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Robust Parameters Design of Categorical Responses under Modeling and Implementation Errors

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Abstract

Nowadays, improving quality is advocated as a strategy to increase market share, and failing to address this crucial issue results in exclusion from the competitive landscape. The importance of quality engineering subjects can be attributed to this. The majority of studies undertaken in recent years have looked into and optimized continuous response variables while ignoring categorical characteristics. This necessitates a change in statistical methods in this discipline to ones that take categorical responses into account. Statistical techniques have always provided researchers with estimates of parameters that are subject to uncertainty. Hence, considering uncertainty in modeling is essential for reducing errors and minimizing costs while increasing quality. In this study, we deal with the robust design of quality characteristics in categorical response problems to reach optimal levels of control variables, which can minimize the error caused by modeling and implementation and provide more accurate estimates. A portion of the uncertainty is taken into account while estimating the model parameters. However, the proposed approach assumes that the optimal settings of design variables during the implementation phase will also experience oscillations, which introduce a type of error into decision-making. Finally, in the optimization phase, multiple equations relating to response levels are modeled and solved using the goal programming approach. The results showed that our approaches could achieve solutions with robustness against the two main source of errors.

Keywords: Quality Engineering, Robust parameter Design, Dual Response Modeling, Logistic Regression, Model Iimprecision, Iimplementation Error.

1.Introduction

Consumers prioritize quality, manufacturers, and service providers must consistently deliver high-quality goods and services to dominate the market and satisfy customers. To maintain their position and compete globally, businesses must monitor and evaluate quality indicators. Success depends on raising quality while lowering production costs and prices. Decision-makers must use forecasting and improvement methods, such as the response level method, to identify and improve factors affecting quality characteristics. Response variables express qualities related to goods and services. Finding the relationship between the variables and simultaneously optimizing these properties is crucial (Hejazi & Bashiri, 2009)because the response variables are influenced by one or more independent variables. Robust design, introduced by Japanese quality consultant Taguchi in the 1980s, considers the variability of a characteristic to minimize it. This method, known as robustness, enables the creation of products with minimal variability in random variables. Customers will be more satisfied as a result.

The response surface method (RSM) is a robust design technique that considers scattering effects from two angles, aiming to bring a response variable to the ideal value. it is a statistical approach to experiment design and process variable effect optimization (Veza et al., 2023). It takes into account environmental error and modeling error (prediction error) as criteria for dispersion. Additionally, Myers and Carter (Myers & Carter, 1973) introduced the dual response modeling approach, which models the mean and variance of a process and uses regression relationships derived from the mean and variance of a process. This approach reduces process variability and brings it closer to the desired target level. However, most studies consider response variables to be continuous, as many problems fall into the discrete category and can be defined using these variables. This research aims to minimize the uncertainty of models of location effects and dispersion of problems with discrete output. The main objective is to provide a more accurate model than previous studies using the dual response method for quality characteristics in the categorical type, reducing model uncertainty-related variability and deviations from the recommended course of action. Due to this, Brenneman and Myers (Brenneman & Myers, 2003) incorporated categorical (nominal) uncontrollable variables into their research, reducing prediction variance and bringing the average closer to the desired value. They compared the impact of continuous and categorical noise assumptions on the robust design of quality characteristics and examined outcomes. Hejazi and Akbari (Hejazi & Akbari, 2017) developed a mathematical programming model to optimize multi-response systems using the response method, using dummy variables and binary and ordinal logistic regression as follows:

$$\text{Max } \pi_{j_{max}} \quad (1)$$

Subject to:

$$\pi_{j+1} > \pi_j \qquad \forall j = 1.2. \dots . j-1 \quad (2)$$

$$\sum_{j=1}^{2} D_{ij} \le 1 \qquad \text{for all i} \quad (3)$$

$$D_{ij}: \text{Binary} \quad (4)$$

D_{ij}: Represents dummy variables used as an indicator for each level of the categorical variables

$\sum_{j=1}^{2} D_{ij} \le 1$: This limitation is to prevent linear relationship between dummy variables

The aim of this model is to maximize the probability of placing the response variable in the category with the highest priority. The response variables are categorized from the lowest to the highest favorability, and the priority order of the categories must be observed. Hejazi and Akbari (Hejazi & Akbari, 2017) investigated a case example of the friction stir welding process, using the binomial response method with the error approach. Sohn (Sohn, 1999) optimized the process of making windows using nine independent variables, using the binomial response method with the error approach. Lawson and Montgomery (Lawson & Montgomery, 2006) discussed the applications of logistic regression in marketing and sales, investigating effective factors in customer satisfaction using binary, nominal, and ordinal logistic regression. In real-world experiments, data simultaneously have a quantitative and qualitative nature (ordinal). Wu (Wu, 2008) presented a novel approach for robust design of quantitative and ordinal response and independent variables using the Taguchi loss function method. Zhou and Wu's research (Zhou et al., 2008) investigated the role of customer emotions in product design, examining the needs and feelings of customers about a product. Demirtas, Anagun (Demirtas et al., 2009) used ordinal logistic regression to determine the type of optimal design of kitchen appliances, using mutual analysis to improve product design. Mehrjoo and Bashiri (Mehrjoo & Bashiri, 2013) The aim of this model is to maximize the probability of placing the response variable in the category with the highest priority. The response variables are categorized from the lowest to the highest favorability, and the priority order of the categories must be observed to predict daily production planning in Iran's automotive industry. In recent years, the study of quality design with model uncertainty has gained significant attention, particularly in the area of parameter design and tolerance design. The correlation between response variables is considered, and the group decision technique is employed to identify the importance of each response variable. The variance-based and mean-based models provided by Díaz-García, Ramos-Quiroga (Díaz-García et al., 2005) have been used to convert their probabilistic model into a deterministic model, reducing the variance of the model. Sadjadi, Habibian [12] identified influencing factors on increasing the durability and robustness of a chemical product using the Delphi method. Kovach and Cho [13] explored output and input variables using the qualitative function expansion method and estimated regression equations related to location and dispersion effects using the binomial response modeling method. Hejazi, Bashiri [14] presented a model with a correlation between auxiliary variables and found the best combination of factors affecting their problem using location and dispersion effects. Hejazi, Bashiri [15] optimized the multi-response problem by considering the correlation between the response variables and used the dual response method for the robustness of their model results.

Designing and developing high-quality products is crucial for factories, as it involves parameter design and tolerance design, which affect product performance and production costs. These factors increase the complexity of multi-response problems, which require the optimal values of parameters and tolerances for all responses. Hazrati-Marangaloo and Shahriari's research (Hazrati-Marangaloo & Shahriari, 2017) proposed a new approach for robust design of parameters and tolerance design simultaneously in multi-response problems, using the loss function approach and one-way multivariate analysis of variance. Ouyang, Ma (Ouyang et al., 2017) presented a new loss function method for multi-response optimization to deal with model parameter uncertainty and implementation error. The parametric modeling method is widely used to construct response functions for mean and variance because it is simple and easy to use. They showed that the robust design of quality characteristics is considered despite the categorical responses and states that in this research, the uncertainty of the model has been taken into account with the help of the Bayesian approach and the binary response method. Wang, Ma (Wang et al., 2019) presented that in the study of simultaneous optimization of parameter design and robust design with several qualitative features, the existing modeling methods rarely consider the change of predicted responses related to the uncertainty of model parameters and other random errors. take, to ignore its impact on the strength and economy of the product or process. In recent years, quality design with model uncertainty has attracted the research interest of many researchers. Ouyang, Zhou (Ouyang et al., 2019) stated that the uncertainty modeling method in quality design is mainly examined from the following two aspects: (1) Uncertainty of the model parameters (2) Uncertainty of the model structure itself. Some researchers consider the effect of model parameters and model structure uncertainty on optimization results by using some uncertainty analysis methods such as fuzzy mathematics with confidence interval, Bayesian statistics and group models. Gu, Tong (Gu et al., 2019) proposed to express parameter and tolerance integration scheme for multivariate quality attributes based on modified process capability index. However, they assume that the response variables are independent of each other and do not consider the correlation between the responses. Lee, Yang (Lee et al., 2020) presented a systematic method to optimize the mean and variance of multiple responses in a multi-stage process. In a multi-stage process, each stage is influenced by its previous stage and affects the next stage. In addition, each step often includes several response variables to optimize. Wang, Mao (Wang et al., 2021) proposed a simultaneous multi-response optimization of parameter and tolerance design method using Bayesian modeling method. The total cost model includes cost tolerance, quality reduction and rejection cost. However, the method only focuses on studying normal responses. Li, He (Li et al., 2022) proposed a robust method for multi-response optimization (MRO) considering location effect, dispersion effect and model uncertainty simultaneously. have proposed a multi-objective optimization model for MRO that simultaneously maximizes the degree of utility of local and dispersion effects.

