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Developing a Decision Model as Budget Assignment Method for Locating Industrial Facilities: Real Case Study

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Abstract

In today's world, due to the existence of several criteria in every decision, budgeting has become a complex issue. Applying decision-making methods significantly helps to make optimal decisions. Multiple criteria decision making (MCDM) is a branch of operational research dealing with finding optimal results in complex scenarios including various indicators, conflicting objectives, criteria, and various indicators. The paper presents a location-allocation decision-making problem resulting in the selection of the most desirable location for the consulting service center in the Qazvin state of Iran. In the first stage, different location criteria are determined. Then, the decision-making matrix preparation is completed based on the criteria dimension and expert opinions. The decision problem is formulated as a multiple criteria ranking problem (MCDM). In the second stage, the decision-making is performed by using all three models of the PROMETHEE method. Finally, considered locations are ranked from the best choice to the worst one with the application of the PROMETHEE MCDM/A method.

Keywords: Locational alocation, multicriteria decision making, PROMETHE, location problems.

1.Introduction

In today's economy, characterized by a dynamic and volatile environment, many researchers stress the significance of location factors(Kaboli & et. al., 2007). Location allocation decisions are made in both private and public sectors. For example, governments need to determine the locations for emergency bases highway patrol vehicles, fire bases, ambulances, television antennas, and exploratory oil wells. In all cases, poor locations can increase the likelihood of property damage and cost life. In private sectors, locations of warehouses and distribution centers, production and assembly facilities, offices, and retail outlets must be onsidered. Facility location applications are concerned with the location of one or more facilities in such a way that a certain objective such as minimizing transportation cost, providing equitable service to consumers, capturing the largest market share, and etc. Facility location problems may rise challenging geometrical and combinational problems. The research on facility location problems spans many research fields such as operations research/management science, industrial engineering, geography, economics, computer science, mathematics problems considering both qualitative and quantitative factors. Kahne (Kahne, 1975) considered 29 attributes and and marketing (Kim & et.al, 1999). Location theory was first introduced by Weber (Tabari & et. al., 2008), who considered the problem of locating a single warehouse in order to minimize the total travel distance between the warehouse and a set of spatially distributed costumers. In fact, he proposed a material index for selecting the location in which if this index is grater than one, the warehouse should be installed in the vicinity of the source of raw material; or otherwise, it should be close to the market. Smithies and Stevens (Stevens & et. al., 1961) extended the Hotelling's problem later. Hakimi (Hakimi, 1964) considered a general problem to locate one or more facilities on a network by minimizing the sum of the distances and the maximum distance between facilities and points on a network. Considerable research and theoretical interest in the location problem have been carried out after this seminal paper. Brown and Gibson (Tabari & et. al., 2008), and Buffa and Sarin proposed a facility location model for a multidimensional location problem based on critical factors, objective factors, and subjective factors. Fortenberry and Mitra (Fortenberry & Mitra, 1986) resented a model for the location-allocation

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used a weighting model to determine the relative mportance with uncertainty in attributes. Kirkwood (Kirkwood, 1982) discussed a multi-disciplinary study conducted to select a site for a nuclear power facility. Linares and Romrero (Liang, 2000) roposed a methodology that combined several multi-criteria methods to address electricity planning problems. Several MCDM methods for the location selection are used such as Liang and Wang (Liang & Wang, 1991) who proposed an algorithm for a site selection based on the concepts of the fuzzy set theory. Lianares (2000) proposed a holistic MCDM model for the facility location selection. Montajabiha and Wang (2016) proposed a fuzzy TOPSIS model in the facility location problem. Finally, this paper concludes with gained results.

approach to select the best facility location under linguistic environment. MCGDM methods if decision makers (DMs) are not able to treat precise data in order to define their preferences, the intuitionistic fuzzy set (IFS) theory enables them (Montajabiha and Wang, 2016). The ranking of options is done by comparing their pairs in each index. The ROMETHEE method provides six generalized criteria to define the superiority function for the decision maker (Figueroa, 2019).The structure of this paper is as follows: First, the **PROMETHE** model is introduced. Second, the case study is described in detail. Analysis of a case study is then discussed in order to verify the practicability and effectiveness of the proposed

P_j: Preference Function of Criteria j

sc: Sign of Criteria (1×n vector)

