Optimized Computational Affine Image Algorithm Using Combination of Update Coefficients and Wavelet Packet Conversion

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Abstract

Updating Optimal Coefficients and Selected Observations Affine Projection is an effective way to reduce the computational and power consumption of this algorithm in the application of adaptive filters. On the other hand, the calculation of this algorithm can be reduced by using subbands and applying the concept of filtering the Set-Membership in each subband. Considering these concepts, the first step is to apply an adaptive algorithm in the packet wavelet transform domain in this Article, and then using multiple decimate and independent filters instead of an adaptive filter in the corresponding packages of this structure is suggested. Then the EFI image algorithm is applied to the corresponding packages of this structure, and the three methods of data selector based on the Set-Membership filtering concept, the method for selecting the optimal coefficients for the update, and the observation selector method in the same time in all corresponding packets in the AFin algorithm apply. The simulation results show the efficiency of the proposed algorithm in terms of the convergence rate and the reduction of computational complexity.

Keywords: Update Selection Coefficients, Data Selector, Set-Membership Filtering, Packet Wavelet

1. Introduction

Adaptive filters have been used extensively in various fields such as telecommunications, control, sonar, radar, acoustic and speech processing. The least-squares mean (LMS) algorithms and the least-average normalized squares (NLMS) are widely used in engineering applications due to their low complexity and low computational volume. Unfortunately, for the color data, the convergence rate of these algorithms is low [1]. For high-correlation input signals, the AP algorithm shows a high convergence rate relative to the LMS algorithms and NLMS algorithms. However, the computational complexity of the AP algorithm is one of the weaknesses of this algorithm. In [2,3] To overcome the problem of computational complexity and increase the convergence rate, the subband processing technique is based on an adaptive algorithm. Also, in recent years, multi-cycle filtering technique has also been used to analyze sub-bands [4]. In the processing of subbands using comparative filters, not only the amount of computations can be reduced, but the convergence rate also increases [6-4]. In addition, in [7], provided an efficient and effective structure using packet wavelet transforms to apply any type of matching algorithm. This structure increases the convergence rate of algorithms almost without any additional calculations. To reduce the complexity of the computational algorithms, several algorithms have been proposed with updating the selected coefficients [11-8]. In these algorithms, only the optimal subset of all coefficients is updated in succession repeats, and thus can greatly reduce the computational complexity.

The computational complexity of the algorithms increases the power consumption and the need for high memory, resulting in high costs imposed on the system [11, 8]. Another effective approach to reduce the complexity of the computational algorithms is to apply Set-Membership filtering to comparative filters [11]. This approach, while reducing average computing, also increases the convergence rate of the algorithm. In [12], an AP-specific algorithm is presented in which part of the input selective observation is involved in the update equation for each occurrence. In this algorithm, the optimization is not performed on the coefficients and all the coefficients in each repetition of the algorithm are updated.

The proposed algorithm proposed in this Article, in addition to the advantages of converting wavelet packets, uses three techniques to reduce computations simultaneously: First, reduce the computational complexity caused by the optimal coefficient selector method, and the second is the scattered update using the Set-Membership along with the variable step coefficient and the third-best visitor selection method.

The process of this article is as follows: Section 2 summarizes AP, SCU-AP [13], Set-Membership Filtering Techniques (SM)] 11 [describes the SM-AP algorithm] 14,1 [and the SR-AP algorithm] 12 [. The proposed algorithm, WP-SM-SR-SCU-AP, will be presented in Section 3. The computational complexity of the proposed algorithm is discussed in Section 4. The simulation results, which show the performance and computational complexity of the algorithm, are presented in Section 5. In Section 6, the results are briefly summarized.

2. Foreground Topics

2-1. AP algorithm

The optimal signal d (k) is assumed to be:

$$d(k) = \mathbf{u}^T(k)\boldsymbol{w} + v(k) \tag{1}$$

Where v (k) is the noise indicator, w is an unknown column vector that should be estimated. Also, u (k) is the input vector M × 1 as u (k) = [u (k), u (k-1), ..., u (k-M + 1)] T.

