

# An Assessment Method for Project Cash Flow under Interval-Valued Fuzzy Environment

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## Abstract

Effective project management requires reliable knowledge of cash required in different stages of project life cycle. Getting this knowledge is highly dependent on sophisticated consideration of project environment. Nature of projects and their environments is associated with uncertain conditions. In this paper, a new project cash flow assessment method based on project scheduling is proposed to foresee projects' cash flow in their different stages. Interval-valued fuzzy sets (IVFSs) are applied to address the uncertainty of activity durations and costs. First, an IVF-project scheduling method is proposed to calculate early start time and early finish time of activities under IVF-environment; accordingly, a new method of cash flow assessment is introduced under IVF-environment. For the purpose of illustration, the proposed method is implemented to generate cash flow of main activities of a large-scale project. The results show the flexibility of the presented assessment method in expressing uncertainty, in addition to its capability in risk evaluation. Furthermore, using alpha-cuts to address different levels of uncertainty and risk provides a comprehensive insight into the cash required in different stages of project life cycle under different levels of risk and uncertainty. Finally, the results are discussed, and the proposed method is believed to be useful in the project evaluation.

*Keywords:* Cost forecasting, Project cash flow, Fuzzy project scheduling, Assessment method, Interval-valued fuzzy sets (IVFSs).

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## 1. Introduction

Project profitability in construction industry is highly influenced by cash (Hwee and Tiong, 2002; Ebrahimnejad et al., 2012). Inability in providing for daily activities of projects caused by ineffective cash flow control could lead to projects failure (Khosrowshahi and Kaka, 2007). Therefore, the project manager is highly dependent on reliable project cash flow to foresee, analyze, and make effective decisions to deal with the potential problems. Critical path method (CPM) assumes durations of project activities to be deterministic and known, while, in reality, they are rarely known in advance (Zammori et al., 2009). Also, CPM is not capable of modeling the unpredictable nature of project activities (Barraza et al., 2000). Moreover, the standard cost flow generation derived from CPM suffers from impractical scheduling techniques. Thus, fuzzy extensions of CPM and program evaluation and review technique (PERT) have been proposed in recent decades as efforts to tackle these problems. While it is widely accepted that effective project management requires managing uncertainty, satisfying this requirement by a more sophisticated approach could be a vital step before getting practical results (Atkinson et al., 2006).

Sophisticated uncertainty management requires sophisticated tools. Most of the previous fuzzy-based

approaches were based on classic fuzzy sets theory (Zadeh, 1965). In fuzzy sets theory, the decision-maker (DM) faces difficulty when expected to give an exact opinion in a number in interval  $[0, 1]$ . Expressing this degree of uncertainty by an interval is a possible solution. Interval-valued fuzzy sets (IVFSs) by Grattan-Guinness (1975) are appropriate tools for this matter. This practical extension of the fuzzy sets theory replaces traditional  $[0, 1]$  - valued membership degrees by intervals in  $[0, 1]$ . This is how the DM can express unknown and vague membership degrees. IVFSs allow for addressing the lack of information and vagueness based on feelings rather than mere facts or proof. Another advantage of the IVFS is its ease in application in comparison with type-2 fuzzy sets (Cornelis et al., 2006).

In this paper, project cash flow is generated based on IVF-project scheduling. This approach simultaneously gives the advantages of IVF-sets and fuzzy project scheduling. Fuzzy scheduling is firstly introduced to calculate early start time and early finish time of activities with IVF-numbers; then, cash flow generation approach is presented based on the IVF-scheduling. Moreover, alpha-cuts of IVF-numbers are applied in the proposed assessment method to consider different levels of

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confidence in fuzzy parameters. Finally, an application example based on the main activities of a large-scale project is solved to illustrate the application of the IVF-assessment method, and the computational results are discussed.

The rest of the paper is organized as follows. In the next section, a brief overview of the related studies on project cash flow analysis is given. In section 3, the IVF-CPM is introduced. In section 4, the proposed IVF-cash flow generation approach is proposed. The method is implemented for an activity network of the main activities of a large-scale project, and the corresponding results are discussed in Section 5. Finally, Section 6 concludes this paper.

## 2. Literature Review of Fuzzy Project Cash Flow

A large number of cash flow analysis methods are based on cost *S*-curves in which receipts and payments in different stages of the project are analyzed. Usually, the appreciation in projects cost is assumed as follows:

- In the first third of project duration, the cost achieves one quarter of the costs incurred at one third of project duration in a parabolic pattern.
- In the second third of project duration, the cost accumulation is linear and reaches three quarters of the total project costs.
- Finally, in the third duration, cost accumulation reaches 100% of the total cost in the mirror image of the first third duration shape.

