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Geometric Analysis of the Historical Periods of the Mausoleum of Shah *Nematollah Valli* in Mahan

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Abstract

This paper analyses the mausoleum of Shah Nematollah Valli in Mahan. Its construction started in the Timurid era and gradually transformed into its current form. This complex was built during three periods (Timurid, Safavid and Qajar) and in various Iranian architectural styles. This article tries to recognize the architecture of different periods from the point of view of proportions and geometric order. The method used in the present study is analytical-descriptive by the research hypothesis. To achieve the theoretical foundations of the research, which includes information about geometry and proportions as well as similar examples, data is gathered from previous research and field works then data is analyzed through descriptive and analytical methods of building geometry. Different spaces of this complex are evaluated in terms of compatibility with golden ratios and geometric patterns using software such as phi Matrix 1.16 pro and Phi Matrix Golden Ratio. According to the obtained outputs, the initial geometry of the building in the Timurid era, in comparison with its later geometry in the Safavid and Qajar periods, has undergone ruptures and changes in pattern (paradigm shifts), although there are similarities. The results of this study show that the initial geometry of the building follows a precise geometrical regularity in terms of form in the dome part, has the form of a Chalipa enclosed in a regular octagon in the plan, and has golden rectangular proportions in the facade and section. Furthermore, the geometry of the added parts in later periods is based on square shapes and a quadruple system.

Keywords: Mausoleum of Shah Nematollah Valli; Golden Proportions; Chalipa Form; Quadruple System.

1. Introduction

Iranian architecture is abundant with works that the architects of this land have created using mathematical knowledge and geometric patterns, where geometry is used as a common basis to meet the structural, aesthetic, and functional needs of the buildings (Bilalan, and Hassanpour Lemar, 2019). Many of Iran's historical monuments have not yet been studied mathematically. Through the geometric analysis of traditional architecture, it can be shown that various proportions, such as the golden ratio, have been widely used in the design of plans, sections, and architectural and geometric patterns (Najafgholi Pourkalantari et al, 2017, 477). Contemporary architects have a minimal and extremely limited knowledge of the science of geometry and how it is used in past architecture, because this knowledge has been passed down from master to student and little writing has been left behind. One of the most important obstacles to understanding and recognizing the texts relating to these ideas is the writings of Western scholars who, with the dominance of modernity and a purely scientific approach, have flipped through these texts and spelled them out for us (Taheri, 2013). The consequence of this defect is observed in contemporary architecture as a sharp decline in geometric concepts, as today's architects ignore the role of mathematical sciences focused on geometry and even numerical proportions. In contemporary buildings, the application of these sciences is mostly observed in structural and static calculations (Gomez, 1984, 8). The tomb of Shah Nematullah Vali, known as the "Astaneh of Mahan", is one of the garden tombs in Islamic architecture. Its construction started in the Timurid era with a dome house, which is the central building of the complex. It has been gradually transformed into its recent form, as the footprints of all the rulers of different dynasties can be observed in it. In the history of post-Islamic architecture, a small number of geometric patterns from the Timurid era have come down to us through scroll drawings. These intertwined geometric patterns, such as knots, were used as geometric grammar to systematize the decorations of architectural coverings and domed spaces (Sarhangi, 2012).

This study tries to analyze the geometry of the Astaneh of Mahan as a work from the history of Islamic architecture and mausoleums in Iran. This complex has been built during three periods and styles in Iranian architectural history (Timurid, Safavid, and Qajar periods) with a combination of consecutive open and closed spaces. In the application of geometry, Iranian architecture should introduce a suitable geometric pattern for architects in line with plans for the expansion and development of contemporary buildings. The original building of the tomb tower was built with an octagonal plan in the middle of an

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open area (probably a garden) on top of the tomb of the famous mystic Shah Nematullah Vali. Over six centuries, the building gradually turned into its present form with an approximate area of 14,000 square meters and a combination of central courtyards and closed porticos. This study seeks to answer the question that with consideration to the addition of adjunct parts, what geometric order and numerical pattern in the hidden layers have connected the architectural structures of the different parts of the complex in successive periods and caused coherence and harmony in the entire complex. What message and concept do the geometric patterns convey? In this article, after introducing and describing the architecture of this garden mausoleum, using library resources and field studies, we discuss the maps made during the three main construction periods of this complex, namely Timurid, Safavid, and Qajar eras. Next, the geometric and numeric structure of the building's architecture is analyzed and examined in each era.

2. Methodology

The method used in the present study is analytical-descriptive by the research hypothesis. To obtain the theoretical foundations of the research, which includes information about geometry and proportions as well as similar examples, data was collected using the archival method and field works, and then the geometry of the building was analyzed using a descriptive-analytical method. In this manner, first, based on inscriptions and reliable sources, the date of construction of each part of the building was determined and its drawings were separated from the newer added parts, and in the next step, the geometric proportions of each part were analyzed. To achieve more accurate results, golden proportions and geometric patterns were applied to the drawings of the plans and separate views of the complex in analytical software environments such as Phi Matrix 1.16 Pro and Phi Matrix Golden Ratio Design within the environment of AutoCAD 2019, and the resulting output images were analyzed and presented. The method of working with the above software is that after activation, it draws the golden proportions on the images and background photos, and the user finds the golden proportions on the pictures and the master of the building faster by matching these drawings to the golden proportions and identifies possible geometric proportions. In addition, due to the large size of the building under study and the limited format of this article, it is not possible to provide all the output images. Therefore, to achieve the goals and answer the research questions, some of the items have been selected and presented.

Explanation that the images presented in this research were obtained and presented in the following two ways:

1. The figures which were taken and drawn by the Cultural Heritage and Tourism Organization of Kerman province, were modified by the author. In such a way that after the studies, the construction time of each part of the collection was determined and the author has chronology according to the period of construction of this complex so that the architectural documents of each era can be analyzed separately. These drawings with their geometric analysis were made in AutoCAD software (figures: 1, 4, 6, 7, 8, 10).

2. In preparing the other figures, in addition to the separation of the chronology of the construction period and the steps described in the previous paragraph, which were done in the AutoCAD software, after screening the image, geometric analysis software such as “phiMatrix1.16 pro” and “Golden Ratio1.618” were used. The image is executed on it and after fixing the proportions, a screenshot is retaken from the final image and presented. Explanation that although such geometric analysis can be done with AutoCAD drawing software, the use of these auxiliary software increases the speed and accuracy of the work (figures: 9, 11, 12, 14).

