right)\right)}{(\lambda-K)\sqrt{\Delta^2+2\sigma^2\lambda}}, \quad D \geq 0 \quad (24)$$

$$= \frac{K\lambda \exp\left(D/\sigma^2\left(\sqrt{\Delta^2+2\sigma^2\lambda}+\Delta\right)\right)}{(\lambda-K)\sqrt{\Delta^2+2\sigma^2\lambda}}, \quad D \leq 0 \quad (25)$$

Distribution of demand in lead time

The marginal distribution of demand in lead time:

$$f(D) = \frac{K\lambda}{(\lambda-K)} \frac{\exp\left(-D/\sigma^2\left(\sqrt{\Delta^2+2\sigma^2 K}+\Delta\right)\right)}{\sqrt{\Delta^2+2\sigma^2 K}} - \frac{\exp\left(-D/\sigma^2\left(\sqrt{\Delta^2+2\sigma^2\lambda}-\Delta\right)\right)}{\sqrt{\Delta^2+2\sigma^2\lambda}}$$

when D ≥ 0. (26)

$$= \frac{K\lambda}{(\lambda-K)}\left(\frac{\exp\left(D/\sigma^2\left(\sqrt{\Delta^2+2\sigma^2 K}+\Delta\right)\right)}{\sqrt{\Delta^2+2\sigma^2 K}} - \frac{\exp\left(D/\sigma^2\left(\sqrt{\Delta^2+2\sigma^2\lambda}-\Delta\right)\right)}{\sqrt{\Delta^2+2\sigma^2\lambda}}\right)$$

when D ≤ 0 (27)

3.2.3. Continuity

The lead time demand function f (D) is expected to be continuous at every point including D = 0

Differentiability of f (D) at D = 0

$$\frac{df_+(D)}{dD} = \frac{-K\lambda}{(\lambda-K)}\left(\frac{\exp\left(-D/\sigma^2\left(\sqrt{\Delta^2+2\sigma^2 K}-\Delta\right)\right)}{\sqrt{\Delta^2+2\sigma^2 K}\,.\,\sigma^2} \times \left(\sqrt{\Delta^2+2\sigma^2 K}-\Delta\right)\right.$$

$$= \left.\frac{\exp\left(-D/\sigma^2\left(\sqrt{\Delta^2+2\sigma^2\lambda}-\Delta\right)\right)}{\sqrt{\Delta^2+2\sigma^2\lambda}\,.\,\sigma^2} \times \left(\sqrt{\Delta^2+2\sigma^2\lambda}-\Delta\right)\right) \quad (28)$$

$$\left.\frac{df_+(D)}{dD}\right|_{D=0} = \frac{-K\lambda}{(\lambda-K)\sigma^2}\left(\frac{\sqrt{\Delta^2+2\sigma^2 K}-\Delta}{\sqrt{\Delta^2+2\sigma^2 K}} - \frac{\sqrt{\Delta^2+2\sigma^2\lambda}-\Delta}{\sqrt{\Delta^2+2\sigma^2\lambda}}\right)$$

$$= \frac{K\lambda}{(\lambda-K)\sigma^2}\left(\frac{-\Delta\sqrt{\Delta^2+2\sigma^2 K}-\sqrt{\Delta^2+2\sigma^2\lambda}}{\sqrt{\Delta^2+2\sigma^2\lambda}\,.\sqrt{\Delta^2+2\sigma^2 K}}\right)$$

