Process Capability Analysis in the Presence of Autocorrelation

Mohsen Mohamadi[a], Mehdi Foumani[b,*], Babak Abbasi[c]

[a] *Faculty of Industrial and Mechanical Engineering, Qazvin Branch, Islamic Azad University, Qazvin, Iran*

[b] *School of Applied Sciences and Engineering, Monash University, Gippsland Campus, Churchill, VIC 3842, Australia*

[c] *School of Mathematical and Geospatial Sciences, RMIT University, Melbourne, Australia*

Received 23 June, 2011; Revised 27 July, 2011; Accepted 19 August, 2011

Abstract

The classical method of process capability analysis necessarily assumes that collected data are independent; nonetheless, some processes such as biological and chemical processes are autocorrelated and violate the independency assumption. Many processes exhibit a certain degree of correlation and can be treated by autoregressive models among which the autoregressive model of order one (AR (1)) is the most commonly used one. In this paper, we discuss the effect of autocorrelation on the process capability analysis when a set of observations are produced by an autoregressive model of order one. We employ the multivariate regression model to modify the process capability estimated from the classical method where AR (1) parameters are utilized as regression explanatory variables. Finally, the performance of the method developed in this paper is investigated using a Monte Carlo simulation.
Keywords: Process capability analysis; Statistical process control; Autocorrelation; AR (1).

1. Introduction

Process Capability Indices (PCIs) are introduced to give a clear indication of the capability of a manufacturing process. They are formulated to quantify the relation between the desired engineering specifications and the actual performance of the process. In fact, PCIs are organized to determine whether the process is capable of visiting specification limits on the quality features of interest or not. The quantitative measure of PCI indicates the amount of customer requirements that are obtained from quality characteristics. Generally, a larger amount of PCI shows a better process performance and a smaller amount of PCI demonstrates a worse process performance. PCIs have been applied extensively in different production systems and can be regarded as effective and superior means of determining product quality.

Basic assumptions of PCIs are: 1) The observations collected are assumed to be identically distributed 2) the observations are always assumed independent 3) the observations are normally distributed with mean μ and variance σ^2. According to the definitions and assumptions mentioned, we can use the following well-known capability indices:

$$C_p = \frac{USL - LSL}{6\sigma} \tag{1}$$

$$C_{pl} = \frac{\mu - LSL}{3\sigma} \qquad C_{pu} = \frac{USL - \mu}{3\sigma}$$

$$C_{pk} = \min\left\{C_{pl}, C_{pu}\right\} \tag{2}$$

Eq (1) determines the capability of a central process. According to Eq (2), we can also calculate the capability of a process when its specification interval mean is not equal to the process mean. The *USL* and *LSL* indicate the upper specification limit and the lower specification limit, respectively. Furthermore, μ is the process mean and σ is the standard deviation. In general, products with multiple features could usually contain huge non-central specifications and central specifications. In fact, whenever all process capabilities of each characteristic satisfy preset specifications, consumers will not reject products. It is clear that a single PCI is not able to visit the consumer requirements stated above, and it seems that many crucial problems relate to central quality characteristics.

Despite the fact that PCIs are predominantly defined under the independence assumption, most of the processes in the real world produce auto correlated data. For instance, data exhibit some degree of autocorrelation for chemical processes such as the production of pig iron. In addition, some biological processes are auto correlated and violate the independency assumption. For these processes it is proved that autocorrelation dramatically affects the amount of PCIs defined under the

* Corresponding author e-mail: mehdi.foumani@monash.edu

independency assumption (Mingoti and Oliveira, 2011). Nevertheless, the impacts of autocorrelation on the amount of PCIs have not been studied extensively. Therefore, the present study is an attempt to address this issue. Moreover, a robust estimation method for process capability will be developed.