DC: Differences on Criteria (Deviation between alternatives over j^{th} criteria)

PFC: Preference Function on Criteria

PFV: Preference Function Value

PI: Preference Index Value

WPI: Weighted Preference Index

pfv : Preference Function Value Vector

PI: Preference Index

2. PROMETHEE Decision-Making Process

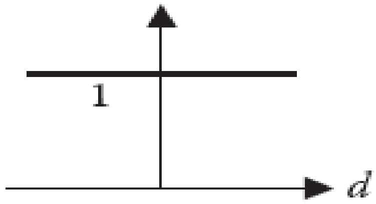
2.1

Before describing the detailed process of the algorithm some notations are summarized for more simplification.

The PROMETHEE method is well-known as a multi-criteria decision-making method in which alternatives ranking is obtained based on preference function, criteria, and their criteria. For this purpose, determined preference function $P_j(f_j a_1 a_2)$ has been used to show preference of alternative $\boldsymbol{a_1}$ over alternative $\boldsymbol{a_2}$ for specific criteria, *fj*. In other words, $P_j(f_j a_1 a_2)$ is a type of function that can convert alternative differences on j^{th} criteria $f_j a_1 a_2 = \pm 1*(x_1 - x_2)$ to find required values for further calculation. Sign of criteria (c_j) can be stored in ***sc***1×n vector (***sc***=[±1 ±1 …±1]) and criteria weights are demonstrated in weight wector (***w***). The **PROMETHEE** method encompasses six types of preference function for criteria as mentioned in table 1.

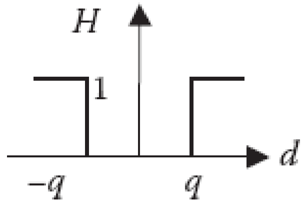
Table 1.
Preference function types in the PROMETHEE method

Type I (Usual criterion)



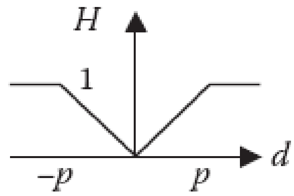
$$P(d)=\begin{cases}0 & d\le 0\\ 1 & d>0\end{cases}$$

Type II (Quasi-criterion)



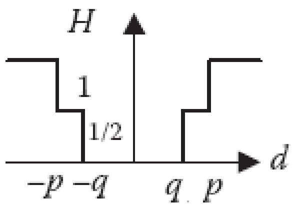
$$P(d)=\begin{cases}0 & d\le q\\ 1 & d>q\end{cases}$$

Type III (V-sharp criterion)



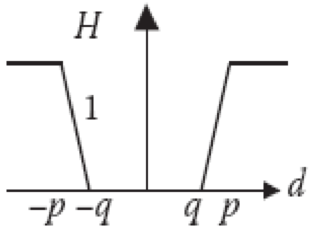
$$P(d)=\begin{cases}\dfrac{d}{p} & d\le p\\ 1 & d>p\end{cases}$$

Type IV (Level-criterion)



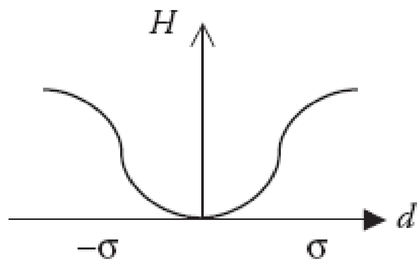
$$P(d)=\begin{cases}0 & d\le q\\ \dfrac{1}{2} & q<d\le q+p\\ 1 & q+p<d\end{cases}$$

Type V (Linear criterion)



$$P(d)=\begin{cases}0 & d\le q\\ \dfrac{d-q}{p-q} & q<d\le q+p\\ 1 & q+p<d\end{cases}$$

Type VI (Gaussian criterion)



$$P(d)=\begin{cases}0 & d\le 0\\ 1-e^{-\frac{d^2}{2\sigma^2}} & d>0\end{cases}$$

2.1.1

PROMETHEE as a powerful decision-making tool can calculate alternatives ranking by various criteria consideration, in 2 phases. In the method, w_j is the weight of the j^{th} criteria and normalized weights aggregation should be equal to on ($\sum_{j=1}^{n} w_j = 1$).

Phase 1 started by gathering and preparing data in the shape of a matrix, which introduces alternatives and their criteria values in every row as x_{ij}.