Using the LMS algorithm, the following recursive formula can easily be obtained for the AP algorithm [12].

$$\boldsymbol{w}(k) = \boldsymbol{w}(k-1) + \mu \boldsymbol{U}^*(k)(\boldsymbol{U}(k)\boldsymbol{U}^*(k))^{-1}\boldsymbol{e}(k) \tag{2}$$

Where e (k) = d (k) -UT (k) w (k-1) is an error vector and U (k) = [u (k), u (k-1), ..., u (k-K + 1)] matrix M × K and the desired signal vector as d (k) = [d (k), d (k-1), ..., d (k-K + 1)] T.

2-2. Updates the selected coefficients of the AP algorithm

In this algorithm, the input vector u (k) and vector of coefficients associated with the filter are divided into P blocks with L = M / P lengths as follows:

$$\mathbf{u}(k) = \left[\mathbf{u}_1^T(k),\ \mathbf{u}_2^T(k),\ ...,\mathbf{u}_P^T(k)\right]^T$$

And: $\boldsymbol{w}(k) = \left[\boldsymbol{w}_1^T(k),\ \boldsymbol{w}_2^T(k),\ ...,\boldsymbol{w}_P^T(k)\right]^T$

The optimization problem with multiple constraints is considered as follows:

$$\min_{1\le i\le P}\ \min_{w_i(k+1)} \left\| \boldsymbol{w}_i(k+1)-\boldsymbol{w}_i(k) \right\|_2^2 \qquad (3)$$

The condition is also applied with limitation 1≤k≤L for solving equation (3).

For each block, a value function is defined as the following equation.

$$J_i(k)=\left\| \boldsymbol{w}_i(k+1)-\boldsymbol{w}_i(k) \right\|_2^2+\boldsymbol{\lambda}^T(\mathbf{d}(k)-\boldsymbol{U}^T(k)\boldsymbol{w}(k+1))$$

In which $\boldsymbol{\lambda}=[\lambda_1 \quad \lambda_2 \quad ... \quad \lambda_K]$ is a vector of Lagrange coefficients. By solving the equation, we obtain the relation (4).

$$2(\boldsymbol{w}_i(k+1)-\boldsymbol{w}_i(k))=\boldsymbol{U}_i(k)\boldsymbol{\lambda} \qquad (4)$$

Where is it

$$\boldsymbol{U}_i(k)=\left[\mathbf{u}_i(k) \quad \mathbf{u}_i(k-1) \quad ... \quad \mathbf{u}_i(k-K+1)\right]_{L\times K}$$

By multiplying the sides of relation (4) in the matrix and assuming that the matrix has a perfect order size, the relation (5) is obtained as follows:

$$\boldsymbol{\lambda}=2(\boldsymbol{U}_i^T(k)\boldsymbol{U}_i(k))^{-1}\boldsymbol{U}_i^T(k)(\boldsymbol{w}_i(k+1)-\boldsymbol{w}_i(k)) \qquad (5)$$

By placing the relation (5) in (4) and using the restriction (3), the relation (6) will be obtained:

$$\boldsymbol{w}_i(k+1)=\boldsymbol{w}_i(k)+\boldsymbol{U}_i(k)(\boldsymbol{U}_i^T(k)\boldsymbol{U}_i(k))^{-1}\boldsymbol{e}(k) \qquad (6)$$

Where $\boldsymbol{e}(k)=\boldsymbol{U}^T(k)\boldsymbol{w}(k)$ the error vector is dimensioned. After entering the positive and small step coefficient, the AP algorithm is obtained with a constant block for updating as follows:.

$$\boldsymbol{w}_i(k+1)=\boldsymbol{w}_i(k)+\mu\boldsymbol{U}_i(k)(\boldsymbol{U}_i^T(k)\boldsymbol{U}_i(k))^{-1}\boldsymbol{e}(k) \qquad (7)$$

The blocks with the smallest Euclidean square squares in successive repetitions, similar to Equation (3), are calculated as follows and selected for updating, which can be obtained according to (6).

$$\begin{aligned} i &= \arg\min_{1\le j\le P} \left\| \boldsymbol{w}_j(k+1)-\boldsymbol{w}_j(k) \right\|_2^2 \\ &= \arg\min_{1\le j\le P} \boldsymbol{e}^T(k)(\boldsymbol{U}_j^T(k)\boldsymbol{U}_j(k))^{-1}\boldsymbol{e}(k) \end{aligned}$$