2. Literature Review

There are quite a few studies which have dealt with process capability analysis when one of its three basic assumptions is not met. For a comprehensive discussion of these studies, see the review paper with discussion by Kotz and Johnson (2002). Furthermore, a comprehensive literature review of classified studies on process capability indices in the time span of 2000–2009 can be found in Yum and Kim (2010). However, the non-normality assumption is more investigated (for example see Clement 1989; Somerville and Montgomery 1996; Tang and Than 1999; Hosseinifard et al. 2008). In this paper, we present a new way of calculating PCI when data display an inner dependent behavior. More specifically, we explore the process capability estimation in AR (1) processes.

There are a few studies dealing with PCI estimation for auto correlated processes. Shore (1997) described some of the undesirable effects that autocorrelations may have on the sampling distribution of estimates of the mean and the standard deviation, and thus on the calculated PCIs via the Monte Carlo simulation. He stated that both C_p and C_{pk} are biased upwards when autocorrelation is present and therefore critical values and confidence intervals extracted under the assumption of independent data should not be used as the rate of type I and type II errors may be high. Besides, he carefully studied differences between the two approaches suggested in the presence of autocorrelation and also demonstrated them numerically for an AR(1) model when autocorrelation existed. The first approach was *model-free* and estimated μ and σ, by $\overline{\mathrm{X}}$ and MR/d_2, where MR was the average range observed in subsamples of size n while the process was in control. This procedure appraised the instantaneous capability but customers wished to be aware of the long-term capability of the process which also produced changes in the mean while the process was engaged producing. The second procedure estimated long-term capability. It was *a model-dependent* approach and estimated μ and σ, by $\bar{X}$ and S^2 of all data. As a result, Shore believed that the *model-free* approach is better than the *model-dependent* approach when both performance and convenience in applications are considered, and thus the process capability analysis should be used along the lines of the *model-free* approach. Still it seemed that more research was needed. Therefore, in other studies, Noorossana (2002); Scagliarini (2002); Chen et al. (2003) and Vannman and Kulahci (2008) evaluated the effect of autocorrelation on the process capability. Noorossana (2002) showed through an example that auto correlated data could lead to biased estimates of PCI, and eventually to wrong decisions about performance. He employed a two-step procedure based on multivariate regression and time series modeling to remove autocorrelation that may exist in the data and also to estimate model parameters correctly. Scagliarini (2002) described properties of the estimator of C_p for auto correlated observations in the presence of measurement errors. He assessed the performances of the estimator of C_p in the case of measurement errors for an autoregressive model of order one (AR (1)) and compared them with those results achieved in the error free case. Vannman and Kulahci (2008) devised a new way called the "iterative skipping" strategy to perform process capability analysis when observations are auto correlated. In this method, the data set was separated into subsamples by skipping a pre-determined number of observations. Clearly, as the independence assumption may be valid for the obtained sets, Eq(1) and (2) could be used to estimate PCI for each subsample of data.

Jing et al. (2009) also used the Taguchi method in order to estimate the PCI of auto correlated observations. They evaluated the impacts of autocorrelation on μ, σ, and probability density function (pdf) for models of order one. Moreover, Jing et al. (2010) developed a comparison method for five different estimation strategies of process capability when the observations were not independent. Eventually, they succeeded in finding an estimation method for process capability analysis when observations were auto correlated.

Considering the aforementioned studies, it is clear that existing studies on process capability analysis mostly do not check the autocorrelation of observations although it is common in industries and should not be overlooked. To fill in this gap, in the present paper the researchers propose a robust estimation method for the process capability analysis when a set of observations are auto correlated and produced by an autoregressive model of order one. In this regard, the rest of the paper is organized as follows: section 3 includes the suggested model and then Monte Carlo simulation studies. In section 4, the performance of the proposed method is investigated by applying statistical analyses, and finally conclusions and final remarks are provided in section 5.