Table 2.
Decision matrix (alternatives and criteria)

		$\pm c1$	$\pm c2$	...	$\pm cn$	
	a_1	x_{11}	x_{12}	...	x_{1n}	
	a_2	x_{21}	x_{22}	...	x_{2n}	
$D=$	⋮	⋮	⋮	⋮	⋮	$i=1,\dots,m$ $j=1,\dots,n$
	a_m	x_{m1}	x_{m2}	...	x_{mn}	

By considering the criteria sign vector (***sc***), the elements ($f_j a_1 a_2$) of matrix ***DC*** can be interpreted as the deviation values between alternatives and all pairwise comparisons over j_{th} criteria *(size of DC matrix $m^2 \times n$).* In table 3, $f_{3\,12} = \pm 1 * (x_{13} - x_{23})$ illustrates mentioned comparison between alternative a_1 and a_2 on 3^{rd} criteria considering c_j sign for instance.

Table 3.
Matrix of differences on criteria

		$\pm c_1$	$\pm c_2$	...	$\pm c_n$	
	a_1a_1	$f_{1\,11}$	$f_{2\,11}$	...	$f_{n\,11}$	
	a_1a_2	$f_{1\,12}$	$f_{2\,12}$	...	$f_{n\,12}$	
$DC=$	⋮	⋮	⋮	⋮	⋮	$i=1,\dots,m$ $j=1,\dots,n$
	a_ma_{m-1}	$f_{1\,m\,(m-1)}$	$f_{2\,m\,(m-1)}$	...	$f_{n\,m\,(m-1)}$	
	a_ma_m	$f_{1\,m\,m}$	$f_{2\,m\,m}$	...	$f_{n\,m\,m}$	

2.1.2

Preference function value and weighted preference index show a relation between alternatives. Based on criteria features, elements of ***DC*** have to adjust with the preference function of j^{th} criteria. Here preference functions convert ***DC*** matrix to ***PFC*** which is a fundamental matrix for further calculation, including ***PFV*** , ***PI*** and finally alternative ranking can be obtained.

Table 4.
Matrix of preference function on DC

		$\pm c_1$	$\pm c_2$	...	$\pm c_n$	
	a_1a_1	$P_1(f_{1\,11})$	$P_2(f_{2\,11})$	...	$P_n(f_{n\,11})$	
	a_1a_2	$P_1(f_{1\,12})$	$P_2(f_{2\,12})$	...	$P_n(f_{n\,12})$	
PFC=	⋮	⋮	⋮	⋮	⋮	$i=1,\dots,m$ $j=1,\dots,n$
	a_ma_{m-1}	$P_1(f_{1\,m\,(m-1)})$	$P_2(f_{2\,m\,(m-1)})$	...	$P_n(f_{n\,m\,(m-1)})$	
	a_ma_m	$P_1(f_{1\,m\,m})$	$P_2(f_{2\,m\,m})$	...	$P_n(f_{n\,m\,m})$	

2.1.3 Preference Function Value

Horizontal summation of *PFC* expresses ***pfv*** vector of all pair alternatives within $m^2\times 1$ vector which could be elaborately reshaped to matrix *PFV* $m\times m$ as table 5

Table 5.
Matrix of preference function value

		a_1	a_2	…	a_m	
	a_1	$\sum P_j(f_{j11})$	$\sum P_j(f_{j11})$	⋯	$\sum P_j(f_{jm1})$	
	a_2	$\sum P_j(f_{j12})$	$\sum P_j(f_{j21})$	⋯	$\sum P_j(f_{jm2})$	$i=1,\ldots,m$
PFV =	⋮	⋮	⋮	⋮	⋮	$j=1,\ldots,n$
	a_m	$\sum P_j(f_{j1m})$	$\sum P_j(f_{j2m})$	⋯	$\sum P_j(f_{jmm})$	

2.1.4.Weighted Preference Index

By applying criteria weights on the *PFC* matrix, needed matrixes like *WPI* and *PI* are obtainable. According to Eq.1, the ***PI*** matrix is obtained from the multiplication of w_j to c_j value of *PFC*. In this study, w_j values of all criteria follow the normalization condition which means criteria weights summation is equal to 1 ($\sum_{j=1}^{n} w_j = 1$).