(29)

$$\frac{df_-(D)}{dD} = \frac{K\lambda}{(\lambda-K)}\left(\frac{\exp(D/\sigma^2\left(\sqrt{\Delta^2+2\sigma^2 K}+\Delta\right)}{\sqrt{\Delta^2+2\sigma^2 K}} \times \frac{\sqrt{\Delta^2+2\sigma^2 K}+\Delta}{2}\right.$$

$$-\left.\frac{\exp\left(D/\sigma^2\left(\sqrt{\Delta^2+2\sigma^2\lambda}+\Delta\right)\right)}{\sqrt{\Delta^2+2\sigma^2\lambda}} \times \frac{\sqrt{\Delta^2+2\sigma^2\lambda}+\Delta}{2}\right) \quad (30)$$

$$\left.\frac{df_-(D)}{dD}\right|_{D=0} = \frac{K\lambda}{(\lambda-K)\sigma^2}\left(\frac{\sqrt{\Delta^2+2\sigma^2 K}+\Delta}{\sqrt{\Delta^2+2\sigma^2 K}} \times \frac{\sqrt{\Delta^2+2\sigma^2\lambda}+\Delta}{\sqrt{\Delta^2+2\sigma^2\lambda}}\right)$$

$$= \frac{K\lambda}{(\lambda-K)\sigma^2}\left(\frac{\Delta\sqrt{\Delta^2+2\sigma^2\lambda}-\sqrt{\Delta^2+2\sigma^2 K}}{\sqrt{\Delta^2+2\sigma^2 K}\,.\,\sqrt{\Delta^2+2\sigma^2\lambda}}\right) \quad (31)$$

$$\text{Thus} \quad \frac{df_+(D)}{dD} = \frac{df_-(D)}{dD} \quad (32)$$

Hence, the function f(D) is differentiable at D = 0.

3.2.4. Mode of f (D)

$$\frac{df_+(D)}{dD} = 0 \text{ or,}$$

$$D = \frac{\sigma^2}{\sqrt{\Delta^2+2\sigma^2 K}-\left(\sqrt{\Delta^2+2\sigma^2\lambda}\right)} \log_e \frac{\sqrt{\Delta^2+2\sigma^2\lambda}\left(\sqrt{\Delta^2+2\sigma^2 K}-\Delta\right)}{\sqrt{\Delta^2+2\sigma^2 K}\left(\sqrt{\Delta^2+2\sigma^2\lambda}-\Delta\right)} \quad (33)$$

Again,

$$\frac{df_{-}(D)}{dD}=0$$

or, $$\frac{\exp\left(D/\sigma^2\left(\sqrt{\Delta^2+2\sigma^2 K}+\Delta\right)\right)}{\sqrt{\Delta^2+2\sigma^2 K}}\times\frac{\left(\sqrt{\Delta^2+2\sigma^2 K}+\Delta\right)}{2}$$

$$D=\frac{\sigma^2}{\left(\sqrt{\Delta^2+2\sigma^2 K}-\sqrt{\Delta^2+2\sigma^2\lambda}\right)}$$

or, $$\log_e\frac{\sqrt{\Delta^2+2\sigma^2 K}\left(\sqrt{\Delta^2+2\sigma^2\lambda}+\Delta\right)}{\sqrt{\Delta^2+2\sigma^2\lambda}\left(\sqrt{\Delta^2+2\sigma^2\lambda}+\Delta\right)} \quad (34)$$

So, it can be seen that the above value of D is positive; hence, no model value exists in the negative region. So, there is only one mode for the distribution of D and mode lies in the positive region.

3.2.5. Distribution function of D

For D < 0, distribution function of demand in lead time is

$$F(D)=\frac{K\lambda}{\lambda-K}\int_{\propto}^{o}\frac{\exp D/\sigma^2(A+\Delta)dD}{A}-\frac{K\lambda}{\lambda-K}\int_{-\propto}^{o}\frac{\exp(D/\sigma^2(B+\Delta))}{B}.dD$$

where A $=\sqrt{\Delta^2+2\sigma^2 K}$ and B $=\sqrt{\Delta^2+2\sigma^2\lambda}$

or

$$F(D)=\frac{\frac{K\lambda\sigma^2}{\lambda-K}\sigma^2\exp\left(D/\sigma^2(A+\Delta)\right)}{(A+\Delta)A}-\frac{K\lambda\sigma^2\exp\left(D/\sigma^2(B+\Delta)\right)}{(\lambda-K)(\Delta+B)B}$$

$$=\frac{K\lambda\sigma^2}{\lambda-K}\frac{\exp\left(D/\sigma^2(A+\Delta)\right)}{A(A+\Delta)}-\frac{\exp\left(D/\sigma^2(B+\Delta)\right)}{B(B+\Delta)}$$