In Table 1, a comparison of the studies conducted using the response modeling method in terms of environmental factors, modeling error, implementation error, and also the examination of the specific characteristics of each study is shown.

Table 1
Related studies using Response Surface Methodology

Special feature of the research	Implementation error	Multiple response	Modeling error	Environmental error	Paper
Considering noise variables categorically			*		(Brenneman & Myers, 2003)
Using probabilistic programming method and converting probabilistic model to deterministic model		*	*		(Díaz-García et al., 2005)
Combination of two approaches of goal programming and utility function		*		*	(Kazemzadeh et al., 2008)
Using Delphi methods and utility function		*		*	(Sadjadi et al., 2008)
Considering the noise variable and using goal programming		*		*	(Kovach & Cho, 2008)
Considering the correlation between covariates		*		*	(Hejazi et al., 2011)
Considering the correlation between response variables and using the group decision-making		*	*		(Hejazi et al., 2012)
Presenting a new approach by combining the two concepts of robust parameter design and tolerance design in multi-response equations		*		*	(Hazrati‐Marangaloo & Shahriari, 2017)
A method for model parameter uncertainty and implementation error	*	*			(Ouyang et al., 2017)
Presenting a combined model with the help of Pareto diagram and optimization method to reduce modeling error		*	*		(Ouyang et al., 2019)
New economic parameter design under Bayesian modeling and optimization framework and its combination with multi-objective genetic algorithm		*	*		(Wang et al., 2019)
Using the PRIM1 algorithm method and multi-response mean and variance optimization method		*			(Lee et al., 2020)
Considering implementation error and considering location effects, dispersion effects and model uncertainty	*				(Gu et al., 2019)
Bayesian modeling and optimization and uncertainty and considering the implementation error	*	*	*		(Wang et al., 2021)
A method for the uncertainty of model parameters and utility function		*	*	*	(Li et al., 2022)

Special feature of the research	Implementation error	Multiple response	Modeling error	Environmental error	Paper
Considering the implementation error in the design based on the effective parameters on categorical responses	*	*	*		Proposed method

1. Proposed approach

As mentioned in Section 1, the Dual Response Surface Method (DRSM) is one of the techniques used for designing sustainable quality features. It consists of three stages: experimental design, modeling, and optimization. Now, with the aid of these three stages, the steps involved in conducting this research are described:

1.1. Design of experiments

Since experiments often deal with two or more factors. One of the best methods for conducting such tests is using full factorial tests It should be noted that the proposed optimization model can work the data from any regular or non-regular design of experiments.

1.2. Modeling

After identifying the independent and the response variables in the experimental design phase, the next step is to obtain the relationship between the inputs and outputs of the problem. As mentioned, when the response variables in a problem are defined qualitatively, logistic regression can be employed to fit the regression equation. Therefore, in this section, we first explore various logistic regression models. Subsequently, the location and dispersion effects are calculated, and finally, the proposed model of this study is presented.

1.2.1. Binary logistic regression

If the response variables have only two categories (labeled by zero and one), binary logistic regression can be used to fit the regression equation. If $\pi(X)$ represents the probability of the desired event (Y=1) in terms of m independent variables and Y has two states 0 and 1, then the regression equation related to $\pi(X)$ is obtained from formula (5) (Agresti, 2018) and (Gujarati Damodar, 2004).

$$\pi(X) = E(Y = 1|X_i) = \frac{1}{1+e^{-(\beta_1+\beta_2 X_2+\cdots+\beta_m X_m)}} \quad (5)$$

According to the formula (5), it is obvious that as the value of the basic power e changes between -∞ and +∞, $\pi(X)$ can only take values between zero and one.

If the probability of occurrence Y=0 is represented by 1-$\pi(X)$, the formula (6) is obtained:

$$1 - \pi(X) = \frac{1}{1+e^{(\beta_1+\beta_2 X_2+\cdots+\beta_m X_m)}} \quad (6)$$

Therefore, by dividing the two equations (5) by (6), the probability ratio equation will be as follows.

$$\frac{\pi(X)}{1-\pi(X)} = \frac{1+e^{(\beta_1+\beta_2 X_2+\cdots+\beta_m X_m)}}{1+e^{-(\beta_1+\beta_2 X_2+\cdots+\beta_m X_m)}} = e^{(\beta_1+\beta_2 X_2+\cdots+\beta_m X_m)} \quad (7)$$

As you can see, the above model is non-linear, so by applying a logarithmic transformation to both sides of the equation (7), it can be written in a linear form.

$$\text{logit}\left(\frac{\pi(X)}{1-\pi(X)}\right) = \beta_1 + \beta_2 X_2 + \cdots + \beta_m X_m \quad (8)$$

The model (7) is also called the logistic regression odds ratio model, and the maximum likelihood method (MLE) can be used to estimate the β coefficients in this model. It should be noted that these coefficients can be obtained by entering the data of a problem with the help of various statistical software such as MINITAB and SPSS.

1.2.2. Ordinal Logistic Regression

If the response variables in an experiment are defined as ranked categories, ordinal logistic regression is used. If j represents the number of categories of response variables, p represents the cumulative probabilities of responses and π represents the individual probabilities of occurrence of each category of response variables according to the following relationship (Hejazi & Bashiri, 2009) and (Agresti, 2018):

$$P_j = p(Y \leq j) = \pi_1 + \pi_2 + \cdots + \pi_j \quad j = 1.\ldots.J \qquad (9)$$

$$P(Y \leq j) = \frac{\exp(Z_j)}{1+\exp(Z_j)} \qquad (10)$$

The value of Z_j in formula (10) is equal to:

$$Z_j = \alpha_j + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_m x_m \quad j = 1\ldots J-1 \qquad (11)$$

In this equation, α_j is intercept associated with the j^{th} category, and βs are the values of the coefficients related to the independent variables. Another index in the ordinal logistic regression method is the probability ratio. This ratio shows the cumulative probability value for category j to j-1. Equation (12) represents the probability ratio for the j^{th} category [2].

$$\text{Odds } (Y \leq j) = \frac{p(Y\leq j)}{1-p(Y\leq j)} = \frac{p_j}{1-p_j} \qquad j = 1.\ldots.J-1 \qquad (12)$$

In the formula (12), the value of the probability ratio is equal to:

$$\frac{P_j}{1-P_j} = \exp(Z_j) \qquad (13)$$

Model (13) is of nonlinear nature. Therefore, in order to calculate probability values in ordinal logistic regression, the natural logarithm function is utilized as the model (Hejazi & Bashiri, 2009).

In other words, by applying the natural logarithm to both sides of the equation, it can be converted into a linear form.

$$\text{logit}(Y \leq j) = \ln\left(\frac{p_j}{1-p_j}\right) = \alpha_j + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_m x_m \qquad (14)$$

If the number of categories of the response variable is equal to j, then there are j-1 equations, which together with the equation $\sum_{j=1}^{J} \pi_j = 1$, the j equation of unknown j is created, that is, for each line of the response variable should be written as possible equations (13). Equation (13) cannot be written for the j^{th} row. As it was said, p_j is the cumulative probability of row j and on the other hand, $\sum_{j=1}^{J} \pi_j = 1$, as a result, the value of the denominator of the equation (13) will be equal to zero, which is meaningless. Because of this, equation $\sum_{j=1}^{J} \pi_j = 1$ is added to j-1 of the equation (Hejazi & Bashiri, 2009). Also, in this model, the relationship (15) can be used to calculate the individual probabilities of each category (Agresti, 2018).

$$\pi_j = p(Y = j) = p(Y \leq j) - p(Y \leq j-1) \qquad (15)$$

1.2.3. Multinomial Logistic Regression

If the response variables in an experiment are defined nominally and the categories do not have priority, nominal logistic regression can be used to fit the regression equation. Individual probabilities related to the j^{th} category in nominal logistic regression is calculated through formula (16) (Agresti, 2018).

$$\pi_j = \frac{e^{\alpha_j + x\beta_j}}{\sum_h^J e^{\alpha_h + x\beta_h}} \quad j = 1 \ldots. J \qquad (16)$$

In this regression, like two binary and ordinal regressions, the sum of the probabilities of different categories is equal to one, and the parameters α and β are equal to zero for the base category. For example, if j = 3, the values of α_3 and β_3 are equal to zero, and since these values reach the base e, the numerical value of one is obtained for this category.