3. Description of the Proposed Model

Hereinafter the estimated PCI for auto correlated processes is called $C_{p\text{-}au}$. Since AR (1) parameters impact on the bias in $C_{p\text{-}au}$ in comparison to the PCI which is known for independent observations, we propose to use these parameters to diminish autocorrelation effects on the PCI estimation. To this end, AR (1) parameters should be incorporated in the PCI formula. Here, we use the multivariate regression analysis and nominate PCI as follows:

$$C_{p-au} = C_{pu} + bias \quad (3)$$

We apply a multivariate regression model as shown in Eq(4) to calculate $C_{\text{p-au}}$. It has been known that $C_{\text{p-au}}$ is a linear combination of C_{pu}, φ and ε, where C_{pu} denotes the PCI in Eq(2) based on the independence assumption. φ and ε give us the correlation coefficient and model parameter, respectively:

$$C_{p-au} = C_{pu} + \beta_1\varphi + \beta_2\varepsilon \quad (4)$$

The model coefficients β_1 and β_2 are estimated from the observations of the process by the multivariate regression. In fact, the effects that φ and ε may have on the PCIs are the main motivation for using the proposed model in the presence of autocorrelation. We follow a two-step procedure in order to calculate β_1 and β_2. The first step is generating sets of data which are auto correlated and calculating C_{pu} for each set of data. Then, β_1 and β_2 for each set of data are estimated by the multivariate regression in the second step. To check the performance of the model, a Monte Carlo simulation is also performed.

First, we consider an AR(1) process in Eq(5) to generate sets of auto correlated data where e_{t} is a random variable that represents the amount by which the t^{th} measurement will differ from the mean due to the effect of common causes. Typically, $e_1,e_2,e_3,\ldots,e_{\text{n}}$ are regarded as a sequence of independent and identically distributed random variables with mean zero and standard deviation σ, and parameters μ and σ are estimated from the data of the process.

$$x_t = \varepsilon + \varphi x_{t-1} + e_t \quad (5)$$

Consider a situation where $C_{\text{p-au}}$=1 in this simulation, and then make simulation for certain φ, ε and a pre-determined sample size (N=10000). Afterwards, this procedure must be repeated for twenty iterations with different φ and ε in each iteration. Eq(5) can be used to obtain the upper specification limit of each set of 10000 observations.

$$C_{p-au} = \frac{\phi^{-1}\left[\int_{-\infty}^{usl} f(x)dx\right]}{3} = \frac{\phi^{-1}[\Psi_x(usl)]}{3} \quad (6)$$

where $f(x)$ and Ψ_x are the probability density function and the cumulative density function of x, respectively. It is clear that Eq(6) can be rewritten as $\phi(3C_{\text{p-au}})=p$. Because we initially set $C_{\text{p-au}}$=1 in each iteration, the value of p can be easily determined. Hence, it suffices to organize the observation of each set in the increasing order, and then choose the $p\times10000^{\text{th}}$ observation as USL since we have $\Psi_x(USL)=p$ according to Eq(6). σ must be also calculated in order to estimate $C_{\text{p-au}}$, but its calculation is inaccurate when data are auto correlated. Therefore, a new technique is devised to transform the auto correlated data into independent data. In this technique, $x_t-\varphi x_{t-1}$ is used instead of x_t ($\forall t \in 2,3,\ldots,10000$), and it is proved that $x_t-\varphi x_{t-1}$ ($\forall t \in 2,3,\ldots,10000$) is always independent. Thus, firstly Eq(5) is rewritten as $x_t - \varphi x_{t-1}=\varepsilon+e_{\text{t}}$. Obviously, the independence assumption for $\varepsilon+e_{\text{t}}$ is valid because ε is $N(\varepsilon,0)$ and e_{t} is $N(0, \sigma^2)$. The summation of these two independent parameters is independent too. Since $x_t-\varphi x_{t-1}$ is equal to $\varepsilon+e_{\text{t}}$, it seems reasonable to conclude that $x_t-\varphi x_{t-1}$ is $N(\varepsilon, \sigma^2)$, t=2,3,…,10000, and it is subsequently an independent variable.

