

Sliding Mode Controller Designed for a Class of Under Actuated Systems

Sara Rafiei Lak , Saeed Barghandan*

Department of Electrical Engineering, Ahar Branch, Islamic Azad University, Ahar, Iran,

Email: saeed_barghandan@yahoo.com

Abstract

Many real systems are among underactuated stimulation systems. The number of actuators in such systems is less than their freedom degree. Translational oscillator with rotational actuator (TORA) is an underactuated system in cascaded form. In the present paper, this system has been investigated as a case study and then a sliding mode controller has been designed for it. This is due to the fact that among non-linear controlling methods, sliding mode control is appropriate to achieve global stabilization. Also it is robust against external disturbances and parametric uncertainties in the system under control. In this paper the efficiency of simulation has been investigated.

Keywords: under actuated systems, sliding mode control, TORA system

1. Introduction

Over the last decade, there has been a strong interest in underactuated systems. The control of underactuated systems is a challenging issue. The fact is, the irritability of these systems is less than the degrees of freedom that there is need to control. Due to the large use of underactuated systems in recent decades, efforts have been made to design controllers for these types of systems. The underactuated systems have very important applications, such as robotic rockets, marine robots, and underwater boats, weight reduce and significant cost saving and energy savings from these systems. Some advantages of underactuated systems, including their resistance to actuator defects, are noteworthy. In underactuated systems, the use of smooth feedback to stabilize around equilibrium points is entirely impossible even in

localized conditions. In [1] there is limited provision for a stabilizer feedback law. In some underactuated systems, these conditions may not be satisfactory. For example, if a linear system has uncontrollable mode and positive eigenvalues, then the main system with the feedback control rule cannot be stabilized, even locally. In the reference [2] there is a stabilizer continues controller for a class of local controllable systems, even if the linear system has non-controllable modes (eigenvalues with positive real parts). In the reference [3], Global stabilizer non-Lipchitz continuous controller is proposed to satisfy two systems that may not provide the required Brockett conditions. These results indicate that continuous stabilizing feedback may overcome the difficulties of stabilizing smooth feedback. Researchers in Control Science have paid much attention to many

control issues in relation to mechanical systems, and various control strategies such as backstepping control technology techniques [4], energy based technique[3] or comparative control, intelligent control, hybrid control [5] have been suggested for this purpose .

The main discussion of this article is to develop sliding mode control method that can global stabilize all degrees of freedom in the under control system. This underactuated systems class can be described as external disturbance cascade forms. Since the slider model control is not smooth, the proposed sliding mode controllers can stabilize systems that they do not meet Brockett's the necessary conditions. Advantages of sliding mode control are the lack of sensitivity to errors of model and parametric uncertainty and other disturbances. When the system modes are on the sliding surface, the system behavior is determined by the structure of the sliding surface, this advantage gives us little freedom in controller design. So that we can modify the system model with virtual disturbances.

2. The Statement of Problem

The underactuated system is written as follows:

$$\begin{aligned} m_{\gamma}(q)\ddot{q}_{\gamma} + m_{\gamma\gamma}(q)\dot{q}_{\gamma} + h_{\gamma}(q, \dot{q}) &= \cdot \\ m_{\gamma\gamma}(q)\ddot{q}_{\gamma} + m_{\gamma\gamma\gamma}(q)\dot{q}_{\gamma} + h_{\gamma\gamma}(q, \dot{q}) &= \tau \end{aligned} \quad (1)$$

Where $q = [q_{\gamma}, q_{\tau}]^T$ and \dot{q}, q are the states of under controlled system, and τ is input control, h_i include Coriolis centrifugal, centrifugal, and gravitational. In the

reference [6], a systematic approach is proposed (1) for transforming an underactuated system to normal cascade form (2) as follows:

$$\begin{aligned} \dot{x}_1 &= x_2 + d_1, \\ \dot{x}_2 &= f_1(x_1, x_2, x_3, x_4) + d_2 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= f_2(x_1, x_2, x_3, x_4) + d(x_1, x_2, x_3, x_4)u + d_3 \end{aligned} \quad (2)$$

where $x_i \in R^n (i = 1, 2, 3, 4)$ system states, $u \in R^n$ is input control and $f_1, f_2 : R^{4n} \rightarrow R^n, b : R^{4n} \rightarrow R^{n \times n}$ is nonlinear vector functions and b is inverse able and $d_i \in R^n (i = 1, 2, 3)$ represents disturbances. Many of the underactuated systems are convertible to relationship 2. Examples are: reverse pendulum [7], N-Transitional oscillator Rotary actuator (TORA) for example in [3] [7], airplane with vertical Takeoff capability [9]

In this paper, our control goal is to stabilize all case in relation (2) and converge it to zero with relation (2), we have the following assumptions:

Assumption 1: $f_1(0, 0, 0, 0) = 0$

Assumption 2: if $\partial f_1 / \partial x_3$ is reversible or $\partial f_1 / \partial x_4$ is reversible

Assumption 3: $f_1(0, 0, x_3, x_4) = 0$ is asymptotic stable for example x_3 and x_4 converges to 0 if $f_1(0, 0, x_3, x_4) = 0$.

Assumption 1 is essential condition for the origin to have the closed loop system reach equilibrium regardless of disturbances.

