Control of a Hyperchaotic System Via Generalized Backstepping Method

Zinat Asadi [a *], Ahmad Fakharian [b]

[a]*Faculty of Mechanical Engineering Takestan Branch, Islamic Azad University Qazvin, Iran*

[b]*Faculty of Electrical, Biomedical and Mechatronic Engineering Qazvin Branch, Islamic Azad University Qazvin, Iran*

Abstract

This paper investigates on control and stabilization of a new hyperchaotic system. The hyperchaotic system is stabilized using a new technique which called Generalized Backstepping Method (GBM). Because of its similarity to Backstepping approach, this method is called GBM. But, this method is more applicable in comparison with conventional Backstepping. Backstepping method is used only for systems with strictly feedback form, but GBM works for a wide range form of the nonlinear dynamical systems. In Design procedure, two cases is considered that their difference is in the number of control inputs. Numerical simulation results are presented to show the effectiveness of the proposed controller.

Keywords: hyperchaotic systems; generalized backstepping method; stabilization.

1. Introduction

Chaos and its applications have been studied and developed by scientists in past decades, considerably. A regular chaotic system has one positive Lyapunov exponent, while the system with more than one positive Lyapunov exponent is called “hyperchaotic” which has more complicated dynamics than a chaotic system. In recent years, many of researchers have focused on study and analysis of hyperchaotic systems dynamics, synchronization of hyperchaotic systems, and proposition and applying new control design techniques on them.

Authors in [1-6] have proposed new hyperchaotic systems and investigate their chaotic nature. Synchronization between chaotic systems is one of major interesting problem in researches [7-10], while in [11] and [12] uncertainty in models parameters have been considered, too. Stabilization and control of hyperchaotic systems in order to achieve desire objective have been attracted attention of researchers [13-15].

One of the well-known nonlinear controller design technique is backstepping approach which works for nonlinear systems with strict-feedback form. It could not obtain good performance in non strict-feedback form and in some MIMO nonlinear systems. Hence, this method was improved by Ali Reza Sahab and Mohammad Haddad Zarif and introduced in [16]. This technique is called generalized backstepping method (GBM) because of its similarity to backstepping approach and applications in nonlinear dynamical systems. However backstepping method is applied only on systems with strict-feedback form but GBM expands this class.

In [17], generalized backstepping method has been used to control three chaotic systems Lorenz, Chen, and Lu. Newton-Leipink chaotic system has been stabilized using GBM in [18]. Hybrid generalized backstepping method with Genetic Algorithm has been utilized to control Chua Circuit in [19]. In [20], a novel fractional-order hyperchaotic system with a quadratic exponential nonlinear term has been proposed and its synchronization has been done using GBM. In [21-23], generalized backstepping method has been used to stabilize parameters perturbation in chaotic systems by hybrid of Adaptive Neuro-Fuzzy Inference System. Our contribution in this paper is applying GBM as a new and comprehensive controller design technique than regular backstepping approach to a new four-state hyperchaotic system, [1], and evaluation of its performance.

The rest of the paper is organized as follows: In Section II, the hyperchaotic system is presented. The Generalized

* Corresponding author. Email: elec.research@qiau.ac.ir

Backstopping Method is described In Section III. In Section IV, the stability conditions in hyperchaotic systems are derived by generalized backstepping method. In Section V, designed controller for stability conditions in hyperchaotic systems is simulated. Finally, in Section VI, conclusions are drawn.

2. A Novel Hyperchaotic System

The state space model of the novel hyperchaotic system is described by [1]:

$$\begin{aligned} \dot{x} &= a(y-x) + gyz - hw \\ \dot{y} &= cx - dy - xz \\ \dot{z} &= xy - bz \\ \dot{w} &= ry + fyz \end{aligned} \tag{1}$$

Where a, b, c, d, f, g, r and h are constant positive parameters. It exhibits extremely rich dynamical behaviors, including 3-tori (triple tori), 2-tori (quasi periodic), limit cycles (periodic), chaotic and hyperchaotic attractors [1].