1.2.4. Calculation of modeling error

After estimating the parameters of the logistic regression (model), the next step to reduce the uncertainty of the model is to obtain the equation related to the modeling error (variance). To obtain this equation, the variance of the model coefficients (β) must be obtained first. To calculate the variance of the coefficients in the MLE method, we use the inverse of the Fisher matrix, which is actually the variance-covariance matrix of the coefficients, and this matrix can be calculated from formula (17) (McCullagh, 2019).

$$I^{-1} = \sum = (X'\hat{V}X)^{-1} \qquad (17)$$

In this equation, $\sum$ represents the variance-covariance matrix, and the main diagonal units of this matrix represent the variance of the coefficients, and the other units represent the covariance values. Also, I^{-1} is the inverse of Fisher's matrix and X and X' are respectively the matrix and transhed of the matrix of independent variables, which are defined as follows:

$$X = \begin{bmatrix} 1 & x_{1.1} & \cdots & x_{1.m} \\ 1 & x_{2.1} & \cdots & x_{2.m} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n.1} & \cdots & x_{n.m} \end{bmatrix}$$

In this matrix, $x_{n.m}$ is the value of the m^{th} independent variable for the nth observation. Also, in formula (17), since the residuals in binary logistic regression have a binomial distribution, so the probability of each observation being in the desired category also follows the binomial distribution (Agresti, 2018) and the related variance matrix is equal to:

$$\hat{V} = \begin{bmatrix} \hat{\pi}_1(1-\hat{\pi}_1) & 0 & \cdots & 0 \\ 0 & \hat{\pi}_2(1-\hat{\pi}_2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \hat{\pi}_n(1-\hat{\pi}_n) \end{bmatrix}$$

$\hat{V}$ is a diagonal matrix in which π_n indicates the probability of placing the nth observation in the desired category. By replacing the above two matrices in equation (17), the variance-covariance matrix of the model coefficients can be obtained. As mentioned in this research, it deals with the reduction of forecast variance and since logistic regression is a function of random variables which are actually the coefficients of the model, for this purpose after obtaining the variance-covariance matrix of the coefficients with the help of formula (19) Modeling error can be obtained. If f is a function of the estimators of B, the following equation is obtained by using Taylor expansion with degree two approximation (Alexander et al., 1974).

$$f(B) \approx f(\beta) + \nabla f(\beta)^T(B-\beta) \qquad (18)$$

Then the variance of the desired function can be approximated with the help of the following relations.

$$\mathrm{Var}(f(B)) \approx \mathrm{Var}\big(f(\beta) + \nabla f(\beta)^T(B-\beta)\big) \qquad (19)$$

$$= \mathrm{Var}(f(\beta) + \nabla f(\beta)^T.B - \nabla f(\beta)^T.\beta)$$

$$= \mathrm{Var}(\nabla f(\beta)^T.B)$$

$$= \nabla f(\beta)^T.\mathrm{Cov}(B).\nabla f(\beta)$$

In formula (19), β represents the average coefficients of the model and Cov (B) represents the variance-covariance matrix of the coefficients.

1.3. Optimization

In this section, the method of optimization of the proposed model is discussed. Because the presented model is a multi-response type, the goal programming method is used to solve it, which will be explained in the following.

1.3.1. Location effects model

In the ordinal logistic regression model, the response levels are categorized from the least to the most favorable, so in this model, the objective of maximization is a priority function. Meanwhile, in the binary logistic regression model, the last category is generally considered as the base category, so the li effects model for binary and ordinal response variables is generally defined as follows:

$$\max\big(f_{i.j}(x)\big) = \pi_{i.j_{max}} \qquad i=1,\ldots,n \qquad (20)$$

Subject to:

$$\pi_{i.j+1} > \pi_{i.j} \qquad i=1,\ldots,n\ ,\ j=1,\ldots,J-1 \qquad (21)$$

$$-1 \le x_k \le 1 \qquad k=1,\ldots,m \qquad (22)$$

In this model, i is the index of each response variable, j represents the number of categories of each response variable, and k represents the number of independent variables. Also, the purpose of this model is to maximize the probability of each response variable being placed in the desired category. Because in the ordinal logistic regression model, the response levels are categorized from the least to the most favorable, therefore, if there are ordinal responses, the constraint (21) should be used to establish this condition. Constraint (22) also determines the area related to the changes of the independent variables of the test, and since these variables are defined in coded form, the range of changes is defined between -1 and 1. After estimating the logistic regression coefficients with the help of MINITAB and SPSS software, the objective function and limitations of the location effects model can be written as follows.

π in the objective function (20), which indicates the individual probability of each category, can be calculated according to the formula (15).

$$\pi_{j+1} = p(Y = j + 1) = p(Y \leq j + 1) - p(Y \leq j)$$

$$j = 1, \ldots, J-1 \qquad (23)$$

Also, the constraint (21) can be written in the form of relation (23):

$$\pi_{j+1} > \pi_j = p(Y \leq j + 1) - p(Y \leq j) > p(Y \leq j)$$

$$= p(Y \leq j + 1) > 2p(Y \leq j) = \frac{e^{z_{i,j+1}}}{1+e^{z_{i,j+1}}} > 2\frac{e^{z_{ij}}}{1+e^{z_{ij}}} \qquad (24)$$

1.3.2. Robust design model

In this model, in addition to location effects, dispersion effects are also considered, and the goal of this model is to maximize the probability of the output variable being placed in the desired category, and at the same time, reduce the variability of the process. The robust design model in this research is generally defined as follows:

$$\max(f_{i.j}(x)) = \pi_{i.j_{max}} \qquad i=1,\ldots,n \qquad (25)$$

$$\min \operatorname{var}\left(f_{i.j}(x)\right) = \operatorname{var}(\pi_{i.j_{max}}) \qquad i=1,\ldots,n \qquad (26)$$

Subject to:

$$\pi_{i.j+1} > \pi_{i.j} \qquad i=1,\ldots,n\ ,\ j=1,\ldots,J-1 \qquad (27)$$

$$-1 \leq x_k \leq 1 \qquad k=1,\ldots,m \qquad (28)$$

In this model, i is the index of each response variable, j represents the number of categories of each response variable, and k represents the number of independent variables. Also, the purpose of this model is to maximize the probability of each response variable being placed in the desired category. Since the dispersion effects in this research are of the type of modeling error, the objective function (19) can be calculated with the help of the formula (26). In the ordinal logistic regression model, the answer levels are categorized from the least to the most favorable, so if there are ordinal answers to establish this condition, the restriction (27) should be used. Constraint (28) also determines the area related to the changes of the independent variables of the test and since these variables are defined in coded form, the range of changes is defined between -1 and 1.

1.3.3. Implementation error checking

In most industrial systems today, there is more than one response variable, and therefore an operating condition must be found that simultaneously satisfies all quality characteristics. Existing issues for quality improvement are influenced by various factors, and it is important to identify the influencing factors and how to deal with them in the implementation of the issue. The existence of errors in modeling and implementation can change the results of these programs. Also, the implementation of decisions may fluctuate.

This research is trying to find a solid solution that the control factors or design variables of a process, their values are determined and the results obtained are implementable and reliable. Also, in this study, the impact of modeling and implementation error is investigated simultaneously. The defaults of this model are as follows:

1. Control variables fluctuate or change during execution

2. The experimenter cannot set precise control values. Because it may be possible to control during testing, but it may change during executions after optimization

3. Quality variables or characteristics are predetermined

4. Continuous type control variables can be measured

$$\text{Max } \pi_{i.j_{max}} = f(\beta.x_k + \delta_k) \qquad i=1,\ldots,n \qquad (29)$$

Min $Var\left(\pi_{i.j_{max}}(x_k + \delta_k)\right)$ i=1,…,n (30)

Subject to:

$\pi_{i.j+1}\ (x_k + \delta_k) > \pi_{i.j}\ (x_k + \delta_k)$ j=1,…,J , i=1,…,n (31)

$\delta_k \in S$ k=1,…,m (32)

$-1 \leq x_k \leq 1$ k=1,…,m

In the model (31), S here represents the set of possible scenarios of δ that oscillates around the zero value and leads to the value of the design parameters decreasing or increasing from their nominal value.