The assumptions 2, 3 are necessary to improve the overall control of the sliding mode controller. However, the existence of d_2 in equation 2 gives us the freedom of choosing f_1 and finally these assumptions are obtained

3. Slider Module Controller Design

In this section, we will refer to a general sliding mode controller that controls all states simultaneously. First, the error variable is expressed as:

$$\begin{aligned} e_1 &= x_1, & e_3 &= f(x_1, x_2, x_3, x_4) \\ e_2 &= x_2, & e_4 &= \frac{\partial f_1}{\partial x_1} x_2 + \frac{\partial f_1}{\partial x_2} f + \frac{\partial f_1}{\partial x_3} x_4, \\ E_1 &= [e_1, e_2, e_3]^T, & E_2 &= [e_1, e_2]^T, \end{aligned}$$

That E_2 and E_1 is just for analyzes of stability of the system. It is better to delete definitions and proofs of e_4 or e_3 from E_2 or E_1 . The following assumptions present the required conditions for in the design of the sliding mode controller offer [10]:

Assumption 4: If $d_1 \neq 0$, then the maximum absolute value of sum of the rows $\partial f_1 / \partial x_1$ is finite. For example, $\|\partial f_1 / \partial x_1\|_\infty \leq \beta_1$ in which β_1 is a constant non-negative integer.

Assumption 5: If $d_2 \neq 0$, then the maximum absolute value of sum of the rows $\partial f_1 / \partial x_2$ is finite. For example, $\|\partial f_1 / \partial x_2\|_\infty \leq \beta_2$ in which β_2 is constant non-negative integer.

Assumption 6: If $d_3 \neq 0$, and if $\partial f_1 / \partial x_3$ is inverse able, $\partial f_1 / \partial x_3$ is limited, for example,

$\|\partial f_1 / \partial x_3\|_\infty \leq \beta_3$ where β_3 is a positive integer.

Assumption 7: If $d_3 \neq 0$, and $\partial f_1 / \partial x_4$ is inverse able, so the maximum absolute value of the sum of the rows $\partial f_1 / \partial x_4$ is limited, for example, $\|\partial f_1 / \partial x_4\|_\infty \leq \beta_4$ where β_4 is a positive integer

Assumption 8: Disturbance d_i are bounded.

If $\partial f_1 / \partial x_4$ is reversible then we will have:

$$\|d_3\| < d_3 + d_4 \|\xi(x)\|_2, \quad \|d_1\| < d_1 \|E_2\|_2, \quad \|d_2\| < d_2 \|E_2\|_2$$

If $\partial f_1 / \partial x_3$ is reversible then we will have:

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($i = 1, 2, 3, 4$) $\geq i$ are fixed numbers, and ($\xi(x)$) is also the vector field for the states $x = [x_1, x_2, x_3, x_4]^T$. If $\partial f_1 / \partial x_4 = 0$ and $\partial f_1 / \partial x_3$ invers able. Therefore, the switching level is expressed as follows:

$$s = c_1 e_1 + c_2 e_2 + c_3 e_3 + e_4$$

If $\partial f_1 / \partial x_4$ is invers able therefore, the switching level is expressed as follows:

$$s = c_1 e_1 + c_2 e_2 + c_3 e_3,$$

In which C_i ($i=1,2,3$) are constant positive integers, so that the dynamics of the system on the sliding manifold $S = 0$ is stabled asymptotically, then the conditions C_i will be expressed, because we use two different levels S should have a relative degree of 1 with respect to the u control input.

Note that the system expressed in relation (2) and assumptions (1) to (8) will not meet the bracket requirements.

For example, consider the underactuated system:

$$\begin{aligned} \dot{x}_1 &= x_2 & \dot{x}_2 &= g \sin x_1 / l + x^3 \\ \dot{x}_3 &= x_4, & \dot{x}_4 &= u \end{aligned} \quad (3)$$

Which relates to an unstable two-degree freedom mechanical system of in this excitation region in which x_i are system's states and u is input, g is gravity acceleration and l is length of the pendulum. It is clear that the linear system (3) has an uncontrollable mode around the origin.

Therefore, this underactuated system is not stabilized by the smooth feedback, because it is opposite to the first requirement of the bracket. However, continuous state feedback law exists and the system (3) is global stable.

If we include x_3 in equation 3 and reduce x_3 from second equation of 3, then system (3) can be rewritten in the general form in which we will have:

$$\begin{aligned} f_1 &= g \sin x_1 = /l + x^3 + x_3 f_2 = 0, \\ b &= 1, d_1 = 0, \quad d_2 = -x_3 \text{ and } d_3 = 0 \end{aligned} \quad (4)$$

We have defined the error variables before, so we have:

$$\begin{aligned} d_2 &= -x_3 = \frac{g \sin x_1}{l} + x_3^3 - e_3 \\ &\Rightarrow |(1+x_3^2)x_3| = \left| e_3 - \frac{g \sin x_1}{l} \right| \\ &\Rightarrow |x_3| \leq |(1+x_3^2)x_3| = \left| e_3 - \frac{g \sin x_1}{l} \right| \leq |e_3| + \frac{g|e_1|}{l} \\ &\Rightarrow |d_2| = |x_3| \leq \frac{(1+g)}{l} \|E_1\|_2. \end{aligned}$$

It is clear that assumptions 1 through 8 are met.