Usually, shape of letter “S” of cost-time curve makes it be termed as the “S-Curve” (Cooke and Jepson, 1979). Accuracy of any S-curve based method depends on how well the assumptions of conditions represent the real-world situation of the project (Boussabaine and Kaka, 1998). An approach to model the imprecise data of a project is applied to the fuzzy sets theory.

Several studies based on fuzzy sets theory focusing on cash flow analysis and generation have been carried out in the recent decades. Boussabaine and Elhag (1999) used fuzzy averaging techniques to analyze cash flow. Kumar et al. (2000) used fuzzy sets theory to assess working capital requirements under uncertainty. Lam et al. (2001) proposed a method to find the optimum cash flow while consuming minimum resources. Their method was based on fuzzy reasoning and fuzzy optimization. Barraza et al. (2004) used stochastic S-Curve generated by simulation. By proposing a fuzzy, stochastic, single-period model of cash management, Yao et al. (2006) tried to provide managers with a comprehensive insight into the real situations in cash management problems. Khosrowshahi and Kaka (2007) proposed a decision support model to manage cash flow of construction projects. Maravas and Pantouvakis (2012) proposed a cash flow calculation method that was represented by an S-surface for projects with fuzzy duration and costs. Their method was based on fuzzy project scheduling. Rostami et al. (2013) proposed a

fuzzy statistical expert system for cash flow analysis, designed to handle the uncertain environment of projects.

It is concluded from the above that despite using fuzzy sets theory to address the uncertainty in project cash flow generation, most of the studies were based on the classical fuzzy sets theory and were deprived of the advantages existing in new extensions of fuzzy sets theory. For instance, when a new project is being planted, lack of knowledge and expertise could not be perfectly handled by classical fuzzy set. Another shortcoming of the existing literature lies in ignoring fuzzy project scheduling in cash flow analysis. Using fuzzy project scheduling to generate cash flow results in a method close to what practitioners use to generate cash flow. Also, this approach has more accuracy in modeling uncertainty according to the recent literature (Maravas and Pantouvakis, 2012). In this paper, a new method of cash flow analysis is proposed based on IVFSs and fuzzy project scheduling. This method simultaneously has the merits of IVFSs and IVF-project scheduling. In other words, the DM can express uncertainty with more flexibility, and consequently achieve more reliable results. Additionally, the results are in a form close to what practitioners use in project management.

## 3. IVF-CPM Algorithm

In order to enable project scheduling to model real-world projects, fuzzy project scheduling has been introduced. Fuzzy project scheduling can consider activities with fuzzy durations. By calculating early start and early finish time, forward pass can be obtained (Chanas and Kamburowski, 1981; McCahon and Lee, 1988; Prade, 1979).

$$\tilde{E}S_{start} = (0) \quad (1)$$

$$\tilde{E}S = \tilde{m}\tilde{a}x(\tilde{E}F_p)_{p \in P} \quad (2)$$

$$= \left[ (ef_{p_1}^U, ef_{p_1}^L), ef_{p_2}, (ef_{p_3}^L, ef_{p_3}^U) \right] \quad (3)$$

$$\tilde{d} = [(d_1^U, d_1^L), d_2, (d_3^U, d_3^L)] \quad (4)$$

$$\tilde{E}F = \tilde{E}S + \tilde{d} = \left[ (ef_{p_1}^U, ef_{p_1}^L), ef_{p_2}, (ef_{p_3}^L, ef_{p_3}^U) \right] + [(d_1^U, d_1^L), d_2, (d_3^L, d_3^U)] = \left[ (ef_{p_1}^U + d_1^U, ef_{p_1}^L + d_1^L), ef_{p_2} + d_2, (ef_{p_3}^L + d_3^L, ef_{p_3}^U + d_3^U) \right].$$

where  $\tilde{E}S$  is the IVF-early start time,  $\tilde{E}F$  is the IVF-early finish time,  $P$  is the set of proceeding activities, and  $\tilde{d}$  is the IVF-activity duration.

Notably, the IVF-operators are presented in Appendix A. Also, the  $\tilde{m}\tilde{a}x$  operator is based on ranking and signed distance of IVF-numbers from  $\tilde{0}$  introduced by Su (2007) and presented in detail in Appendix B.