1.3.4. Minimax/Maximin model

Objective Function: $Gp = \sum_i \left(we_i \frac{de_i^-}{te_i}\right) + \sum_i \left(wd_i \frac{dd_i^+}{td_i}\right)$ (35)

Subject to

$zz_{ei} = {}^{Min}_{S}\ \pi_{i.j_{max}}$ i=1,…,n

$zz_{di} = {}^{Max}_{S}\ var(\pi_{i.j_{max}})$ i=1,…,n

$\pi_{i.j+1}\ (x_k + \delta_k) > \pi_{i.j}\ (x_k + \delta_k)$ i=1,…,n , j=1,…,J-1

$zz_{ei} + de_i^- - de_i^+ = te_i$ i=1,…,n

$zz_{di} + dd_i^- - dd_i^+ = td_i$ i=1,…,n

$\delta_k \in S$ k=1,…,m

$-1 < x_k < 1$ k=1,…,m

In the above model, the objective functions depend on the value of the implementation error scenario. In this part, by using the robust design model in the above relationship, by obtaining the worst value of δ scenarios, the Minimax/Maximin model can be reached (Hejazi et al., 2013). Maximin is obtained from equation (29) and minimax is obtained from equation (30).

Maximum mean response:

Max ${}^{Min}_{S}\ \pi_{i.j_{max}}$ i=1,…,n (33)

Subject to:

$\pi_{i.j+1}\ (x_k + \delta_k) > \pi_{i.j}\ (x_k + \delta_k)$ i=1,…,n , j=1,…,J-1

$\delta_k \in S$ k=1,…, m

$-1 \leq x_k \leq 1$ k=1,…,m

Minimum response variance:

Min ${}^{Max}_{S}\ var(\pi_{i.j_{max}})$ i=1,…,n (34)

Subject to:

$\pi_{i.j+1}\ (x_k + \delta_k) > \pi_{i.j}\ (x_k + \delta_k)$ i=1,…,n , j=1,…,J-1

$\delta_k \in S$ k=1,…, m

$-1 \leq x_k \leq 1$ k=1,…,m

Model (33) shows that we maximize the lowest value obtained from $\pi_{i.j_{max}}$.

Similarly, for relation (34), we minimize the maximum value obtained from the variance $\pi_{i.j_{max}}$.

1.3.5. Goal Programming approach

In this section, using the above minimax/maxim models which are obtained as multi-objective objective function, the ideal planning model is developed as follows.

Here, d^+ and d^- respectively indicate the positive and negative deviations of the objective function of the problem from the objectives in question.

$$h(d^+.d^-) = \begin{cases} d^+ \ ; \ \text{If f is the objective function of a minimization problem} \\ d^- \ ; \text{If f is the objective function of a maximization problem} \\ d^+ + d^- \ ; \ \text{otherwise} \end{cases}$$

td_i and te_i respectively specify the ideal value of the i^{th} response variable caused by the effects of dispersion (robustness) and location effects. wd_i and

we_i also show the relative weight of the i^{th} response variable caused by dispersion effects (robustness) and location effects, respectively (Hejazi et al., 2017).

In general, in ideal planning, the logic of optimal mathematical models is taken into consideration at the same time as the desire of the decision-maker to provide certain objectives from various goals. In the next section, a numerical example is examined using some of the approaches mentioned above, and the results are contrasted with those of earlier studies.

2. Numerical example

The model (29) that was stated in part 2 is the proposed model of this research. Therefore, in this section, in order to validate and apply the current proposed approach, a numerical example taken from the real test data presented in the article by Hejazi and Akbari (Hejazi & Akbari, 2017) will be solved. In addition, it was shown in the last chapter that in order to make a decision, due to the existence of different goals in the presented model, a set of answers is needed. Therefore, in this section, the ideal programming method is used to solve the multi-objective model. One of the best methods of producing patchwork sheets is to use the friction stir welding process. The friction stir welding process was first invented in 1991 at TWI Institute as one of the solid-state welding methods. Friction stir welding is a new method in welding metals in the solid state because in this process, due to the intense deformation and heat caused by friction in the welding zone, a microstructure is obtained, which has superior mechanical properties compared to other welding processes (Ghaffarpour et al., 2017). In the research conducted by Hejazi and Akbari (Hejazi & Akbari, 2017), the friction stirs welding process of thin sheets of 5038-H12 and 6061-T6 alloy with the same thickness of 1.5 mm has been investigated experimentally. Also, in his experiment, the independent variables and test answers are considered in groups. In the current research, in order to improve the quality of the friction stir welding process, the robust model proposed in the third chapter is implemented based on the mentioned phases. Also, in this section, MINITAB and SPSS software were used to estimate the regression equations and GAMS software was used to optimize the proposed model.

2.1. First phase: design of experiments

As mentioned, in this research, in order to identify the variables of the problem, the experimental design presented in the research of Hejazi and Akbari (Hejazi & Akbari, 2017) is used. with the difference that in the current research, the independent variables of the problem are defined as continuous and coded. In the proposed approach for the welding process, Hejazi and Akbari (Hejazi & Akbari, 2017) stated that the two output characteristics of final strength and elongation are affected by two independent linear variables (X_1) and rotational speed (X_2). So that the output characteristics of elongation are binary and ultimate strength are defined sequentially. Also, the independent variables x_1 and x_2 can be measured in three and two levels, respectively. In this example, each of the experiments is performed at 6 levels with 8 replications and a confidence level of 0.1. The test plan table of this problem according to the relative frequency of response variable levels is given below (Table 2).

Table 2
Relative frequencies of response levels of response variables

Number	The optimal value	The optimal value	Y_1		Y_2		
	X_1	X_2	The first category	The second category	The first category	The second category	The third category
1	-1	-1	1	0	0.75	0.125	0.125

2	-1	1	0.25	0.75	0	0.25	0.75
3	0	-1	0.5	0.5	0	0.875	0.125
4	0	1	0.5	1	0	0.25	0.75
5	1	-1	0.875	0.125	0.5	0.25	0.25
6	1	1	1	0	0	0	1

2.2. The second phase: modeling

In this phase, according to the definitions provided for logistic regression in the third chapter and with the help of MINITAB and SPSS software, the logistic regression estimation of the data in Table 2 for the two response variables of ultimate strength and elongation is discussed, the results of which are as follows:

2.2.1. Binary logistic regression estimation

Fig 1 shows the output of SPSS software for the regression equation related to the variable of length increase. In order to find out the appropriateness of binary logistic regression, the obtained p-value is used. In this way, if the p-value is less than 0.1, the null hypothesis (based on the zero coefficient of the independent variables) will be rejected and the coefficient of the opposite independent variable will be zero. The results of the modeling indicate that the y-intercept of this model is not significant under alpha 0.1, so the y-intercept is removed under these conditions and the modeling is done again. Also, this model rejects the existence of effects. The output results related to the existence of mutual effects are given in Appendix (b). After removing the meaningless effects, the following final model is obtained.

Variables in the Equation

		B	S.E.	Wald	df	Sig.	Exp(B)
Step 1[a]	x2	.861	.333	6.682	1	.010	2.367
	x1	-.767	.414	3.442	1	.064	.464

a. Variable(s) entered on step 1: x2, x1.

