Computer & Robotics

An Optimal Defect-Free Synthesis of Four-Bar Mechanisms Using Constrained APT-FPSO Algorithm

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Abstract

Four-bar mechanisms are one of the most common and useful components in the industry. In practical applications, they are designed to generate the desired output motion. This paper analyses the nonlinear problem of optimal defect-free synthesis of four-bar mechanisms by a constrained version of the newly developed adaptive particularly tuneable fuzzy particle swarm optimization (APT-FPSO) algorithm. To evaluate the algorithm, we considered designing a four-bar mechanism to generate a path that included three loops with 90 precision points in a case study. The results obtained from the case study analysis support the superior performance of APT-FPSO compared to the standard PSO in solving the path generation problem.

Keywords: (Optimization, PSO, Inverse Kinematics)

1.Introduction

The applications of four-bar mechanisms are very wide. For instance, they can be found in automotive suspension [1-3] and steering systems [4]. Additionally, they are used in automatic door closers, pantographs, bicycle suspension, double-wishbone suspensions in vehicles, windshield wipers, car window crank, and the like. However, the synthesis of these geometrically simple machines may become a bit challenging when expected to perform a specific task with high accuracy. On the other hand, it becomes difficult to solve the problem of the synthesis of four-bar mechanisms by using deterministic methods [5]. Therefore, strong intelligent optimization algorithms can be used to surmount such highly nonlinear, constrained, multi-objective problems [6-8]. Some studies have analyzed the effects of misalignments and clearances in mechanical tools and linkages [9-12]. Varedi et al. have used Particle Swarm Optimization (PSO) algorithm to optimize the mass distribution of the links and to alleviate the harsh impacts of clearance in joints in mechanisms [13]. Daniali et al. have presented a novel optimization algorithm based on PSO to conquer the highly nonlinear problem of simultaneous kinematic and dynamic synthesis of four-bar mechanisms with joint clearance [14]. Sardashti et al. have taken advantage of the PSO algorithm to solve this problem with existing clearance at one, two, three, all, and none of the joints [15]. Singh et al. proposed a defect-free optimal synthesis of a human knee exoskeleton with the aid of nature-inspired optimization algorithms [16]. For the optimal path synthesis of a four-bar linkage, Sleesongsom and Bureerat have proposed a new variant of the Teaching-learning Based Optimization algorithm, namely Self-adaptive Population size TLBO [17]. Kafash and Nahvi introduced a new objective function,

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namely as Circular Proximity Function, towards solving the optimal synthesis of four-bar linkages [18]. By taking advantage of the Differential Evolution (DE) algorithm, they demonstrated the efficacy of the proposed method by solving the optimal path-generation problems for several case studies.
Optimization algorithms are being used in a wide spectrum of engineering applications [19-23]. PSO algorithm is among swarm-based, metaheuristic, optimization algorithms which mimics the social behavior of animals and insects in a stochastic, yet intelligent, manner [24-26]. However, this powerful algorithm, by itself, is not fully exempt from premature convergence, and trapping in local extrema sometimes becomes ineluctable. Bakhshinezhad et al. have recently developed a variant of the PSO algorithm, namely adaptive particularly tunable fuzzy particle swarm optimization (APT-FPSO) algorithm [27]. Having been run seven benchmark functions all in four dimensions over 1000 times, they statistically proved the enhanced exploitation of APT-FPSO compared to PSO. Nasouri Gilvaei et al. have combined APT-FPSO with the firefly algorithm (FA) to solve the reactive power dispatch problem [28].
This work's major contribution is the development of the constrained version of the APT-FPSO algorithm and using it to solve the practical engineering optimization problem of optimal synthesis of four-bar mechanisms. The path generation problem needs to be solved free of any defects, so it necessitates the recruitment of a *constrained* optimization algorithm. To the authors' best knowledge, this would be the first time that a fuzzy-aided variant of the PSO is recruited to solve the optimal defect-free synthesis of four-bar mechanisms. The results obtained from using the APT-FPSO were compared to those from the standard PSO algorithm.
The organization of the rest of the paper is as follows: In the next section, the APT-FPSO algorithm is briefly introduced and shown how it performs. Section 3 formulates the problem of the optimal defect-free design of four-bar linkages. Besides, this section covers how this problem can be solved using optimization algorithms, and it includes the results obtained. Finally, Section 4 concludes the paper.