#### 4. Interval-valued Fuzzy Cash Flow Generation Approach

Cost per unit of time calculation is required in order to generate project cash flow. By introducing IVF-project scheduling, uncertainty in activities' durations will be considered in the cash flow. Therefore, durations differ for activities in different scenarios, and this approach leads to indifferent results for the cash flow. In order to get an insight into the longest and shortest durations, activities starting in the earliest possible time and lasting the least duration (*Min Duration<sub>α</sub>*) and activities starting in the latest possible time and lasting the longest duration (*Max Duration<sub>α</sub>*) should be considered in cash flow generation. Duration calculation for activities with IVF-time method can be extended based on (Maravas and Pantouvakis, 2010). The resulting IVF-CPM not only enables the project managers to consider uncertainty in a more practical way, but also provides a more thorough understanding of the activities durations. These durations for activities with early start  $\tilde{E}S = [(es_1^U, es_1^L), es_2, (es_3^L, es_3^U)]$  and early finish  $\tilde{E}F = [(ef_1^U, ef_1^L), ef_2, (ef_3^L, ef_3^U)]$  are calculated as follows:

$$Min D_{\alpha} = [min\ duration_{\alpha}^L, min\ duration_{\alpha}^U] \quad (5)$$

$$Min D_{\alpha}^L = [inf ES_{\alpha}^L, inf ES_{\alpha}^L + inf D_{\alpha}^L] \quad (6)$$

$$= [\frac{\alpha}{\lambda}(es_2 - es_1^L) + es_1^L, \frac{\alpha}{\lambda}(ef_2 - ef_1^L) + ef_1^L]$$

$$Min D_{\alpha}^U = [inf ES_{\alpha}^U, inf ES_{\alpha}^U + inf D_{\alpha}^U] \quad (7)$$

$$= [\frac{\alpha}{\rho}(es_2 - es_1^U) + es_1^U, \frac{\alpha}{\rho}(ef_2 - ef_1^U) + ef_1^U]$$

$$Max D_{\alpha} = [max\ duration_{\alpha}^L, max\ duration_{\alpha}^U] \quad (8)$$

$$Max D_{\alpha}^L = [sup ES_{\alpha}^L, sup ES_{\alpha}^L + sup D_{\alpha}^L] \quad (9)$$

$$= [\frac{\alpha}{\lambda}(es_2 - es_3^L) + es_3^L, \frac{\alpha}{\lambda}(ef_2 - ef_3^L) + ef_3^L]$$

$$Max D_{\alpha}^U = [sup ES_{\alpha}^U, sup ES_{\alpha}^U + sup D_{\alpha}^U] \quad (10)$$

$$= [\frac{\alpha}{\rho}(es_2 - es_3^U) + es_3^U, \frac{\alpha}{\rho}(ef_2 - ef_3^U) + ef_3^U]$$

where  $Min D_{\alpha}^L$  and  $Min D_{\alpha}^U$  represent  $\alpha$ -cuts of minimum duration in the lower and upper IVF-numbers,  $Max D_{\alpha}^L$  and  $Max D_{\alpha}^U$  denote  $\alpha$ -cuts of maximum duration in the lower and upper IVF-numbers, respectively.  $ES_{\alpha}^L$  and  $ES_{\alpha}^U$  are  $\alpha$ -cuts of IVF-early start of the lower and upper numbers,  $sup$  is supremum,  $inf$  is infimum, and  $\lambda$  and  $\rho$  are membership degrees of the lower and upper numbers, respectively. It should be noted that  $0 \leq \alpha \leq 1$ .

Fig. 1 displays an activity with early start of [(1,2), 3, (4,5)], duration of [(5,6), 7, (8,9)], and early finish of [(6,8), 10, (12,14)]. This activity has durations of  $min D_0^L = [2, 8]$ ,  $max D_0^L = [4, 12]$ ,  $min D_0^U = [1, 6]$ , and  $max D_0^U = [5, 14]$ . Each one of the maximum and minimum durations is separately calculated for the upper and lower fuzzy numbers. Consequently, uncertainty can play a more vital role in duration calculations, and since activity cost is distributed in these intervals, uncertainty is also calculated in cash distribution. In the best possible scenario, the activity can start at interval of [1, 2] and end at [6, 8]; in the worst case, it can start at interval of [4, 5] and last till [12, 14]. All the calculations for different  $\alpha$ -levels can be made.