System (1) has three equilibrium points which are as following:

$$\begin{aligned} s_1 &= \begin{pmatrix} 0 & 0 & 0 & 0 \end{pmatrix}^T \\ s_2 &= \begin{pmatrix} -\dfrac{bdr}{\sqrt{m}} & \dfrac{\sqrt{m}}{df} & -\dfrac{r}{f} & \dfrac{bnr}{kf^2\sqrt{m}} \end{pmatrix}^T \\ s_3 &= \begin{pmatrix} \dfrac{bdr}{\sqrt{m}} & -\dfrac{\sqrt{m}}{df} & \dfrac{r}{f} & -\dfrac{bnr}{kf^2\sqrt{m}} \end{pmatrix}^T \end{aligned} \tag{2}$$

Where $m = -bdr(cf + r)$ and $n = adf^2 + cgfr - afr + gr^2$

Considering $a = 71$, $b = 13$, $c = 52$, $d = 5.8$, $f = 30$, $g = 25$, $r = 24$ and $, h = 2$ as values of parameters in the novel hyperchaotic system (1), hyperchaotic behavior is occurred and shown in Fig. 1.

3. Generalized Backstepping Method

Generalized Backstepping Method will be applied to a certain class of autonomous nonlinear systems which are expressed as follow [16], [17], and [19]:

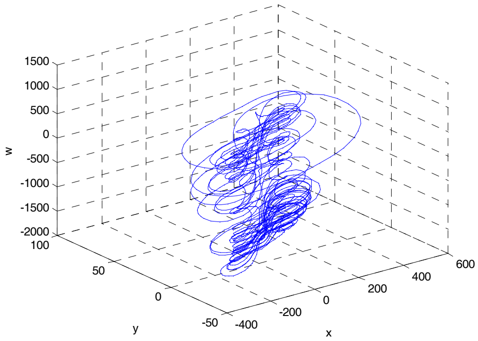
$$\begin{aligned} \dot{X} &= F(X) + G(X)\eta \\ \dot{\eta} &= f_0(X,\eta) + g_0(X,\eta)u \end{aligned} \tag{3}$$

In which $\eta \in \Re$ and $X = [x_1 \quad x_2 \quad \ldots \quad x_n]^T \in \Re^n$. In order to obtain an approach to control these systems, we may need to prove a new theorem as follows.

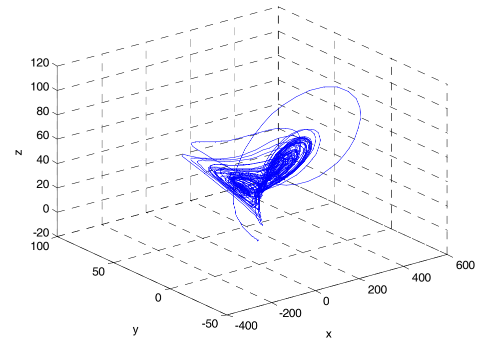
Theorem: suppose equation (3) is available, then suppose the scalar function $\varphi_i(x)$ for the i_{th} state could be determined i a manner which by inserting the ith term for η, the function $v(x)$ would be a positive definite equation (4) with negative definite derivative.

$$v(x) = \frac{1}{2}\sum_{i=1}^{n} x_i^2 \tag{4}$$

Therefore, the control signal and also the general control lyapunov function of this system can be obtained by following equations [16]:



(a)



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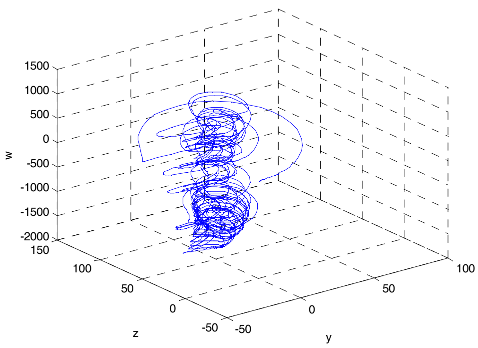
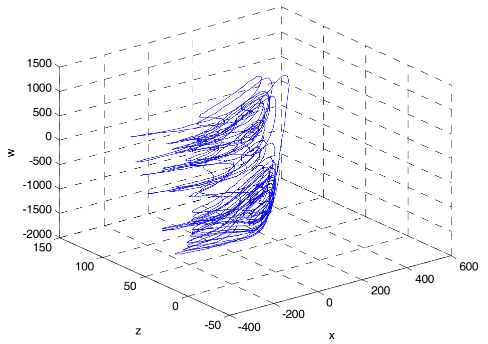


