# An Uncertain Time Horizon Inventory Management Model with Partial Backlogging Considering Deterioration Cost

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**Abstract.** A new mathematical model for the optimal production is formulated for the inventory management system under time-varying and stochastic inflation environment for deteriorating items. The time horizon is finite and demand rate is dependent to the inflation. In the real situation, some but not all customers will wait for backlogged items during a shortage period, such as for fashionable commodities or high-tech products with the short product life cycle. The longer the waiting time is, the smaller the backlogging rate would be. According to such phenomenon, taking the backlogging rate into account is necessary. Thus, the model incorporates partial backlogging. The total system cost including deterioration cost, production cost, inventory holding cost, backordering cost, lost sale cost and ordering cost is formulated as an optimal control problem. The numerical example has been provided for evaluation and validation of the theoretical results.

**Keywords:** Inventory management; stochastic; deterioration items; inflation-dependent demand rate; shortages.

## 1.  Introduction

During the past decades, the replenishment scheduling problems were typically considered by researchers which had attended parameters in real world situations, such as uncertain conditions, physical characteristics of inventoried goods, effects of inflation and time value of money, partial backlogging of unsatisfied demand, etc. Inventoried goods can be broadly classified into four meta-categories based on: Obsolescence, Deterioration, Amelioration and the goods which are categorized in any of groups above.

Since 1975 the effects of inflation on inventory systems were considerably marked in several papers. There are a few papers for obsolescing and ameliorating items. For example, Moon et al. [30] considered ameliorating/deteriorating items with a time-varying demand pattern. Another research for ameliorating items has been done by Sana [31].

There are also some researches on inventory system for no obsolescing, deteriorating and ameliorating products. Buzacott [3] made the first attempt in this field that dealt with an economic order quantity (EOQ) model with inflation subject to different types of pricing policies. Sarker and Pan [34] surveyed the effects of inflation and the time value of money on order quantity with finite replenishment rate. Vrat and Padmanabhan [36] determined optimal ordering quantity for stock-dependent consumption-rate items, and showed that as the inflation rate increases, ordering quantity and the total system cost increase.

Misra [29] developed a discounted cost model and included internal (company) and external (general economy) inflation rates for various costs associated with an inventory system. Other efforts were extended the previous works to consider more complex and realistic assumption, such as Hariga and Ben-Daya [15], Hariga [14], Uthayakumar and Geetha [43], Datta and Pal [10], Maity [24] and Chung [8].

Against obsolescing and ameliorating items, the deteriorating inventory models under inflationary conditions are studied greatly. For example, Chen and Lin [6] discussed an inventory model for deteriorating items with a normally distributed shelf life, continuous time-varying demand, and shortages under an inflationary and time discounting en-

vironment. Chung and Tsai [9] studied an inventory model with the demand of linear trend considering the time-value of money and deterioration. Wee and Law [38] proposed an inventory model with deterioration and inflation considering the demand rate as a linear decreasing function of the selling price. Liao and Chen [21] surveyed a retailer's inventory control system for the optimal delay in payment time for initial stock dependent consumption rate when a wholesaler permits delay in payment. The effect of inflation rate, deterioration rate, initial stock-dependent consumption rate and a wholesaler's permissible delay in payment is discussed.

Hou [18] prepared an inventory model for deteriorating items with stock-dependent consumption rate. Yang [39] studied the two-warehouse inventory problem with a constant demand rate, deterioration and shortages. Lo et al. [22] developed an integrated production-inventory model with assumptions of varying rate of deterioration, partial backordering, inflation, imperfect production processes and multiple deliveries. A Two storage inventory problem with dynamic demand and interval valued lead-time over a finite time horizon under inflation and time-value of money considered by Dey et al. [11]. Balkhi [2] presented a production lot-size inventory model where the production, demand and deterioration rates are known, continuous and differentiable functions of time. Shortages are allowed, but only a fraction of the stock out is backordered, and the rest is lost. Other efforts inventory systems with inflation and deterioration have been done by Balkhi [1], Hou and Lin [19], Maiti et al. [23], Chang[14], Su et al. [35], Hsieh and Dye [16], Wee and Law [37], Yang et al. [41] and [42], Sarker et al. [33], Chen [5], Jaggi et al. [201], Chern et al. [7] and Sarkar and Moon [32].