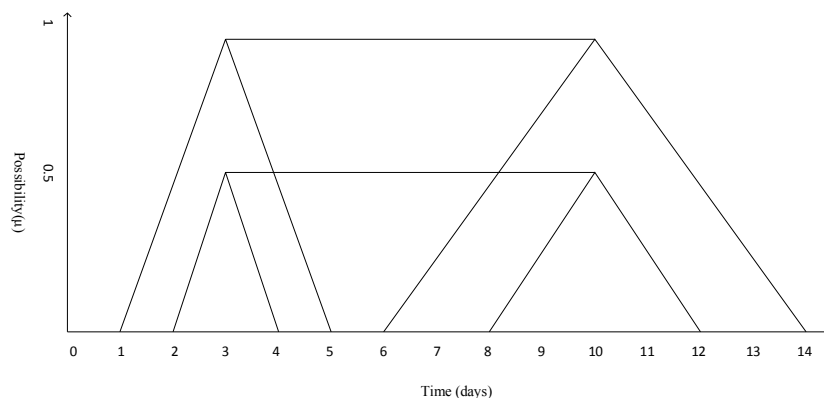


Fig. 1. IVF activity start time and completion time

Uncertainty in cost and duration is responsible for project cash flow changes. Correlation of time and cost uncertainties can follow different patterns. Sometimes, activities have fixed costs with uncertain durations, or vice versa. There are cases where time and cost uncertainties are correlated, which means if one of them changes unfavorably, it causes unfavorable changes in another (Maravas and Pantouvakis, 2012). Thus, if they are positively correlated, the best and worst cases of cash distribution of the upper and lower interval-valued numbers for activity  $i$  with  $cost_i = [(ci_1^U, ci_1^L), ci_2, (ci_3^L, ci_3^U)]$  and duration of  $d_i = [(di_1^U, di_1^L), di_2, (di_3^L, di_3^U)]$  per unit of time  $t$  at level  $\alpha$  can be defined as follows:

$$\begin{aligned} \min \text{ cash distribution } i, t_\alpha^L &= \frac{\inf \text{ cost } i_\alpha^L}{\sup \text{ duration } i_\alpha^L} & (11) \\ &= \frac{\frac{\alpha_c}{\lambda_c} (ci_2 - ci_1^L) + ci_1^L}{\frac{\alpha_d}{\lambda_d} (di_2 - di_3^L) + di_3^L}, 0 \leq \alpha \\ &\leq \lambda \end{aligned}$$

$$\begin{aligned} \min \text{ cash distribution } i, t_\alpha^U &= \frac{\inf \text{ cost } i_\alpha^U}{\sup \text{ duration } i_\alpha^U} & (12) \\ &= \frac{\frac{\alpha_c}{\rho_c} (ci_2 - ci_1^U) + ci_1^U}{\frac{\alpha_d}{\rho_d} (di_2 - di_3^U) + di_3^U} \end{aligned}$$

$$\begin{aligned} \max \text{ cash distribution } i, t_\alpha^L &= \frac{\sup \text{ cost } i_\alpha^L}{\inf \text{ duration } i_\alpha^L} & (13) \\ &= \frac{\frac{\alpha_c}{\lambda_c} (ci_2 - ci_3^L) + ci_3^L}{\frac{\alpha_d}{\lambda_d} (di_2 - di_1^L) + di_1^L}, 0 \leq \alpha \\ &\leq \lambda \end{aligned}$$

$$\begin{aligned} \max \text{ cash distribution } i, t_\alpha^U &= \frac{\sup \text{ cost } i_\alpha^U}{\inf \text{ duration } i_\alpha^U} & (14) \\ &= \frac{\frac{\alpha_c}{\rho_c} (ci_2 - ci_3^U) + ci_3^U}{\frac{\alpha_d}{\rho_d} (di_2 - di_1^U) + di_1^U} \end{aligned}$$

Moving from activities to the entire activity network, these formulas can be extended to calculate the sum of direct cash distribution of all activities in any time period.

$$\min Ct_\alpha^L = \sum_{i=1}^n \frac{\frac{\alpha_c}{\lambda_c} (ci_2 - ci_1^L) + ci_1^L}{\frac{\alpha_d}{\lambda_d} (di_2 - di_3^L) + di_3^L}, 0 \leq \alpha \leq \lambda \quad (15)$$

$$\min Ct_\alpha^U = \sum_{i=1}^n \frac{\frac{\alpha_c}{\rho_c} (ci_2 - ci_1^U) + ci_1^U}{\frac{\alpha_d}{\rho_d} (di_2 - di_3^U) + di_3^U} \quad (16)$$

$$\begin{aligned} \max Ct_\alpha^L &= \sum_{i=1}^n \frac{\frac{\alpha_c}{\lambda_c} (ci_2 - ci_3^L) + ci_3^L}{\frac{\alpha_d}{\lambda_d} (di_2 - di_1^L) + di_1^L}, 0 \leq \alpha \\ &\leq \lambda \end{aligned} \quad (17)$$

$$\max Ct_\alpha^U = \sum_{i=1}^n \frac{\frac{\alpha_c}{\rho_c} (ci_2 - ci_3^U) + ci_3^U}{\frac{\alpha_d}{\rho_d} (di_2 - di_1^U) + di_1^U} \quad (18)$$

where  $Ct$  is the total direct cash distribution of activities ( $i = 0, 1, 2, \dots, n$ ) occurring in  $t$  time period. In order to calculate the total cash flow of the project, the sum of direct cash distribution of activities in different time periods should be calculated. These calculations are presented as follows:

$$\min CFt_\alpha^L = \sum_{t=0}^T \sum_{i=1}^n \frac{\frac{\alpha_c}{\lambda_c} (ci_2 - ci_1^L) + ci_1^L}{\frac{\alpha_d}{\lambda_d} (di_2 - di_3^L) + di_3^L}, 0 \leq \alpha \leq \lambda \quad (19)$$

$$\min CFt_\alpha^U = \sum_{t=0}^T \sum_{i=1}^n \frac{\frac{\alpha_c}{\rho_c} (ci_2 - ci_1^U) + ci_1^U}{\frac{\alpha_d}{\rho_d} (di_2 - di_3^U) + di_3^U} \quad (20)$$

$$\max CFt_\alpha^L = \sum_{t=0}^T \sum_{i=1}^n \frac{\frac{\alpha_c}{\lambda_c} (ci_2 - ci_3^L) + ci_3^L}{\frac{\alpha_d}{\lambda_d} (di_2 - di_1^L) + di_1^L}, 0 \leq \alpha \leq \lambda \quad (21)$$

$$\max CFt_\alpha^U = \sum_{t=0}^T \sum_{i=1}^n \frac{\frac{\alpha_c}{\rho_c} (ci_2 - ci_3^U) + ci_3^U}{\frac{\alpha_d}{\rho_d} (di_2 - di_1^U) + di_1^U} \quad (22)$$

The minimum and maximum values show the limits of cash flow within the best and worst scenarios. This approach would provide the project manager with a better understanding of uncertainty in different periods of project and would enable the manager to make the proper tactical decisions. Uncertainty in the cash flow in different  $\alpha$ -levels can be obtained by the following:

$$CFU_\alpha = [CFU_\alpha^L, CFU_\alpha^U] \quad (23)$$

$$\begin{aligned} CFU_\alpha^L &= \max CFt_\alpha^L - \min CFt_\alpha^L & (24) \\ &= \sum_{t=0}^T \sum_{i=1}^n \frac{\frac{\alpha_c}{\lambda_c} (ci_2 - ci_3^L) + ci_3^L}{\frac{\alpha_d}{\lambda_d} (di_2 - di_1^L) + di_1^L} \\ &\quad - \sum_{t=0}^T \sum_{i=1}^n \frac{\frac{\alpha_c}{\lambda_c} (ci_2 - ci_1^L) + ci_1^L}{\frac{\alpha_d}{\lambda_d} (di_2 - di_3^L) + di_3^L}, \\ &0 \leq \alpha \leq \lambda \end{aligned}$$

$$CFU_{\alpha}^U = \max CFt_{\alpha}^U - \min CFt_{\alpha}^U \tag{25}$$

$$= \sum_{t=0}^T \sum_{i=1}^n \frac{\frac{\alpha_c}{\rho_d} (ci_2 - ci_3^U) + ci_3^U}{\frac{\alpha_d}{\rho_d} (di_2 - di_1^U) + di_1^U}$$

$$- \sum_{t=0}^T \sum_{i=1}^n \frac{\frac{\alpha_c}{\rho_d} (ci_2 - ci_1^U) + ci_1^U}{\frac{\alpha_d}{\rho_d} (di_2 - di_3^U) + di_3^U}$$

$CFU_{\alpha}$  displays uncertainty in working capital requirement throughout the project implementation. This measure helps the project manager to manage working requirement capitals more efficiently. The main purpose of cash flow management is to make a balance between investment capital and working capital; thus, having a comprehensive understanding of uncertainty enables the manager to achieve this goal, in addition to avoiding any unpleasant surprises.

**5. Application Example**

For the purpose of illustration, the proposed method is applied to generate cash flow for a network of main activities in a construction project. The activity network is displayed in Fig. 2, and the adopted data consisting of activities with IVF-durations and costs are displayed in Table 1. The membership degrees of the lower and upper numbers for all durations and costs are considered as 1 and 0.6, respectively.