First case: $\partial f_1 / \partial x_3$ and $\partial f_1 / \partial x_4 = 0$ are reversible:

The switching level is defined as $s = c_1 e_1 + c_2 e_2 + c_3 e_3 + e_4$ consider the following matrix:

$$A_{n1} = \begin{bmatrix} 0 & I_n & 0 \\ 0 & 0 & I_n \\ -c_1 I_n & -c_2 I_n & -c_3 I_n \end{bmatrix},$$

Where in I_n is a $n \times n$ matrix and also $\lambda_{left}(-A_{n1})$ is the real part of the eigenvalues of the matrix $-A_{n1}$. The c_i must be chosen in a way that the matrix A_n is a Hurwitz and:

$$\max\{d_1, d_2, \beta_1 \bar{d}_1 + \beta_2 d_2\} < \lambda_{left}(-A_{n1})$$

The slider mode controller has two parts: the u_{eq} equivalent control section and the u_{sw} control switch section. Equation control on Manifold $S=0$ can be calculated by inserting $\dot{s} = 0$ since that's mean:

$$\begin{aligned} u_{eq} &= - \left[\frac{\partial f_1}{\partial x_3} b \right]^{-1} \{ c_1 x l 2 + c_2 f_1 + c_3 \frac{\partial f_1}{\partial x_1} x_2 + c_3 \frac{\partial f_1}{\partial x_2} f_1 \\ &\quad + c_3 \frac{\partial f_1}{\partial x_3} x_4 + \frac{d}{dt} \left[\frac{\partial f_1}{\partial x_1} x_2 \right] + \frac{d}{dt} \left[\frac{\partial f_1}{\partial x_2} f_1 \right] \\ &\quad + \frac{d}{dt} \left[\frac{\partial f_1}{\partial x_3} \right] x_4 + \frac{\partial f_1}{\partial x_3} f_2 \}. \end{aligned}$$

The switching control unit is designed to converge S to Manifold $S = 0$:

$$u_{sw} = - \left[\frac{\partial f_1}{\partial x_3} \right]^{-1} [M \sin g(s) + \lambda s], \quad (5)$$

Where in:

$$M = (c_1 \bar{d}_1 + c_2 \bar{d}_2 + c_3 \beta_1 \bar{d}_1 + c_3 \beta_2 \bar{d}_2) \|E_1\|_2 + \beta_3 (\bar{d}_3 + \bar{d}_4) \|\xi(x)\|_2 + \rho,$$

And, ρ, λ are positive constants. Sliding mode controller is

$$u = u_{eq} + u_{sw} \quad (6)$$

Theorem 1

If $\partial f_1 / \partial x_4 = 0$ and the matrix $\partial f_1 / \partial x_3$ is reversible, then in accordance with the control law (6), all states (2) converge asymptotically to zero.

Proof: First, we prove that there is a sliding mode. We consider the Lyapunov function as follows:

$$v = \frac{1}{2} s^2$$

Now, by derivation of V and with placement (6) to (4), we will have:

$$\begin{aligned} \dot{V} &= s^T \dot{s} = s^T [c_1 \dot{e}_1 + c_2 \dot{e}_2 + c_3 \dot{e}_3 + \dot{e}_4] \\ &= s^T \left[-M \operatorname{sign}(s) - \lambda s + c_1 d_1 + c_2 d_2 + \right. \\ &\quad \left. c_3 \frac{\partial f_1}{\partial x_1} d_1 + c_3 \frac{\partial f_1}{\partial x_2} d_2 + \frac{\partial f_1}{\partial x_3} d_3 \right] \\ &< -\lambda s^T s - (M - c_1 \bar{d}_1 \|E_1\|_2 - c_2 \bar{d}_2 \|E_1\|_2 - \\ &\quad c_3 \beta_1 \bar{d}_1 \|E_1\|_2 - c_3 \beta_2 \bar{d}_2 \|E_1\|_2 - \beta_3 \bar{d}_3 \\ &\quad - \beta_3 \bar{d}_4 \|\xi(x)\|_2) \|s\|_1 \\ &= -\lambda s^T s - \rho \|s\|_1 \leq 0. \end{aligned} \quad (7)$$

So, with the control rule (6), the system can reach the sliding surface of $s = 0$ and stay on it. In manifold, $e_4 = c_1 e_1 - c_2 e_2 - c_3 e_3$, or $S = 0$, the main system can be reduced to the following system:

$$\dot{E}_1 = A_{n1} E_1 + D_1 \quad (8)$$

where in:

$$D_1 = [d_1, d_2, (\partial f_1 / \partial x_1) d_1 + (\partial f_1 / \partial x_2) d_2]^T$$

$$\|d_1\| < \bar{d}_1 \|E_1\|_2, \|d_2\| < \bar{d}_2 \|E_1\|_2,$$

$$\|(\partial f_1 / \partial x_1) d_1 + (\partial f_1 / \partial x_2) d_2\| < (\beta_1 \bar{d}_1 + \beta_2 \bar{d}_2) \|E_1\|_2.$$

This is a disrupted system with disturbance D_1 , which:

$$\|D_1\|_2 < \gamma \|E_1\|_2, \gamma = \max\{\bar{d}_1, \bar{d}_2, \beta_1 \bar{d}_1 + \beta_2 \bar{d}_2\}$$

by assuming $Q = Q^T > 0$, Lyapunov equation $P_{n1} + A_{n1} P^T = -Q^T > 0$ has unique response and $P = P^T > 0$. Because matrix A_{n1} is a Horvitz.