D < 0 (35)

For D > 0, the distribution function of demand in lead time is

$$F(D)=\int_{\propto}^{o}f_{-}(D)dD+\int_{o}^{D}f_{+}(D)dD$$

$$=\frac{K\lambda\sigma^2}{\lambda-K}\quad\frac{1}{A(A+\Delta)}-\frac{1}{B(B+\Delta)}-\frac{K\lambda\sigma^2}{(\lambda-K)}\frac{\exp\left(-D/\sigma^2(A-\Delta)\right)}{A(A-\Delta)}$$

$$-\frac{1}{A(A-\Delta)}-\frac{\exp\left(-D/\sigma^2(B-\Delta)\right)}{B(B-\Delta)}+\frac{1}{B(B-\Delta)} \quad (36)$$

$$F(D)=\frac{K\lambda\sigma^2}{\lambda-K}\begin{pmatrix}\frac{1}{A(A+\Delta)}-\frac{1}{B(B+\Delta)}+\\ \frac{1}{A(A-\Delta)}-\frac{1}{B(B-\Delta)}\end{pmatrix}$$

$$=\frac{K\lambda\sigma^2}{\lambda-K}x\begin{pmatrix}\frac{1}{\sigma^2 K}-\\ \frac{1}{\sigma^2\lambda}\end{pmatrix}-\frac{K\lambda\sigma^2}{\lambda-K}\begin{pmatrix}\frac{\exp\left(-D/\sigma^2(A-\Delta)\right)}{B(B-\Delta)}-\\ \frac{\exp\left(-D/\sigma^2(B-\Delta)\right)}{B(B-\Delta)}\end{pmatrix}$$

$$=1-\frac{K\lambda\sigma^2}{\lambda-K}\begin{pmatrix}\frac{\exp\left(-D/\sigma^2(A-\Delta)\right)}{A(A-\Delta)}-\\ \frac{\exp\left(-D/\sigma^2(B-\Delta)\right)}{B(B-\Delta)}\end{pmatrix} \quad (37)$$

$$\underset{D\to\propto}{F(D)}=1-0=1$$

3.2.6. Total cost

The total variable cost can be expressed in mathematical form:

$$\text{T.C.}=\frac{\frac{12\times\Delta}{Q}C_s+\left(D-\Delta\left(\frac{1}{K}+\frac{1}{\lambda}\right)\right)}{C_u\times C_c+\frac{1}{2}Q\times C_u\times C_c} \quad (38)$$

3.2.7. Order quantity

If α be the percentage of stock out per cycle then α = 1 - F(D)

$$=\frac{K\lambda\sigma^2}{(\lambda-K)}\left(\frac{\exp(-D/\sigma^2(A-\Delta)}{A(A-\Delta)}\right)-\frac{\exp(-D/\sigma^2(B-\Delta))}{B(B-\Delta)} \quad (39)$$

If we take β = permissible total number of stock outs per year them

$$\beta=\frac{12\Delta}{Q}\times\alpha \quad (40)$$

$$\text{So } Q=\frac{\frac{12\Delta}{\beta}\times\frac{K\lambda\sigma^2}{(\lambda-K)}\left(\frac{\exp(-D/\sigma^2(A-\Delta)}{A(A-\Delta)}\right)-}{\frac{\exp(-D/\sigma^2(B-\Delta)}{B(B-\Delta)}} \quad (41)$$

3.3. Modification of the two models when total lot is divided in sub lots

In the two models discussed before it is assumed that the total quantity is delivered at one time but in actual case the total quantity comes in lots after the procurement lead time.