The optimization problem (3) can also be generalized to several blocks rather than a block. This process leads to the update of the AP (SCU-AP) coefficients update algorithm as follows:

$$\begin{aligned} &\boldsymbol{w}_{I_B}(k+1)=\boldsymbol{w}_{I_B}(k)+\mu\boldsymbol{U}_{I_B}(k)(\boldsymbol{U}_{I_B}^T(k)\boldsymbol{U}_{I_B}(k))^{-1}\boldsymbol{e}(k) \\ &I_B=\arg\min_{I_B\in S} \boldsymbol{e}^T(k)(\sum_{I_B\in S}\boldsymbol{U}_{I_B}^T(k)\boldsymbol{U}_{I_B}(k))^{-1}\boldsymbol{e}(k) \end{aligned} \qquad (8)$$

In relation (8), S contains all of the subsets of B. Using equation (8) is massively computable. Because choosing a subset of B is complex with this criterion. To reduce the computational complexity of relation (8), it is necessary to simplify the relationship. Assuming that the diameter of the matrix of its non-trivial elements is much larger, one can ignore the non-essential elements. This approach is similar to the simplification of the algorithm in reference [10]. Therefore, the expression to be minimized in (8) can be approximated as a relation (9):

$$\boldsymbol{e}^T(k)(\sum_{I_B\in S}\boldsymbol{U}_{I_B}^T(k)\boldsymbol{U}_{I_B}(k))^{-1}\boldsymbol{e}(k)\approx\sum_{I_B\in S}\frac{\left\|\boldsymbol{e}(k)\right\|^2}{\left\|\boldsymbol{U}_{I_B}(k)\right\|^2} \qquad (9)$$

2-3. Set-Membership Affine Projection (SM-AP) algorithm

The update equation for the SM-AP algorithm [14, 1] is as follows:

$$w(k+1)=\begin{cases} w(k)+\mu U^*(k)(U(k)U^*(k))^{-1}e(k)\mathbf{u}_1 & if\ \left|e(k)\mathbf{u}_1\right|>\gamma \\ w(k) & otherwise \end{cases} \qquad (10)$$

Where is it $\mathbf{u}_1=[1,0,0,...,0]_{K\times 1}^T$

3. Selective Regressor Affine Projection (SR-AP) algorithm

Suppose that we choose Q from the observation L in the AP algorithm and use the smaller AP matrices in the AP update algorithm [12]. The update equation for this algorithm will be as follows.

$$\boldsymbol{w}_i(k+1) = \boldsymbol{w}_i(k) + \boldsymbol{U}_{i,\tau_Q}(k)(\boldsymbol{U}^T_{i,\tau_Q}(k)\boldsymbol{U}_{i,\tau_Q}(k))^{-1}\boldsymbol{e}_{i,,\tau_Q}(k) \qquad (11)$$

An optimal set of observational selections will also be obtained through the criterion (12).

$$\tau_Q^{opt}(\mathrm{k}) = \arg\max_{\tau_Q \in S} \boldsymbol{e}^*_{i,\tau_Q}(\mathrm{k})\left(\boldsymbol{U}_{i,\tau_Q}(\mathrm{k})\boldsymbol{U}^*_{i,\tau_Q}(\mathrm{k})\right)^{-1}\boldsymbol{e}_{i,\tau_Q}(\mathrm{k}) \qquad (12)$$

Finally, the equation for updating the SR-AP algorithm with optimal observations is given by equation (13).

$$\boldsymbol{w}_i(\mathrm{k}) = \boldsymbol{w}_{i-1}(\mathrm{k}) + \mu\boldsymbol{U}^*_{i,\tau_Q^{opt}}(\mathrm{k})\left(\boldsymbol{U}_{i,\tau_Q^{opt}}(\mathrm{k})\boldsymbol{U}^*_{i,\tau_Q^{opt}}(\mathrm{k})\right)^{-1}\boldsymbol{e}_{i,\tau_Q^{opt}}(\mathrm{k}) \qquad (13)$$

To evaluate the SR-AP algorithm $r = Q/L$, a criterion is defined that indicates the ratio of the observed observations to the total observations. For the WP-SCU-AP algorithm with j-level parsing, we want to update Block B of the block P. The set of B-block indices determines the block P. At this point we consider the optimization problem with a set of constraints as expressed in the equation:

$$\mathbf{d}_{l,D}(q) = \boldsymbol{U}^T_{l,I_B}(q)\boldsymbol{w}_{I_B}(q+1) \qquad l = 0,1,\ldots,2^j - 1$$