By using $V_1 = E^T P E_1$ as a Lyapunov function, we have:

$$\dot{V}_1 \leq -\lambda_{\min}(Q) \|E_1\|_2^2 + 2\lambda_{\max}(P) \gamma \|E_1\|_2^2, \quad (9)$$

Where $\lambda_{\max}(P)$ is the largest and $\lambda_{\min}(P)$ is the smallest eigenvalues of a definite positive matrix P . If $\gamma < \lambda_{\min}(Q) / 2\lambda_{\max}(P)$,

therefore $\dot{V} < 0$ and the main system is asymptotically stable. So ratio of $\lambda_{\min}(Q) / 2\lambda_{\max}(P)$, is very important and should be so big that it will repel the big disturbances. In fact, this ratio can be maximized by selecting $Q = I_{3n}$ [11]. If $Q = I_{3n}$ then $\lambda_{\min}(Q) = 1$ and $\lambda_{\max}(P) = 1 / (2\lambda_{\text{ift}}(-A_{n1}))$. If $\gamma < \lambda_{\text{ift}}(-A_{na})$ then the reduced system at the origin is asymptotically stable, and e_1, e_2, e_3 , all three converge to zero.

Since $s = c_1e_1 + c_2e_2 + c_3e_3 + e_4 = 0$ so e_4 converges to zero. By recalling $e_3 = f_1$ and assumption 3, it can easily be proved that after stabilization $e_1 = x_1, e_2 = x_2, e_3 = f_1$ both x_3 and x_4 converge to zero.

The switching level is defined as follows:

$$s = c_1e_1 + c_2e_2 + e_3$$

The constants c_i can be chosen according to the following matrix.

$$A_{n2} = \begin{bmatrix} 0 & I_n \\ -c_1I_n & -c_2I_n \end{bmatrix}$$

That's the A_{n2} matrix is Horvitz and $\max\{\bar{d}_1, \bar{d}_2\} < \lambda_{left}(-A_{n2})$. Equivalent control and switching control are as follows:

$$u_{eq} = - \left[\frac{\partial f}{\partial x_r} d \right]^{-1} \left[c_1x_r + c_2f_1 + \frac{\partial f}{\partial x_r} x_r + \frac{\partial f}{\partial x_r} f_1 + \frac{\partial f}{\partial x_r} x_r + \frac{\partial f}{\partial x_r} f_r \right], \quad (10)$$

$$u_{sw} = - \left[\frac{\partial f}{\partial x_r} b \right]^{-1} [M \text{sign}(s) + \lambda s]$$

Where:

$$M = (c_1\bar{d}_1 + c_2\bar{d}_2 + \beta_1\bar{d}_1 + \beta_2\bar{d}_2) \|E_2\|_2 + \beta_4(\bar{d}_3 + \bar{d}_4) \|\xi(x)\|_2 + \rho, \quad (11)$$

Slider mode control is a combination of u_{eq} and u_{sw} so we have

$$u = u_{eq} + u_{sw} \quad (12)$$

Theorem 2

If the matrix $\partial f_1 / \partial x_4$ is reversible, and in accordance with the control rule (12), all states of system (2) converge asymptotically to zero.

Proof: As in the proof of theorem (1) we can prove that the convergence to the sliding surface $S = 0$ will occur in a finite time, and in sliding manifold $S = 0$ or $e_3 = -c_1e_1 - c_2e_2$, the main system will be reduced to the following system.

$$E_2 = A_2E_2 + D_2, \quad (13)$$

That:

$$D_2 = [d_1, d] ^T, \quad \|D_2\|_2 < \max\{\bar{d}_1, \bar{d}_2\} \|E_2\|_2$$

Since A_2 is Horvitz and $\max\{\bar{d}_1, \bar{d}_2\} < \lambda_{left}(-A_{n2})$, infiltration system E_2 is asymptotically stable. So, $e_2 = x_2, e_1 = x_1$ will converge to zero. Since $e_3 = f_1 = 0$ and $s = 0$ using the assumption (3) we conclude that all states of the original system converge to zero.

4. TORA System

The TORA rotary motion oscillator system as a benchmarking system is used to evaluate the performance of various nonlinear controllers in [12], [3] and [13]. A general outline of the TORA system can be found in [3]. Let Z_1 be the normal displacement of the platform from the balance position. $\theta_1, z_2 = \dot{z}_1$ is the angle of the rotor, and $\theta_2 = \theta_1$. The dynamics of the TORA system is expressed as follows:

$$\begin{aligned} \dot{z}_1 = z_2, \quad \dot{z}_2 &= \frac{-z_1 + \varepsilon\theta_2^2 \sin \theta_1}{1 - \varepsilon^2 \cos^2 \theta_1} - \frac{\varepsilon \cos \theta_1}{1 - \varepsilon^2 \cos^2 \theta_1}, \\ \dot{\theta}_1 = \theta_2, \quad \dot{\theta}_2 &= \frac{\varepsilon \cos \theta_1 (z_1 - \varepsilon\theta_2 - \varepsilon\theta_2^2 \sin \theta_1)}{1 - \varepsilon^2 \cos^2 \theta_1} + \frac{1}{1 - \varepsilon^2 \cos^2 \theta_1} v \end{aligned} \quad (14)$$

Where v is input control and ε is a constant parameter that depends on the mass of the rotor platform. All values have been normalized to non-dimensional units. Using the following coordinates:

$$\begin{aligned} x_1 &= z_1 + \varepsilon \sin \theta_1, & x_3 &= \theta_1, \\ x_2 &= z_2 = \varepsilon \theta_2 \cos \theta_1, & x_4 &= \theta_2, \\ v &= \frac{\varepsilon \cos x_3 [x_1 - (1 + x_4^2) \varepsilon \sin x_3] + u}{1 - \varepsilon^2 \cos^2 x_3}, \end{aligned}$$

We are now able to change the TORA system to the following cascading form:

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -x_1 + \varepsilon \sin x_3 - 11\varepsilon x_3 + 11\varepsilon x_3, \\ \dot{x}_3 &= x_4, \\ \dot{x}_4 &= u. \end{aligned} \quad (15)$$