Fig. 1. SPSS software output for binary logistic regression without considering the y-intercept

In normal regression, the significance test of the coefficient of an independent variable is performed through the t statistic; But in logistic regression, another statistic called parent is used. The parent statistic for each of the coefficients shows the significance of the corresponding coefficient, like normal regression. This statistic has a χ^2 distribution with one degree of freedom. According to the results, it can be seen that the value of sig related to the significance test of the coefficients is lower than the desired alpha, so the null hypothesis that the coefficients of the model are meaningless is rejected. In this way, the equation of binary logistic regression in case of removing the y-intercept and re-modeling is as follows.

$$z_{11} = -0.767x_1 + 0.861x_2 \quad (36)$$

The probability of levels one and two in the binary logistic regression model is as follows.

$$P(y_1 = 1) = \pi_{12} = \frac{e^{z_{11}}}{1+e^{z_{11}}} = \frac{e^{-0.767x_1+0.861x_2}}{1+e^{-0.767x_1+0.861x_2}} \quad (37)$$

$$P(y_1 = 0) = \pi_{11} = 1 - \pi_{12} = \frac{1}{1+e^{-0.767x_1+0.861x_2}}$$
(38)

2.2.2. Estimation of ordinal logistic regression

In ordinal logistic regression, if j categories are defined, there are j-1 regression equations. Since in this example, the number of categories related to the ultimate strength variable is equal to 3, so the number of 2 regression equations is obtained, and the coefficients of these two equations are shown in Fig

2. The results of modeling indicate that the y-intercept of the quadratic equation of this model is not significant under alpha 0.1, so the y-intercept is removed in this condition. In this model, the coefficient related to the mutual effect between two independent variables is meaningless, in other words, the existence of mutual effects between independent variables is rejected. The output results related to the existence of mutual effects are given in Appendix (b).

Logistic Regression Table

Predictor	Coef	SE Coef	Z	P	Odds Ratio	95% CI Lower	95% CI Upper
Const(1)	-2.21650	0.506665	-4.37	0.000			
Const(2)	0.0355829	0.398991	0.09	0.929			
x1	-0.727435	0.413606	-1.76	0.079	0.48	0.21	1.09
x2	-1.79457	0.405075	-4.43	0.000	0.17	0.08	0.37

Log-Likelihood = -34.624
Test that all slopes are zero: G = 29.896, DF = 2, P-Value = 0.000

Goodness-of-Fit Tests

Method	Chi-Square	DF	P
Pearson	13.3858	8	0.10

Fig. 2. MINITAB software output for ordinal logistic regression

Also, goodness of fit tests such as Pearson and logarithm of likelihood are used to evaluate the fit of the whole model. These tests are calculated using the χ^2 statistic and the null hypothesis and its opposite are defined as follows:

$$\begin{cases} H_0: & \text{The model has a good fit} \\ H_1: & \text{The model lacks proper fit} \end{cases}$$

The log-likelihood test is equivalent to the F-statistic in linear regression analysis. This statistic shows the contribution of each variable to the model in determining the changes of the dependent variable. If the contribution of each variable in the model is greater, the inclusion of that variable in the model will increase the logarithm of the likelihood. Hejazi and Akbari (Hejazi & Akbari, 2017) stated in their article that this statistic is dependent on the sample size and cannot be used as a fit index alone, therefore, in addition to the log-likelihood test, the Pearson test can also be used. In this test, large values of χ^2 and small values of p-value show that the model may not fit the data well, so according to the results of the log-likelihood test in Fig 2, it can be seen that the model fits well. has not been but according to the Pearson test results in Fig 2, it can be seen that the p-value is equal to the alpha value of 0.1, so it can be concluded that based on this test, the model is well fitted. Finally, ordinal logistic regression equations can be written in the following form.

$$Z_{21} = -2.2165 - 0.7274x_1 - 1.7946x_2 \quad (39)$$

$$Z_{22} = -0.7274x_1 - 1.7946x_2 \quad (40)$$

The probability of different levels in the ordinal logistic regression model is calculated as follows:

$$P(y_2 \le 1) = \pi_{21} = \frac{e^{Z_{21}}}{1+e^{Z_{21}}} = \frac{e^{-2.2165-0.7274x_1-1.7946x_2}}{1+e^{-2.2165-0.7274x_1-1.7946x_2}} \quad (41)$$

$$P(y_2 \le 2) = \pi_{21} + \pi_{22} = \frac{e^{Z_{22}}}{1+e^{Z_{22}}} = \frac{e^{-0.7274x_1-1.7946x_2}}{1+e^{-0.7274x_1-1.7946x_2}} \quad (42)$$

As can be seen, equation (42) shows cumulative probability. Now, with the help of formula (15) which was presented in chapter 3, we can calculate the individual probability of placing the response variable in the second category.

$$\pi_{22} = P(y_2 \le 2) - P(y_2 \le 1) \quad (43)$$

Also, according to the formula (15), the individual probability of the third category can be calculated as follows:

$$\pi_{23} = 1 - P(y_2 \le 2) = 1 - \frac{e^{z_{22}}}{1+e^{z_{22}}} = \frac{1}{1+e^{z_{22}}} = \frac{1}{1+e^{-0.7274x_1-1.7946x_2}} \quad (44)$$

2.2.3. Presentation of location effects model

In this part, the location effects model is presented with the help of equations obtained in sections 3.2.1 and 3.2.2.

- **Non-linear model**

In this model, the goal is to maximize the probability of final strength and elongation variables being placed in the most desirable category. According to the basic model presented in section 3, the objective function of the location effects model can be written in the following form.

In the objective function of the main model of this research, the relation (33) is stated that a parameter named δ_k is considered with the values of -0.4, 0, and 0.4 respectively for the variable x_k due to considering the constancy of x.

$$\max \pi_{12} = \frac{\exp(-0.767(x_1+\delta_1)+0.861(x_2+\delta_2))}{1+\exp(-0.767(x_1+\delta_1)+0.861(x_2+\delta_2))} \quad (45)$$

$$max\, \pi_{23} = \frac{1}{1+exp(-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))} \quad (46)$$

Subject to:

$$\pi_{22} > \pi_{21} \quad (47)$$

$$\pi_{23} > \pi_{22} \quad (48)$$

$$-1 \le x_1 \le 1 \quad (49)$$

$$-1 \le x_2 \le 1 \quad (50)$$

$$\delta_k \in \{-0.4, 0, 0.4\} \qquad k=1,2 \quad (51)$$

Equation (51) will create nine scenarios, which respectively refer to three different states of δ for two decision variables.

In the model, with the help of two formulas (21), the constraints (47) and (48) can be simplified as follows.

$$\frac{exp(-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}{1+exp(-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))} > 2\frac{exp(-2.2165-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}{1+exp(-2.2165-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))} \quad (52)$$

$$1+\frac{exp(-2.2165-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}{1+exp(-2.2165-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))} > 2\frac{exp(-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}{1+exp(-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))} \quad (53)$$

2.2.4. Calculation of modeling and implementation error

In this part, with the help of formula (19), the variance related to binary and ordinal logistic regression can be calculated.

•Variance related to binary logistic regression

To obtain the inverse of the Fisher matrix, it is enough to calculate the variance-covariance matrix of the model coefficients. This matrix can be calculated using MINITAB software.

$$I^{-1} = \Sigma = \begin{bmatrix} 0.17102 & -0.03225 \\ -0.03225 & 0.11104 \end{bmatrix} \quad (54)$$

The principal diameter columns of this matrix represent the variance of the model coefficients and the other columns represent the covariance values. By putting the variance-covariance matrix in the formula (19), the variance of the binary logistic regression or in other words, the modeling error can be calculated. The calculations for this part have been done with the help of MATLAB software, and the result is as follows.

$$\mathrm{var}(\pi_{12}) = \mathrm{var}\left(\frac{1}{1+e^{-z_{11}}}\right) =$$

$$(x_1+\delta_1)\exp(-2z_{11})\left(\frac{0.17102(x_1+\delta_1)-0.03225(x_2+\delta_2)}{(\exp(-z_{11})+1)^4}\right.$$

$$-$$

$(x_2+\delta_2)\exp(-2z_{11})\left(\frac{0.03225(x_1+\delta_1)-0.11104(x_2+\delta_2)}{(\exp(-z_{11})+1)^4}\right)$

(55)

• Variance related to ordinal logistic regression

Since y-intercept related to the equation of the third category is meaningless, the variance-covariance matrix contains only two coefficients. Finally, the matrix calculated with the help of MINITAB software for two coefficients β_1 , β_2 is as follows.

$$I^{-1}=\Sigma=\begin{bmatrix}0.17107 & 0.04522\\ 0.04522 & 0.16409\end{bmatrix} \quad (56)$$

Now, in order to reduce the variability of the regression equation related to the probability of placing the final strength variable in the third category, the obtained variance-covariance matrix is put in the formula (19) to calculate the modeling error. As in the previous part, these calculations were done with the help of MATLAB software, and the result is as follows.