C_{pu} can be obtained for each set by using the sample standard deviation of x_t-x_{t-1} , t=2,3,…,N instead of σ in Eq(2) and $p\times10000^{\text{th}}$ data as USL. At the end, a multivariate regression is applied to estimate β_1 and β_2 when the generation of 10000 observations is repeated twenty times. It should be noted that as we use different $C_{\text{p-au}}$=0.5,0.6,0.7,0.8,0.85,0.9,0.95,1,1.05,1.1,1.2,1.25,1.3, 1.35,1.4,1.45,1.5,1.6,1.7,1.8 for each time, 400 sets of observations are created. For instance, the generated observations for $C_{\text{p-au}}$=0.9, 1 are shown in Table 1 in order to make it easy for the readers to understand how $\hat{C}_{\text{p-au}}$ can be estimated. The estimated $C_{\text{p-au}}$ is shown in the table, and it can be used for estimating β_1 and β_2 when $0<\varphi<0.2$.

Table 1
The estimated $C_{\text{p-au}}$ when $C_{\text{p-au}}$=0.9, 1 and $0<\varphi<0.2$

Iter.	$C_{\text{p-au}}$	$\hat{C}_{\text{p-au}}$	φ	ε	μ		Iter.	$C_{\text{p-au}}$	$\hat{C}_{\text{p-au}}$	φ	ε	μ
1	0.9	0.76	0.03	3	3.1		1	1	0.98	0.12	10.39	11.83
2	0.9	0.97	0.16	11.27	13.4		2	1	0.88	0.004	7.36	7.67
3	0.9	0.99	0.08	17.9	19.6		3	1	1.26	0.004	21.8	21.92
4	0.9	0.78	0.18	22.3	27.1		4	1	0.71	0.038	0.38	0.37
5	0.9	0.88	0.14	6.7	8		5	1	1.11	0.11	19.2	21.77
6	0.9	0.82	0.13	6.37	7.4		6	1	0.9	0.012	24.3	24.55
7	0.9	0.87	0.06	21.65	23.2		7	1	1.08	0.07	24.7	26.7
8	0.9	0.77	0.034	5.8	6		8	1	1.38	0.12	19.7	22.5
9	0.9	0.89	0.03	20.13	20.7		9	1	0.95	0.14	10.9	12.8
10	0.9	1.11	0.04	22.7	26.3		10	1	1.06	0.13	12.4	14.4
11	0.9	0.53	0.04	5.8	6.35		11	1	1.13	0.017	5.4	5.4
12	0.9	0.79	0.17	6	7.2		12	1	0.87	0.09	16	17.7
13	0.9	0.69	0.1	1.25	1.38		13	1	1	0.08	8	8.7
14	0.9	0.46	0.16	1.9	2.31		14	1	1.06	0.07	24	25.8
15	0.9	0.89	0.09	16	17.73		15	1	0.86	0.03	18.2	18.8
16	0.9	0.77	0.092	4.8	5.27		16	1	0.83	0.13	10.3	11.9
17	0.9	0.91	0.09	21.1	23.2		17	1	0.87	0.14	18.7	21.7
18	0.9	0.67	0.08	4.35	4.7		18	1	0.94	0.145	6.7	7.9
19	0.9	0.57	0.18	4.27	5.2		19	1	1.02	0.096	11	12.2
20	0.9	0.78	0.002	24.86	24.9		20	1	1.11	0.11	23.34	26.2

