Shiraz Journal of System Management
Vol 3, No. 3, Ser. 11 (Year 2015), 077-090

Dynamic Facility Location with Stochastic Demand

Ali Gholinejad Devin
Sadjad institution of higher education, Mashhad, Iran

Katayun Abedzade Ghuchani
Sadjad institution of higher education, Mashhad, Iran

Reza Sadeghi
Imam Reza international university, Mashhad, Iran

Hamidreza Koosha
Department of Industrial Engineering,
Ferdowsi University of Mashhad, Mashhad, Iran

Abstract. Determination of facilities, such as factories or warehouses, location and availability conditions is one of the important and strategic decisions for an organization to make. Transportation costs that form a major part of goods price are dependent to this decision making. There are verity of methods have been presented to achieve the optimal locations of these facilities which are generally deterministic. In real world accurate estimation of the effective parameters on this optimal location for single or multiple time periods is difficult and merely impossible.In this research, we try to achieve an efficient model with consideration of uncertainty demand over different time periods on the basis of previously presented models that we call stochastic dynamic facilities location problem. In order to do so we use stochastic constrain programming which convert the stochastic model to a deterministic one. According to this conversion we can use the cutting edge standard algorithm and methods of programing to solve the model. Next we implement the presented model for a case study and solve the problem with MATLAB software and examine the efficiency of model. Implementation the presented model and analyzing the results cause better management and better control of changing facility status

Keywords: Location Problem, Stochastic Dynamic Location, Stochastic Modeling, Stochastic Constrains, Stochastic Demand.

1. Introduction

We have some facilities for supplying demand points with stochastic amount, generally to Minimize total costs as the objective function in stochastic facility location problem. Facilities service levels are deterministic from the beginning and modeled with stochastic constrains. Since the demand is stochastic in this model and we cannot determine the value of demand in any time, Demands are divided in to two groups' normal distribution and Poisson distribution on the basis of previous information.

1. Deterministic problem for demand with Poisson distribution
2. Deterministic non-linear integer problem for Normal distribution

In stochastic facility location problem with uncertain demands, the aim is to reach to the optimal facility allocation, supply demand with facilities and minimizing costs and distances[1], minimizing the cost for satisfying stochastic demands in each facility[2-5] and minimizing the cost for satisfying stochastic demand related to customer's location [6]. Dynamic facility location was presented by Wesolowsky [7] and Truscott [8]. Wenrouy and Orlencutter [9] studied facility location problem in dynamic condition and un-capacitated facility in which goods are transported to customer's demands from facilities. In each planning horizon new facilities can be installed and existing facilities can be removed. The objective function is minimizing operational costs such as production, transportation and location. They solved the problem by using branch and bound algorithm with lower bound calculated by heuristic solving of LP relaxation. Luss [10] categorized location models according to capacity into two groups. The first group is single facility that its capacity can be increase and decrease in each planning horizon and the second one is considering limited number of facilities. Snyder [11] studied single facility location problem in probabilistic transportation time. They assume that facility can be relocated and transportation time is discreet and different according to each scenario. The aim is to choose the appropriate location for each facility to minimize the transportation and relocation costs. Drezner and Wesolowsky [12] investigate on the location problem of single facility in growing city in which the amount of demand varies over time but it is predetermined. The objective function of this problem is to minimize the total expected cost. In addition they considered the possibility of relocation in their problem and found the time periods that relocation with occur. Cem Canel [13] presented an optimization algorithm to solve the capacitated location problem with dynamic conditions. In these problems the objective function is based on minimizing the costs. Drezner and Wesolowsky [14] studied the location problem with single facility and some demand points with specific weights changing over the time. They assume that the location of facility can be changed one or more than one time in a period. Its objective function was minimizing the total costs and minimizing the maximum cost of location. Roberto Gigues Galva [15] solved dynamic p-median problem with Lagrangian heuristic algorithm. In that research two relaxation Lagrangian methods and a heuristic algorithm based on Lagrangian relaxation was presented. Pierre Chardaire and Alain Sutter [16] modeled un-capacitated dynamic location problem in QAP. This kind of problems is NP-hard and they presented a heuristic algorithm with simulated annealing and a lower bound obtained by Lagrangian relaxation. Current et al [17] studied the dynamic p-median location problem with uncertain amount of facilities in two state with different objective function the first one was to minimize the average loss opportunities and the second one was to minimize the maximum total loss. He called these problems as unlimited facilities location problems and solved them with Lagrangian relaxation heuristic algorithm. Canel et al [18] assumed that facilities can be installed, closed and reopened more than one time. In both investigations binary variables were used to show the facility state. They use a bilinear statement in objective functions in order to show the changes of facilities state from open to closed or closed to reopen. Brotcorne et al. [19]