In the second equation, we introduce $-11\varepsilon x_3 + 11\varepsilon x_3$ in which $11\varepsilon x_3$ is the same disturbance as in assumptions 2 and 3. Then equation (15) can be expressed as equation (2). While:

$$\begin{aligned} f_1 &= -x_1 + \varepsilon \sin x_3 - 11\varepsilon x_3, & f_2 &= 0 \\ d_2 &= 11\varepsilon x_3, & d_3 &= 0 \end{aligned}$$

Notice that:

$$\begin{aligned} \partial f_1 / \partial x_3 &= \varepsilon \cos x_3 - 11\varepsilon \neq 0, \\ f_1(0, 0, x_3, x_4) &= 0 \end{aligned}$$

There is only one answer, and it is $x_3=x_4=0$. Therefore, assumptions 1 to 3 are observed. Since this system has only one disturbance, and $d_2 = 11\varepsilon x_3$ and $\partial f_1 / \partial x_1 = \partial f_1 / \partial x_2 = 0$. Assumptions 6 and 8 are true. It is clear that:

$$|\partial f_1 / \partial x_3| < 12\varepsilon$$

We remind that $e_1 = x_1$ and $e_3 = f_1 = x_1 + \varepsilon \sin x_3 - 11\varepsilon x_3$ so we have:

$$\begin{aligned} |d_2| &= |e_3 + x_1 - \varepsilon \sin x_3| \leq |e_3| + |e_1| + \varepsilon |x_3| \\ \Rightarrow 10\varepsilon |x_3| &\leq 2\|e\|_2 \quad \Rightarrow |d_2| = 11\varepsilon |x_3| \leq 2.2\|e\|_2. \end{aligned}$$

So the assumptions 6 and 8 are also well established. In the next section, the TORA system is simulated with the proposed controller in the simulink environment of MATLAB software, and then it will be shown that the proposed controller is capable of stabilizing the TORA system.

5. Numerical Simulations

The fig 1 shows simulator program written in the simulink environment of MATLAB Software. The blocks in this simulator are written in the environment of m-file programming. In the simulation done, five blocks have been designed that include: The first block is related to TORA robot system which its equations are mentioned in the previous section of this article. There is 4 modes in this equation, so the simulator block will also have four outputs. The second block is defined as sliding surfaces and the error signals generator. The third and fourth blocks are related to the entrance control, which is a combination of two U_{eq} and U_{sw} parts. The sum of these two parts gives us control system entrance which is sliding mode controller. The fifth block is related to given changed variable and is obtained θ and z variables through it. further to examine the controller capabilities discussed in the previous section, we will examine the effect of changing the value of each of the controller parameters.

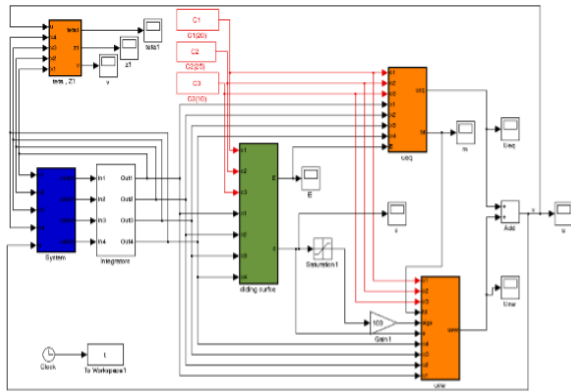


Fig.1. The Simulator

Analysis of effect parameter c_1 :

The nominal value of the parameter c_1 is selected 20. To examine its effect on the closed loop system response and control signal, we increased its quantities to %100 and then decreased its quantities to %100. According to Figures 2 and 3, choosing this nominal value is desirable because by its reduction to 10 fluctuation of plotted graphs are reduced while when it is increased to 30 fluctuation of plotted graphs are increased. So $C_1 = 20$ is more appropriate

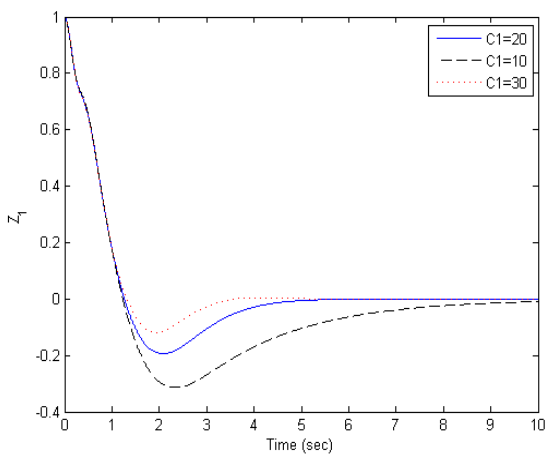


Fig.2. The normal displacement of the platform from the balance position: Z_1

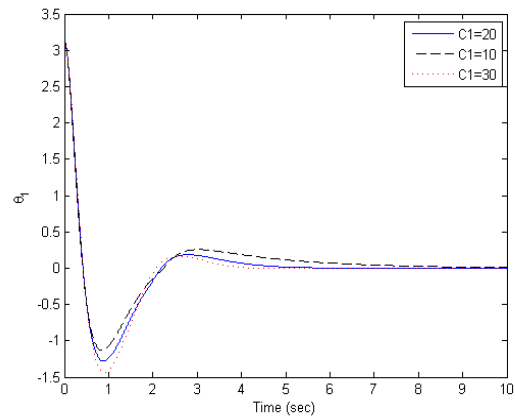


Fig.3. behavior of θ_1

Although the dynamic behavior of the sliding surface is appropriate for the value 30 according to Fig. 4. Fig. 5 and 6 are drawn U_{eq} and U_{sw} . Apparently, choosing value 30 for parameter C_1 is better because the signal control range is low. But according the behavior of θ_1 shown in Fig.3, the choice of nominal value 20 is conservative and appropriate. Sum result of the signals U_{eq} and U_{sw} is plotted in Fig. 7. The $v(t)$ is also presented in Fig. 8.