(c)



(d)

Fig. 1. Phase portraits of the four-scroll hyperchaotic attractors (1), (a):xyw, (b):xyz, (c):yzw, (d):xzw

$$
\begin{aligned}
u =& \frac{1}{g_0(X,\eta)}\left\{\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial\varphi_i(x)}{\partial x_j}\left[f_i(x)+g_i(x)\eta\right]\right\} \\
&-\frac{1}{g_0(X,\eta)}\left\{\sum_{i=1}^{n}x_i g_i(x)+\sum_{i=1}^{n}k_i\left[\eta-\varphi_i(x)\right]\right\} \\
&-\frac{1}{g_0(X,\eta)}\left\{f_0(X,\eta)\right\} \\
&ST:\quad k_i \succ 0\,;\, for\ i=1,2,\cdots,n
\end{aligned}
\tag{5}
$$

$$
\dot v(X,\eta)=\frac{1}{2}\sum_{i=1}^{n}x_i^2+\frac{1}{2}\sum_{i=1}^{n}\left[\eta-\varphi_i(x)\right]^2 \tag{6}
$$

4. Control of Hyperchaotic System

The generalized backstepping method is used to design two controllers. First controller is designed by two control inputs, then second controller is designed by three control inputs.

Case I

In order to control hyperchaotic system we add two control inputs u_1 to the second equation and u_2 to fourth equation of system (1).

$$
\begin{aligned}
\dot x &= a(y-x)+gyz-hw \\
\dot y &= cx-dy-xz+u_1 \\
\dot z &= xy-bz \\
\dot w &= ry+fyz+u_2
\end{aligned}
\tag{7}
$$

In order to stabilize states of system, the theorem 1 is utilized, hence, it is sufficient to consider

$$
\begin{cases}
\eta_1 = y \\
\eta_2 = w \\
u_{a1} = cx-dy-xz+u_1 \\
u_{a2} = ry+fzy+u_2
\end{cases}
\tag{8}
$$

Hence, we rewrite (7) as:

$$
\begin{aligned}
\dot x &= a(\eta_1-x)+g\eta_1 z-h\eta_2 \\
\dot\eta_1 &= u_{a1} \\
\dot z &= x\eta_1-bz \\
\dot\eta_2 &= u_{a2}
\end{aligned}
\tag{9}
$$

The terms u_{a1} and u_{a2} are produced in the procedure of proof, for more details see [16]. Now, establish following virtual control inputs equations.

$$
\begin{aligned}
\varphi_{11}(x,y,z,w) &= 0 \\
\varphi_{12}(x,y,z,w) &= k_1 x
\end{aligned}
\tag{10}
$$

$$
\varphi_{21}(x,y,z,w)=\varphi_{22}(x,y,z,w)=0 \tag{11}
$$

According to the theorem, the control signals will be obtained from the equations (12) and (13).

$$
u_1 = -k_2 y-(a+c+gz)x+dy \tag{12}
$$

$$
u_2 = k_1\dot x-k_3 w-k_4(w-\varphi_{12})+hx-ry-fyz \tag{13}
$$

And Lyapunov function is given by:

$$
v(x,y,z,w)=\frac{1}{2}x^2+\frac{3}{2}y^2+\frac{1}{2}z^2+w^2+\frac{1}{2}(w-\varphi_{12})^2 \tag{14}
$$

Case II

In order to control the hyperchaotic system, we add three control inputs, u_1 to the second equation, u_2 to the third equation and u_3 to fourth equation of system (1).