investigated on ambulance location and relocation models and divided them into deterministic and probabilistic models. Deterministic models are used in production planning in which stochastic state is not considered and total ambulances are available in each time. In probabilistic models, since all ambulances are not available at the time and waiting in queues is considered, the models are more practical in real world. Romauch, and Hartl [20] modeled dynamic facility location problem with probabilistic demand. They solved the problem globally by using probabilistic dynamic programming. Since this method is useful for small problems they locally solved it by Monte Carlo method. Dias et al. [21] presented a mimetic algorithm that combine genetic algorithm and local searching, for capacitated and un-capacitated dynamic location problems. In this problem facilities can be installed, closed or reopened more than one time over a planning horizon. Installation and reopening costs considered different from each other in this model. Gourdin and Klopfenstein [22] studied dynamic facility location problem with limited capacity in growing network environment with different demands and costs in each period and presented integer programming model for this problem. Dias et al. [23] presented dynamic location problem. In this problem facilities can be installed, closed and reopened. They solved it by initial dual heuristic algorithm. Thanh et al. [24] presented mixed integer programming model for designing and planning a distribution system. This paper could be useful for strategic decision such as installing, closing or improvement a facility and choosing suppliers. Uncertainty can be largely controlled by considering some periods and stochastic demands in stochastic dynamic location problem. This trend has been used in household location problems with stochastic demands related to product, inventory and delivery to customers in some periods [25] and location of empty container problem over planning horizon [26]. The presented model in [26] controlled uncertain condition with considering stochastic demands, customers and facilities location and distances between them. The rest of this paper is organized as follows, section 2 defines problem and the mathematical model to minimize the total costs of changes in facility operation states and cost of supplying customer stochastic demands, section 3 will present Implementation the presented model and analyzing the results cause.

2. Mathematical Formulation

In this research, we try to achieve an efficient model with consideration of uncertainty demand over different time periods on the basis of previously presented models that we call stochastic dynamic facilities location problem. In order to do so we use stochastic constrain programming which convert the stochastic model to a deterministic model. Objective is to minimize the cost of supplying each customer demand from facilities by considering stochastic demands and Euclidean distance between facilities and customers. Indexes, parameters and variables that are used in model are described in table below.

Table 1. Indexes, parameters and decision variables in the model

	Indexes
$\boldsymbol{j}$	Number of facilities $j = 1, \dots, n$
$\boldsymbol{i}$	Number of customers $i = 1, \dots, m$
$\boldsymbol{t}$	Planning horizons(number of periods) $t = 1, \dots, T$
$\boldsymbol{\xi}$	Number of studied scenarios $\boldsymbol{\xi} = 1, \dots, B$
	Parameters
$\boldsymbol{g_{ijt}}$	cost of supplying customer i demand from facility j in period t
$\boldsymbol{u_j}$	Capacity of facility j
$\boldsymbol{d_{it}}$	Demand of customer i in period t
$\boldsymbol{C^c}_{\boldsymbol{j}}$	temporarily closing cost of facility j
$\boldsymbol{C^o}_{\boldsymbol{j}}$	Reopening cost of facility j
$\boldsymbol{f^o}_{\boldsymbol{j}}$	Opening cost of facility j
$\boldsymbol{F^o}_{\boldsymbol{j}}$	holding cost of facility j
$(\boldsymbol{x_j}, \boldsymbol{y_j})$	Location of facility j
$(\boldsymbol{a_i}, \boldsymbol{b_i})$	Location of customer i
$\boldsymbol{\beta}$	Cost ratio coordinated with the distance between facility and customer
	Decision variables
$\boldsymbol{x_{ijt}}$	[1 or 0] if demand of customer i in period t supplied from facility j or not
$\boldsymbol{s_{jt}}$	[1 or 0] If facility j is installed at beginning of period t or not
$\boldsymbol{y_{jt}}$	[1 or 0] If facility j is available in period t or not
$\boldsymbol{v^o}_{\boldsymbol{jt}}$	[1 or 0] If facility j is reopened at beginning of period t or not
$\boldsymbol{v^c}_{\boldsymbol{jt}}$	[1 or 0] If facility j is closed at beginning of period t or not
$\boldsymbol{z_{ijt}}$	Supplied amount with facility j for customer i in period t