2.Overview Of Apt-Fpso Algorithm

This algorithm begins with initializing some random positions, i.e., candidate solutions. Next, these positions are evaluated, and the initial values for *personal* and *global fittest* are selected. Thereafter, the algorithm's main loop begins in which the particles' positions and velocities are updated, and the new *personal* and *global fittest* is stored. The previously mentioned steps are reiterated until the desired termination criterion is met, and the latest *global fittest* is selected as the final answer to the problem. Note that the termination criterion in this work is for the algorithm to reach 50 iterations.
In this algorithm, Eqs. 1 and 2 represent, respectively, the relationships for updating position and velocity of the particles:

$$x_{t+1}{}^{i} = x_t{}^{i} + v_{t+1}{}^{i}, \quad (1)$$

$$v_{t+1}{}^{i} = w_i \times v_t{}^{i} + c_{1,t}{}^{i} \times r1 \times \left(p_t{}^{i} - x_t{}^{i}\right) + c_{2,t}{}^{i} \times r2 \times \left(p_t{}^{g} - x_t{}^{i}\right), \quad (2)$$

where $p_t{}^{i}$ denotes the personal best record of the i^{th} particle in the t^{th} iteration, and $p_t{}^{g}$ indicates the global best in the t^{th} iteration; $r1$ and $r2$ are two normally distributed random numbers within the range [0, 1]. w is the inertia weight. $c_{1,t}{}^{i}$ and $c_{2,t}{}^{i}$ denote, respectively, the personal and global learning coefficients of the i^{th} particle in the t^{th} iteration.

A great deal of endeavor has been being made to extricate the standard PSO from the premature convergence. All these algorithms pursue is achieving a trade-off between exploration and exploitation by tuning the algorithm's parameters. For instance, Fuzzy Adaptive PSO algorithms can substantially improve the trade-off compared to the standard PSO.

Regarding the APT-FPSO algorithm, the first input for the developed fuzzy inference system (FIS) is dedicated to normalized iteration (NIt) from the beginning (NIt≈0) to the end of the algorithm execution (NIt =1). Eq. 3 gives the normalized iteration.

$$\text{NIt} = \frac{Current\ iteration}{Maximum\ number\ of\ iterations}. \quad (3)$$

The normalized iteration has been fuzzified by using three linguistic variables of Low, Medium, and High and assigning three corresponding Gaussian membership functions (MFs).

The second input of the FIS has to do with each particle's fitness value in every iteration. Eq. 4 formulates the normalized fitness index.

$$\text{NFI} = \frac{fitness_{t,i} - min(fitness_t)}{max(fitness_t) - min(fitness_t)}, \quad (4)$$

where $fitness_{t,i}$ indicates the fitness value of the i^{th} particle in the t^{th} iteration; besides, $max(fitness_t)$ and $min(fitness_t)$ denote, respectively, maximum and minimum fitness values of the swarm in the t^{th} iteration. Similarly, three gaussian MFs are used to define the second input NItm, named Low, Medium, and High.

The outputs of the designed FIS, however, are personal and global learning coefficients, i.e. ${c_{1,t}}^i$ and ${c_{2,t}}^i$. The learning coefficients' optimal values have been proved to be in the range [0.5, 2.5] [29, 30]. Meanwhile, each of the outputs ${c_{1,t}}^i$ and ${c_{2,t}}^i$ is described via five triangular MFs that are linguistically named as Very low, Low, Medium, High, and Very High.

In short, the designed FIS has two inputs, i.e. normalized iteration and normalized fitness index, and two outputs, i.e. c_1 and c_2. The adaptiveness and tunability in this algorithm mean that the algorithm's learning coefficients are able to be tuned during the execution of the algorithm adaptive to the two indices: the current iteration number and the fitness value for each particle. The general idea behind this approach is to, respectively, decrease and increase the particles' exploration and exploitation abilities as the algorithm approaches the last iterations. For more elaborate explanations on the APT-FPSO algorithm, please refer to [27].

Given the mentioned inputs and outputs, one can form many rule-based structures for the designed FIS, any one of which provides a unique performance. Different combinations of input-output linguistic variables of a fuzzy inference system lead to various rule-based structures. The performance of the fuzzy inference system depends highly on the rule-based structure. Different rule-based structures control the trade-off between exploitation and exploration during the execution of the algorithm. However, the surfaces of the four most principal of them are shown in Figure 1. Table 1 describes the four primary rule-based structures used in this study.