3. Research Background

In recent years, several researchers have studied Iranian architecture from the perspective of geometry (Najafgholi Pourkalantari et al., 2017; Ziaeinia and Hashemi Zarjabad, 2016; Rezazadeh Ardabili and Sabetfard, 2013; Silvayeh et al., 2012). Among these works, we can observe two different approaches; The first approach can be called form-oriented because the researchers have only been engaged in analyzing the geometry and discovering the superficial forms of past architectural works and have studied the geometry only from the view pointof golden and aesthetic proportions. Among them are the following;

Ahmadi engages in the geometrical analysis of the tomb of Sheikh Zahed Gilani (belonging to the fifteenth century) in northern Iran, and reaches a complex geometry by analyzing the facade of the tomb through a combination of squares, isosceles triangles, and regular octagons in a regular grid. The results of this study showed that geometry has played a decisive role in the design of Timurid-style architecture (Ahmadi, 2012). Another study on the Hague Church (twentieth century) designed by Aldo van Eyck examines the presence of Fibonacci numbers and a golden rectangle in the composition of the design plan. Based on a detailed analysis of this work, the church appears to have been designed according to the Fibonacci-Golden Rectangle relationship (Fernández-Llebrez & Fran, 2013). In an article, Dewitte examines the geometric proportions of the Dom Paul Bellot Chapel by the Benedictine architect, and shows the golden proportions in a combination of 60-degree angles. The geometry of this building demonstrates beautiful harmony using golden proportions (Dewitte, 2015). In another study, a geometric analysis is carried out on Dolatabad Garden in Yazd, and the results show a diverse system of proportions that create unity in this eye-catching architecture (Garofalo, 2016).

The second group consists of researchers who have pointed to the symbolic and semantic aspect of geometric shapes and patterns in architecture using the tools of geometric analysis, and have expressed their symbolic, mythological, and metaphysical concepts as geometric shapes and motifs. Among them are the following: By analyzing the geometric arrays in the Kharqan Tomb Tower (11th century AD), Carl Bier proved that although geometric patterns in Islamic art are often regarded as decorative and void of basic meaning, there is a real connection between geometric patterns and metaphysical concepts. Then he put forth an argument based on interpretations of Qur'anic verses stating that the patterns of these two historical works strengthen each other in both visual and verbal expression, and that these patterns may represent the Arabic concept of "Alam Al-Mithal", the world of images. This concept plays an important role in depicting the intermediate space between philosophy and mysticism in Iran at that time, and the geometric motifs of these two historical works express the sacred geometry of early Islamic Iran (Bier, 2002). In another study, Bier studied the evolution of geometric shapes and motifs in the tombs of the Iranian plateau from the Seljuk period (thirteenth century), and the development of geometric patterns in a ten-sided tomb tower called the Blue Dome of Maragheh, due to its unique patterns, using a special combination of pentagons and decagons (Bier, 2012). Furthermore, Hassan Bolkhari claimed that: "According to the sages, numerical and geometric calculations not only serve to create religious buildings but also are the most important tool for the portrayal of abstract meanings". Documenting the views of Seyyed Haidar Amoli in his book "Jami' al-Asrar wa Manba' al-Anwar", Bolkhari explores the wide and collateral ratio of wisdom, geometry, and architecture in the Islamic civilization, and states that there is a clear relationship between mystical allegories and some artistic and architectural forms and decorations. Therefore, based on the obvious similarity between Seyed Haidar's allegory of mirrors and the allegorical radiance of light in them, he hypothesizes that this allegory has played a role in the emergence of such an industry (mirror work) in Iranian architecture (Bolkhari, 2013, 15).

Nader Ardalan and Laleh Bakhtiar stated that all elements of Islamic architecture in Iran, from decorative geometric patterns and shapes in architecture to plans for the establishment of urban complexes, indicate the principle of "unity in plurality" in Sufism. They interpreted the geometric patterns as eternal examples that can, through spiritual interpretation, lead the deep-thinking mind from the appearance of the patterns to the mysterious and mystical truths embedded in them (Ardalan, 2012, 40). Hossein Sultanzadeh examined the Taj Mahal tomb garden, which was built during the Gurkanid period in India, in terms of symbolism. He considered the architecture of the Taj Mahal to be based on the four-garden pattern of Iranian architecture, as the number four has a symbolic role in the formation of the different parts of this complex (Sultanzadeh, 2012). Culturally and historically, this tomb is very similar to the tomb garden of Shah Nematullah Vali in Mahan, the dome of which was originally built in the Timurid era by the order of a Gurkanid king.

In this study, both points of view s are considered. Geometric patterns are extracted from the perspective of both formalism and golden and beautiful proportions, and in the next step, the symbolic meanings of these patterns are outlined within the framework of this article.

4. Introduction of the Astaneh of Mahan

The Astaneh of Mahan is the mausoleum of the famous mystic Shah Nematullah Vali. He spent the last twenty-five years of his life in Kerman and Mahan engaging in pietism and worship until the year 834 AH /1429 AD when he passed away in Kerman and was buried in Mahan (36 km southeast of Kerman). Immediately after his death, Ahmad Shah Bahmani ordered the construction of a high dome on top of his tomb, and the building was completed in 840 AH / 1435 AD according to the inscription on the north side of the dome (Farzam, 2000). The construction of the Astaneh continued during the 6th century. The complex consists of four consecutive courtyards located on one axis and a dome house with surrounding porticos in the middle of this axis. The main part of the complex is the dome house, which was originally built as a single tomb tower in the middle of the garden. The total area of the enclosed spaces (the building) reaches 6000 m^2, and if the central courtyards are also included, the area reaches about 14000 m^2. This dome house, which has been built and developed during six centuries, is a display of Iranian architecture with different methods while in harmony at the same time. (Nourizadeh and Saidaei, 2016).

Three periods in the history of Iranian architecture have played a major role in the construction and completion of this complex, namely the Timurid, Safavid, and Qajar eras. The original dome was built in the Timurid period, and the Shah Abbasi portico and courtyard on the Qibla side were added to it during the reign of the Safavid Shah Abbas the Great. The rest of the present complex includes three large central courtyards with rooms around them, a roofed portico on the east side, four tall minarets, a cistern, a caravanserai, a bathhouse , and a caretaker residence were added to the previous building in the Qajar era.

In the picture below, the whole plan of Shah Nematullah Wali's tomb complex is shown in different colors by the period of construction, so that the red color shows the initial core of the tomb, which is the dome house built in the Timurid period, and the yellow color of the sections, the addition of the Safavid period and the white color also show the rest of the additions of this complex in the Qajar period (Figure 1).