Using the Lagrangian coefficients method [6] again, for the proposed criterion, the return relation for updating the coefficients in the WP-SCU-AP algorithm is obtained as the following equation.

$$\boldsymbol{w}_{l,I_B}(q+1) = \boldsymbol{w}_{l,I_B}(q) + \mu\boldsymbol{U}_{l,I_B}(q)(\boldsymbol{U}^T_{l,I_B}(q)\boldsymbol{U}_{l,I_B}(q))^{-1}\boldsymbol{e}_{l,D,I_B}(q) \qquad (14)$$
$$l = 0,1,\ldots,2^j - 1$$

$$\boldsymbol{U}_{l,I_B}(q) = \left[\boldsymbol{U}^T_{l,1}(q) \quad \boldsymbol{U}^T_{l,2}(q) \quad \ldots \quad \boldsymbol{U}^T_{l,B}(q)\right]^T \qquad (15)$$

and $\boldsymbol{e}_{l,D}(q) = \mathbf{d}_{l,D}(q) - \boldsymbol{U}^T_{l,I_B}(q)\boldsymbol{w}_{l,I_B}(q)$

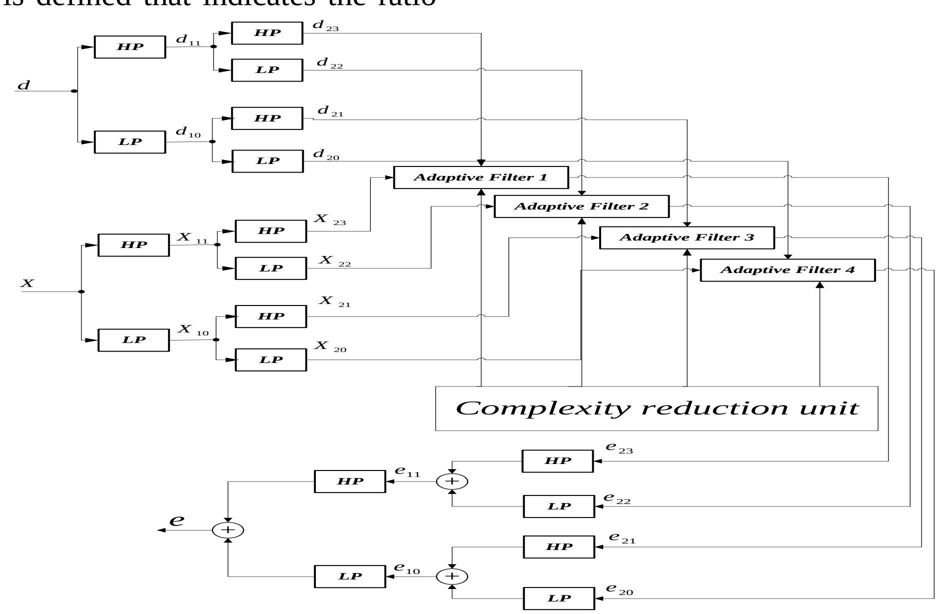

Fig.1. The proposed structure of applying adaptive filters in the domain of packet wavelet transform

Now consider the optimization for the 'B' block and then the criteria for selecting the blocks are computed. In the following, the selected blocks are updated at the output of each pair of filters in the same repeat. Using the criteria below, the blocks that must be selected in each sub-row and each iteration are updated.

$$l, I_B = \arg\min_{I_B} \left\| \mathbf{w}_{l,I_B}(q+1) - \mathbf{w}_{l,I_B}(q) \right\|_2^2 \quad , \quad l = 0,1,\ldots,2^j - 1 \qquad (16)$$

Relationship (16) is solved with the aid of (14) and (15) and yields the following relation:

$$l, I_B = \arg\min_{I_B} \mathbf{e}_{l,D,I_B}^T \Big(\sum_{l,I_B \in S} \mathbf{U}_{l,I_B}^T(q)\mathbf{U}_{l,I_B}(q) \Big)^{-1} \mathbf{e}_{l,D,I_B}(q) \quad , \quad l = 0,1,\ldots,2^j - 1 \qquad (17)$$