$$var(\pi_{23})=var\left(\frac{1}{1+e^{z_{22}}}\right)=$$

$$(x_2+\delta_2)\,exp(2z_{22})\left(\frac{0.04522(x_1+\delta_1)+0.16409(x_2+\delta_2)}{(exp(z_{22})+1)^4}\right)$$

$$+$$

$$(x_1+\delta_1)\,exp(2z_{22})\left(\frac{0.17107(x_1+\delta_1)+0.04522(x_2+\delta_2)}{(exp(z_{22})+1)^4}\right)$$

(57)

2.2.5. Presentation of Robust design model

In this part, with the help of equations obtained in sections 3.2.3 and 3.2.4, the final model of this research is presented as follows:

$$\max \pi_{12}=\frac{\exp(-0.767(x_1+\delta_1)+0.861(x_2+\delta_2))}{1+\exp(-0.767(x_1+\delta_1)+0.861(x_2+\delta_2))}$$

(58)

$$\max \pi_{23}=\frac{1}{1+\exp(-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}$$

$$\min var(\pi_{12})=$$

$$(x_1+\delta_1)\exp(-2z_{11})\left(\frac{0.17102(x_1+\delta_1)-0.03225(x_2+\delta_2)}{(\exp(z_{22})+1)^4}\right)-$$

$$(x_2+\delta_2)\exp(-2z_{11})\left(\frac{0.03225(x_1+\delta_1)-0.11104(x_2+\delta_2)}{(\exp(z_{11})+1)^4}\right)$$

$$\min var(\pi_{23})=$$

$$(x_2+\delta_2)\ \exp(2z_{22})\left(\frac{0.04522(x_1+\delta_1)+0.16409(x_1+\delta_1)}{(\exp(z_{22})+1)^4}\right)+$$

$$(x_1+\delta_1)\exp(2z_{22})\left(\frac{0.17107(x_1+\delta_1)-0.04522(x_2+\delta_2)}{(\exp(z_{22})+1)^4}\right)$$

Subject to:

$$\frac{\exp(-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}{1+\exp(-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}>2\frac{\exp(-2.2165-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}{1+\exp(-2.2165-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}$$

$$1+\frac{\exp(-2.2165-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}{1+\exp(-2.2165-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}>2\frac{\exp(-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}{1+\exp(-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}$$

$$-1<x_1<1$$

$$-1<x_2<1$$

$\delta_k \in${-0.4,0,0.4} k=1,2

As it is obvious, in the multi-objective model (58), the goal is to maximize the probability of placement of the response variables of final strength and length increase in the desired categories and simultaneously reduce the variability of the process; the intended variability is caused by the modeling and implementation error in logistic regression. Finally, the reduction of modeling and implementation error in binary and ordinal logistic regression leads to the reduction of model inaccuracies, improves the welding process, and provides stable results. Constraints (49) and (50) are related to the order of priority of the three existing categories of ultimate strength variables, and two constraints (48) and (49) are the test area and the change interval of independent variables that are coded in They show that they have been considered.

2.3. The third phase: optimization

In this section, first the linear and non-linear models of location effects and then the proposed research model is solved by the ideal programming method, taking into account the modeling error presented in subsection 3.2.5.

2.3.1. Solving the nonlinear model

The model presented in section 3.2.3 is of the four-objective type. Therefore, each objective function is solved separately by considering other constraints. The results obtained from GAMS software can be seen in Table 3.

Table 3

Optimal values of objective functions (45) and (46)

The objective function	The value of the objective function	The optimal value of $\mathbf{x_1}$	The optimal value of $\mathbf{x_2}$
Probability of average increase in Elongation	0.8359	-1	1
Probability related to average ultimate strength	0.9257	1	1

The calculated values in the above table are the optimal values of each objective function separately, and the effect of each objective function on the other objective function has not been calculated. This effect is shown with the help of the pay-off Table below.

Table 4

Pay-off matrix for goals (45) and (46)

The objective function	Probability of average objective function of increasing length	The probability related to the objective function of the average ultimate strength	The optimal value of $\mathbf{x_1}$	The optimal value of $\mathbf{x_2}$
Using the optimal values of the objective function of the average increase in length	0.8359	0.744	-1	1
Using the optimal values of the objective function of the average ultimate strength	0.5234	0.9257	1	1

By examining Table 4, it can be seen that the use of the objective function of the average length increase and the optimal values of the variables in this objective function will lead to a decrease in the average final strength and vice versa. This problem indicates that the above four objectives are in conflict with each other and the increase of one lead to the decrease of the other objective function. Therefore, in this problem, the multi-objective method of ideal planning can be used to simultaneously optimize these four conflicting objectives. For this purpose, one of the objective functions is considered as the main objective and since the priority in this problem is to increase the probability of the final strength variable being placed in the third category, this objective is considered as the main objective and the average increase in length is added to the constraints. Moreover, with the help of formula (35), we can calculate the t values for the four models we have, and finally, we can write the non-deterministic model of location effects.

2.3.2. Solving the robust design model

The proposed model of this numerical example that is presented in the subsection 3.2.5 has four goals. To solve this model with the help of the ideal programming method, like solving the location effects model, first each objective function is solved separately by considering other constraints. The results are using GAMS software.

2.3.3. Minimax / Maximin model

Now, by optimizing the worst-case scenario for each of these two objective functions, the optimal value of the average final strength and elongation for rotational and rotational speed settings is obtained. These results will be considered ideal values.

- **Maximum average of ultimate strength**

max zz_1 (59)

Subject to:

$$z_1(\delta) = \frac{1}{1+\exp(-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))} \geq \mathbf{zz_1}$$

$$z_2(\delta) = \frac{\exp(-0.767(x_{1+}\delta_1)+0.861(x_2+\delta_2))}{1+\exp(-0.767(x_{1+}\delta_1)+0.861(x_2+\delta_2))}$$

$$z_3(\delta) = (x_1+\delta_1)\exp(-2z_{11})\left(\frac{0.17102(x_1+\delta_1)-0.03225(x_2+\delta_2)}{(\exp(z_{22})+1)^4}\right) - (x_2+\delta_2)\exp(-2z_{11})\left(\frac{0.03225(x_1+\delta_1)-0.11104(x_2+\delta_2)}{(\exp(z_{11})+1)^4}\right)$$

$$z_4(\delta) = (x_2+\delta_2)\exp(2z_{22})\left(\frac{0.04522(x_1+\delta_1)+0.16409(x_2+\delta_2)}{(\exp(z_{22})+1)^4}\right) + (x_1+\delta_1)\exp(2z_{22})\left(\frac{0.17107(x_1+\delta_1)-0.04522(x_2+\delta_2)}{(\exp(z_{22})+1)^4}\right)$$

$$\frac{\exp(-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2)}{1+\exp(-0.7274((x_1+\delta_1)-1.7946(x_2+\delta_2)} > 2\frac{\exp(-2.2165-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}{1+\exp(-2.2165-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}$$

$$1 + \frac{\exp(-2.2165-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}{1+\exp(-2.2165-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))} > 2\frac{\exp(-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}{1+\exp(-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}$$

$-1 \leq x_1 \leq 1$

$-1 \leq x_2 \leq 1$

$\delta_k \in \{-0.4, 0, 0.4\}$ k=1,2

- **Maximum average increase in length**

Max zz_2 (60)

Subject to:

$$z_1(\delta) = \frac{1}{1+exp(-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}$$

$$z_2(\delta) = \frac{exp(-0.767(x_{1+}\delta_1)+0.861(x_2+\delta_2))}{1+exp(-0.767(x_{1+}\delta_1)+0.861(x_2+\delta_2))} \geq \mathbf{zz_2}$$

$$z_3(\delta) = (x_1+\delta_1)\exp(-2z_{11})\left(\frac{0.17102(x_1+\delta_1)-0.03225(x_2+\delta_2)}{(exp(z_{22})+1)^4}\right) - (x_2+\delta_2)\,exp(-2z_{11})\left(\frac{0.03225(x_1+\delta_1)-0.11104(x_2+\delta_2)}{(exp(z_{11})+1)^4}\right)$$

$$z_4(\delta) = (x_2+\delta_2)\exp(2z_{22})\left(\frac{0.04522(x_1+\delta_1)+0.16409(x_2+\delta_2)}{(exp(z_{22})+1)^4}\right) + (x_1+\delta_1)\,exp(2z_{22})\left(\frac{0.17107(x_1+\delta_1)-0.04522(x_2+\delta_2)}{(exp(z_{22})+1)^4}\right)$$

$$\frac{exp(-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}{1+exp\left(-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2)\right)} > 2\frac{exp(-2.2165-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}{1+exp(-2.2165-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}$$

$$1 + \frac{exp(-2.2165-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2)}{1+exp(-2.2165-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2)} > 2\frac{exp(-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2)}{1+exp(-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2)}$$

$\delta_k = \{-0.4, 0, 0.4\}$ k=1,2

$-1 < x_1 < 1$

$-1 < x_2 < 1$

Now, by optimizing the worst-case scenario of each of these two objective functions, the optimal value of the variance of the final strength and the increase in length are obtained for the rotational and rotational speed settings. These results will be considered goal values.