$$
\begin{aligned}
\dot x &= a(y-x)+gyz-hw \\
\dot y &= cx-dy-xz+u_1 \\
\dot z &= xy-bz+u_2 \\
\dot w &= ry+fyz+u_3
\end{aligned}
\tag{15}
$$

For stabilization of the states, in order to use the theorem 1, we use some changes in variables as following:

$$
\begin{cases}
\eta_1 = y \\
\eta_2 = z \\
\eta_3 = w \\
u_{a1} = cx-dy-xz+u_1 \\
u_{a2} = xy-bz+u_2 \\
u_{a3} = ry+fzy+u_3
\end{cases}
\tag{16}
$$

Hence, we rewrite (15) as following:

$$\begin{aligned} \dot{x} &= a(\eta_1 - x) + g\eta_1\eta_2 - h\eta_3 \\ \dot{\eta}_1 &= u_{a1} \\ \dot{\eta}_2 &= u_{a2} \\ \dot{\eta}_2 &= u_{a3} \end{aligned} \tag{17}$$

Now, it is sufficient to consider following equations:

$$\varphi_1(x,y,z,w) = -k_1 x \tag{18}$$

$$\varphi_2(x,y,z,w) = 0 \tag{19}$$

$$\varphi_3(x,y,z,w) = k_2 x \tag{20}$$

According to the theorem, the control signals will be obtained by following relations:

$$u_1 = -k_1\dot{x} - k_3(y - \varphi_1) - (a + c + gz) + dy + xz \tag{21}$$

$$u_2 = (b - k_4)z - (1+g)xy \tag{22}$$

$$u_3 = -k_2\dot{x} - k_5(w - \varphi_3) + hx - ry - fyz \tag{23}$$

And Lyapunov function is considered as:

$$\begin{aligned} v(x,y,z,w) &= \frac{1}{2}x^2 + \frac{1}{2}y^2 + z^2 + \frac{1}{2}w^2 \\ &+ \frac{1}{2}(y - \varphi_1)^2 + \frac{1}{2}(w - \varphi_3)^2 \end{aligned} \tag{24}$$

It is necessary to mention that in these two cases system is considered as a MIMO system and GBM is designed for it.

5. Numerical Simulations

This section presents numerical simulations of controlled hyperchaotic system. The Generalized Backstepping Method (GBM) is used as an approach to control Chaos in hyperchaotic system. We select the Gain of first controllers $k_1 = 1, k_2 = 7, k_3 = 5$, and $k_4 = 10$ and for second controllers $k_1 = 1, k_2 = 1, k_3 = 7, k_4 = 10$, and $k_5 = 8$. In order to select values of parameters, it is important to note that the derivate of Lyapunov function should be negative to ensure stability of system. The initial values of the hyperchaotic system are considered $x(0) = -1$, $y(0) = 1$, $z(0) = -1$, and $w(0) = 1$. Responses of states x, y, z, and w of hyperchaotic system in cases I and II which two and three control signal inputs are applied to system, respectively, are shown in Fig. 2, 3, 4 ,and 5. It is clear that states of system converge to origin and system is stabilized using two and three control inputs. Figure 6 and 7 show the control law u_1 in (12) and u_2 in (13), respectively, which are applied to system to stabilize it toward origin point. Figure 8 shows the control law u_1 in (21) and u_2 and u_3 in (22) and (23) are shown in Fig. 9 and Fig. 10, respectively.

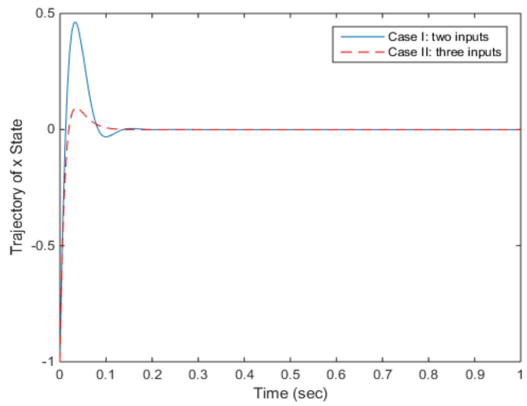


Fig. 2. The time response of the state x for the controlled hyperchaotic system (7) in cases I: two control signal inputs (12), (13), and case II: three control signal inputs (21), (22), (23)