Now if the inter-arrival time is taken constant and is equal to T_1 (month) then the demand during inter-arrival time of the item (CRGO) from store will be $N(d, \Delta, T_i, \sigma\sqrt{T_i})$.

If the quantity receipt (a) is normally distributed with a mean of Δ a and standard deviation σ and denoted by N (a, Δa, σ) then the net inventory building up in each inter arrival time (T_1) will be distributed with a mean (Δa - Δ, T_i) and standard deviation $\sqrt{\sigma^2 + (\sigma\sqrt{T_i})^2}$. If the net accumulation is denoted by $f(a_n)$ then:

$$f(a_n) = N(a_n, \Delta_n, \sigma_n) \tag{42}$$

Where $\Delta_n = \Delta_a - \Delta T_i$ (42')

and $$\sigma_n = \sqrt{\sigma^2 + (\sigma\sqrt{(T_i)})^2} \quad (42'')$$

Now if $\Delta_n - 3\sigma_n \geq 0$ (43)

If $\Delta_n - 3\sigma_n < 0$ (44)

then at α cyclic probability of stock out and number of sub lots the probability of stock out in inter-arrival time, P(s) is expressed by:

$$\alpha = 1 - (1 - p(s))^n \tag{45}$$

So after getting p(s) one can get the value of required extra buffer stock that should be kept between the arrival of first and last sub lots.

If this extra buffer is denoted by B then the R.O.P. calculated as before by any of the two models should be increased by $B_{ST.}$

So New R.O.P. = R.O.P. + B_{ST} (46)

3.4. Procedural steps to the solution

In the first model the economic lot size is derived in equation (13) in the form of equation (47)

$$Q_0 = \frac{1}{L}\left(1 + \left(\sqrt{1 + \frac{24C_s\Delta L^2}{C_u C_c}}\right)\right) \cong \frac{2}{L} \tag{47}$$

Where $L = -\frac{\sqrt{\Delta^2 + 2\sigma^2\lambda} - \Delta^2}{\sigma^2}$

The R.O.P. derived from equation (8) can be shown as equation (48)

$$\text{R.O.P.} = 2.3\left(\log\left(\frac{\frac{\alpha}{\lambda\sigma^2}(\Delta^2 + 2\sigma^2\lambda -}{\Delta\sqrt{\Delta^2 + 2\sigma^2\lambda})}\right)\right) \times \frac{\sigma^2}{\left(\Delta - \sqrt{\Delta^2 + 2\sigma^2\lambda}\right)} \tag{48}$$

So one can see that in the first model for a particular service level the R.O.P. and the Economic order quantity is directly available from the mode.

But in the 2nd model there is scope of taking the effect of any administrative delay. Hence the problem is much more completed and no direct Economic Order quantity can be obtained directly from the equation.

In this case the author has fixed the permissible stock out per year (β) which is equal to number of cycles (order) into the probability of stock out cycle (α).

After fixing the permissible stock out per year one can see with different β how the ordering quantity is related with re-order point by equation (41). The author also obtained the cost of carrying ordering quantity and buffer stock, cost of order per year and the total cost per year. So minimizing total cost for any re-order point with a fixed β one is able to find it out the economic order quantity. This re-order point is the optimum R.O.P. ($R.O.P_0$) which corresponds to economic order quantity (Q_0).

Now the $R.O.P_0$ and (Q_0) might have been obtained by the computer by finding out the point at which the slope of the total cost curve is zero.

In the second model the relation between ordering point and ordering quantity is related by the equation (41).

$$Q = \frac{12\Delta}{\beta} \times \frac{\lambda K\sigma^2}{(\lambda - K)}\left(\frac{\exp(-D/\sigma^2(A-\Delta))}{A(A-\Delta)}\right) - \left(\frac{\exp(-D/\sigma^2(B-\Delta))}{B(B-\Delta)}\right)$$

and the total cost, TC = Number of orders × ordering cost + (Order Level, R.O.P. - average consumption in the two average lead time) × Unit cost × carrying cost + ½ × order quantity × unit cost × carrying cost..