- **Minimum variance of length increase**

min zz_3 (61)

Subject to:

$$z_1(\delta) = \frac{1}{1+\exp(-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}$$

$$z_2(\delta) = \frac{\exp(-0.767(x_{1+}\delta_1)+0.861(x_2+\delta_2))}{1+\exp(-0.767(x_{1+}\delta_1)+0.861(x_2+\delta_2))}$$

$$z_3(\delta) = (x_1+\delta_1)\exp(-2z_{11})\left(\frac{0.17102(x_1+\delta_1)-0.03225(x_2+\delta_2)}{(\exp(z_{22})+1)^4}\right) -$$

$(x_2 + \delta_2)\exp(-2z_{11})\left(\frac{0.03225(x_1+\delta_1)-0.11104(x_2+\delta_2)}{(\exp(z_{11})+1)^4}\right) \le \mathbf{zz_3}$

$z_4(\delta) = (x_2 + \delta_2)\exp(2z_{22})\left(\frac{0.04522(x_1+\delta_1)+0.16409(x_2+\delta_2)}{(\exp(z_{22})+1)^4}\right) + (x_1 + \delta_1)\exp(2z_{22})\left(\frac{0.17107(x_1+\delta_1)-0.04522(x_2+\delta_2)}{(\exp(z_{22})+1)^4}\right)$

$\frac{\exp(-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}{1+\exp(-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))} > 2\frac{\exp(-2.2165-0.7274(x_1+\delta_1(i))-1.7946(x_2+\delta_2\ ))}{1+\exp(-2.2165-0.7274(x_1-1.7946x_2))}$

$1 + \frac{\exp(-2.2165-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}{1+\exp(-2.2165-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))} > 2\frac{\exp(-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}{1+\exp(-0.7274(x_1+\delta_2)-1.7946(x_2+\delta_2))}$

$\delta_k = \{-0.4, 0, 0.4\}$ k=1,2

$-1 < x_1 < 1$

$-1 < x_2 < 1$

- **Minimum final strength variance**

$\min zz_4$ (62)

Subject to:

$z_1(\delta) = \frac{1}{1+exp(-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}$

$z_2(\delta) = \frac{exp(-0.767(x_{1+}\delta_1) + 0.861(x_2+\delta_2))}{1+exp(-0.767(x_{1+}\delta_1) + 0.861(x_2+\delta_2))}$

$z_3(\delta) = (x_1 + \delta_1\ ) exp(-2z_{11})\left(\frac{0.17102(x_1+\delta_1)-0.03225(x_2+\delta_2)}{(exp(z_{22})+1)^4}\right) - (x_2+\delta_2) exp(-2z_{11})\left(\frac{0.03225(x_1+\delta_1)-0.11104(x_2+\delta_2)}{(exp(z_{11})+1)^4}\right)$

$z_4(\delta) = (x_2 + \delta_2\ ) \exp(2z_{22})\left(\frac{0.04522(x_1+\delta_1)+0.16409(x_2+\delta_2)}{(exp(z_{22})+1)^4}\right) + (x_1+\delta_1) exp(2z_{22})\left(\frac{0.17107(x_1+\delta_1)-0.04522(x_2+\delta_2)}{(exp(z_{22})+1)^4}\right) \le \mathbf{zz_4}$

$\frac{exp(-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}{1+exp(-0.7274((x_1+\delta_1)-1.7946(x_2+\delta_2))} > 2\frac{exp(-2.2165-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}{1+exp(-2.2165-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}$

$1 + \frac{exp(-2.2165-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}{1+exp(-2.2165-0.7274(x_1+\delta)-1.7946(x_2+\delta_2))} > 2\frac{exp(-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}{1+exp(-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}$

$\delta_k \in \{-0.4, 0, 0.4\}$ k=1,2

$-1 < x_1 < 1$

$-1 < x_2 < 1$

Tables 5-8 provide the optimal results of the optimization Models (59-62) with respect to the scenarios of implementation errors ($\delta_k \in \{-0.4, 0, 0.4\}, \forall k=1,2$). After running the model in GAMS software, the values of $x_1 = 1$ and $x_2 = 1$ and the variance value equal to 0.814 were obtained in order to maximize the average strength, the results of which are as follows.

Table 5
The results of the quantification of the implementation error for the objective function of average ultimate strength z_1

Z_1	-0.4	0	0.4
-0.4	0.814	0.9	0.948
0	0.854	0.923	0.961
0.4	0.887	0.941	0.971

Z_2	-0.4	0	00.4
-0.4	0.514	0.599	0..687
0	0.438	0.523	0.608
-0.4	0.364	0.447	0.533

Z_3	-0.4	0	0.4
-0.4	0.005	0.008	0.011
0	0.01	0.014	0.017
0.4	0.017	0.022	0.026

Z_4	-0.4	0	0.4
-0.4	0.004	0.002	0.001
0	0.005	0.002	0.0008
0.4	0.005	0.002	0.0006

After running the model in GAMS software, the values of x_1=-0.078 and x_2=1 and the variance value equal to 0.567 were obtained in order to maximize the average strength, and the results are as follows.

Table 6
The results of the quantification of the implementation error for the objective function of the average increase in length z_2

Z_1	-0.4	0	0.4
-0.4	0.667	0.804	0.849
0	0.728	0.846	0.918
0.4	0.782	0.88	0.938

Z_2	-0.4	0	0.4
-0.4	0.707	0.773	0.828
0	0.64	0.715	0.780
0.4	0.567	0.649	0.723

Z_3	-0.4	0	0.4
-0.4	0.004	0.006	0.006
0	0.002	0.002	0.007
0.4	0.003	0.006	0.008

Z_4	-0.4	0	0.4
-0.4	0.004	0.004	0.003
0	0.002	0.002	0.002
0.4	0.003	0.002	0.001

After running the model in Gems software, the values of x_1=-0.078 and x_2=1 and the variance value equal to 0.008 were obtained in order to maximize the average strength, the results of which are as follows.

Table 7
The results of quantification of the implementation error for the final strength variance objective function Z_3

Z_1	-0.4	0	0.4
-0.4	0.667	0.804	0.894
0	0.728	0.846	0.918
0.4	0.782	0.88	0.938

Z_2	-0.4	0	0.4
-0.4	0.707	0.773	0.828
0	0.64	0.715	0.78
0.4	0.567	0.649	0.723

Z_3	-0.4	0	0.4
-0.4	0.004	0.006	0.006
0	0.002	0.005	0.007
0.4	0.003	0.006	0.008

Z_4	-0.4	0	0.4
-0.4	0.004	0.004	0.003
0	0.002	0.003	0.002
0.4	0.003	0.002	0.001

After running the model in Gems software, the values of x_1 =1 and x_1 =0.005 the variance value equal to 0.003 were obtained in order to maximize the average strength, the results of which are as follows.

Table 8
The results of valuing the implementation error for the objective function of the variance of length increase Z_4

Z_1	-0.4	0	0.4
-0.4	0.687	0.818	0.902
0	0.746	0.858	0.925
0.4	0.797	0.89	0.943

Z_2	-0.4	0	0.4
-0.4	0.687	0.756	0..814
0	0.617	0.695	0.763
0.4	0.543	0.626	0.703

Z_4	-0.4	0	0.4
-0.4	0.003	0.003	0.002
0	0.002	0.003	0.002

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0.4	0.003	0.002	0.001

Z_3	**-0.4**	**0**	**0.4**
-0.4	0.003	0.005	0.006
0	0.002	0.005	0.007
0.4	0.004	0.006	0.009

Now we have optimal results for each response variable separately. In the next section, a Goal programming model is applied to aggregate the four optimization models into a single one.