Horizontal summation on *WPI* matrix results in ***pi*** vector with the size of $m^2\times 1$. As each element of ***pi*** vector expresses a relation of two alternatives, it could be reshaped to *PI* matrix $m\times m$ *as* it is shown in table 6.

$$\pi(a_i, a_{i'}) = \sum_{j=1}^{n} w_j \times P_j(f_j ii') \qquad i',\ i = 1,2,\ldots,m \quad (1)$$

Table 6.
preference index Matrix

		a_1	a_2	…	a_m	
	a_1	π (a_1 a_1)	π (a_1 a_2)	…	π (a_1 a_m)	$i=1,\ldots,m$
	a_2	π (a_2 a_1)	π (a_2 a_2)	…	π (a_2 a_m)	$j=1,\ldots,n$
PI =	⋮	⋮	⋮	⋮	⋮	
	a_m	π (a_n a_1)	π (a_n a_2)	…	π ($a_m a_m$)	

2.2.

In the second step, based on PROMETHEE method, pertinent calculations on alternatives obtain alternatives out ranking**.**

2.2.1 PROMETHEE I

As PROMETHEE basis is on a relation between alternatives,the entering and leaving flows are determined through Eq.

$$a_1 P^+ a_2 \quad if \quad \phi^+(a_1) > \phi^+(a_2) \qquad (4)$$

$$a_1 I^+ a_2 \quad if \quad \phi^+(a_1) = \phi^+(a_2) \qquad (5)$$

lternatives flow calculations based on PROMETHEE I are determined through Eq.4 to Eq. 7.(2) and Eq.(3).

$$a_1 P^- a_2 \quad if \quad \phi^-(a_2) > \phi^-(a_1) \qquad (6)$$

$$a_1 I^- a_2 \quad if \quad \phi^-(a_1) = \phi^-(a_2) \qquad (7)$$

In this method, the a_1 alternative is superior to alternative a_2 , as Eq.8..

$$a_1 \; P^I \; a_2 \quad if : \begin{matrix} a_1 P^+ a_2 \text{ and } a_1 P^- a_2 \\ a_1 P^+ a_2 \text{ and } a_1 I^- a_2 \\ a_1 I^+ a_2 \text{ and } a_1 P^- a_2 \end{matrix} \qquad (8)$$

In addition, the alternatives a_1 and a_2 are indifferent to each other under the condition of Eq.9.

$$a_1 I^I a_2 \quad if : \; a_1 I^- a_2 \;\; and \;\; a_1 I^+ a_2 \qquad (9)$$

2.2.2. PROMETHEE II

In this method, net flow of alternatives is obtained from Eq. 10 and then a comparison between pairs of *alternatives is accomplished by Eq.11.The a_1 alternative is better than the alternative a_2 under the condition of Eq.9 and Eq. 10.*

$$\phi(a_i) = \phi^+(a_i) - \phi^-(a_i), \qquad i = 1, 2, ..., m \qquad 10)$$

$$a_1 P^{II} a_2 \;\; if : \;\; \phi(a_1) > \phi(a_2) \qquad (11)$$

Furthermore, two alternatives are indifferent to each other under the condition of Eq.12.

$$a_1 I^{II} a_2 if \; : \phi(a_1) = \phi(a_2) \qquad (12)$$

In 3rd method, for every alternative $\boldsymbol{a_i}$ *an interval value* $\left[X_{a_i}, Y_{a_i} \right]$ *is determined, and alternative* $\boldsymbol{a_1}$ *is superior to alternative* $\boldsymbol{a_2}$ *, considering conditions of Eq.13:*

$$a_1 \; P^{III} \; a_2 \quad if \qquad X_{a_1} > X_{a_2} \;\; \text{and} \;\; Y_{a_1} > Y_{a_2} \qquad (13)$$

And, they are indifferent to each other under conditions of Eq.14.

$$a_1 \; P^{III} \; a_2 \quad if \qquad X_{a_1} \geq X_{a_2} \;\; \text{and} \;\; Y_{a_1} \leq Y_{a_2} \qquad (14)$$

2.2.3 PROMETHEE III

In this technique, final alternatives ranking is calculated by using interval values. The interval values are obtained from Eqs.15. and Eq.16.