Let us recall that $C_{p\text{-}au}=C_{pu}+\beta_1\varphi+\beta_1\varepsilon$. Before assuming β_1, β_2 as the model coefficients, it is vital to determine whether $C_{p\text{-}au}$, φ and ε are related or not. In this regard, it is necessary to test the null hypothesis H_0: $\beta_1=0$ and H_1: $\beta_1\neq0$ for the former, which shows whether $C_{p\text{-}au}$ and φ are related or not. In testing this null hypothesis, the statistic $t=\frac{\hat{B}_1-B_1}{S\hat{B}_1}$ and confidence interval $A=\left[-t_{\frac{\alpha}{2},n-3},t_{\frac{\alpha}{2},n-3}\right]$ are used at the significance level α. Using 400 sets of generated observations, statistics are equal to -9.19, -11.81, -10.14 and -48.22 when $0<\varphi<0.2$, $0.2<\varphi<0.4$, $0.4<\varphi<0.6$ and $0.6<\varphi<0.8$, respectively. As a consequence, it seems reasonable to reject the null hypothesis for $0<\varphi<0.2$, $0.2<\varphi<0.4$, $0.4<\varphi<0.6$ and $0.6<\varphi<0.8$ in that the confidence interval is $A=[-1.96,1.96]$ at 5% significance level. Note that $\beta_1\neq0$ means $C_{p\text{-}au}$ relates to φ. Likewise, the null hypothesis H_0: $\beta_2=0$ and H_1: $\beta_2\neq0$ should be tested for the latter, which shows whether $C_{p\text{-}au}$ relates to ε or not. Here, statistics are correspondingly equal to 0.0014, 0.0008, 0.0009 and -0.0001, so the null hypothesis H_0: $\beta_2=0$ is not rejected for $0<\varphi<0.2$, $0.2<\varphi<0.4$, $0.4<\varphi<0.6$ and $0.6<\varphi<0.8$. As a result, the importance of φ and ε are considered as 'more important' and 'important' in the present study, and we presume that both of them are the model coefficients to increase performance.

Since $C_{p\text{-}au}=C_{pu}+\beta_1\varphi+\beta_1\varepsilon$, we can assume $C_{p\text{-}au}$-C_{pu} as a response variable of the multivariate regression, and β_1, β_2 can be also assumed as the model coefficients that are estimated from the observations of the process by the multivariate regression. The regression equations of $C_{p\text{-}au}$-C_{pu} are estimated for different intervals of φ and displayed in Figure 1 below:

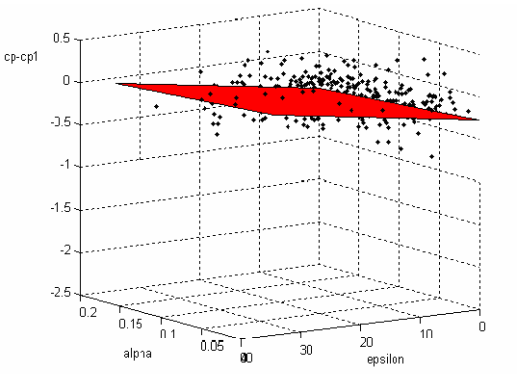

Fig. 1(a). Regression equations of $C_{p\text{-}au}$-C_{pu} when $0<\varphi<0.2$

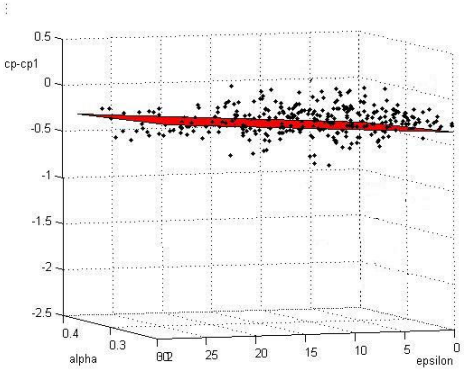

Fig. 1(b). Regression equations of $C_{p\text{-}au}$-C_{pu} when $0.2<\varphi<0.4$

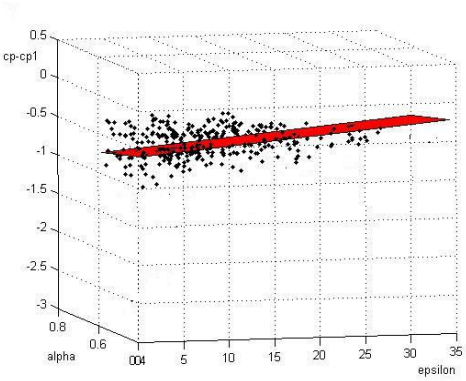

Fig. 1(c). Regression equations of $C_{p\text{-}au}$-C_{pu} when $0.4<\varphi<0.6$