Relationships (17) can be approximated in the same way as the assumptions made in the previous section: Equation (18):

$$l, I_B = \mathbf{e}_{l,D,I_B}^T \Big(\sum_{l,I_B \in S} \mathbf{U}_{l,I_B}^T(q)\mathbf{U}_{l,I_B}(q) \Big)^{-1} \mathbf{e}_{l,D,I_B}(q) \approx \sum_{l,I_B \in S} \frac{\left\| \mathbf{e}_{l,D,I_B}(q) \right\|^2}{\left\| \mathbf{U}_{l,I_B} \right\|^2} \quad for \quad l = 0,1,\ldots,2^j - 1 \qquad (18)$$

By applying the Set-Membership filtering in each subband with variable step coefficient and assuming that the best-looking chooser's method is also applied after the data selector method, the following update equation for the proposed WP-SM-SR- SCU-AP will have:

$$\begin{aligned} &\mathbf{w}_{l,I_B}(q+1) = \\ &\quad \mathbf{w}_{l,I_B}(q) + \alpha_{l,q} \mathbf{U}_{l,I_B,\tau_Q^{opt}}(q) \times \\ &\qquad (\mathbf{U}_{l,I_B,\tau_Q^{opt}}^T(q)\mathbf{U}_{l,I_B,\tau_Q^{opt}}(q))^{-1} \mathbf{e}_{l,D,I_B,\tau_Q^{opt}}(q).\mathbf{u}_1 \end{aligned} \qquad (19)$$

Where the parameter $\alpha_{l,q}$ is defined as follows.

$$\alpha_{l,q} = \begin{cases} 1 - \gamma \Big/ \left| \mathbf{e}_{l,D,I_B,\tau_Q^{opt}}(q)\mathbf{u}_1 \right| & if \ \left| \mathbf{e}_{l,D,I_B,\tau_Q^{opt}}(q)\mathbf{u}_1 \right| > \gamma \\ 0 & otherwise \end{cases}$$

$$l = 0,1,\ldots,2^j - 1$$

In general, in the proposed algorithm, the set-membership filtering technique is used independently at the output of each of the corresponding vowels packets, and then, if needed, to update the coefficients, the best blocks are selected in a predetermined amount

4. Computational Complexity

Calculations of the SCU-AP algorithm are calculated based on blocked matrices. Considering the amount of additional calculations for the NSAF-NLMS algorithm in reference [6], due to NSAF, we note that the NSAF-AP algorithm is 2N × k × K multiplication for the analysis section of the input signal and the desired signal. Also, this algorithm requires a multiplication factor of N × k to synthesize the first line of the error vector. Therefore, the NSAF algorithm requires an additional multiplication (2NkK + Nk multiplication) for applying bank filters in comparison with the usual algorithm. For a WP item with j level, the parity is N equal to For example, if Shannon wavelet filter coefficients (db1) are used, the k value (length of filters) will be 2.Table 1 shows the computational complexity of the computational complexity of the proposed WP-SM-SCU-AP algorithm with j decompositional level. This table shows the amount of computation that the entire sub-band can get an update. If the membership filtering does not allow the update of the coefficients, then the calculation in the corresponding sub-band in that repeat will be zeroed and only a comparison of the threshold of the error occurs.

Table 1. The computational complexity of the proposed WP-SM-SCU-AP algorithm for when the update is performed in all sub-bands simultaneously.

	WP-SM-SCU-APA (j level decomposition)	
	Computations for weight update	Additional Computations
Multiplications	$(K^2+K)BL+KM+K^3+K^2$	$(1-r)K^2L+2^{(j+1)}kK+2^jk$
Divisions	-	$P+2^j$
Comparisons	-	$Plog_2B+O(P)+2^j$

Table 2. The computational complexity of the proposed WP-SM-SR-AP algorithm for when the update occurs in all sub-bands at the same time.

	WP-SM-SR-APA (j level decomposition)	
	Computations for weight update	Additional Computations
Multiplications	$r(rL^2+2L)M+r^2(rL^3+L^2)$	$(1-r)LM$ $+L+1+2^{(j+1)}kK+2^jk$
Divisions	-	$L+2^j$
Comparisons	-	$Llog_2rL$ $+O(L)+2^j$

In the proposed algorithm, provided that the filtering of the membership set authorizes the update, the calculation will be reduced in two steps in accordance with Tables (1) and (2). This decline will naturally be dramatic. If the filtering does not allow the update in the subroutine to be updated, the amount of computation in the substrate in that repetition of the algorithm will be zero. The use of variable step coefficient and dependent on the variance of noise in the proposed algorithm as well as its application in the structure of packet wavelet transform, compensates for the slight decrease in the convergence rate of the algorithm due to the application of simultaneously applied SR and SCU methods and converges the algorithm's rate of convergence Increases