$$\begin{cases} X_{a_i} = \bar{\phi}(a_i) - \alpha\sigma_{a_i} \\ Y_{a_i} = \bar{\phi}(a_i) + \alpha\sigma_{a_i} \end{cases} \quad i = 1, 2, \ldots, m \tag{15}$$

$$\begin{cases} \bar{\phi}(a_i) = \frac{1}{m}\sum_{A_i \in A}\left[\pi(a_i, a_{i'}) - \pi(a_{i'}, a_i)\right] = \frac{1}{m}\phi(a_i) \\ \sigma^2_{a_i} = \frac{1}{m}\sum_{A_i \in A}\left[\pi(a_i, a_{i'}) - \pi(a_{i'}, a_i) - \bar{\phi}(a_i)\right]^2 \end{cases} \tag{16}$$

3. Problem Definition and Decision-Making Process

Industrial Estates Management Organization of Ghazvin province tends to establish a consulting service department in one of five main industrial estates. This department provides companies with engineering and financial consulting. All cities are located in the province with different specifications which are defined as their criteria. Included cities or alternatives in this decision-making case are Abeyek ($\boldsymbol{a_1}$), Arasanj ($\boldsymbol{a_2}$), Heydariyeh ($\boldsymbol{a_3}$), Khorram Dasht ($\boldsymbol{a_4}$), Lia ($\boldsymbol{a_5}$).

Problem features include 17 criteria which are defined by experts after a deep field survey as mentioned in table 7. By conducting meetings with informants and converting qualitative elements to quantitative, matrix (***D***) values have been prepared as mentioned in table 8.

Table 7
criteria of the location budget assignment case study

c_1:	General conditions of land
c_2:	Subside supporting
c_3:	Welfare and healthcare facilities
c_4:	Public infrastructure facilities
c_5:	Number of industrial units that can be installed in the state
c_6:	Counselors willingness to settle in the center
c_7:	Special features and infrastructure
c_8:	Employable people population in the state
c_9:	Cultural and social contexts in the state
c_{10}:	Recruiting potential of the state
c_{11}:	Distance to the capital of the province
c_{12}:	Distance to roads and highways
c_{13}:	Distance to the railway
c_{14}:	Distance to the airport
c_{15}:	Distance to research centers and universities
c_{16}:	Distance to other industrial states
c_{17}:	Distance to administrative centers

Table 8.
Decision matrix for location- budget assignment

		c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}
	a_1	85	77	88	104	91	92	92	85	76	99	72	103	93	67	70	78	75
	a_2	110	77	89	115	93	80	81	87	74	89	74	106	100	83	82	92	81
$D=$	a_3	112	70	69	100	84	58	69	76	65	74	96	122	97	104	99	104	108
	a_4	113	72	68	104	85	57	74	85	65	74	132	124	103	104	105	119	91
	a_5	121	88	116	126	123	112	101	120	92	108	69	100	79	78	74	77	74

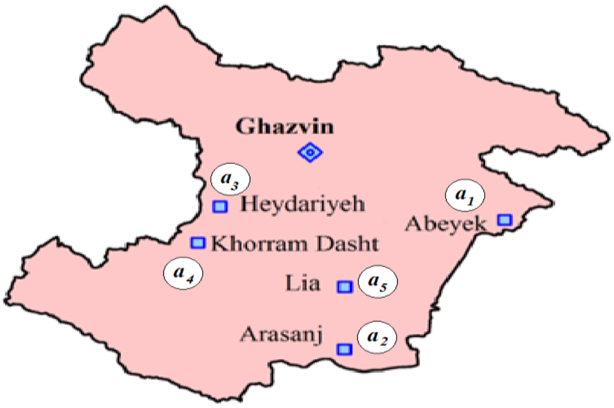


Fig.1. Alternatives of the research in Ghazvin province case study.

Criteria sign vector is introduced based on criteria characteristics as shown in the table.9. The positive signs express the fact that larger amounts are more favorable in the criteria, and in the negative criteria smaller values play this role.

Table 9
Sign of criteria

c_j	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}
Sign	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1

With the respect to criteria features, 17 different values were gathered for every alternative by expert informant questionnaire and field survey as shown in table.3. So, alternatives comparisons and calculations have been accomplished based on thier ***D*** matrix.