$(X, Y), (a, b)$ show the facilities location matrix and the customer's location matrix.

$$(X, Y) = \begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \\ \cdots & \cdots \\ x_n & y_n \end{pmatrix}, (a, b) = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \\ \cdots & \cdots \\ a_m & b_m \end{pmatrix}$$

Proposed model is as follow.

$$\min z = \sum_{j=1}^{n} \sum_{t=1}^{T} (f^{o}{}_{j} S_{jt} + F^{o}{}_{j} . y_{jt} + C^{o}{}_{j} v^{o}{}_{jt} + C^{c}{}_{j} v^{c}{}_{jt}) + \sum_{\xi \in B} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{t=1}^{T} \phi(\xi) d_{it}{}^{\xi} g_{ijt}{}^{\xi} x_{ijt}$$

Subject to:

$$\sum_{j=1}^{n} x_{ijt} = 1 \quad \forall i \in m, \forall t \in T \tag{1}$$

$$y_{jt} = y_{j(t-1)} + s_{jt} + v^{o}{}_{jt} - v^{c}{}_{jt} \ \forall j \in n, \forall t \in T \tag{2}$$

$$\sum_{t=1}^{T} v^{o}{}_{jt} \leq \sum_{t=1}^{T} v^{c}{}_{jt} \quad \forall j \in n, \forall t \in T \tag{3}$$

$$\sum_{t=1}^{T} s_{jt} \leq 1 \quad \forall j \in n \tag{4}$$

$$\sum_{i=1}^{m} d_{it}{}^{\xi} x_{ijt} \leq u_{j}{}^{\xi} y_{jt} \quad \forall j \in n, \forall t \in T, \forall \xi \in B \tag{5}$$

$$\sum_{t=1}^{T} d_{it}{}^{\xi} = \sum_{t=1}^{T} \sum_{j=1}^{n} z_{ijt}{}^{\xi} \quad \forall i \in m, \forall \xi \in B \tag{6}$$

$$g_{ijt}{}^{\xi} = \beta \sqrt{\left(x_j - a_i\right)^2 + \left(y_j - b_i\right)^2} \quad \forall j \in n, \forall t \in T, \forall i \in m, \forall \xi \in B \tag{7}$$

$$x_{ijt}, s_{jt}, y_{jt}, v_{jt}^{o}, v_{jt}^{c} \in [0 or 1], z_{ijt} \geq$$

The objective function is to minimize the total costs of changes in facility operation states and total cost of supplying customer demands. (1) Fulfil the demand of each customer in period t, (2) specify the availability of each facility in period t, (3) ensures that a facility has to be temporarily closed before it can be reopened. (4) Shows that at most one facility can be constructed at location j. (5) Guarantees that the total demand of customers would not exceed the related facility capacity. (6) Is about the dedication of orders of each customer in each scenario. (7) Calculate the cost of supplying customer's demand which should be coordinated with the distance between the facility and customer.

3. Numerical Example

This model implemented in dairy factory. In this case there were 8 facilities and 12 customers and the planning horizon was 5 periods. The capacity of each 8 facilities is as following.

$$u_1 = 45, \quad u_2 = 80, \quad u_3 = 80, \quad u_4 = 140,$$
$$u_5 = 180, \quad u_6 = 140, \quad u_7 = 80, \quad u_8 = 95$$

The distribution of customers' demands is discreet uniform ($d_{it} \sim DU[65, 75]$) according to investigation on historical data. Other cots will be considered as following.

$$C^c{}_j = 1000000, \quad C^o{}_j = 2500000,$$
$$f^o{}_j = 11000000, \quad F^o{}_j = 4000000, \quad \beta = 100000$$

Location vectors of customers and facilities are estimated as following.