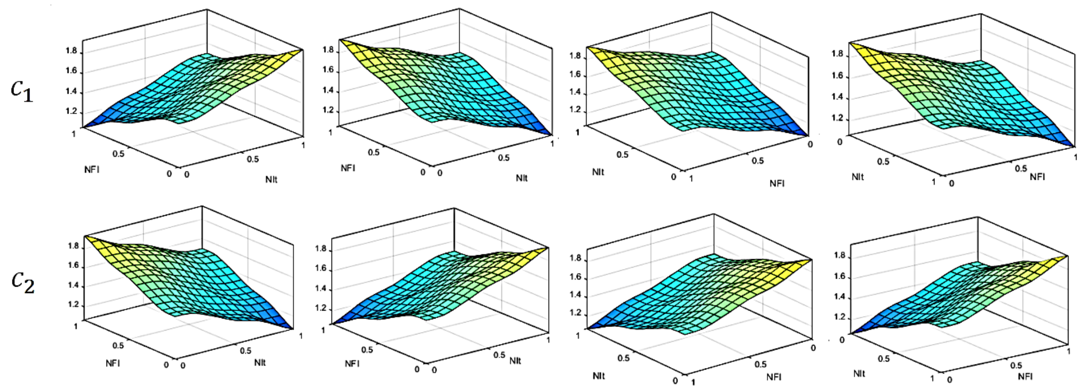

Fig. 1. Four most principal rule-base structures are associated with the inputs and the outputs of the designed fuzzy inference system.

Table 1

The four principal rule-base structures

Rule-base structure No. 1

	C_1			C_2		
NFI / NIt	**Bad**	**Normal**	**Good**	**Bad**	**Normal**	**Good**
Start	Medium	Low	Very Low	Medium	High	Very High
Middle	High	Medium	Low	Low	Medium	High
End	Very High	High	Medium	Very Low	Low	Medium

Rule-base structure No. 2

	C_1			C_2		
NFI / NIt	**Bad**	**Normal**	**Good**	**Bad**	**Normal**	**Good**
Start	Medium	High	Very High	Medium	Low	Very Low
Middle	Low	Medium	High	High	Medium	Low
End	Very Low	Low	Medium	Very High	High	Medium

Rule-base structure No. 3

	C_1			C_2		
NFI / NIt	**Bad**	**Normal**	**Good**	**Bad**	**Normal**	**Good**
Start	Very Low	Low	Medium	Very High	High	Medium
Middle	Low	Medium	High	High	Medium	Low
End	Medium	High	Very High	Medium	Low	Very Low

Rule-base structure No. 4

	C_1			C_2		
NFI / NIt	**Bad**	**Normal**	**Good**	**Bad**	**Normal**	**Good**
Start	Very High	High	Medium	Very Low	Low	Medium
Middle	High	Medium	Low	Low	Medium	High
End	Medium	Low	Very Low	Medium	High	Very High

3. Optimal Defect-Free Synthesis Of Four-Bar Mechanisms Using Apt-Fpso Algorithm

In this section, the APT-FPSO algorithm is applied to solve the nonlinear, constrained problem of optimal defect-free synthesis of a four-bar mechanism. In other words, the problem is designing a four-bar mechanism free of Grashof, order, branch, and circuit defects. To avoid the mentioned defects in the design process, some constraints must be applied to the optimization problem. According to Figure 2, x_0 and y_0 specify the position of the pivot A_0 in the XOY plane, and the symbols $\theta_1, \theta_2, \theta_3$, and θ_4 denote, respectively, the angles of the *ground link (frame), input link (driver), floating link (coupler)*, and *output link (driven)* with respect to the X-axis. The parameters a_1, a_2, a_3, and a_4 denote the lengths of the corresponding links, respectively. Besides, the point of the coupler (P) is at the distance a_5 from the joint A, with an angular position of β with respect to the coupler a_3. Therefore, the coordinates of the point P can be obtained using Eqs. 5 and 6.

$$P_x = x_0 + a_2 \cos\theta_2 + a_5 \cos(\theta_3 + \beta) \quad (5)$$

$$P_y = y_0 + a_2 \sin\theta_2 + a_5 \sin(\theta_3 + \beta) \quad (6)$$

3.1. Decision (design) variables

Similar to any other optimization problem, optimal synthesis of mechanisms requires some decision variables to be found. The vector of the decision variables is given in Eq. 7.