As mentioned in 2.1.1 deviation between alternatives over j^{th} criteria is demonstrated in ***DC*** matrix in table 10. The values of the matrix are obtained based on a pairwise comparison of alternatives in all criteria. Obviously, a_{ii} values of the matrix are zero as there is no difference between one alternative with itself.

Table 10
DC matrix

a_1a_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_1a_2	0	0	0	0	0	0.165	0.236	0	0.014	0.118	0	0	0	0.5	0.5	0.5	0
a_1a_3	0	0.092	0	0	0	0.764	0.691	0.063	0.343	0.542	0.5	0.5	0	0.5	0.5	0.5	0.5
a_1a_4	0	0.066	0	0	0	0.784	0.513	0	0.343	0.542	0.5	0.5	0	0.5	0.5	0.5	0.5
a_1a_5	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	0	0	0
a_2a_1	0.227	0	0	0	0	0	0	0.003	0	0	0	0	0	0	0	0	0
a_2a_2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_2a_3	0	0.092	0	0	0	0.454	0.274	0.092	0.245	0.245	0.5	0.5	0	0.5	0.5	0	0.5
a_2a_4	0	0.066	0	0	0	0.484	0.103	0.003	0.245	0.245	0.5	0.5	0	0.5	0.5	0.5	0
a_2a_5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_3a_1	0.245	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_3a_2	0.018	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_3a_3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_3a_4	0	0	0	0	0	0.001	0	0	0	0	0.5	0	0	0	0	0.5	0
a_3a_5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_4a_1	0.255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_4a_2	0.027	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_4a_3	0.009	0.026	0	0	0	0	0.054	0.063	0	0	0	0	0	0	0	0	0.5
a_4a_4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_4a_5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_5a_1	0.327	0.145	1	1	0	0.393	0.165	0.625	0.589	0.096	0	0	0.5	0	0	0	0
a_5a_2	0.1	0.145	1	0	0	0.722	0.589	0.582	0.675	0.363	0	0	0.5	0	0	0.5	0
a_5a_3	0.082	0.237	1	1	1	0.974	0.897	0.787	0.92	0.764	0.5	0.5	0.5	0.5	0.5	0.5	0.5
a_5a_4	0.073	0.211	1	1	1	0.977	0.802	0.625	0.92	0.764	0.5	0.5	0.5	0.5	0.5	0.5	0.5
a_5a_5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

According to 2.1.2, applying appropriate preference function on ***DC*** matrix can transform it into ***PFC*** matrix. In this case study, preference function type 2 has been used for c_1 and c_2 and type1 for criteria c_3, c_4, and c_5.

Criteria c_6 to c_{10} meet the type 6 preference function. The remaining criteria from c_{11} to c_{17} were compatible with preference type 4 and their result is demonstrated in table 11.

Table 11
PFC matrix

a_1a_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_1a_2	0	0	0	0	0	0.165	0.236	0	0.014	0.118	0	0	0	0.5	0.5	0.5	0
a_1a_3	0	0.092	0	0	0	0.764	0.691	0.063	0.343	0.542	0.5	0.5	0	0.5	0.5	0.5	0.5
a_1a_4	0	0.066	0	0	0	0.784	0.513	0	0.343	0.542	0.5	0.5	0	0.5	0.5	0.5	0.5
a_1a_5	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	0	0	0
a_2a_1	0.227	0	0	0	0	0	0	0.003	0	0	0	0	0	0	0	0	0
a_2a_2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_2a_3	0	0.092	0	0	0	0.454	0.274	0.092	0.245	0.245	0.5	0.5	0	0.5	0.5	0	0.5
a_2a_4	0	0.066	0	0	0	0.484	0.103	0.003	0.245	0.245	0.5	0.5	0	0.5	0.5	0.5	0
a_2a_5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_3a_1	0.245	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_3a_2	0.018	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_3a_3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_3a_4	0	0	0	0	0	0.001	0	0	0	0	0.5	0	0	0	0	0.5	0
a_3a_5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_4a_1	0.255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_4a_2	0.027	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_4a_3	0.009	0.026	0	0	0	0	0.054	0.063	0	0	0	0	0	0	0	0	0.5
a_4a_4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_4a_5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_5a_1	0.327	0.145	1	1	0	0.393	0.165	0.625	0.589	0.096	0	0	0.5	0	0	0	0
a_5a_2	0.1	0.145	1	0	0	0.722	0.589	0.582	0.675	0.363	0	0	0.5	0	0	0.5	0
a_5a_3	0.082	0.237	1	1	1	0.974	0.897	0.787	0.92	0.764	0.5	0.5	0.5	0.5	0.5	0.5	0.5
a_5a_4	0.073	0.211	1	1	1	0.977	0.802	0.625	0.92	0.764	0.5	0.5	0.5	0.5	0.5	0.5	0.5
a_5a_5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