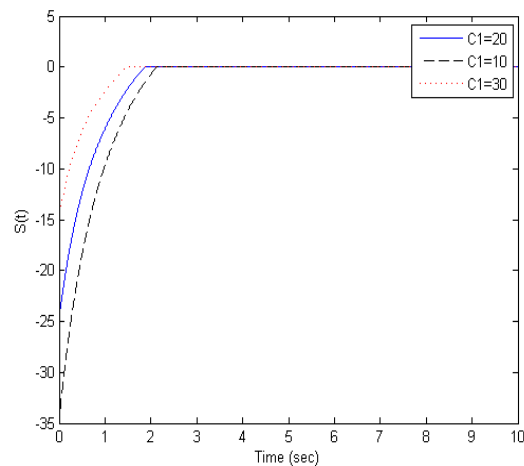


Fig.4. The dynamic behavior of the sliding surface: $S(t)$

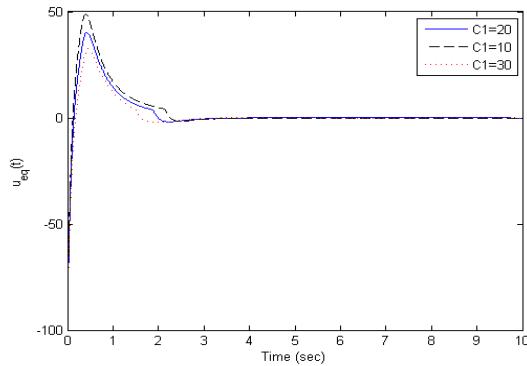


Fig.5. The equivalent control: U_{eq}

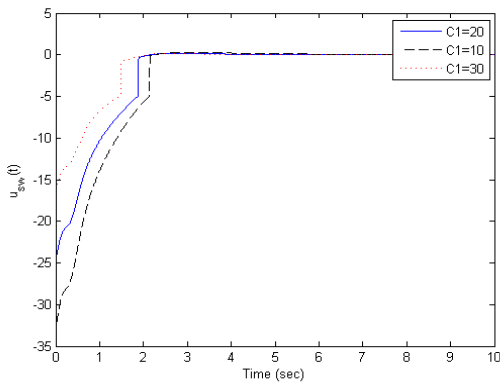


Fig.6. The control switch: U_{sw}

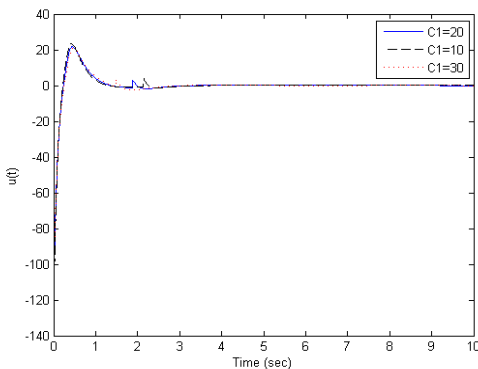


Fig.7. Control signal: $U(t)$

Analysis of effect parameter c_2 :

The nominal value of parameter c_2 is selected 25. To examine its effect on the closed loop system response and control signal, we increased its quantities to %60 and then decreased its quantities to %40.

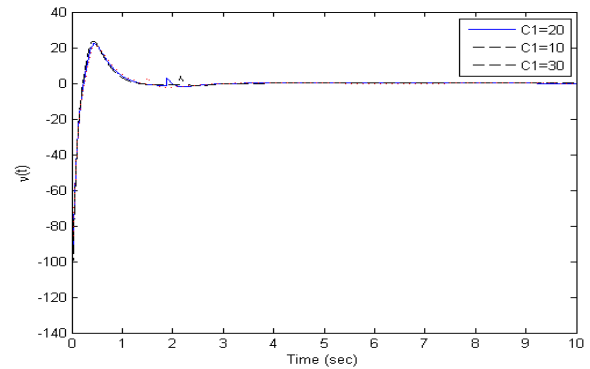


Fig.8. The input control: $v(t)$

The nominal value of parameter c_2 is selected 25. To examine its effect on the closed loop system response and control signal, we increased its quantities to %60 and then decreased its quantities to %40. Then according to figures 9 and 10, choosing this nominal value is desirable because while decreasing it to 15 fluctuations range are amplified although by decreasing it to 40 system response is slowed. So the selected nominal value is more suitable. However, according to figure 11 the dynamic behavior of sliding surface is equal for all values and doesn't show a significant difference. Graphs of U_{eq} and U_{sw} was drawn in figures 12 and 13. In figure 12 when parameter $c_2 = 40$, the U_{eq} signal control range is very high, when it decreases to 15 U_{eq} signal control range decreases too although no great difference is shown in graph U_{sw} . Decrease or increase in the U_{sw} signal control range makes increase the range of oscillations or slows down the charts plotted in figures 9 and 10. So choosing nominal value 25 seems to be appropriate. Sum result of the signals U_{eq} and U_{sw} is plotted in Fig. 14. The $v(t)$ graph is also presented in Fig. 15.