$$[x_0, y_0, a_1, a_2, a_3, a_4, a_5, \theta_1, \beta, \theta_2{}^1, \theta_2{}^2, \ldots, \theta_2{}^N], \quad (7)$$

where the superscript N is the number of the target points to be tracked,

3.2. The Objective Function And The Corresponding Constraints

In the path generation problem, the error area between the desired path and that generated by the coupler point (P) must be minimized. To do so, the mean squared error (MSE), so-called Euclidean distance, between the desired and the generated trajectories is considered as the cost function to be minimized. On the other hand, there are, in general, four constraints imposed on the problem of optimal defect-free synthesis of four-bar mechanisms, namely Grashof, order, branch, and circuit defects [14-16]. If none of the links in a mechanism can rotate completely, the mechanism is said to have Grashof defect. This occurs when the shortest link is neither a driving link nor a ground link [31]. On the hand, the order defect arises if the sequence of occurrence of the designed linkage's several plane positions is not in desired order [32]. The circuit defect happens in mechanism synthesis if a potential solution linkage cannot move between all precision positions without being disassembled. A branch defect takes place if the sign of the transmission angle changes in at least one of the design positions [33]. For more detailed information on different kinds of defects in mechanism and their corresponding rectification, please refer to [31].

Firstly, Grashof conditions for crank-rocker four-bar mechanisms may be fulfilled if the inequality given in Eq. 8 applies:

$$g_1(\boldsymbol{x}) = a_2 + a_1 - a_3 - a_4 < 0;\ \text{where}\ a_2 < a_3 < a_4 < a_1 \tag{8}$$

Moreover, the order, branch, and circuit defects can be satisfied, respectively, with the conditions written in Eqs. 9, 10, and 11.

$$g_2(\boldsymbol{x}) = {\theta_2}^i - {\theta_2}^{i+1} < 0;\ (\mathrm{i} = 1, 2,\ldots, \mathrm{N}) \tag{9}$$

$$g_3(\boldsymbol{x}) = {\theta_3}^i - {\theta_4}^i < 0;\ (\mathrm{i} = 1, 2,\ldots, \mathrm{N}) \tag{10}$$

$$g_4(\boldsymbol{x}) = {\theta_3}^i - {\theta_2}^i < 0;\ (\mathrm{i} = 1,2,\ldots, \mathrm{N}) \tag{11}$$