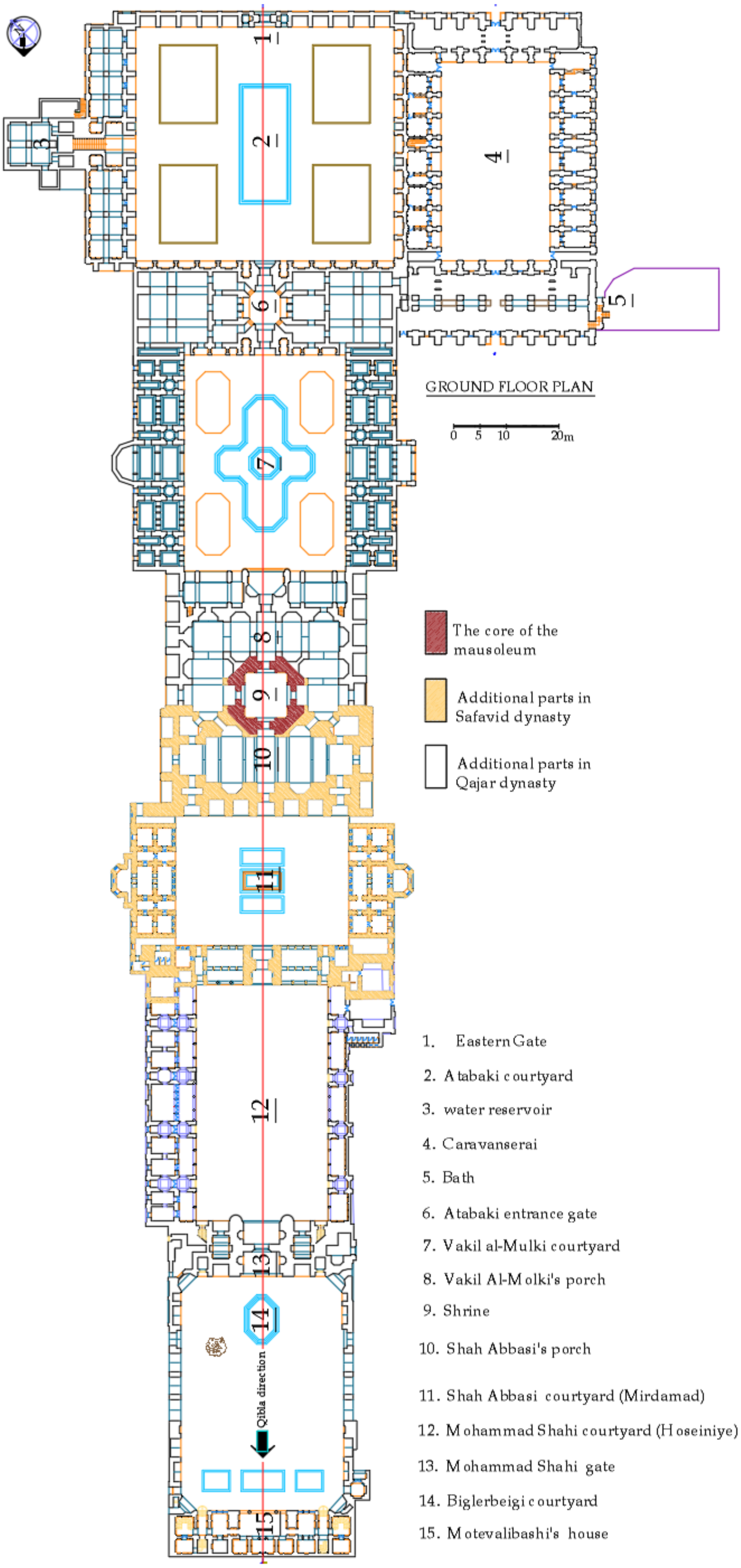


Fig. 1. Plan of the Astaneh of Mahan complex in three construction periods (Timurid, Safavid and Qajar) (Source: Authors' modifications on a drawing provided by the Cultural Heritage and Tourism Organization)

5. Golden Proportions in Geometry

The human body has divine order and proportions both in appearance and in the systems of the internal organs. For example, golden ratios are found in measuring the proportions of the human cardiovascular system, and this knowledge is consistent with the ancient knowledge that linked the human heart to its esoteric and spiritual characteristics and the human spirit. Thus, the physical structure of man is a manifestation of the divine aesthetics that can be measured by the "golden ratio" as a tool (Jowers & et al, 2010). As it is stated in the Qur'an: "Indeed, we created man with the fairest stature." Man, as the master of all creatures, has the most beautiful proportions. Therefore, it can be inferred that man is influenced by the art of his Lord in creating his works, and uses those sacred proportions to create of architectural works. In Islamic art and architecture, the main proportional systems are: the golden mean ratio and the three main proportional roots √2, √3, "Phi number" and √5, upon which the design of the geometric patterns in all Islamic arts and architectural forms is based (Figure 2). The golden ratio is a geometric system in which there is a relation between two parts in a set. Two parts of a line are not equal to each other, but are related to each other: a / b = (a + b) / a

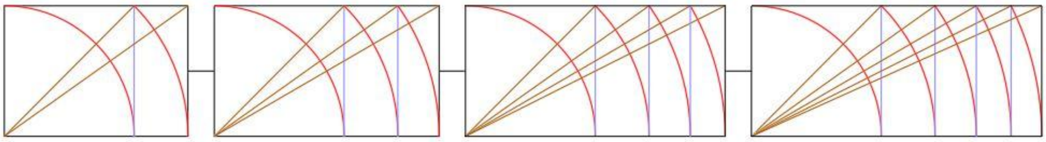


Fig. 2. Golden ratios in a rectangle, from left to the right, √2, √3, √4, and √5, respectively

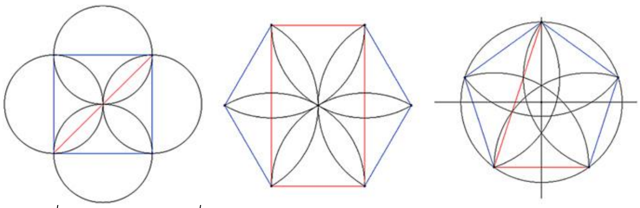


Fig. 3. Root of proportions (a): √2 ratio, (b): √3 ratio, and (c): the golden proportion (phi) (Source: Dabbour, 2012)

Proportional rectangles or proportional roots based on the geometry of polygons. (Figure 3). For example, in a square with dimensions of the same size, its diameter is equal to √2, and the ratio of the larger side to the smaller side of the rectangle enclosed in a regular hexagon is equal to √3, and in pentagons it is equal to the golden mean, which is a number equaling (√5 + 1) / 2 = 1.61803

6. Original Core of the Building: Octagonal Tomb Tower

The construction of the tomb of Shah Nematullah Vali, according to its inscription, was started by order of Sultan Shahab al-Din Ahmad Bahmani and completed by his son in 840 AH (Farzam, 2000). The first building built on top of the tomb of Shah Nematullah (Timurid period) was a single dome in the middle of open land, which according to chronological studies, the plan, has a geometry of eight and a half-eight outside the building and a Chalipa form inside it (Khajeh Hassani, 2015).