Table 2. location of facilities

Facility number	Facility location
1	$(2.2, 1.8)$
2	$(2, 1.7)$
3	$(3.2, 4.1)$
4	$(1.5, 2.1)$
5	$(4, 2.6)$
6	$(6, 4.2)$
7	$(4, 3.6)$
8	$(8, 2.5)$

Table 3. location of customers

Customer number	Customer location
1	$(0.8, 1.2)$
2	$(1.3, 2)$
3	$(2.5, 1.2)$
4	$(1, 4, 3.7)$
5	$(2.1, 2, 7)$
6	$(4, 2, 1)$
7	$(3.4, 1, 7)$
8	$(1.1, 3, 1)$
9	$(3.1, 0, 7)$
10	$(2.6, 3)$
11	$(4, 8, 3.9)$
12	$(2.7, 3, 1)$

The implementation of presented model had been done with 240 different scenarios in each period. The total costs include the costs for changing status facilities over planning horizon (first part of objective function) and for customers supplement (second part of objective function). Facilities status over 5 periods, average demand in each period, average cost in scenarios over planning horizon and average all facilities status has been calculated.

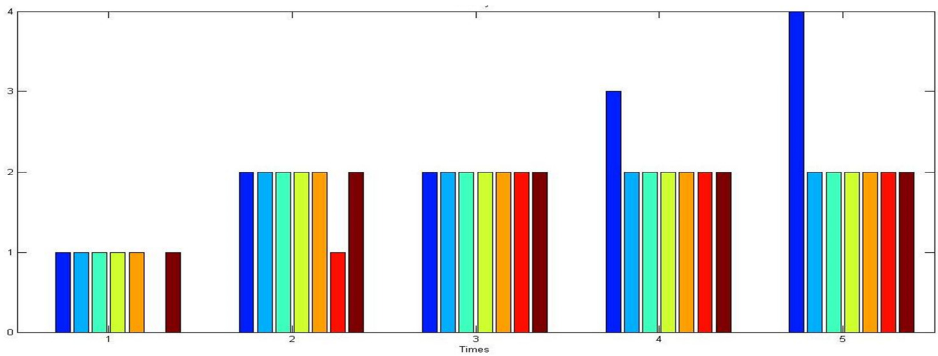

Figure 1. facilities status over 5 periods

In Figure 1, 0 till 4 number in the vertical axe show: not existing facility, open facility, closed facility, reopen facility. Figure 2 illustrate the average of total facilities status in all scenarios over planning horizon.

The average amount of total costs in all scenarios over planning horizon is shown in Figure 3 Some facilities are closed in period 4, because the demand is lower rather other periods and costs decrease incredibly.