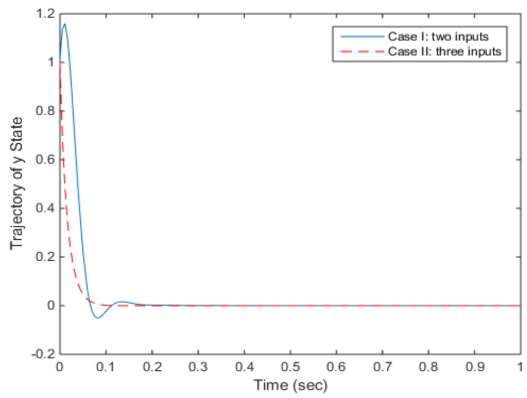


Fig. 3. The time response of the state y for the controlled hyperchaotic system (7) in cases I: two control signal inputs (12), (13), and case II: three control signal inputs (21), (22), (23)

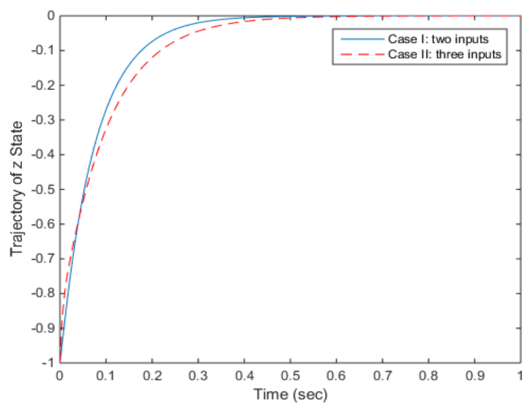


Fig. 4. The time response of the state z for the controlled hyperchaotic system (7) in cases I: two control signal inputs (12), (13), and case II: three control signal inputs (21), (22), (23)

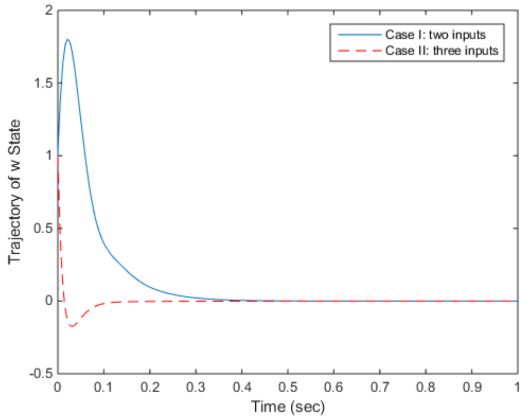


Fig. 5. The time response of the state w for the controlled hyperchaotic system (7) in cases I: two control signal inputs (12), (13), and case II: three control signal inputs (21), (22), (23)

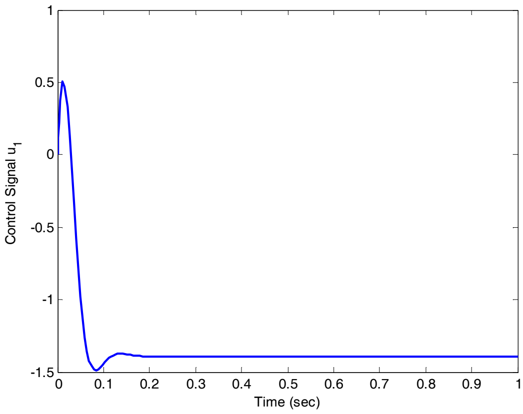


Fig. 6. The time response of the control input u_1 (12) for controlled hyperchaotic system (7).