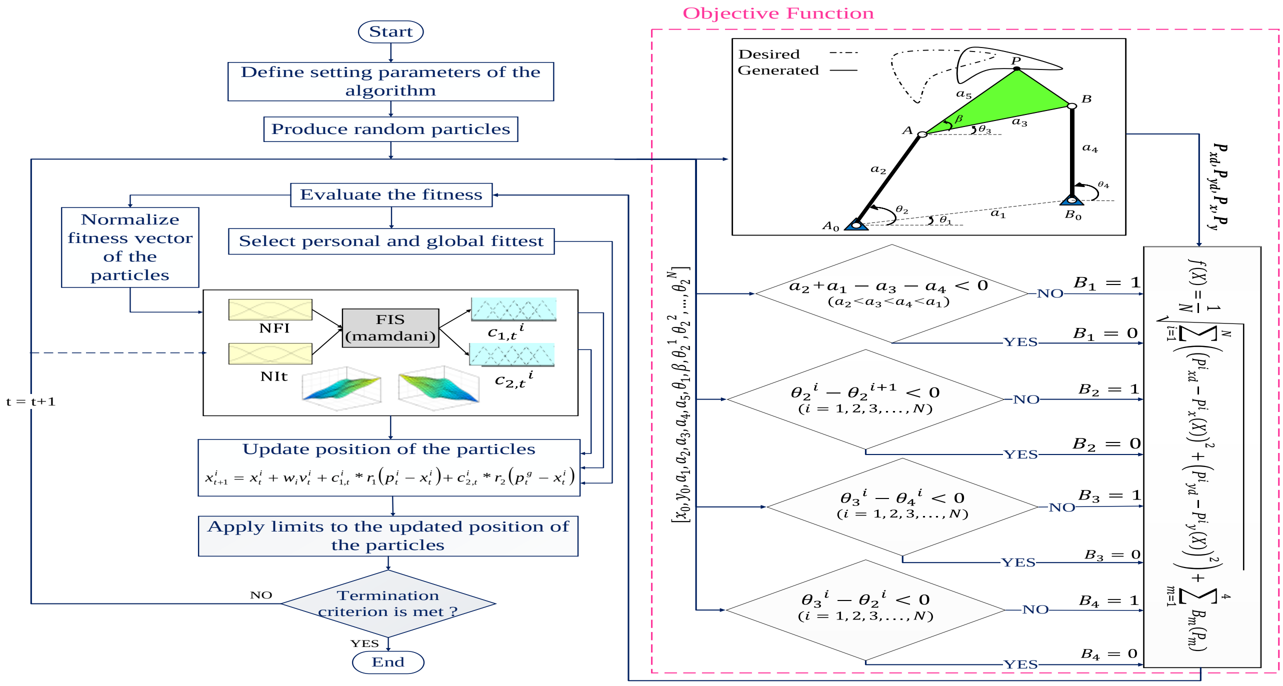

Fig..2. Flowchart of optimal synthesis path-generation problem using APT-FPSO

To satisfy the constraints, the penalty-function method was used. This method converts a constrained problem to an unconstraint one by adding a violation term to the objective function. Whenever a candidate solution circumvents either of the constraints, a large number will be added to its corresponding fitness value, ensuring the candidate solution's unfeasibility. As a result, the objective function with the associated constraints is as written in Eq. 12.

$$f(\boldsymbol{x}) = \frac{1}{N}\sqrt{\sum_{i=1}^{N}\left(\left({P^i}_{xd} - {P^i}_x(\boldsymbol{x})\right)^2 + \left({P^i}_{yd} - {P^i}_y(\boldsymbol{x})\right)^2\right)} \quad (12)$$
$$\sum_{m=1}^{4} B_m(P_m),$$

Where the ${P^i}_d = [{P^i}_{xd}, {P^i}_{yd}]^T$ and $P^i = [{P^i}_x, {P^i}_y]^T$ are the target and generated points, respectively. Besides, the P_m (m=1, 2,…, 4) are large numbers to penalize unfeasible solutions not fulfilling either of the corresponding constraints. Also, B_m is referred to as Boolean Function [14-16] and is defined as

$$B_m = \begin{cases} 0; & if\ g_m(\boldsymbol{x}) \le 0 \\ 1; & otherwise \end{cases} \quad (13)$$

3.3. Case Study: A Closed Path With Three Loops And 90 Precision Points

In this subsection, the developed APT-FPSO algorithms' performance is examined and

compared to that of PSO in an engineering application. The case study included in the optimal synthesis of path generation of four-bar mechanisms selected from the reference [18]. This case study involves three loops and 90 precision points. The standard PSO and the four APT-FPSOs with different structures were set to solve the case study. This problem, however, has an exact solution; that is, a four-bar mechanism is designed, the following parameters: $a_1 = 10.4$, $a_2 = 3.1$, $a_3 = 5$, $a_4 = 8.6$, $a_4 = 6$, $\beta = 1$, $\theta_1 = 0$, and $x_0 = y_0 = 0$. Thereafter, the driver link, a_2, is rotated with the input angles of $\theta_2^N = N * 4^{\circ}$, $N = \{0, 1, \dots, 89\}$. The resulting 90 points produced by the point of the coupler (P) on the mechanism are the target points to be tracked.

The coordinates of the target points are reported in [18]. All the competing algorithms were run with a maximum number of iterations equal to 50 and a population size of 200. Besides, the ranges of the four decision variables were set as $x_A, y_A \in [-5,5]$, $a_3 \in [1, 10]$, and $\beta \in [0,\pi]$. The obtained results are given in Table 2. Figure 3 depicts the convergence diagrams of the examined algorithms. Accordingly, the performance of the examined algorithms can be sorted from the best to the worst: APT-FPSO1, APT-FPSO2, APT-FPSO4, APT-FPSO3, and PSO.

As a result, regardless of the minute intermediate superiority of APT-FPSO algorithms, it can be inferred from Table 2 and Figure 3 that APT-FPSO is superior to the standard PSO. Based on Table 2 and the statistical analysis provided in [27], of all the APT-FPSOs with different structures, the one with the least error is APT-FPSO1, and it was chosen as the best candidate for APT-FPSOs.