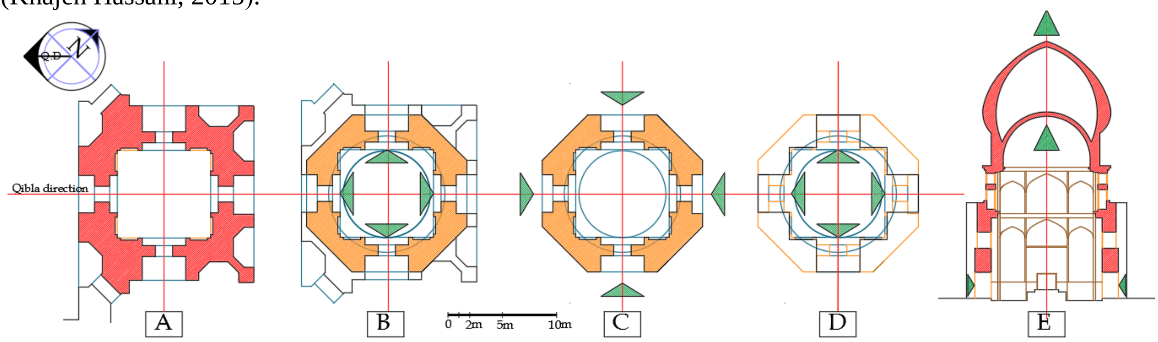


Fig. 4. (A): Plan of the dome house in its current condition; (B) and (C): Plan of the original structure in the Timurid period; (D): Geometric analysis; and (E): Position of the central axis part.

tomb tower with a height of 23.86 m is an octagonal prism with a dome above it. After entering the building, the observer would see a rectangular cube around them, and in the middle of the building, they would observe an octagon that acts as a connecting element between the rectangular cube and the hemisphere of the inner dome.

6.1. -Geometric analysis of the tomb tower

Through a geometric analysis of this tomb tower, we find that in the plan, a square form was used on the inside space, and an octagonal form was used outside the building. The three shapes of square, octagon, and circle can be deduced from the geometric analysis of the dome (Figure 4). From the observer's point of view, the spatial geometry of this

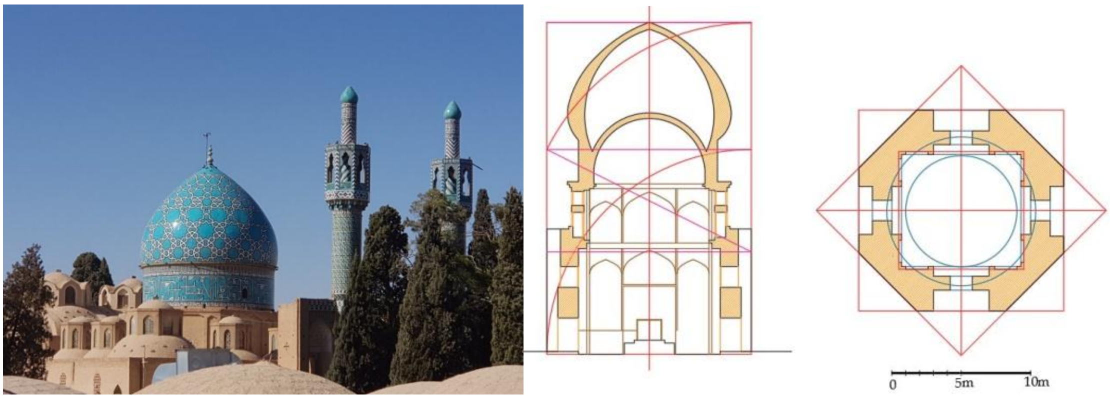


Fig. 5. Geometric proportions in the original tomb tower and the main core of the Astaneh.

6. 2. Added parts in the Safavid era; Shah Abbasi portico and courtyard

The Shah Abbasi portico, known as Darol-Hifaz, according to the bed inscription on its entrance in 998 AH / 1590 AD, was added to the tomb of the Timurid era during the reign of the Safavid Shah Abbas the Great (Farzam, 2000). This roofed space was added to the formerly shaped tower tomb in the direction of the Qibla. The Kerman- Mahan- Bam road used to pass from the west of the complex, so the main entrance of the tomb is located on the west side, and the first added courtyard (Shah Abbasi courtyard or Mirdamad courtyard) was built on the side of the Qibla after the construction of the Shah Abbasi portico (Qayyumi Bidhendi, 2006). This courtyard is similar to four-portico caravanserais, and was probably built to accommodate travelers during the reign of Abbas the Great.

6. 2.1. Geometric study of added parts in the Safavid era

From the geometric analysis of the plan of these two parts, the following points can be found:

1. Both the Shah Abbasi portico and the courtyard are formed in the direction of the axis of the Qibla and the axis of the Qibla is the main axis of symmetry for these two parts.
2. Unlike the Timurid dome, which had a point-like geometry and a closed space in the middle of the playground, this courtyard consists of an open space enclosed between closed spaces.
3. Inward orientation is one of the similarities between this courtyard and the dome of the Timurid era (the principle of introversion in Persian architecture).
4. The square shape is the most common shape used in the proportions of these two parts. The rectangular proportions of the portico are obtained by joining the two squares, and the composition and sequence of the squares can be seen inside the building (Figure 6).
5. The number four and the four-sided geometry of the cruciform shape dominate the courtyard, and the water pool is located at the intersection of the axes of the Chalipa.
6. The proportions of the rectangular Mirdamad courtyard (1.34) are close to the golden ratio φ, and the geometry of the rectangle surrounding the

This tomb tower of the Timurid period, which has an octagonal shape, is located between two intertwined squares, and is inside a square plan with dimensions of 10 * 10 m^2, which in combination with the Chalipa form, has allowed for four entrances. The Qibla axis is the main axis of this mass. The analysis of the proportions of every view produced a golden rectangle with a ratio of 1.618 or the number φ (Figure 5). The graphic roots of these proportions are square (on the ground) and circular (at the height of the sky) forms.