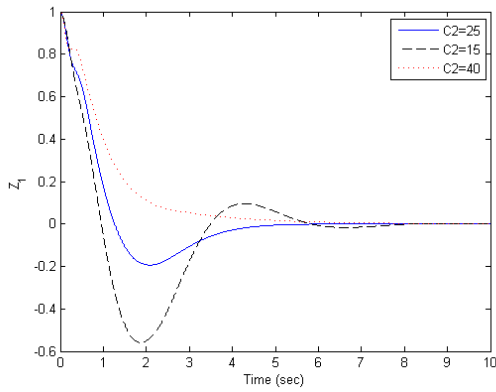


Fig.9. The normal displacement of the platform from the balance position: Z_1

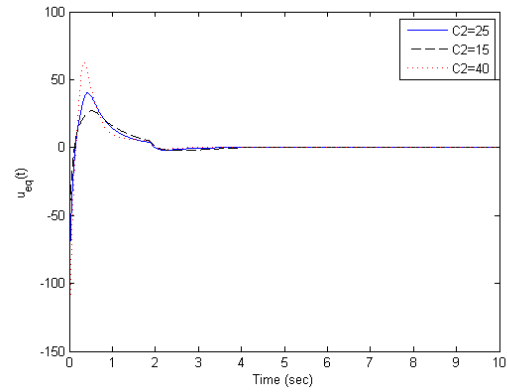


Fig.12. The equivalent control: U_{eq}

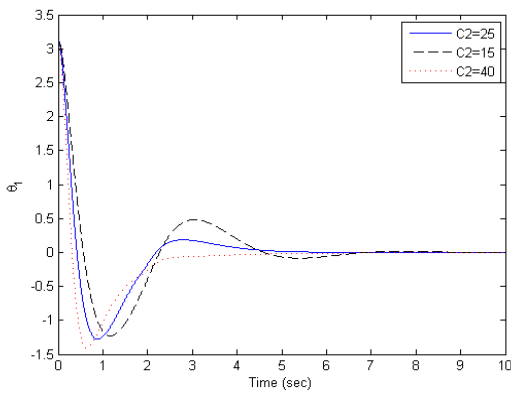


Fig.10. Behavior of θ_1

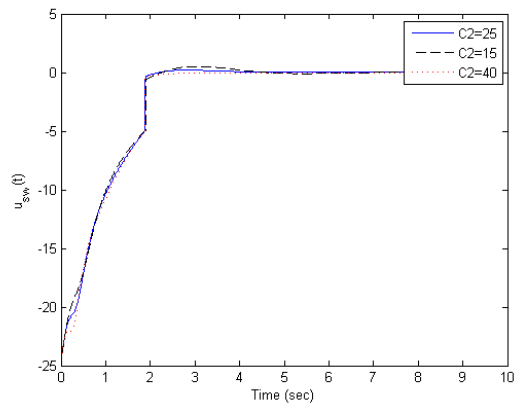


Fig.13. The control switch: U_{sw}

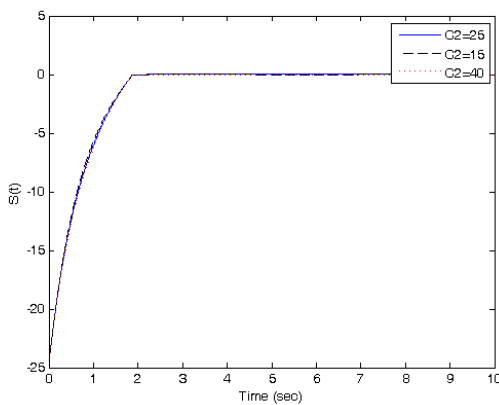


Fig.11. The dynamic behavior of the sliding surface: $S(t)$

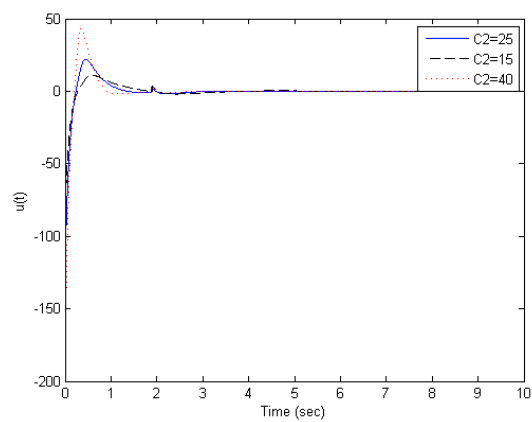


Fig.14. Control signal: $U(t)$

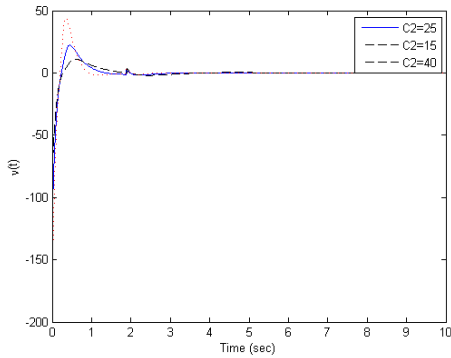


Fig.15. The input control: $v(t)$

Analysis of effect parameter c_3 :

The nominal value of c_3 is selected 10. For examine its effect on the closed loop system response and control signal, we increased its quantities to 50 % and then decreased its quantities to 50 %.