For a particular β (number of stock out per year) with different R.O.P. values, the corresponding ordering quantities and total variable costs are computed by equation (41) and equation (38).

4. Results and Analysis

The variation of optimum re-order point and economic order quantity are investigated in different parametric conditions by the second model. The results are observed in different ways to show the effect of each parameter on another. So, by analytical model, the results are explained in the following steps:

1. It is seen that for any particular Administrative delay lead time, the economic order quantity increases with the demand rate.
2. It may be noted that there is no significant variation of E.O.Q. with the variation of administrative lead time.
3. When procurement lead time increases, the Economic Order Quantity also increases.
4. It may be concluded that with the large variation of standard deviation of demand rate, it has little effect on designing the R.O.P. and E.O.Q.
5. It can be concluded that with the increase of demand rate, there is also increase in optimal re-order point.
6. It is seen that as procurement Lead Time is increasing, the R.O.Po is also increasing, and there exists a relationship between R.O.Po and $1/\lambda$. So, one may

conclude that R.O.Po is dependent on procurement lead time.

7. If there is larger variation of demand rate, the R.O.Po and Qo will practically differ on nothing.
8. It is seen that as the permissible stock out/ year decreases, the R.O.Po increases rapidly.
9. When Demand rate increases, more inventories are to be purchased and R.O.Po and Qo are also increased. So, the total variable cost will also increase.
10. With increase of Administrative delay lead time, the total variable cost (T.C) is increased only slightly.
11. It is seen that with the increase of procurement lead time, the total variable cost increases.
12. There is very little variation of total variable cost with the variation of standard deviation of demand rate.
13. When permissible stock out decreases, the total variable cost increases rapidly. This is due to high level of R.O.Po and economic order quantity.

5. Conclusions

For probabilities of stock out between inter-arrival of sub lots, the modification of the results obtained by the models generates a better result. Arrival of sub lots can be dictated by the application of model. The demand rate is the constant inter-arrival time of one day.

The net accumulation (a_n) is dictated by the normal distribution with mean inter-arrival time of one day. So, there is no probability of stock out between inter-arrival of sub lots. Consequently, no correction of the results obtained by model 2 is necessary, because the critical portion is taken into account by the model.

5.1. Limitations of the Model

In an actual situation, in the future period, the demand, lead times, and the sub lots quantity may change their respective probability distribution patterns. So, in that case, the correct prediction of optimum re-order point and economic ordering quantity may be wrong by the model. In this model, the inter-arrival time between receiving of sub lots is taken constant. Actually, if it varies at an extended period randomly, the model is not correct to produce the results.

The model does not take the interaction of the other production units of the company. It does not distinguish between the different suppliers. The model does not take the effect of any rejection of sub lots due to bad qualities.

5.2. Scope of further Work

It is considered that there is scope for further work in relation to the present study with regard to the following aspects:

(i) To test the effectiveness of the inventory control system developed in this study, the actual distribution of inter-arrival time of sub lots as well as the sub lots quantity distribution can be introduced in Monte Carlo simulation model.

(ii) Taking different statistical distribution systems for demand and different lead times if conditions of the distribution change in future.

(iii) Other components may be also considered for other industries.

References

Abdelsalam, H. M., & Elassal M. M. (2014). Joint economic lot sizing problem for a three—layer supply chain with stochastic demand. *Int. J. Production Economics*, 155, 272–283

Abdul-Jalbar, B., Gutiérrez, J., Puerto, J., & Sicilia, J.(2003). Policies for inventory /distribution systems: The effect of centralization vs decentralization. *International Journal of Production economics*, 81-82,281-93

Almeder, C., Klabjan, D., Traxler, R., & Almada-Lobo, B. (2015). Lead time considerations for the multi-level capacitated lot-sizing problem. *European Journal of Operational Research*, 241, 727-738

Axsater, S.(2001). A Framework for Decentralized Multi-Echelon Inventory Control. IEEE Transactions, 33,91-97.