The following conceptual and symbolic results can be obtained from the geometric study of the original tomb tower:

The original building with the shape of a Chalipa in the plan looks neutral without direction, but from four sides (four entrance doors) it is inclined toward the center point at the intersection of the two axes of the Chalipa (the direction of the Qibla and the axis perpendicular to the direction of Muslim burials). This point is in the dimension of the productive height of the axis of the sky and is moving and ascending toward the planets (infinite universe) from the four sides of the material world (center of the Chalipa) as a starting point. Therefore, it can be said that the geometry of the dome house, while still and stretching inwards, attracts the audience to the upper world (Figure 4, Section E). Considering the purpose of this building, which is the burial place of a Muslim mystic whose body is in a cubic space, this form is the symbol of leaving the realm of property and walking towards the realm of the kingdom, towards the Lord, which is the manifestation of the holy verse "indeed, we are returning to Him". The central point of the Chalipa at the height and at the junction of the cube to the hemisphere of the dome is the center of the corresponding circle that forms the octagon of the dome, and at higher altitudes, is the center of the circles from which the hemispheres of the dome are built. This point, which slides in a vacuum on an invisible axis, is the generator of all the forms around it that are embodied and dissolved in the body of the architecture. This single point, which is the basis of the geometry of many forms, strengthens the principles of "plurality in unity" and "unity in plurality" in Islamic art and architecture. Therefore, it seems that the architecture of this building, in addition to observing regular geometric proportions, has focused on and emphasized the symbolic concepts of these forms and shapes.

can be observed, as well as the repetition of the square form in the divisions of the facade and section (Figure 7).

Similar to the outer rectangular proportions of the dome, the annex portico also has the golden proportion φ.

whole (closed and open spaces of the courtyard) has the golden ratio φ and the number 1.618 (Figure 6).

7. Moreover, from the geometric analysis of the parts, using proportions based on the square form

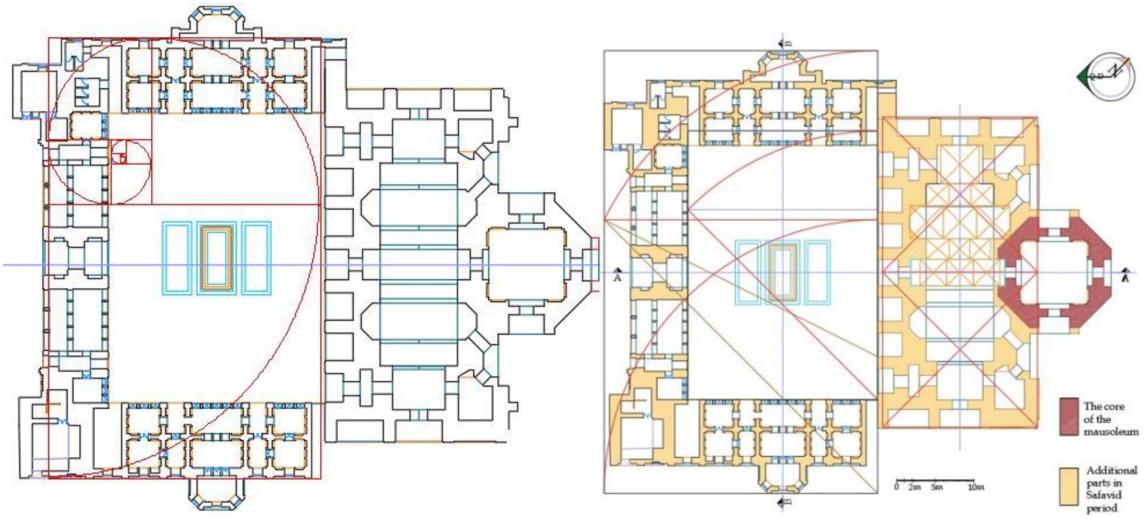


Fig. 6. Geometric analysis of the plan of added parts in the Safavid era.

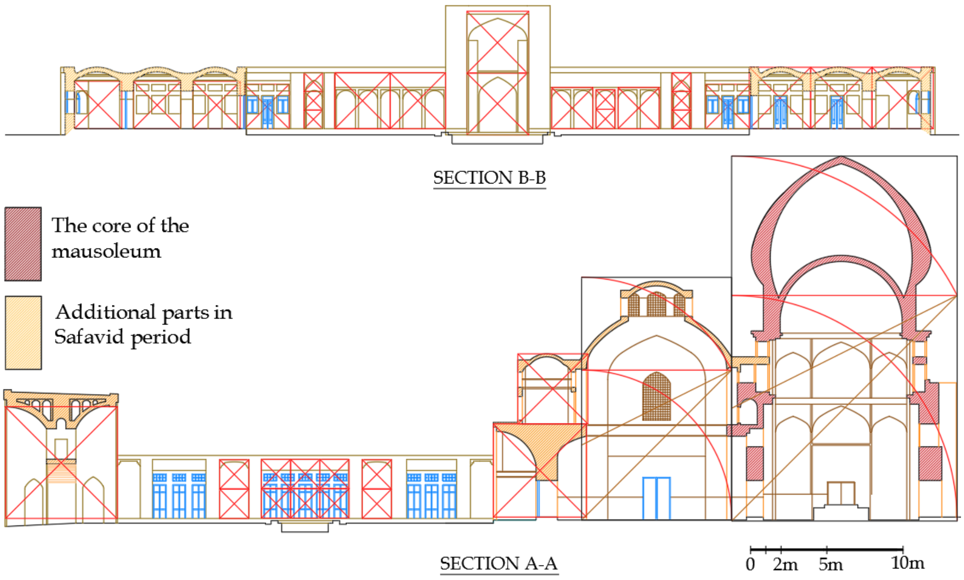


Fig. 7. Geometric analysis of added parts in the Safavid era.

During the Qajar period, there were many additions to the Astaneh complex, including the north, south, and east porticos and three central courtyards. For a more accurate analysis, these can be divided into two periods in terms of time and location: In the first period, additions were made to the previous complex in the direction of the Qibla by order of Mohammad Shah Qajar, and in the second period, in the time of Nasser al-Din Shah Qajar onwards, some parts were added in the opposite direction to the Qibla and in the eastern courtyard.

6.3. Introduction of added parts in the first Qajar period (Mohammad Shah Qajar)

The Mohammad Shahi courtyard, known as the Hosseinieh, was built in the era of Mohammad Shah Qajar (1250-1264 AH / 1848-1834 AD) on the western front of the previous complex. The Mohammad Shahi entrance was built by order of Mohammad Shah Qajar in the Qibla direction of the tomb, and its tall minarets were built during the reign of Mohammad Ismail Khan Nouri, known as Vakil al-Mulk (1277-1284 AH) (Farzam, 2000).