2.3.4. Goal programming model

Finally, the main model is obtained from the relation (35) and from the combination of the above four objective functions, as follows.

$$\text{Min Gp} = \left(w_1 \frac{d_1^-}{t_1}\right) + \left(w_2 \frac{d_2^-}{t_2}\right) + \left(w_3 \frac{d_3^+}{t_3}\right) + \left(w_4 \frac{d_4^+}{t_4}\right) \quad (63)$$

Subject to:

$zz_1 \leq z_1(\delta)$ $\quad \delta_k \in \{-0.4, 0, 0.4\}$ $\quad$ k=1,2

$zz_2 \leq z_2(\delta)$ $\quad \delta_k \in \{-0.4, 0, 0.4\}$ $\quad$ k=1,2

$zz_3 \geq z_3(\delta)$ $\quad \delta_k \in \{-0.4, 0, 0.4\}$ $\quad$ k=1,2

$zz_4 \geq z_4(\delta)$ $\quad \delta_k \in \{-0.4, 0, 0.4\}$ $\quad$ k=1,2

$$z_1(\delta) = \frac{1}{1+exp(-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2)}$$

$$z_2(\delta) = \frac{exp(-0.767(x_{1+}\delta_1\ )+0.861(x_2+\delta_2))}{1+exp(-0.767(x_{1+}\delta_1\ )+0.861(x_2+\delta_2))}$$

$$z_3(\delta) = (x_1 + \delta_1)\, exp(-2z_{11}) \left(\frac{0.17102(x_1+\delta_1)-0.03225(x_2+\delta_2)}{(exp(z_{22})+1)^4}\right) - (x_2+\delta_2)\, exp(-2z_{11}) \left(\frac{0.03225(x_1+\delta_1)-0.11104(x_2+\delta_2)}{(exp(z_{11})+1)^4}\right)$$

$$z_4(\delta) = (x_2 + \delta_2)\, exp(\,2z_{22}) \left(\frac{0.04522(x_1+\delta_1)+0.16409(x_2+\delta_2)}{(exp(z_{22})+1)^4}\right) + (x_1 + \delta_1)\, exp(2z_{22}) \left(\frac{0.17107(x_1+\delta_1)-0.04522(x_2+\delta_2)}{(exp(z_{22})+1)^4}\right)$$

$$\frac{exp(-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}{1+exp(-0.7274((x_1+\delta_1)-1.7946(x_2+\delta_2))} > 2\,\frac{exp(-2.2165-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}{1+exp(-2.2165-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}$$

$$1 + \frac{exp(-2.2165-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}{1+exp(-2.2165-0.7274(x_1+\delta)-1.7946(x_2+\delta_2))} > 2\,\frac{exp(-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}{1+exp(-0.7274(x_1+\delta_1)-1.7946(x_2+\delta_2))}$$

$$z_5(\delta) = \frac{exp\left(-0.7274(x_1+\delta_1)-1.7945(x_2+\delta_2)\right)}{1+exp\left(-0.7274(x_1+\delta_1)-1.7945(x_2+\delta_2)\right) - \frac{\left(exp(-2.2165-0.7274(x_1+\delta_1)-1.7945(x_1+\delta_2)\right)}{exp\left(-2.2165-0.7274(x_1+\delta_1)-1.7945(x_2+\delta_2)\right)}}$$

$$z_6(\delta) = \frac{exp\left(-2.2165-0.7274(x_1+\delta_1)-1.7945(x_2+\delta_2)\right)}{1+exp\left(-2.2165-0.7274(x_1+\delta_1)-1.7945(x_2+\delta_2)\right)}$$

$$zz_1 + d_1^- - d_1^+ = t_1$$

$$zz_2 + d_2^- - d_2^+ = t_2$$

$$zz_3 + d_3^- - d_3^+ = t_3$$

$$zz_4 + d_4^- - d_4^+ = t_4$$

$$-1 < x_1 < 1$$

$$-1 < x_2 < 1$$

$\delta_k \in \{-0.4, 0, 0.4\}$ $\quad$ k=1,2

By placing in the formula (35), the sensitivity analysis of the effect of the weight of each factor based on Table 9 is as follows: Table 9 is specified as 4 rows that have specific values for the average weights and variance of the answers. In the first row, the variance weights are equal to the mean. In the second row, the weight of the average is twice the weight of the variance; in this part w_1 is twice the value of w_2, w_2 is equal to w_3, and these two values are twice w_4. In the third row, the value of the average weight is twice the variances, and the weights of the averages and variances are equal to each other. And finally, the fourth row of the average weight is half of the variance weight, but w_1 is twice w_3, which is equal to w_4, and also for the average answers, w_1 is twice w_3.

Table 9
Weight sensitivity analysis

	w_1	w_2	w_3	w_4	Gp	x_1	x_2	zz_1	zz_2	zz_3	zz_4
1	0.25	0.25	0.25	0.25	0.121	0.05	1	0.687	0.543	0.009	**0.003**
2	0.444	0.222	0.222	0.111	0.124	-0.043	1	0.672	0.56	0.009	**0.004**
3	0.333	0.167	0.333	0.167	0.126	-0.043	1	0.672	0.56	0.009	**0.004**
4	0.222	0.444	0.111	0.222	0.1	0.05	1	0.687	0.543	0.009	**0.003**

The mathematical model was investigated considering the test data of Hejazi and Akbari [9], where the implementation error was obtained in two response levels with nine modes and was analyzed in the tables of Section 3.3.2. Since the solutions of the GAMS software are nonlinear modes of local optimality, these results are not necessarily global optimality. For Table 9, it can be said that using different values for δ can lead the researcher to achieve the desired goal with the least possible error, or in other words, reliable results. These values can be different depending on the type of problem and the opinion of each decision-maker. Also, in this numerical example, it can be seen that there is a slight difference between the optimal values of the control variables in some solutions. For example, in rows 1 and 4, as well as 2 and 3 of the table 9, the values are equal. For the values of $\mathbf{x_2}$ and $\mathbf{zz_3}$ in the four rows of Table 9, there has been no change and the same value is observed. Finally, the analyses carried out led to the fact that by entering the relevant scenarios, the decision-maker can also consider the error in the measurement of variable X and know that the optimized solution will be reliable with the applied restrictions.

3. Conclusion

Many engineering systems consist of several different processes in a row, and uncertainty in one part can affect other parts and affect the final output. which ultimately leads to a significant deviation from the expected performance. Therefore, estimating and reducing variability, or, in other words, reducing risk in the design of a system or its constituent processes, is necessary to provide a robust plan. A low-cost way to reduce these risks and increase quality in a process is to use robust design. For this reason, robust design has been the focus of many researchers for years. In most past research, the environmental error has been considered a measure of dispersion, and less attention has been paid to the error related to modeling and implementation, or, in other words, the prediction error. Also, in those studies that have considered modeling and implementation error as measures of dispersion, most of the response variables have been assumed to be continuous. This is despite the fact that in some systems and experiments, it is not possible to define variables continuously. For this reason, not taking into account the real-world conditions in the research have created the main idea in this research. Another innovation of this research that brings the proposed model closer to real-world conditions is the use of the ideal planning method, because quality engineers are often faced with the problem of how to determine the optimal conditions for the controllable factors of the processes. In order to achieve this goal, several robust models have been presented, and in most of these models, deviation from the mean value and zero dispersion are considered. While customers are usually aware of the maximum acceptable deviation from the average and the variability of a product, Therefore, existing models with robust designs become ineffective. To solve this problem, the ideal programming method can be used. With the help of the dual response method as a method for strong design, an approach for robust design of quality characteristics of issues with categorical response variables has been presented in this research. Also, in the proposed model, the dispersion criterion of the type of modeling and implementation error is considered. The stages of this research are based on the phases of the response level method. In this way, in the first phase, the experiments were designed, and in the modeling phase, due to the categorical nature of the response variables, logistic regression was used. In order to estimate the equation of dispersion effects, variance The logistic model is calculated, and at the end of this phase, the robust model proposed in this research is presented. In the optimization phase, the proposed multi-objective model has been solved with the help of the ideal programming method.

The result of this research can be implemented in many services and production centers where accuracy is considered important and vital. This method provides a low-cost way to increase the accuracy of parameter estimation, which ultimately leads to an increase in quality.

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