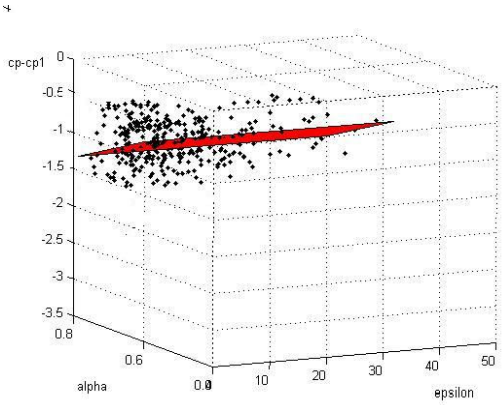

Fig. 1(d). Regression equations of $C_{p\text{-}au}$-C_{pu} when $0.6<\varphi<0.8$

The equations which are usually used to estimate $C_{p\text{-}au}$ are obtained by using the multivariate regression and displayed on the classification of Figure 2. The quality of the output from an auto correlated process can be easily managed by using this classification to monitor the difference between customer requirements and the actual performance of an auto correlated process. On the other hand, this classification is able to create products that meet customer requirements, and then maintain the auto correlated process in a "capable" state.

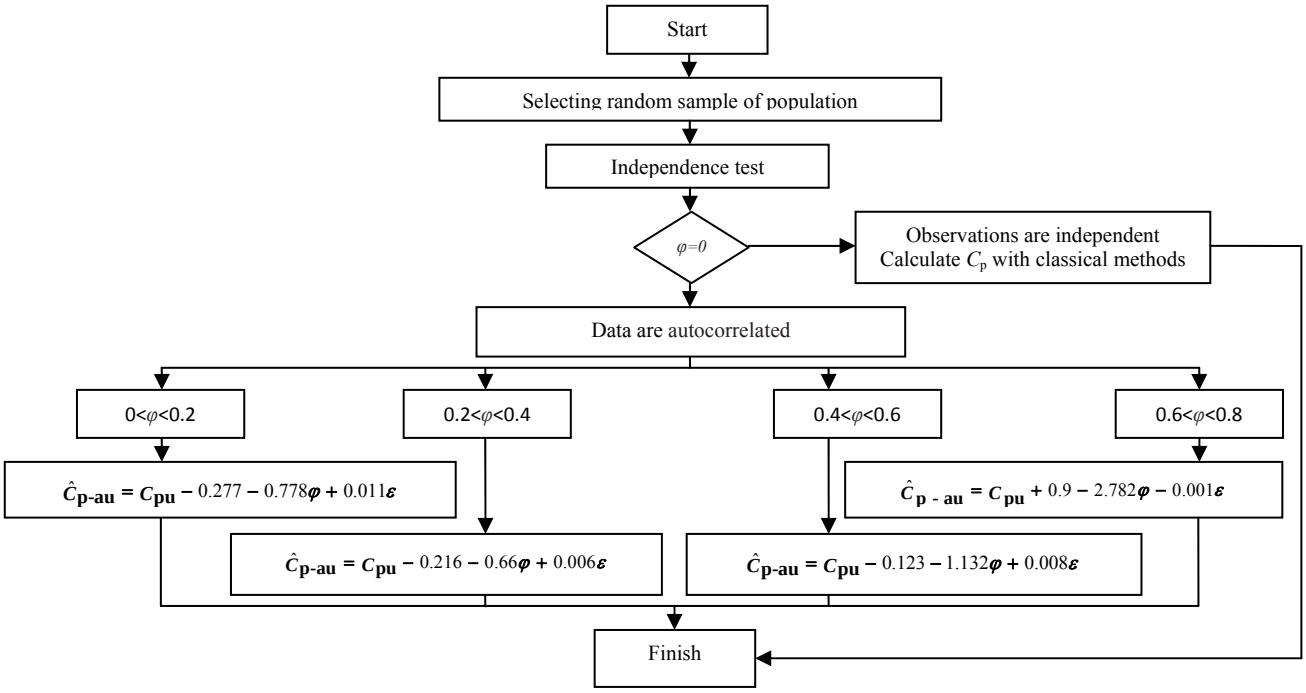

Fig 2. The estimated $C_{p\text{-au}}$ when data are autocorrelated with $0<\varphi<0.2$, $0.2<\varphi<0.4$, $0.4<\varphi<0.6$, $0.6<\varphi<0.8$

4. Statistical Analysis and Model's Validity

We should investigate whether there is a significant difference between the capability index and estimated capability index at a given significance level. To obtain a suitable decision rule, the hypothesis can be formulated as:

$$H_0: C_{p\text{-au}} - \hat{C}_{p\text{-au}} = 0 \qquad H_1: C_{p\text{-au}} - \hat{C}_{p\text{-au}} \neq 0 \quad (7)$$

t statistic where the standard deviation of pair differences is not known is used to test the hypothesis. It is defined as:

$$t = \frac{\bar{d}-0}{s_{\bar{d}}} \quad (8)$$

And the confidence interval is defined as:

$$A = \left[-t_{\frac{\alpha}{2},n-1}, t_{\frac{\alpha}{2},n-1}\right] \quad (9)$$

Note that d_i is obtained by subtracting the i^{th} member of $C_{p\text{-au}}$ from the i^{th} member of $\hat{C}_{p\text{-au}}$. Subsequently, the average of d_i can be determined as $\bar{d} = \frac{\sum_{i=1}^{n} d_i}{n}$ for a population of size n. The null hypothesis will be rejected whenever $t > t_{\frac{\alpha}{2},n-1}$ or $t < -t_{\frac{\alpha}{2},n-1}$, where the constant $t_{\frac{\alpha}{2},n-1}$ is determined so that the significance level of the test is α for a sample size of n observations. If null hypothesis is not rejected, there is no significant difference between $C_{p\text{-au}}$ and $\hat{C}_{p\text{-au}}$. Table 2 shows $C_{p\text{-au}}-\hat{C}_{p\text{-au}}$ for the given $0<\varphi<0.2$.

Table 2
Calculating $C_{p\text{-au}}-\hat{C}_{p\text{-au}}$ when $0<\varphi<0.2$

Iter. No	d= $C_{p\text{-au}}-\hat{C}_{p\text{-au}}$	$\hat{C}_{p\text{-au}}$	$C_{p\text{-au}}$
1	-0.005	1.005	1
2	0.103	0.797	0.9
3	-0.013	0.936	0.95
4	-0.037	1.087	1.05
5	0.004	1.096	1.1
6	0.03	1.17	1.2
7	-0.01	1.26	1.25
8	-0.08	1.38	1.3
9	-0.13	1.48	1.35
10	-0.1	1.5	1.4
11	-0.08	1.53	1.45
12	0.04	1.46	1.5
13	0.14	1.46	1.6
14	0.21	1.49	1.7
15	0.2	1.6	1.8
16	0.07	0.78	0.85
17	0.03	0.77	0.8
18	0.13	0.57	0.7
19	0.1	0.5	0.6
20	0.03	0.47	0.5

According to Table 2, we can calculate the test statistic as follows:

$$t = \frac{\bar{d}-0}{S_d} = \frac{0.0316}{\sqrt{\frac{0.00905}{20}}} = 1.486093 \quad (10)$$

The nominal significance level in this test is 0.05 for a sample of n=20 and 0<φ<0.2; therefore, the confidence interval can be defined as $A = [-t_{0.025,19}, t_{0.025,19}] = [-2.093, 2.093]$. It is known that the null hypothesis will be accepted when $|P - \text{value}| \leq t_{0.025,19}$. Based on these assumptions, the calculated P-value is equal to 0.15. This means that there is no significant difference between $C_{\text{p-au}}$ and $\hat{C}_{\text{p-au}}$ in this situation. According to this procedure, the statistics are also equal to 0.8, 0.151 and 0.1 when 0.2 <φ<0.4, 0.4 <φ<0.6 and 0.6 <φ<0.8, respectively, so it seems reasonable to conclude that there is no significant difference between $C_{\text{p-au}}$ and $\hat{C}_{\text{p-au}}$ for these values of φ.