The creation of a high entrance with a height of 14.13 m with two tall minarets with a height of 38 m are among the obvious civil elements following the dome.

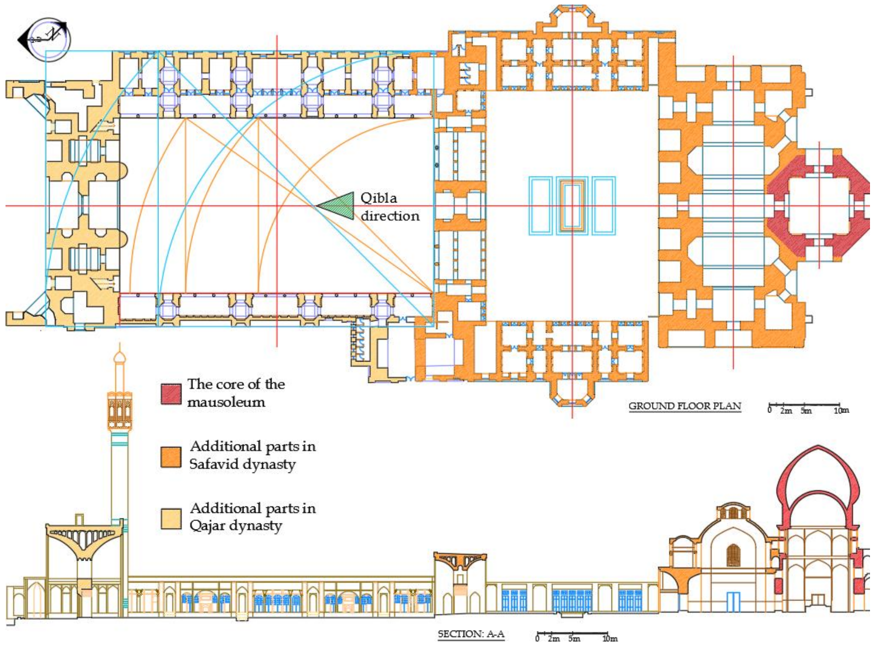


Fig. 8. Geometric analysis of the plan of the Mohammad Shahi courtyard at the top and its longitudinal section at the bottom.

The following results are obtained from the study of the proportions and geometric order of the added parts in the Qajar period (the era of Mohammad Shah Qajar):

1. The proportions of the rectangle in the central courtyard of the Mohammad Shahi courtyard are approximately √3 (Figure 8).
2. The proportions of the rectangle of the added parts in the plan (closed spaces around the central courtyard) are √2.
3. The boundary between open and closed spaces in this courtyard is semi-open spaces (portico), and in this respect, it is different from the Mirdamad courtyard, which was built in the Safavid period (Figure 8).
4. The main geometry of this courtyard, which is symmetrical concerning the axis of the Qibla, is four-sided and cruciform, like the previous courtyard.
5. The geometry in the arrangement of the spaces around the courtyard, as well as the proportions of the rooms, has undergone a paradigm shift in comparison with the courtyard of the Safavid period.
6. The Mohammad Shahi courtyard is stretched in the direction of the Qibla, and the longitudinal axis of this courtyard is perpendicular to the longitudinal axis of the previous courtyard.
7. The design of portico-like spaces that constitute the entrance to the main rooms on the two northern and southern fronts has created a completely different rhythm compared with the portico-like spaces of the Safavid courtyard (paradigm shift).
8. In the front view, the use of the golden ratio φ is evident (Figure 9).

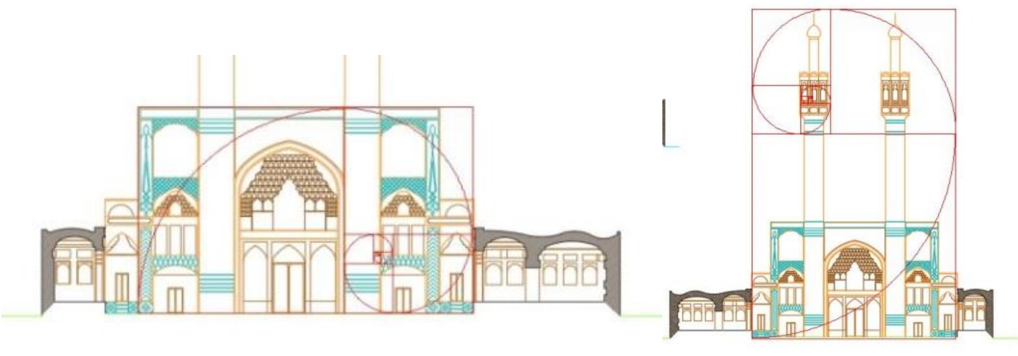


Fig. 9. a) Proportions of the golden spiral in the front view of the Mohammad Shahi entrance (Source: Authors using phi Matrix 1.16 pro software in AutoCAD environment)

6.4. Introduction and geometric analysis of added parts in the second Qajar period (from the time of Nasser al-Din Shah Qajar onwards