5. Simulation Results

In this section, the WP-SM-SR-SCU-AP algorithm is compared and evaluated based on the problem of identifying an unknown system with similar algorithms. In this simulation, we consider an unknown system based on a filter with a finite length of FIR of order M = 32. It should be noted that in the discrete wavelet decomposition structure, the input signal will be decimated at each stage after passing through the corresponding banks to the level j. Therefore, the K value in the AP algorithm should be smaller than in the packet wavelet structure. In this model, the input signal u (k) is filtered through a Gaussian random sequence with a mean of zero through a first-order recursive system with a loss factor of λ = 0.8 and with the relation u (k)

= 0.8u (k-1) + v (k) is obtained. To do fair comparison, the step coefficient in the filters is chosen so that the state of the state of the error error is approximately the same. To obtain learning curves, all simulations have been repeated 200 times and the average of these repetitions has been plotted. The accumulated noise variance and the error threshold in each subband are selected. The AP degree in this simulation is K = 4.

In Figs (2) to (4), in cases where the curve is related to the proposed algorithm, 4 filters are used. In the tables of parameters, P represents the number of blocks and B1 to B4 corresponds to the number of selected blocks associated with the SCU method in each filter. In conventional methods, a filter is used and the number of selected blocks is represented by B. Also, r in all curves represents the ratio of the observed observations in the AP algorithm. In most learning curves, as can be seen, the convergence rate of curves is higher than that of competing algorithms, despite the further reduction in computations due to the use of computational methods. Also, the average number of updates in subclasses related to the proposed algorithms arising from the use of membership filtering in three forms is given below.

In Fig. 2, for the curves (a), (b), (c) and (e), the average number of updates for the WP-SM-SR-SCU-AP algorithm is respectively (511, 496, 442, 385), (547, 502, 486, 401), (563, 523, 491, 398) and (588, 524, 477, 407) instead of 1125 (with two levels of vector decomposition The data is changed due to the decimation from 4500 to 1125, in this set of numbers, the first is attributed to the first and second, third, and fourth matching adaptive filters, respectively, to the second, third, and fourth matching filters. Fig. 3 For the curves (a), (b) and (c) the average number of updates for the WP-SM-SR-SCU-AP algorithm (452, 406, 375, 378), (485, 429 , 432, 404) and (498, 457) , 403, 398) instead of 1125. In this set of numbers, the first is attributed to the first and second, third, and fourth matching adaptive filters, respectively, to the second, third, and fourth matching filters. In Fig. 4, for The curves (a), (b), (c), (e) and (f) the average number of updates for the WP-SM-SR-SCU-AP algorithm (504, 490 501, 430), (492, 485, 431, 406), (557, 502, 506, 435), (601, 550, 542, 491) and (609, 613, 552, 493) instead of 1125 , In this set of numbers, the first corresponds to the first, second, third, and fourth matching filters, respectively, of the second, third, and fourth matching filters. Tables (3) to (5) show the parameters for the curves (2) to (4).

Conclusion

In this Article, a new method is proposed combining set-membership filtering data selector methods and optimal coefficient and observation choices in a packet wavelet-based structure for an AP algorithm. In this way shorter adaptive filters are updated independently and independently in each corresponding subbond pair. The proposed algorithm not only greatly reduces the computational complexity, but in some cases, the convergence rate of the algorithm is considerably increased. The proposed algorithm utilizes 3 degrees of freedom and simultaneously reduces the computations and can be very useful in practical applications.

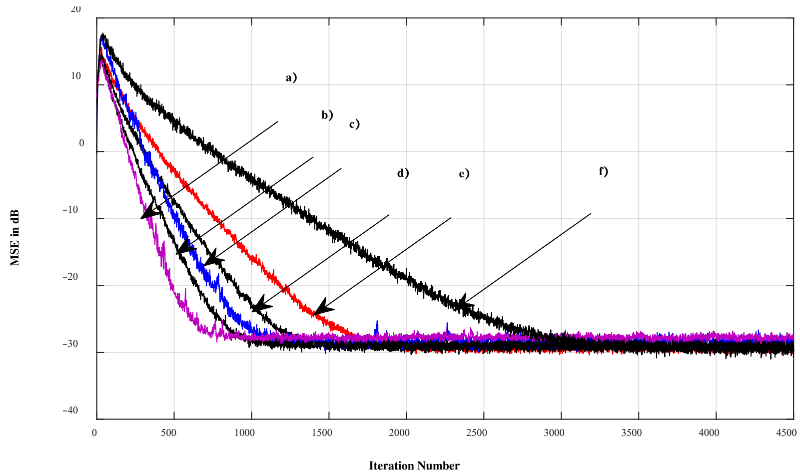

Fig.2. Comparison of learning curves of family-couple algorithms WP-SM-SR-SCU-AP proposed algorithm.