As it is explained in 2.1.3, horizontal aggregation on ***PFC*** matrix for bilateral elements results ***pfv*** vector with m^2 rows. By putting these elements in $m \times m$ matrix respectively, ***PFV*** matrix has resulted as mentioned in table12. So, preference function values of every two alternative are depicted.

Table 12
PFV matrix

	a_1	a_2	a_3	a_4	a_5
a_1	0	2.032	5.496	5.248	0.500
a_2	0.230	0	3.902	3.646	0
a_3	0.245	0.018	0	1.0012	0
a_4	0.255	0.027	0.652	0	0
a_5	4.840	5.176	11.162	10.872	0

By multiplying normalized weights vector into ***PFC*** matrix, *WPI* matrix is calculated which is shown in table 13. If weight vector elements aggregation was equal to one. These weights are called normalized weights. By using this matrix, every weighted pairwise amount of the main matrix is obtained.

Table 13
WPI matrix

a_1a_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_1a_2	0	0.001	0.007	0	0	0	0.029	0.029	0.029	0	0	0.001	0.007	0	0	0	0.029
a_1a_3	0.004	0.020	0.032	0.029	0.029	0	0.029	0.029	0.029	0.029	0.004	0.020	0.032	0.029	0.029	0	0.029
a_1a_4	0	0.020	0.032	0.029	0.029	0	0.029	0.029	0.029	0.029	0	0.020	0.032	0.029	0.029	0	0.029
a_1a_5	0	0	0	0	0	0	0.029	0	0	0	0	0	0	0	0	0	0.029
a_2a_1	0.000	0	0	0	0	0	0	0	0	0	0.000	0	0	0	0	0	0
a_2a_2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_2a_3	0.005	0.014	0.014	0.029	0.029	0	0.029	0.029	0	0.029	0.005	0.014	0.014	0.029	0.029	0	0.029
a_2a_4	0.000	0.014	0.014	0.029	0.029	0	0.029	0.029	0.029	0	0.000	0.014	0.014	0.029	0.029	0	0.029
a_2a_5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_3a_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_3a_2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_3a_3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_3a_4	0	0	0	0.029	0	0	0	0	0.029	0	0	0	0	0.029	0	0	0
a_3a_5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_4a_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_4a_2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_4a_3	0.004	0	0	0	0	0	0	0	0	0.029	0.004	0	0	0	0	0	0
a_4a_4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_4a_5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a_5a_1	0.037	0.035	0.006	0	0	0.029	0	0	0	0	0.037	0.035	0.006	0	0	0.029	0
a_5a_2	0.034	0.040	0.021	0	0	0.029	0	0	0.029	0	0.034	0.040	0.021	0	0	0.029	0
a_5a_3	0.046	0.054	0.045	0.029	0.029	0.029	0.029	0.029	0.029	0.029	0.046	0.054	0.045	0.029	0.029	0.029	0.029
a_5a_4	0.037	0.054	0.045	0.029	0.029	0.029	0.029	0.029	0.029	0.029	0.037	0.054	0.045	0.029	0.029	0.029	0.029
a_5a_5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Based on Eq.1, horizontal summation of every row of the ***WPI*** matrix leads to making a ***pi*** vector with m^2 elements. Then by putting this vector values in an $m \times m$ matrix, a preference index value with the name of ***PI*** is created as depicted in Table14.

Table 14
PI matrix

	a_1	a_2	a_3	a_4	a_5
a_1	0	0.014	0.014	0.015	0.285
a_2	0.120	0	0.001	0.002	0.304
a_3	0.323	0.230	0	0.038	0.657
a_4	0.309	0.214	0.059	0	0.640
a_5	0.029	0	0	0	0

In the first method of PROMETHEE, the priority of alternatives is determined based on the leaving and entering flow based on Eq.8. Alternatives leaving and entering flows are depicted in table 13. So, the fifth city (a_5) with the greatest leaving values is the best alternative among available 5 alternatives and the ranking of alternatives is mentioned in Fig.2.