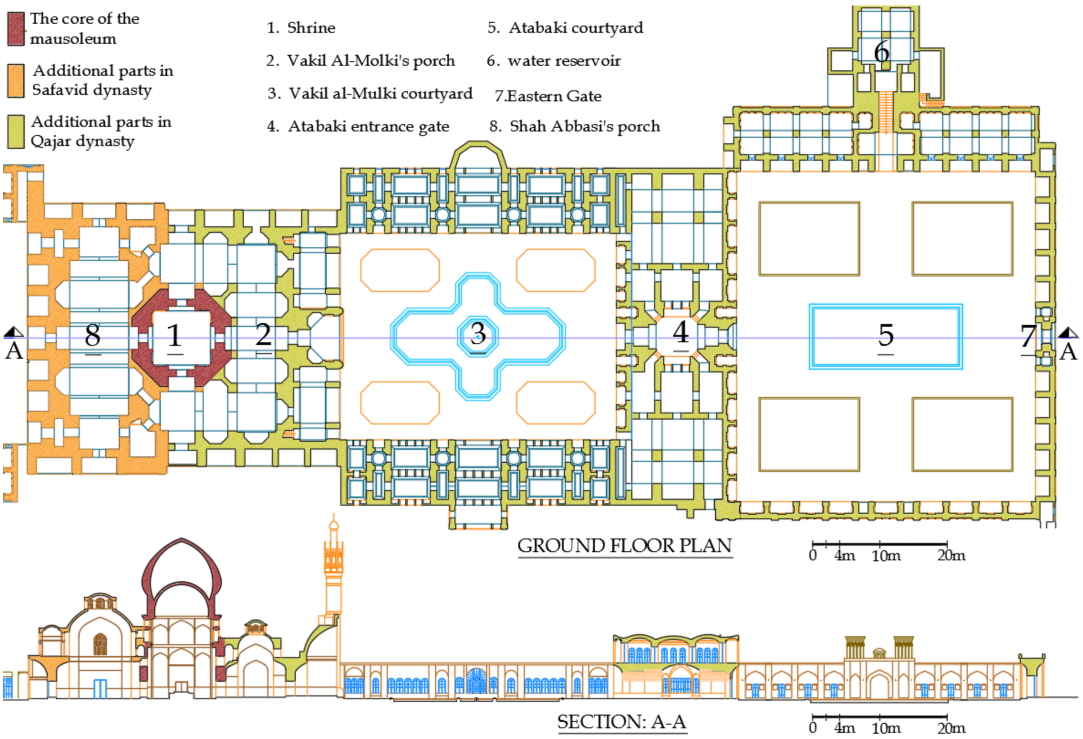
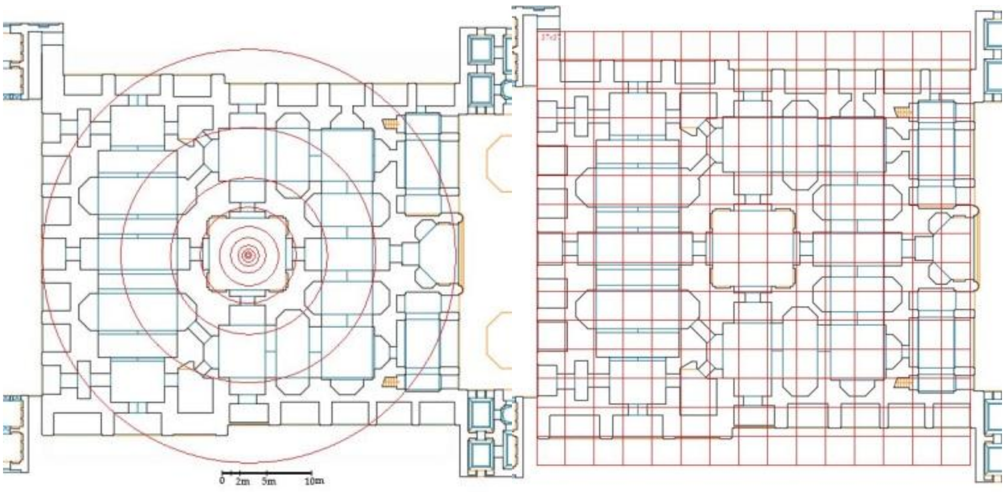


Fig. 10. Plan and section of added parts in the second Qajar period (Source: Authors based on drawings of Cultural Heritage Organization in Kerman Province)

The eastern, northern, and southern porticos of the shrine and the Vakili courtyard, which was added to the previous complex during the reign of Nasser al-Din Shah Qajar (1264-1313 AH) and the rule of Mohammad Ismail Khan (Vakil al-Mulk I) and his son until 1387 AH (Farzam, 2000).



a b

Fig. 11. a) 15*15 modular network of added porticos in Safavid and Qajar eras; b) Conformity of the portico centers to the center of the dome (Source: Authors using phi Matrix 1.16 pro and AutoCAD software)

The following results are obtained from the geometric analysis of Vakil al-Mulki porticos:

1. In terms of plan geometry, these porticos, in combination with the porticos of the Safavid era, complement the square form that surrounds the square of the original dome, and have less strength and height compared with the Shah Abbasi portico.
2. The northern, eastern, and southern additions to the dome are symmetrical to the added part of the Safavid period in terms of size in the plan, and are located in a regular 15 * 15 modulation network, which is exactly the central point of the dome circles as the center of symmetry of the added masses (Figure 11).

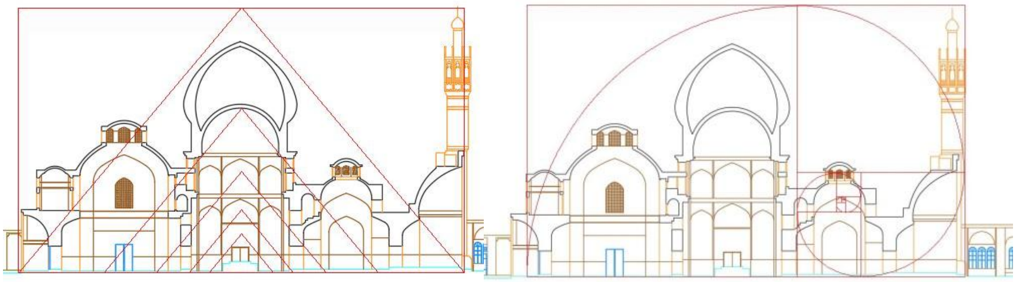


Fig. 12. a) Geometric analysis of the east-west section of the dome and the porticos around it (Source: Authors using phiMatrix1.16 pro and AutoCAD software)

3. The perpendicular rectangle on the golden spiral in the longitudinal section of the added porticos in the Safavid and Qajar periods shows the proportion φ in the geometry of the most central part of this complex, which was built during three historical periods (Timurid, Qajar, and Safavid) (Figure 12). Based on this analysis, we can see the unity and harmony in the geometric order of the architecture in the different eras of this complex's construction
4. .

Fig. 13. Image of the northern front of the Astaneh where the dome and the two minarets of the Vakil al-Mulki courtyard can be seen.

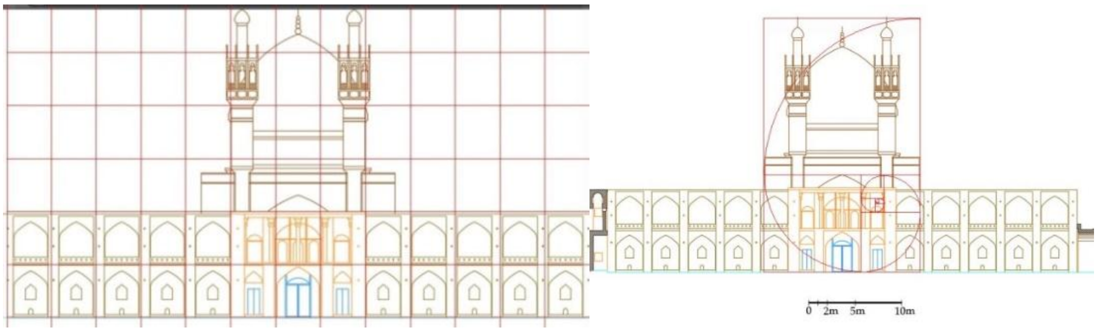
The Vakil al-Mulki courtyard, as the third added courtyard, was built on the eastern front and opposite the Qibla, in contrast to previous additions on the Qibla side (Figure 13). The following results are obtained from the geometric analysis of the Vakil al-Mulki porticos:

This courtyard, which was built in symmetry with the Shah Abbasi courtyard, like the previous courtyards, has an axis of symmetry corresponding to the direction of the Qibla.