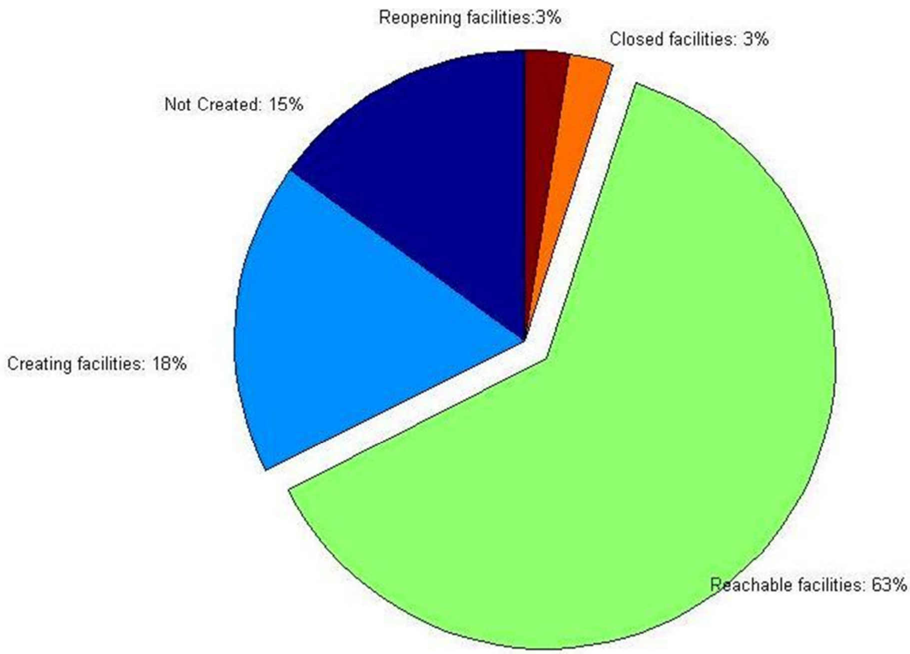

Figure 2. the average facilities status

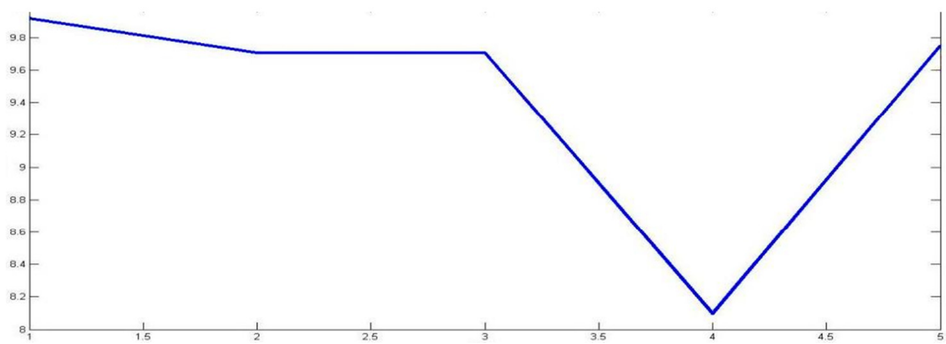

Figure 3. the average of total costs over planning horizon

Figure 4 illustrate the average amount of demand in all scenarios. In period 4 demands is the lowest amount between periods and Figure 5 illustrate the place distribution of client and facilities.

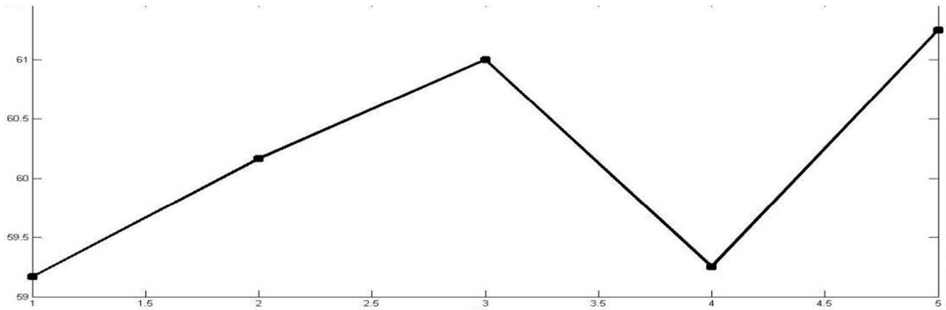

Figure 4. the average amount of demands over planning horizon

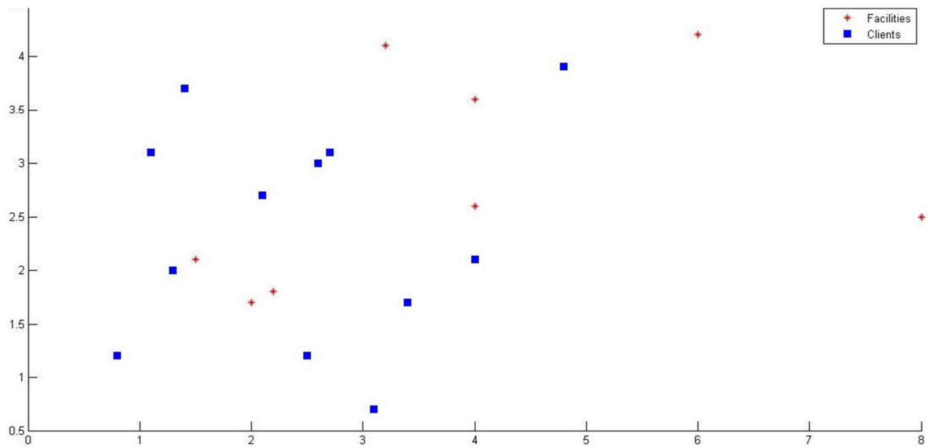

Figure 5. place distribution of client and facilities

This model is useful for management, scheduling facilities in future and minimizing total costs. Implementation the presented model and analyzing the results cause better management and better control of changing facility status, so the costs will be decreased considerably.

4. Conclusions

Accurate estimation parameters cannot be fulfil in real world with high uncertain conditions. Investigation basis on previous information and consideration distribution probability could be helpful. In this research we tried to minimize total costs over periods with considering stochastic demand customer from facilities and distances between customers and facilities. Implement model in according to real information and estimate parameters basis on previous information cause decrease in total costs significantly and show effectiveness of presented model. For future research, investigation on more parameters could be mentioned.