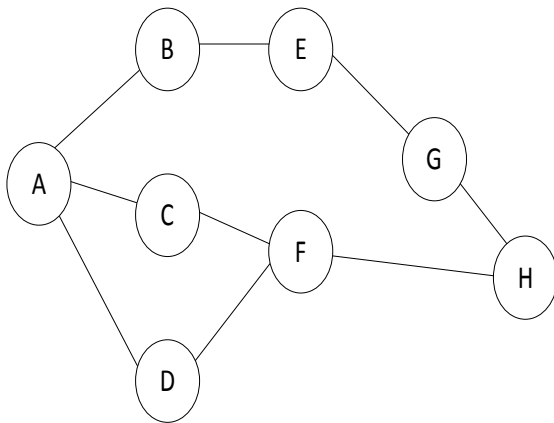


Fig. 2. Sample activity network

**5.1. Computational results**

IVF-project scheduling is applied to calculate early start time and early finish time activities. The corresponding results are presented in Table 2. Based on the achieved results, IVF-Gantt chart is illustrated in Fig. 3. This chart illustrates a two-dimensional Gantt schedule with the project of early start and finish dates, which are presented

by IVF-numbers and calculated from the forward pass. The x-axis denotes time, while the y-axis demonstrates the activity name and the membership function which is the possibility level. Unlike conventional Gantt charts, activity durations are defined by two IVFSs which are the IVF-start date and the IVF-completion date. This approach provides the actual start and completion activity limits. Cash distributions for all activities are calculated at three different  $\alpha$ -cuts, and the results are displayed in Table 3.

Table1  
Activity network data

Activity	Predecessor	Lag	IVF-duration(days)	IVF-cost(\$)
A	-	0	[(5,7),8,(9,11)]	[(20,35),40,(45,60)]
B	-	0	[(2,4),6,(7,9)]	[(10,25),30,(35,50)]
C	A	0	[(16,18),20,(23,25)]	[(220,235),240,(245,260)]
D	A	2	[(10,12),15,(17,19)]	[(130,145),150,(155,170)]
E	A,B	0	[(6,8),10,(11,13)]	[(20,35),40,(45,60)]
F	C,D	0	[(7,9),10,(12,14)]	[(60,75),80,(85,100)]
G	E	0	[(6,8),10,(12,14)]	[(40,55),60,(65,80)]
H	F,G	0	[(1,3),4,(5,7)]	[(10,25),30,(35,50)]

Table 2  
IVF-project scheduling

Activity	Early start	Early finish
A	[(0,0),0,(0,0)]	[(5,7),8,(9,11)]
B	[(0,0),0,(0,0)]	[(2,4),6,(7,9)]
C	[(5,7),8,(9,11)]	[(21,25),28,(32,36)]
D	[(7,9),10,(11,13)]	[(17,21),25,(28,32)]
E	[(5,7),8,(9,11)]	[(11,15),18,(20,24)]
F	[(21,25),28,(32,36)]	[(28,34),38,(44,50)]
G	[(11,15),18,(20,24)]	[(17,23),28,(32,38)]
H	[(28,34),38,(44,50)]	[(29,37),42,(49,57)]

Table 3  
Cash distributions

Activity	$\alpha = 0$		$\alpha = 0.5$		$\alpha = 1$					
	Min l	Max l	Min u	Max u	Min u	Max u				
A	3.8	6.4	1.8	12	4.8	5.2	3.1	7.7	5	5
B	3.5	8.7	1.1	25	4.7 2	5.4	2.6	10	5	5
C	10. 2	13. 6	8.8	16.2	11. 5	12. 6	10. 2	13. 8	12	12
D	8.5	12. 9	6.8	16.2	9.7	10. 4	8.2	12. 8	10	10
E	3.1	5.6	1.5	10	3.8	4.2	2.6	6.2	4	4
F	6.2 5	9.4	4.2	14.2	7.6	8.2	5.8	10. 6	8	8
G	4.5 8	8.1 2	2.8	13.3	5.7	6.2	4.1	8.7	6	6
H	5	11. 6	1.4 2	50	7	8	3.6	16	7. 5	7. 5

5.2. Discussion of results

Cash distribution activities are the basis for further calculations. In any time period, depending on activities implemented in those periods, cumulative amount of the outcome results in the amount of cash needed in those periods. In the project management, different  $\alpha$ -cuts refer to different risk levels (Mon et al., 1995).  $\alpha$ -cut limits the degree of fuzziness and measures prediction robustness. A higher level of  $\alpha$  means a higher confidence in the parameter (Li & Vincent, 1995). The risk level increases from “none” to “high” as the  $\alpha$ -cut moves from 1 to 0. Usually, the absolute worst and best scenarios are

presented in the PERT network diagram, but in the proposed method, applying  $\alpha$ -cut enables the method to consider normal situations where somethings can be better than expected and some can be worse. Actually, this situation is more likely to happen in project environment; therefore, the proposed method is more practical. To illustrate this risk analysis method, calculations for 3 different  $\alpha$ -levels (0, 0.5 and 1) were made. Uncertainty at levels 0, 0.5, and 1 is considered as maximum, medium, and minimum, respectively.