Table 2
The Obtained Results

	Exact Solution	**APT-FPSO1**	**APT-FPSO2**	**APT-FPSO3**	**APT-FPSO4**	**PSO**
a_1	10.4	10.4	10.4	10.4	10.4	10.4
a_2	3.1	3.1	3.1	3.1	3.1	3.1
a_3	5	4.999877	4.999403	5.000232	4.999285	5.002342
a_4	8.6	8.6	8.6	8.6	8.6	8.6
a_5	6	6	6	6	6	6
$\beta(rad)$	1	0.99990	1.000006	1.000242	1.000642	1.001777
x_0	0	-5.768576 e-4	-7.087716 e-4	-1.766042e-4	3.1245364e-3	1.170283e-2
y_0	0	1.504298e-05	5.939275 e-4	6.446192e-4	1.654866e-3	6.952297e-3
$\theta_1(rad)$	0	0	0	0	0	0
MSE (fitness)	0	3.455784e-4	9.100124e-4	1.719827e-3	1.709593e-3	7.444480e-3

Therefore, only the final answer of APT-FPSO1 was shown and compared to that offered by the standard PSO. Figure 4 illustrates the difference between the generated path by APT-FPSO and that by the standard PSO. The numerical coordinates of the generated and the desired paths are given in the appendix.

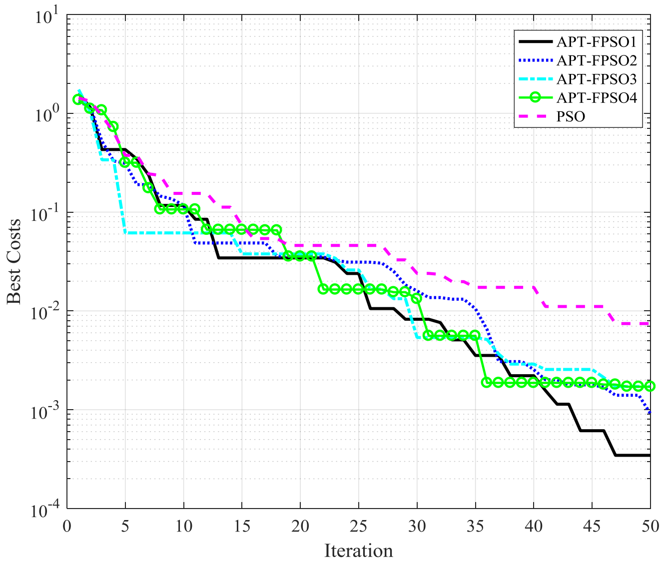

Fig. 3. Comparison of the convergence diagrams of APT-FPSOs, with different rule-base structures, and that of the standard PSO.

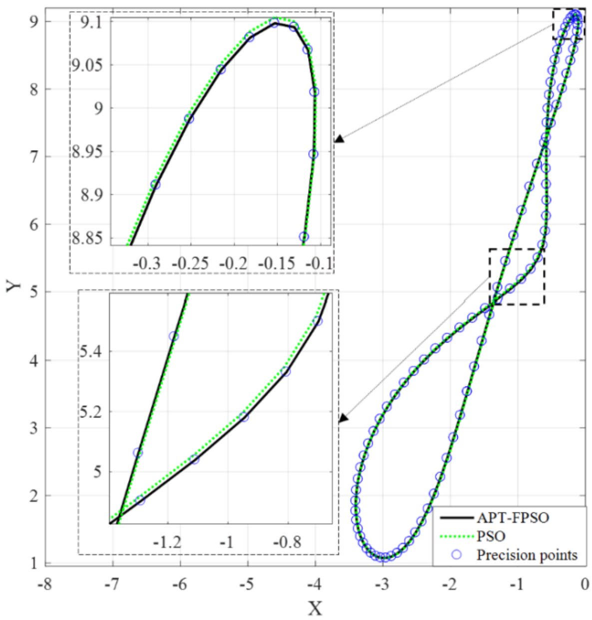

Fig. 4. Comparison of the generated path by APT-FPSO and that by the standard PSO.

The optimally designed four-bar mechanism generated by using APT-FPSO is illustrated in Figure 5.

It should be mentioned that compared to the standard PSO, APT-FPSO requires more CPU time; however, this fact is not far from expectations due to the no free lunch theorem [34].