5. Conclusion

It is common to come across biological and chemical processes which, because of their intrinsic nature, produce auto correlated data. If these auto correlated data are treated independently during capability analyses, the conclusions may lead to incorrect decisions. To prevent such incorrect decisions, we proposed a strategy to handle problems that occur during capability analyses if the observations are not independent. This strategy is based on subtracting consecutive observations from each other in order to obtain samples with independent observations, and then using regression analysis to calculate PCI at different levels of autocorrelation. Using this strategy, we can find powerful decision rules to determine the capability of a process at a given significance level. A Monte Carlo simulation was also employed to evaluate the provided results. For future research, we would recommend the extension of this strategy to estimate the process capability index of AR(P) or non-normal processes.

6. References

[1] Ahmad, S., Abdollahian, M. Zeephongsekul, P. (2007). Process capability analysis for non-quality characteristics using Gamma distribution. 4th international conference on information technology–new generations, USA, 425-430.

[2] Chen, S.M. Hsu, Y.S. Pearn, W.L. (2003). Capability measures for m-dependent stationary process. Statistics, 37(2), 145-168.

[3] Clements, J.A. (1989). Process capability calculations for non-normal distributions. Quality Progress, 22(9), 95-100.

[4] Hosseinifard, S.Z. Abbasi, B. Ahmad, S. Abdollahian, M. (2008). A Transformation Technique to estimate the process capability Index for non-normal process. International Journal of Advanced Manufacturing Technology 40(5-6), 512-517.

[5] Jing, S., Shengxian, W. Zhihui. F. (2009). The Effect of Autocorrelated Data on Taguchi Process Capability Index C_{pm} Based on AR(1) Model. International Conference on Management and Service Science (MASS 2009), Wuhan and Beijing, China, 1-4.

[6] Jing, S. Shengxian, W. Zhihui. F. (2009). Process capability analysis and estimation scheme for autocorrelated data Journal of Systems Science and Systems Engineering, 19(1), 105-127.

[7] Kotz, S. Johnson, N.L. (2002). Process capability indices: a review, 1992-2000 Discussions. Journal of Quality Technology, 34(1), 2-53.

[8] Mingoti, S.A. Oliveira F.L.P. (2011). On Capability Indices for Multivariate Autocorrelated Processes. Brazilian Journal of Operations & Production Management, 8(1), 133-152.

[9] Noorossana, R. (2002). Short communication process capability analysis in the presence of autocorrelation. Quality and Reliability Engineering International, 18(1), 75-77.

[10] Scagliarini, M. (2002). Estimation of c_p for autocorrelated data and measurements errors. Communications in Statistics-Theory and methods, 31(1), 1647-1664.

[11] Shore, H. (1997). Process capability analysis when data are auto correlated. Quality Engineering, 9(4), 615-625.

[12] Somerville, S.E. Montgomery, D.C. (1996). Process capability indices and non-normal distribution. Quality Engineering, 9(2), 305-316.

[13] Tang, L.C. Than, S.E. (1999). Computing process capability indices for non-normal data: a review and comparative study. Quality and Reliability Engineering International, 15(5), 339-353.

[14] Vannman, K. Kulahci, M. (2008). A Model-free Approach to Eliminate Autocorrelation When Testing for Process Capability. Quality and Reliability Engineering International, 24(2), 213-228.

[15] Yum, B.J. Kim, K.W. (2010). A bibliography of the literature on process capability indices: 2000–2009. Quality and Reliability Engineering International, 27(3), 251-268.