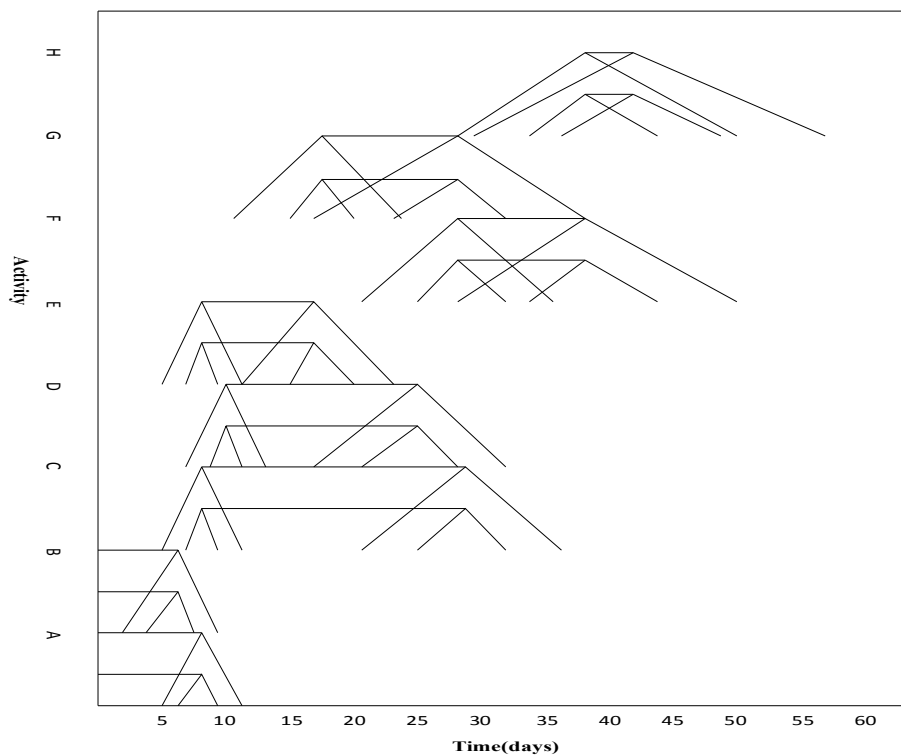


Fig. 3. IVF-Gantt chart

For example, in the first 5 days of project implementation at  $\alpha = 0.5$ , required cash for maximum and minimum of the lower and upper numbers in increasing order is 5.8, 9.5, 10.6, and 17.7. Therefore, at the best case scenario, 5.9 unit of cash is required each day, and 17.7 unit of cash is required in the worst case scenario. Cash flow uncertainty decreases as  $\alpha$ -cuts move from zero to one. Cash flow uncertainty at  $\alpha = 0$  is [31.3, 129.1], while at  $\alpha = 0.5$ , it decreases to [4.9, 45.4]. Project manager based on the risks of the environment at which the project is being implemented can set different  $\alpha$ -cuts to achieve more practical and reliable results.

6. Conclusions

In this paper, an interval-valued fuzzy (IVF) extension of project cash flow generation was introduced for activities with IVF-duration and cost. This assessment method was based on IVF-fuzzy project scheduling. This approach provided managers with thorough and comprehensive insight into cash flowing out of the project in different stages of project implementation. In other words, project cash flow in different stages of project life cycle was presented in an approach close to S-curve method applied by project managers. This comprehensive vision improved project manager’s understanding of

uncertainty and required resources, and consequently avoided unpleasant surprises. Employing IVFSs provided the method with more flexibility in addressing uncertainty along with adding fuzzy type II advantages without inheriting its complexity. IVFSs enabled the method to be effectively applied to projects, like new product development (NPD), in which the existing information was vague and unknown. The proposed method could be useful in feasibility study, in addition to implementation stage. Infact, the results could be used as an input in project evaluation methods such as net present value and rate or return. The assessment method's application was illustrated by a practical example in construction industry. In the application example, IVF-early start time and early finish time were calculated and IVF-Gantt chart was displayed. Cash distribution for activities at different  $\alpha$ -cuts was also presented to illustrate the impacts of different risk levels on the project cash flow. Finally, the results of method application were discussed. Applying this method as an evaluation tool in earned value analysis could be a promising research direction.

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**Appendix A**

In the following, some basic concepts of the IVFSs are introduced.