Axsater, S. (2003). Approximate optimization of a two-level distribution inventory system. *International Journal of Production Economics*, 81-82, 545-53

Cárdenas-Barrón, L. E., González-Velarde, J. L., Treviño-Garza, G. (2015). A new approach to solve the multi-product multi-period inventory lot sizing with supplier selection problem. *Computers & Operations Research*, 64, 225–232

Christina, W.Y.W., Kee-hung, L., & Edward, W.N.B. (2015). The performance of contingencies of supply chain information integration: The roles of product and market complexity. *International Journal of Production Economics*, 165, 1-11

Cramer, H. (1946). Mathematical Methods of Statistics. Princeton University Press.

Eltogral, K., Darwish, M., & Ben-Daya, M. (2007). Production and shipment lot sizing in a vendor-buyer supply chain with transportation cost. *European Journal of Operational Research,* 176, 1592-1606

Forsberg, R. (1996). Exact evaluation of (R, Q)-policies for two-level inventory systems with Poisson demand. *European Journal of Operational Research*, 96,130-38

Ganeshan, R. (1999). Managing supply chains inventories: A multiple retailer, one warehouse, multiple supplier models. *International Journal of Production Economics,* 59, 341-54

Gradshteyn, I.S., & Ryzhik, I.M.(1965). Table of Integrals, Series and Products. New York: Academic Press.

Hadley, G., & Whitin, T.M. (1963). Analysis of Inventory Systems. Prentice- Hall, Englewood Cliffs, N.J.

Lee, W. (2005). A joint economic lot size model for raw material ordering, manufacturing setup, and finished goods delivering. *Omega*, 33, 163-74

Lee, H.T., & Wu, J.C. (2006). A study on inventory replenishment policies in a two-echelon supply chain system. *Computers & Industrial Engineering*, 51 (2), 257-63

Mazdeh, M. M., Emadikhiav M., & Parsa , I. (2015). A heuristic to solve the dynamic lot sizing problem with supplier selection and quantity discounts. *Computers & Industrial Engineering*, 85, 33–43

Moghadam, M. R. S., Afsar, A., & Sohrabi, B. (2008). Inventory lot-sizing with supplier selection using hybrid intelligent algorithm. *Applied Soft Computing*, 8, 1523–1529

Nurshanti, I., & Suparno, I. (2010). Push and pull joint economic lot sizing model with remanufacturing. Proceeding of ICIEBM, Yogakarta-Indonesia, 419-424

Prasertwattana, K., & Chiadamrong, N. (2004a). Purchasing and Inventory Policy in a Supply Chain under the Periodic Review: A Single Manufacturer and Multiple Retrailers' Case. *Industrial Engineering and Management Science*, 3, 38-51

Prasertwattana, K., & Chiadamrong, N. (2004b). Improving Material Ordering and Inventory Control Systems under the Supply Chain Concept Using Genetic Algorithm, *Research and Development Journal*. 15(1), 67-80

Pujawan, I.N. (2004). The effect of lot sizing rules on order variability. European Journal of Operational Research, 159, 617–35.

Rahman, M.A.A., & Sarker, B.R.(2006). Supply chain models for an assembly system with preprocessing of raw materials. *European Journal of Operational Research*, 181(2), 733-752

Seo, Y., Jung, S., & Hahm, J.(2002). Optimal reorder decision utilizing centralized stock information in a two-echelon distribution system. *Computers & Operation Research*, 29, 171-93

Tee, Y., & Rossetti, M.D. (2002). A robustness study of a multi-echelon inventory model via simulation. *International journal of Production Economics*, 80, 265-77

Yaghin R. G., Fatemi Ghomi S.M.T., & Torabi S.A. (2013). A possibilistic multiple objective pricing and lot-sizing model with multiple demand classes. *Fuzzy Sets and Systems*, 231, 26–44

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