Table 15
Leaving& entering flow of alternatives

	a_1	a_2	a_3	a_4	a_5
Lev_flow	0.195	0.114	0.019	0.014	0.471
Ent_flow	0.082	0.107	0.312	0.305	0.007

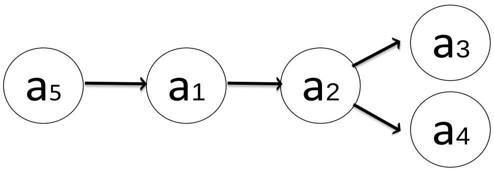


Fig.2. PROMETHEE I

In PROMETHEE II method, regarding Eq.11, alternatives net flow is computed. These gained results have been depicted in table 14. Based on alternative net-flows, the last alternative with the biggest net-flows is the best option for the budget assignment to establish an industrial facilities center. The alternatives ranking are shown in Fig.3.

Table 16
Net flow of alternatives

a_1	0.113
a_2	0.008
a_3	-0.293
a_4	-0.291
a_5	0.463

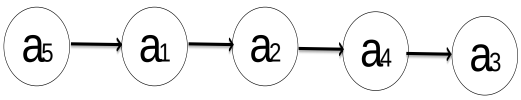


Fig.3. PROMETHEE II

In PROMETHEE III, alternatives' intervals lower and upper bound are obtained based on Eq.15. As it is mentioned before, if alternative lower bound Xa_i was greater than another alternative upper bound Ya_j, alternative a_i is superior to a_j. Alternatives interval bounds are shown in table 17 which is used for alternatives ranking.

Table 17
Interval values of alternatives

	X	Y
a_1	0.057	0.124
a_2	-0.026	0.038
a_3	-0.274	-0.195
a_4	-0.270	-0.196
a_5	0.331	0.411

For better demonstration, alternatives intervals are illustrated in Fig.4. Considering mentioned explanation, a_5 is superior to other alternatives and its interval is greater than others. Alternatives a_1 and a_2 are in second and third positions but for a_3 and a_4 no difference can be expressed. So, alternatives priority is depicted in Fig.5.

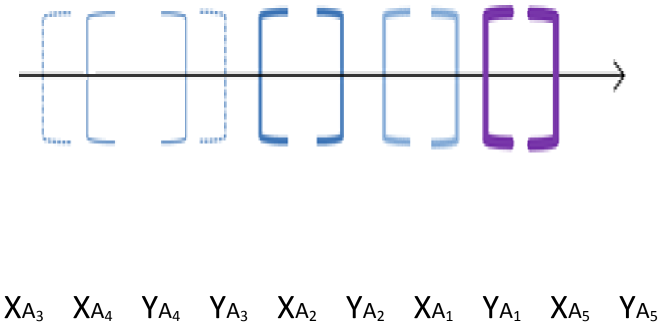


Fig.4. Interval of alternatives

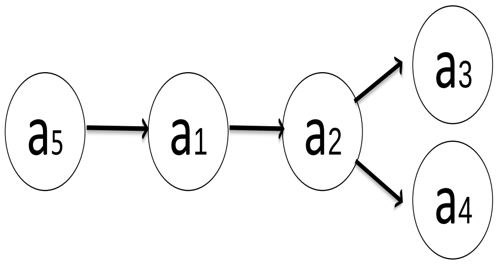


Fig.5. PROMETHEE III

All above, point out that fifth alternative is the best alternative clearly among nominated cities in Qazvin province for a consulting service department establishment.

4.Conclusion

In comparative analysis, the ranking results derived by all types of PROMETHEE illustrates that Lia city (a_5) was the superior than other alternatives. In another word PROMETHEE methods results were to some extent alike. The simplicity of the PROMETHEE methodology as a source for comparing to the other outranking techniques can be considered as one of the main advantages of the PROMETHEE. So based on the method and mentioned nominated city in the province Qazvin, fifthe alternative accuaired the most desirability to allocate the location for consulting service center. The paper presents a location-allocation decision-making problem resulting in the selection of the most desirable location for the consulting service center in the Qazvin state of Iran.

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