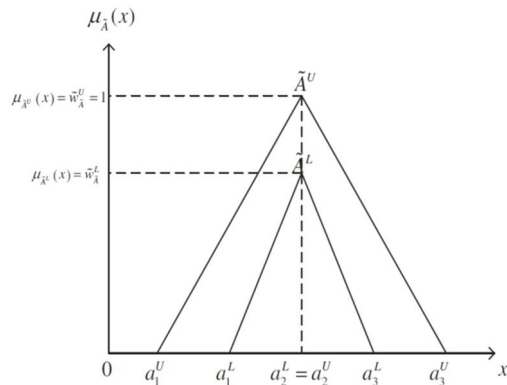


Fig. 1A. An interval-valued triangular fuzzy number  $\tilde{A}$  ( $\tilde{W}_A^L \neq \tilde{W}_A^U$ )

A triangular interval-valued fuzzy number is shown in Fig. 1A, in which  $\tilde{A}^L$  and  $\tilde{A}^U$  represent the lower and upper triangular interval-valued fuzzy numbers, and  $\tilde{W}_A^L$  and  $\tilde{W}_A^U$  are the degrees in which event  $x$  may be a member of the lower and upper numbers, respectively (Yao and Lin, 2002), which can be described as:

$$\tilde{A} = [\tilde{A}_x^L, \tilde{A}_x^U] = [(a_1^L, a_2^L, a_3^L; w_A^L), (a_1^U, a_2^U, a_3^U; w_A^U)] \quad (1A)$$

Arithmetic operations between two triangular interval-valued fuzzy numbers  $\tilde{A}$  and  $\tilde{B}$  displayed as  $\tilde{A} = [(a_1^U, a_1^L), a_2, (a_3^U, a_3^L)]$  and  $\tilde{B} = [(b_1^U, b_1^L), b_2, (b_3^U, b_3^L)]$ , respectively, are as follows (Chen, 1997; Hong and Lee, 2002; Chen and Chen, 2008; Vahdani et al, 2010; Mousavi et al., 2013):

Addition of interval-valued fuzzy numbers  $\oplus$ :

$$\begin{aligned} \tilde{A} \oplus \tilde{B} = & \quad (2A) \\ & [(a_1^U, a_1^L), a_2, (a_3^U, a_3^L)] \oplus [(b_1^U, b_1^L), b_2, (b_3^U, b_3^L)] = \\ & [(a_1^U + b_1^U, a_1^L + b_1^L), a_2 + b_2, (a_3^U + b_3^U, a_3^L + b_3^L)]. \end{aligned}$$

Subtraction of interval-valued fuzzy numbers  $\ominus$ :

$$\begin{aligned} \tilde{A} \ominus \tilde{B} = & \quad (3A) \\ & [(a_1^U, a_1^L), a_2, (a_3^U, a_3^L)] \ominus \\ & [(b_1^U, b_1^L), b_2, (b_3^U, b_3^L)] = [(a_1^U - b_1^U, a_1^L - b_1^L), a_2 - b_2, (a_3^U - b_3^U, a_3^L - b_3^L)]. \end{aligned}$$

**Appendix B**

In the following, the ranking method of IVF-numbers introduced by Su (2007) is presented.

Let  $\tilde{A} = [\tilde{A}_x^L, \tilde{A}_x^U] = [(a_1^L, a_2^L, a_3^L; w_A^L), (a_1^U, a_2^U, a_3^U; w_A^U)]$ , then the signed distance of  $\tilde{A}$  from  $\tilde{0}$  is calculated as follows:

$$\begin{aligned} (d(\tilde{A}, \tilde{0})) & \quad (1B) \\ & = \frac{1}{w_A^L} \int_0^{w_A^L} d^*([A_l^U(\alpha), A_l^L(\alpha)] \\ & \cup [A_r^L(\alpha), A_r^U(\alpha)], 0) d\alpha \\ & + \frac{1}{w_A^U - w_A^L} \int_{w_A^L}^{w_A^U} d^*([A_l^U(\alpha), A_r^L(\alpha)], 0) d\alpha \\ & = \frac{1}{8} [6a_2 + a_1^L + a_3^L + 4a_1^U + 4a_3^U + 3(2a_2 - a_1^U \\ & - a_1^L) \frac{w_A^L}{w_A^U}] \end{aligned}$$

Based on the aforementioned distance calculation method, two IVF-numbers will be ranked as follows:

$$\tilde{A} = [\tilde{A}_x^L, \tilde{A}_x^U] = [(a_1^L, a_2^L, a_3^L; w_A^L), (a_1^U, a_2^U, a_3^U; w_A^U)]$$

$$\tilde{B} = [\tilde{B}_x^L, \tilde{B}_x^U] = [(b_1^L, b_2^L, b_3^L; w_B^L), (b_1^U, b_2^U, b_3^U; w_B^U)]$$

$$\tilde{B} \leq \tilde{A} \text{ if } d(\tilde{B}, \tilde{0}) < d(\tilde{A}, \tilde{0})$$

$$\tilde{B} \approx \tilde{A} \text{ if } d(\tilde{B}, \tilde{0}) = d(\tilde{A}, \tilde{0})$$

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