Table.3. Parameters and algorithms associated with simulated shapes in fig. 2.

Curve	Parameters and the name of the curve algorithm
a	WP-SM-SR-SCU-AP, r=0.75, P=4, B1=4, B2=B3=B4=3
b	WP-SM-SR-SCU-AP, r=0.75, P=4, B1=4, B2=B3=B4=2
c	WP-SM-SR-SCU-AP, r=0.5, P=4, B1=4, B2=B3=B4=2
d	APA
e	WP-SM-SR-SCU-AP, r=0.25, P=4, B1=4, B2=B3=B4=2
f	SCU-SR-AP, r=0.25, P=4, B=2

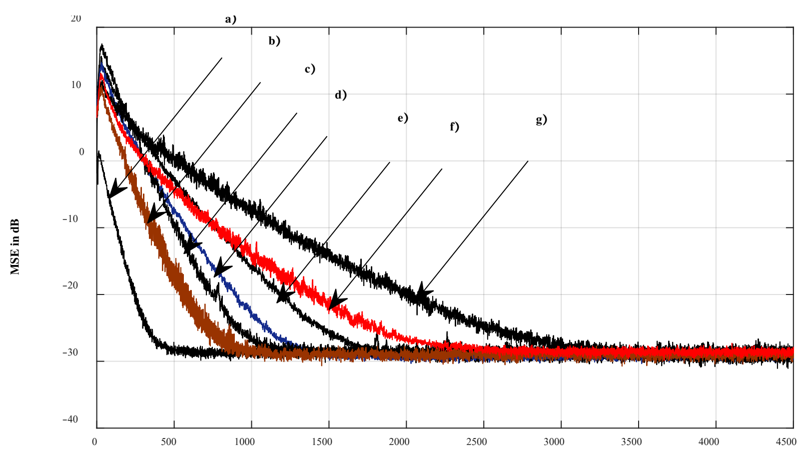

Fig.3. Comparison of learning curves of family-couple algorithms WP-SM-SR-SCU-AP proposed algorithm.

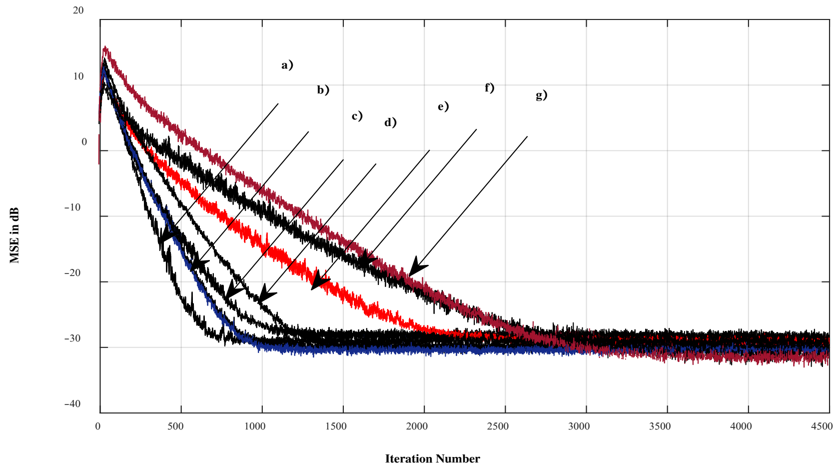

Fig.4. Comparison of learning curves of family-couple algorithms WP-SM-SR-SCU-AP proposed algorithm.