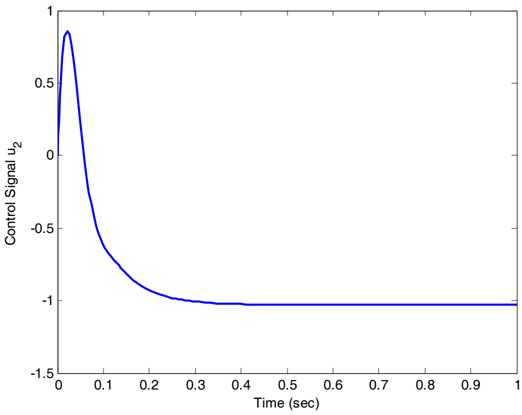


Fig. 7. The time response of the control input u_2 (13) for controlled hyperchaotic system (7).

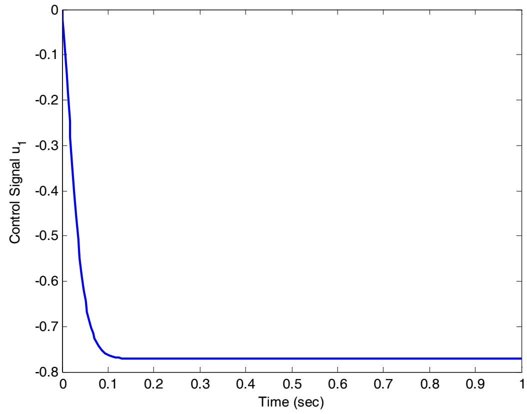


Fig. 8. The time response of the control input u_1 (21) for controlled hyperchaotic system (15).

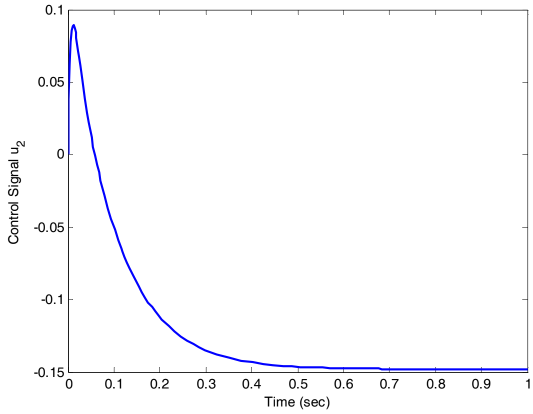


Fig. 9. The time response of the control input u_2 (22) for controlled hyperchaotic system (15).

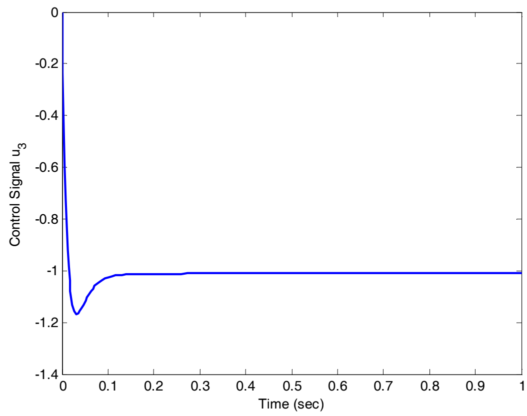


Fig. 10. The time response of the control input u_3 (23) for controlled hyperchaotic system (15).

As it is obvious the designed controllers based on GBM for a MIMO system can stabilize it to the origin while backstepping method is not applicable for this hyperchaotic system because it does not have strict feedback form and is a MIMO system. Based on simulations, In case II, which we consider three input signal control, the response is better than case I. The responses and convergence of states to origin are faster and do not have overshoot. The amplitude of control efforts in each of two cases makes implementation possible; hence practical test can be done.

6. Conclusion

The stability conditions in hyperchaotic systems have been studied. These conditions have been applied to the novel hyperchaotic system. The theoretical conditions for controlling hyperchaos in this system have been obtained using Generalized Backstepping Method. In the proposed method which is called Generalized Backstepping Method, by feedbacking the dynamics of system and without eliminating the nonlinear dynamics, a controller is designed. A theorem has been expressed for this method and the proof has been given. Finally, numerical simulation have been done to verify the effectiveness of the proposed control scheme and Working on combination of intelligent and classic controllers or selecting optimal functions for φ_i in theorem 1 could be interesting areas for future works.

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