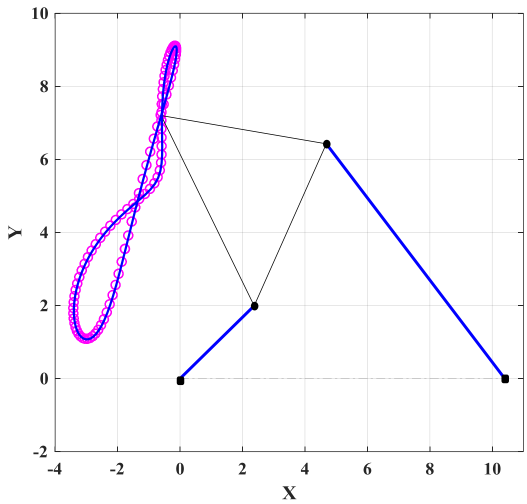

Fig. 5. Optimally designed a four-bar mechanism using APT-FPSO.

There exist a large number of works for the further expansion of this study in the future. For instance, to further improve the optimization algorithm's performance, one can consider other indices to the input, e.g., diversity of the swarm or the output, e.g., inertia weight (*w*), of the corresponding FIS. One may also take advantage of an Adaptive Neuro-fuzzy Inference System (ANFIS) to tune the MFs associated with the developed FIS. On the other hand, APT-FPSOs can optimally synthesize more complex mechanisms and deal with various sophisticated optimization problems from an eclectic realm.

4. Conclusion

In this study, a constrained version of the APT-FPSO algorithm was proposed to surmount the nonlinear problem of optimal defect-free synthesis of four-bar mechanisms. The case study had to with tracking a closed path with three loops that included 90 precision points. The problem has been solved by APT-FPSO algorithms of four different rule-based structures. It was concluded that, regardless of the intermediate superiority among them, all of the four APT-FPSOs appeared to be of higher accuracy than the standard PSO. Because the learning coefficients are being tuned at an individual level, APT-FPSO further enhances the exploitation ability of the standard PSO, without jeopardizing the exploration. It can be concluded that APT-FPSO can be used as a robust constrained, meta-heuristic optimization algorithm, and it could be a proper candidate for solving constrained optimization problems from an eclectic realm.

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Appendix A. Numerical coordinates of the generated and the desired paths.

No	Precision point's coordinates		Generated point's coordinates		No	Precision point's coordinates		Generated point's coordinates	
	x	*y*	*x*	*y*		*x*	*y*	*x*	*y*
1	-1.7476	3.5356	-1.74764808850705	3.53557746485548	46	-0.7003	5.4992	-0.700294160870266	5.49921920690989
2	-1.6409	3.9034	-1.64089532630207	3.90344667602063	47	-0.8066	5.3313	-0.806601576760017	5.33126821108790
3	-1.5302	4.2837	-1.53021041519070	4.28365919974604	48	-0.9461	5.1801	-0.946052779512598	5.18011985689427
4	-1.4158	4.6713	-1.41579024955026	4.67129550034767	49	-1.1105	5.0394	-1.11052228153416	5.03939260999306
5	-1.2983	5.0613	-1.29825000887978	5.06129600119047	50	-1.2908	4.9028	-1.29075650459414	4.90284616304178
6	-1.1786	5.4487	-1.17861803788268	5.44868769711554	51	-1.4789	4.7658	-1.47891216361701	4.76582173926998
7	-1.0583	5.8288	-1.05828198035055	5.82879085736758	52	-1.6691	4.6253	-1.66911227656177	4.62534722624071
8	-0.9389	6.1974	-0.938896190471547	6.19738420098900	53	-1.8571	4.4798	-1.85712473827476	4.47975209467118
9	-0.8223	6.5508	-0.822266051472496	6.55081593622569	54	-2.0399	4.3283	-2.03989839547990	4.32826520882361
10	-0.7102	6.8861	-0.710226339955189	6.88605803538615	55	-2.2152	4.1707	-2.21519292977161	4.17071910564995
11	-0.6045	7.2007	-0.604528619160574	7.20070953894726	56	-2.3813	4.0074	-2.38132664022394	4.00735479220710
12	-0.5067	7.4930	-0.506748201970923	7.49296006960710	57	-2.5370	3.8387	-2.53701359711566	3.83869673555180
13	-0.4182	7.7615	-0.418216097734287	7.76152681970199	58	-2.6813	3.6655	-2.68126002501842	3.66547255237201
14	-0.3400	8.0056	-0.339976812788409	8.00557767625869	59	-2.8133	3.4886	-2.81329822342045	3.48856040306634
15	-0.2728	8.2247	-0.272769600902332	8.22465088490001	60	-2.9325	3.3090	-2.93254401440152	3.30895355878160
16	-0.2170	8.4186	-0.217028904770812	8.41857872031374	61	-3.0386	3.1277	-3.03856897587040	3.12773575551703
17	-0.1729	8.5874	-0.172899085490303	8.58741976226171	62	-3.1311	2.9461	-3.13108203583965	2.94606346599364
18	-0.1403	8.7314	-0.140258742359592	8.73140199119363	63	-3.2099	2.7652	-3.20991703353359	2.76515272572312
19	-0.1188	8.8509	-0.118750614493222	8.85087716999502	64	-3.2750	2.5863	-3.27502408961105	2.58626905304713
20	-0.1078	8.9463	-0.107813936924518	8.94628584711951	65	-3.3265	2.4107	-3.32646338408036	2.4107195501134