The geometry of this courtyard is based on the Iranian four-garden geometry. The cruciform water fountain in the center of this courtyard and the presence of four gardens evoke and strengthen the four-garden geometric pattern.

The use of a modular grid is evident in the views of its northern, southern, eastern, and western fronts (Figure 14: a).

There is no golden ratio in the rectangle of the courtyard, but in the view of the eastern front, the use of the golden ratio φ is evident (Figure 14: b).



A b

Fig. 14: a) Modular grid of the western view of the Vakil al-Mulki courtyard; b) Golden spiral proportions in the tomb entrance from the Vakil al-Mulki courtyard

6.4.3. The Atabaki courtyard was built during the reigns of Vakil al-Mulk I and II, and its eastern entrance was built by order of Mirza Ali Asghar Khan Atabak (deceased in 1325 AH) and is known by his name (Farzam, 2000). This courtyard is the last and largest courtyard of the complex, attributed to Mirza Ali Asghar Khan Atabak, known as Amin Al-Sultan, who was the Prime Minister of Nasser al-Din Shah, Muzaffar al-Din Shah, and Mohammad Ali Shah Qajar. It is a one-story caravanserai with a symmetrical view and a cistern on the northern side (Qayyumi Bidhandi, 2002). The following results are obtained by examining the proportions and geometric order of the Atabaki courtyard:

1. This courtyard, like the previous courtyards, has an axis of symmetry corresponding to the direction of the Qibla.
2. The geometry of this courtyard is based on the Iranian four-garden geometry. The large rectangular pond in the center of this courtyard and the presence of four gardens evoke the four-garden geometric pattern.
3. In the plans and views of its northern, southern, eastern, and western fronts, the use of a modular grid is evident (Figure 9: a).
4. There are no golden proportions in the rectangle of the courtyard.
5. The geometry in the arrangement of the spaces around the courtyard, as well as the proportions of the rooms, has undergone a paradigm shift in comparison with the courtyard of the Safavid period.

7. Discussion

The following results are obtained from a geometric analysis of the Astaneh of Mahan complex, which was built and completed during three eras in the history of Iranian architecture:

First era (Timurid period): In the architecture of the dome, as the original core and the main part of the Astaneh, a geometry based on the number 4 prevails. The invisible axis of the Qbla, which plays a decisive role in the direction of Muslim burials, is the main direction and axis of symmetry for the building in the plan. The center of this octagonal mass revolves around a vertical axis, which is the axis of symmetry of the dome and starts from the central point of the burial ground and climbs to the rotating dome toward the sky. The movement from the ground to the sky indicates the concept of the "death of believers" from the Muslim point of view. Furthermore, symbolically, we can remember the holy verses "Indeed, we are from God" and "Indeed, we are returning to Him", as well as the principles of "plurality in unity" and "unity in plurality" in Islamic art and architecture.

Second era (Safavid period): In this period, two main parts were added to the dome in the direction of the Qibla, a Shah Abbasi portico as a closed space and a Shah Abbasi courtyard consisting of a central courtyard enclosed within a closed space. In the geometric analysis of both the Shah Abbasi portico and courtyard, a four-part geometric pattern based on a square is observed. These parts are formed symmetrically on both sides of the Qibla axis. In the central point of the central courtyard is a large rectangular pool that connects the water on the ground and the light from the sky above, which indicates the concept of purity and the presence of divine light.

Third era (Qajar period): For a better analysis and due to the wide range of additions, this era is divided into two periods; The first period covers the era of Mohammad Shah Qajar, and the second period includes the era of Nasser al-Din Shah Qajar onwards.

In the first period of the Qajar era, the Mohammad Shahi courtyard and high entrance and two high minarets were built in the direction of the Qibla. In the geometric analysis of this courtyard as well, a four-part geometry is observed based on two axes of symmetry perpendicular to each other. It can be seen that the main axis of this part is the direction of the Qibla, which passes through the center of the dome house and points to the Kaaba, and all parts are arranged around this axis.

In the second period of the Qajar era, in contrast to the previous additions, the development plans were implemented in the direction of the east of the dome and opposite to the Qibla. This set of additions included a U-shaped portico that encloses the dome on the eastern, northern, and southern fronts, and two courtyards and closed spaces around them, which were gradually added to the previous building at that time. In the Vakil al-Mulki courtyard, considering the four refined gardens on the four sides of the cruciform pond in the center of the courtyard, a geometry based on the number 4 is highly emphasized. This along with the two perpendicular axes in the center of the central courtyard and the tall evergreen trees bring to mind a Persian garden, and are reminiscent of the Garden of Eden described in the Qur'an. The main axis of symmetry in these added parts is completely consistent with the axis of symmetry in the older parts of the complex (direction of the Qibla).

8. Conclusion

In the architecture of the dome, as the original core of the Astaneh of Mahan, a quadruple geometric system prevails, and its mass is arranged around a vertical axis that begins from the central point of the burial place on the ground and ascends to the revolving dome up to the sky. It seems that the architecture of this building in the dome (Timurid era), in addition to observing regular geometric proportions, symbolically refers to the holy verse "Indeed, we are from God, and indeed, we are returning to Him". In the geometric analysis of later periods in the development of this complex, it can be concluded that although the Golden proportions are recognized and demonstrated in the arrangement, harmony, and aesthetics of the architecture in many parts of this complex, the most important unifying geometric pattern in the development plans of successive historical periods has been the use of a four-part geometric system. Despite changes in the materials and construction techniques used in different periods, the effects of this geometric pattern can be observed in the architectural structure of all historical periods in this complex. In addition, the most important connecting and cohesive element of this complex is the invisible axis that extends from the center of the dome in the direction of the Qibla. All the additions in all three periods of the construction and development of the Astaneh are arranged around this axis in a symmetrical order, which seems to refer to the monotheistic slogan "La ilaha illallah".

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