Table.4. Parameters and algorithms associated with simulated shapes in Figure (3)

	Parameters and the name of the curve algorithm
a	WP-SM-SR-SCU-AP, r=1, P=4, B1=B2=B3=B4=4
b	WP-SM-SR-SCU-AP, r=0.5, P=4, B1=B2=B3=B4=4
c	WP-SM-SR-SCU-AP, r=0.5, P=4, B1=B2=B3=B4=3
d	APA
e	SCU-AP, P=4, B=2
f	SR-AP, r=0.5
g	SR-SCU-AP, r=0.25, P=4, B=3

Table.5. Parameters and algorithms associated with simulated shapes in Fig. 4 Parameters and the name of the curve algorithm

curve	Parameters and the name of the algorithm
a	WP-SM-SR-SCU-AP, r=0.75, P=4, B1=4, B2=B3=B4=3
b	WP-SM-SR-SCU-AP, r=0.75, P=4, B1=4, B2=B3=B4=4
c	WP-SM-SR-SCU-AP, r=0.5, P=4, B1=4, B2=B3=B4=3
d	APA
e	WP-SM-SR-SCU-AP, r=0.25, P=4, B1=4, B2=B3=B4=3
f	WP-SM-SR-SCU-AP, r=0.5, P=4, B1=4, B2=B3=B4=2
g	SCU-AP, P=4, B=1

References

[1] M. Z. A. Bhotto, A. Antoniou, "A set-affiliated projection algorithm with adaptive error bound," IEEE Conference on Electrical and Computer Engineering, pp. 894 - 897, May 2009.

[2] A. Gilloire, M. Vetterl, "Adaptive Filtering in Subbands with Critical Sampling: Analysis, Experiments, and Application to Acoustic Echo Cancellation," IEEE Trans. Signal Processing, vol. 40 pp. 1862-1875, Aug. 1992

[3] M. R. Petraglia, D. B. Haddad, and E. L. Marques, "Affine Projection Subband Adaptive Filter with Low Computational Complexity". IEEE Transactions on Circuits and Systems, Part II, Expression briefs, pp. 1-5, IEEE 2016.

[4] A.O. Abid Young, S.A. Samad and A. Hussain, "Improved, Low Complexity Noise Cancellation Technique for Speech Signals," World Applied Sciences Journal, p. 272-278, 2009.

[5] KA Lee, W. S. Gan, "Improving the Convergence of the NLMS Algorithm Using Constrained Subband Updates," "IEEE Signal Process. Lett., Vol. 11, no. Sep 9th 2004

[6] KA Lee, W. S. Gan and Y. Wen, "Subband adaptive filtering using a multiple-constraint optimization criterion," in Proc. EUSIPCO, pp. 1825-1828, 2004.

[7] J. Rasi and B. M. Tazehkand, "A New Efficient and Fast Adaptive Filtering Structure Based on Wavelet Packet Transform," in Proc. 6th International Symposium on Telecommunications (IST), Iran, pp. 334-337, 2012.

[8] K. Dogançay, O. Tanrikulu, "Adaptive Filtering Algorithms with Selective Partial Updates," IEEE Trans. Circuits Syst. II, Analog Digit. Signal Process., Vol. 48 no. 8, pp. 762-769 Aug. 2001

[9] S. Attallah, "The wavelet transform-domain, adaptive filter with partial subband-coefficient updating," IEEE Trans. Circuits Syst. II, Exp. Briefs, vol. 53, no. 1 pp. 8-12, Jan. 2006

[10] P. A. Naylor and A. W. H. Khong, "Affine Projection and Recursive Least Squares", Adaptive Filters, Employing Partial Updates, "IEEE Conference on Signals, Systems and Computers, vol. 1 pp. 950-954, 2004.

[11] S. Werner, M. L. R. de Campos and P. S. R. Diniz, "Partial-Update NLMS Algorithms with Data-Selective Updating," IEEE Transactions on Signal Processing, Vol. 52, no. 4, pp. 938-949, April 2004.

[12] K. Y. Hwang, W. J. Song, "An Affine Projection Adaptive Filtering Algorithm with Selective Regressors," "IEEE Trans. Circuits And Systems-II: Express Briefs, Vol. 54, No. 1 pp. 43-46, Jan. 2007

[13] K. Dogancay, O. Tanrikulu, "Selective-partial-update of NLMS and Affine Projection Algorithms for Ecoustic Echo Cancellation," IEEE International Conference on Coustics, Speech, and Signal Processing, vol.1, pp. 448-451, 2000.

[14] S. Werner, P. S. R. Diniz, "Set-Membership Affine Projection Algorithm," IEEE Signal Processing Letters, vol. 8