									1
21	-0.1067	9.0181	-0.106717002557294	9.01813169164620	66	-3.3644	2.2398	-3.36440040599991	2.23984660663659
22	-0.1146	9.0670	-0.114588450301011	9.06696361653459	67	-3.3891	2.0750	-3.38910202618426	2.07502283405584
23	-0.1304	9.0934	-0.130446416273108	9.09336413604317	68	-3.4009	1.9176	-3.40093291935380	1.91764698228561
24	-0.1532	9.0979	-0.153225147106573	9.09794253596108	69	-3.4004	1.7691	-3.40035196489061	1.76914066196422
25	-0.1818	9.0813	-0.181799001968478	9.08133163871417	70	-3.3879	1.6309	-3.38790830905105	1.63094572686108
26	-0.2150	9.0442	-0.215003991616372	9.04418717276021	71	-3.3642	1.5045	-3.36423679019210	1.50452217106378
27	-0.2517	8.9872	-0.251657148938978	8.98718898016810	72	-3.3301	1.3913	-3.33005242095202	1.39134636568532
28	-0.2906	8.9110	-0.290574124324580	8.91104350558977	73	-3.2861	1.2929	-3.28614359328409	1.29290939831428
29	-0.3306	8.8165	-0.330585476802655	8.81648720190040	74	-3.2334	1.2107	-3.23336362894363	1.21071518067000
30	-0.3706	8.7043	-0.370552213052046	8.70429066795249	75	-3.1726	1.1463	-3.17262024582128	1.14627784937455
31	-0.4094	8.5753	-0.409381237802124	8.57526351381796	76	-3.1049	1.1011	-3.10486245906054	1.10111779420528
32	-0.4460	8.4303	-0.446041554421475	8.43026014722498	77	-3.0311	1.0768	-3.03106440073557	1.07675540199480
33	-0.4796	8.2702	-0.479582341816483	8.27018692016834	78	-2.9522	1.0747	-2.95220554742614	1.07470130228207
34	-0.5092	8.0960	-0.509154509019150	8.09601141134848	79	-2.8692	1.0964	-2.86924692768677	1.09644155444132
35	-0.5340	7.9088	-0.534038117362406	7.90877511929360	80	-2.7831	1.1434	-2.78310309150627	1.14341585813749
36	-0.5537	7.7096	-0.553679373260721	7.70961161813784	81	-2.6946	1.2170	-2.69461002441868	1.21698656632026
37	-0.5677	7.4998	-0.567743087728454	7.49977346825742	82	-2.6045	1.3184	-2.60448984737595	1.31839614730780
38	-0.5762	7.2807	-0.576190148798007	7.28067315742205	83	-2.5133	1.4487	-2.51331411124183	1.44871095279326
39	-0.5794	7.0539	-0.579395499051303	7.05394642414796	84	-2.4215	1.6087	-2.42146876996368	1.60874992892132
40	-0.5783	6.8216	-0.578331170719090	6.82155061650681	85	-2.3291	1.7990	-2.32912538948355	1.79899850443608
41	-0.5749	6.5859	-0.574850162280510	6.58591489141167	86	-2.2362	2.0195	-2.23622454237942	2.01951048584756
42	-0.5721	6.3502	-0.572110654158345	6.35015558400491	87	-2.1425	2.2698	-2.14247816955099	2.26980436343296
43	-0.5751	6.1183	-0.575135991092565	6.11833237478740	88	-2.0474	2.5488	-2.04739731653732	2.54876458646045
44	-0.5913	5.8956	-0.591299760515161	5.89558181876694	89	-1.9503	2.8546	-1.95034944185411	2.85456220012585
	-0.6300	5.6877	-0.629999961661337	5.68765685362148	90	-1.8506	3.1846	-1.85064514665885	3.18461138950162