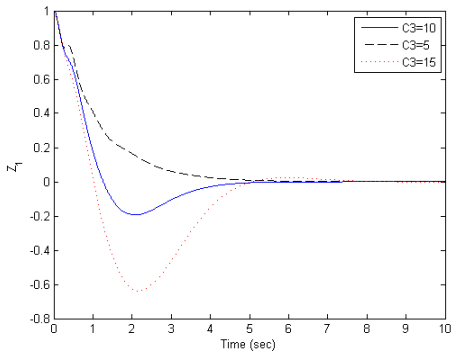


Fig.16. The normal displacement of the platform from the balance position: $Z1$

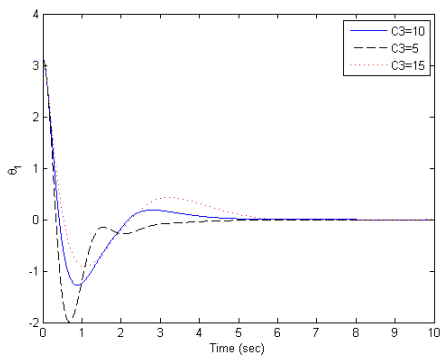


Fig.17. Behavior of θ_1

According to figures 16 and 17, choosing this nominal value is desirable, because by decreasing it to 5 the response of system is slowed while increasing it to 15 fluctuations range are amplified. So the selected nominal value ($c_3 = 10$) is more suitable. Although, the dynamic behavior of the slip surface for $C_3=10$ is more suitable according to fig.18. Due to slow response, value $c_3 = 10$ and greater is more suitable. Graphs are drawn for U_{sw} and U_{eq} in fig 19 and 20. According to these graphs, it seems that selecting the value 10 for C_3 is good because the behavior of U_{sw} and U_{eq} are better. Sum of the signals U_{sw} and U_{eq} that is $u(t)$ has been drawn in Fig. 21. The graph of $v(t)$ is also presented in Fig. 22.

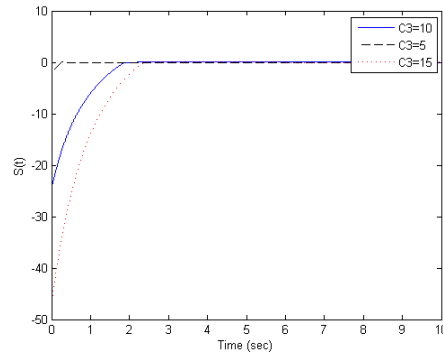


Fig.18. The dynamic behavior of the sliding surface: $S(t)$

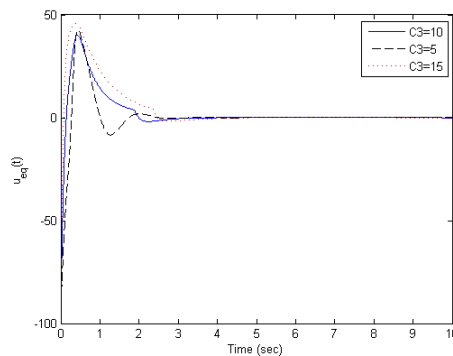


Fig.19. The equivalent control: U_{eq}

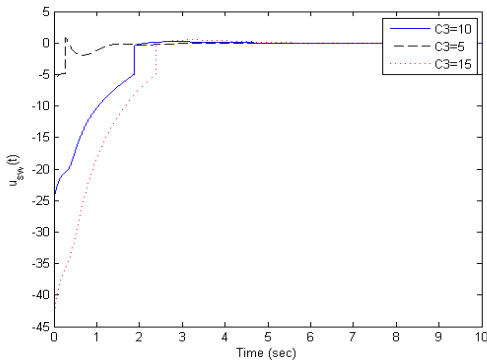


Fig.20. The control switch: U_{sw}

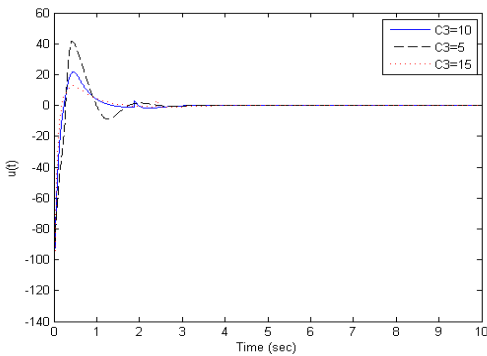


Fig.21. Control signal: $U(t)$

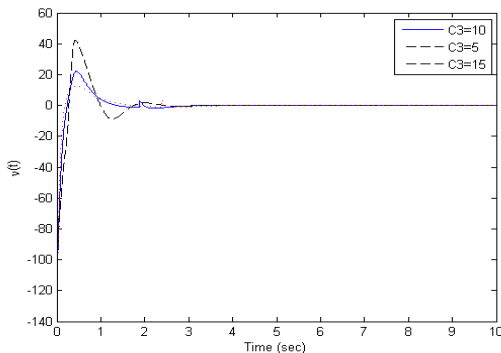


Fig.22. The input control: $v(t)$

6. Results

In this paper, the TORA robot was introduced as a class of underactuated nonlinear systems. Then, sliding mode-based control technique is introduced to investigate the stability of closed loop system, nonlinear

control system in conjunction with the proposed controller, is simulated in the environment Simulink MATLAB software. To investigate the effect of the controller on the behavior of the closed loop system, the control parameters were changed and the resulting graphs were plotted and then examined.

To continue the work, it is suggested to examine the robustness of the proposed control algorithm against the uncertainty and external disturbances entered into the robot. The part devoted to controller design includes fine points in adjusting controller gain and slider mode design coefficients, which makes it difficult to choose an optimally adjusts these coefficients and interest rates. It is suggested to use optimization techniques such as genetic algorithm to adjust these coefficients.

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