References

C.K.Y. Lin, "Stochastic single-source capacitated facility location model with service level requirements" Int. J. Production Economics 117. pp.439–451, 2009.

Canel, C., Khumawala, B. M., Law, J., & Loh, " An algorithm for the capacitated, multi-commodity multi-period facility location problem" Computers and Operations Research, 28(5), pp 411–427.2001

Cem Canel, "multi-period facilities location problem with profit maximization" International Journal of Physical Distribution & Logistics Management, Vol. 29 No. 6, pp. 409-433.1999

Current, J., Ratick, S., & ReVelle, C " Dynamic facility location when the total number of facilities is uncertain: A decision analysis approach" European Journal of Operational Research, 110(3), pp597–609.1997

Drezner, Z, & Wesolowsky, G. O " Facility location when demand is time dependent" Naval Research Logistics, 38(5), 763–777.1991

Eric Gourdin, Olivier Klopfenstein" Multi-period capacitated location with modular equipment's" Computers & Operations Research, pp 661-682, 2008.

Hanan Luss "A Survey Operations Research and Capacity Expansion Problems" institute for operations research and the management science, 1982

Jian Zhou, Baoding Liu" New stochastic models for capacitated location-allocation" Computers & Industrial Engineering, 45, pp111–125, 2003.

Joana Dias, M. Eugénia Captivo, João Clímaco " a memetic algorithm for dynamic location problems" European Journal of Operational Research, pp 313-319. 2004

Joana Dias, M. Eugénia Captivo, João Clímaco" A dynamic location problem with maximum decreasing capacities" Central European Journal of Operations Research, pp 251-280.2008.

Kannan Viswanath, James Ward "Stochastic Location-assignment on an Interval" Newt Spat Econ 10, pp 389–410, 2010

Luce Brotcorne, Gilbert Laporte, Frederic Semet " Ambulance location and relocation models" European Journal of Operational Research, pp 451-463.2003

Martin Romauch and Richard F. Hartl " Dynamic Facility Location with Stochastic demand" lecture notes in computer science 3777: pp180-189.2005.

Neha Mittal, Maria Boile, Alok Baveja, Sotiris Theofanis, "Determining optimal inland-empty-container depot locations under stochastic demand" research in transportation economics 42(1).pp 50-60. 2013.

Owen, S. H., & Dakin, M. S. "Strategic facility location: A review" European Journal of Operational Research, 111(3), pp423–447.1998

Peter Schutza, Leen Stougieb, Asgeir Tomasgardc " Stochastic facility location with general long-run costs and convex short run cost" Operations Research 35, Pp2988 – 3000, 2008.

Phuong Nga Thanh, Nathalie Bostel, Olivier Péton" A dynamic model for facility location in the design of complex supply chains" International Journal of Production Economics, pp 678-693, 2008

Pierre Chardaire and Alain Sutter" Solving the Dynamic Facility Location Problem" NETWORKS, Vole. 28 11 1996.

Roberto Diguez Galv" A Lagrangean heuristic for the p–median dynamic location problem" European Journal of Operational Research 58 .pp250-262.1992

Romauch, M., & Hartl, R. F. "Dynamic facility location with stochastic demands" Lecture Notes in Computer Science, 3777, 180–189.2005

Seyed Mohsen Mousavi, Seyed Taghi Akhavan Niaki "Capacitated location allocation problem with stochastic location and fuzzy demand: A hybrid algorithm "Applied Mathematical Modelling 37, pp. 5109–5119, 2013.

Snyder, L. V. "Facility location under uncertainty: A review" IIE Transactions, 38(7), pp547–564.2006

U. K. Bhattacharya" Location models with stochastic demand points" Operations Research (Jan–Mar) 49(1).pp 62–77, 2012.

Wesolowsky, G. O "Dynamic facility location " Management Science, 19(11), pp1241–1248.1973.

Wesolowsky, G. O., & Truscott, W. G " The multi-period location–allocation problem with relocation of facilities" Management Science, 22(1), pp57–65.1975

Z. Drezner, G.O. Wesolowsky "Facility location when demand is time dependent" Naval Research Logistics 38 pp 763–777.1991.