In all papers mentioned above, it has been assumed that the rate of inflation is known and certain. But inflation is related to many aspects which make it uncertain and unstable in the real world. Horowitz [18] clarified an EOQ model with a normal distribution for the inflation rate and Mirzazadeh and Sarfaraz [27] developed a stochastic mathematical inventory model for multiple items with the internal and external inflation rates. Maity and Maiti [25] developed a numerical approach to a multi-objective optimal inventory control problem for deteriorating

multi-items under fuzzy inflation and discounting. Mirzazadeh [28] compared the average annual cost and the discounted cost methods in the inventory system's modeling with considering stochastic inflation. The results show that there is a negligible difference between two procedures for wide range values of the parameters. Furthermore, Mirzazadeh [27], in another work, proposed an inventory model under time-varying inflationary conditions for deteriorating items.

From the existing literature, it is obvious that inflationary inventory models are usually developed under the assumption of constant and well known time horizon. However, there are many real life situations where these assumptions are not valid, e.g., for a seasonal product, though time horizon is normally assumed as finite and crisp in nature, but, in every year it fluctuates depending upon the environmental effects and it is better to estimate this horizon as a stochastic parameter, which has been considered in this paper.

In this paper the partial backlogging has been considered and replenishment rate is finite. The rest of the paper is organized as follows. Section 2 includes the assumptions and notations. In Section 3, the model formulation is derived. Section 4 explains the solution procedure. Section 5 provides the numerical example to clarify how the proposed model is applied. The final section is devoted to the concluding remarks.

## 2. Assumptions and Notations

The following assumptions and notations have been considered in this model.

(1) The inventory system costs will be increase over time horizon via stochastic inflation rate which is denoted by i with the pdf of $f(i)$.

(2) $H$ is taken to be the stochastic time horizon and $f(h)$ is the pdf of $H$.

(3) $r$ is the discount rate and $R$ is the discount rate net of inflation:

(4) There is no repair or replenishment of the deteriorated items during the inventory cycle. The constant deterioration rate per unit time is denoted by $\tau(0 \leqslant \tau\rangle)$. The deterioration cost per unit of the deteriorated item is $c_6$ at time zero.

(5) The constant annual production (Replenishment) rate, P, is finite and the constant annual demand rate is D. The Replenishment rate is higher than the sum of consumption and deterioration rates.

(6) At time $t = 0, c_1$ is the ordering cost per order, $c_2$ is per unit cost of the item and $c_3$ is the inventory holding cost per unit per unit time.

(7) Lead time is negligible. Also, the initial and final inventory level is zero. Additional notations will be introduced later.

(8) Shortages are allowed. Unsatisfied demand is backlogged, and the fraction of shortages backordered is a differentiable and decreasing function of time $t$, denoted by $\delta(t) = exp(-\alpha t)$, where $t$ is the waiting time up to the next replenishment, $0 \leqslant \delta(t) \leqslant 1$.

## 3.    The Model Formulation

The initial and final inventory levels are both zero. The real time horizon $(H)$ has been divided into n equal and full cycles of length $T$. Each inventory cycle except the last cycle can be divided into four parts (see Figure 1). The production starts at time zero and thereafter, as time passes, the inventory level gradually increasing due to production, demand and deterioration rates. This fact continues till the production stops at time $\lambda_1$. Then the inventory level gradually decreasing mainly due to consumption and partly due to deterioration and reaches zero at time $kT$. Then, shortages occur and are accumulated until time $\lambda_2$. During the time interval [kT, T], we do not have any deterioration and therefore, shortages level linearly change. At time $\lambda_2$ the production starts again and shortages level linearly decreases until the moment of $T$. The partially backordered quantity is supplied to customers during the time interval $[\lambda_2, T]$. At time $T$, the second cycle starts and this behavior continue till the end of the $(n-1) - th$ cycle.

The shortages are not allowed in the last cycle and the inventory cycle can be divided into two parts. The production stops at time $(n-1)T+\lambda_3$ and then the inventory level decreases to lead zero at the end of the time horizon. '[Insert Figure 1 about here]'
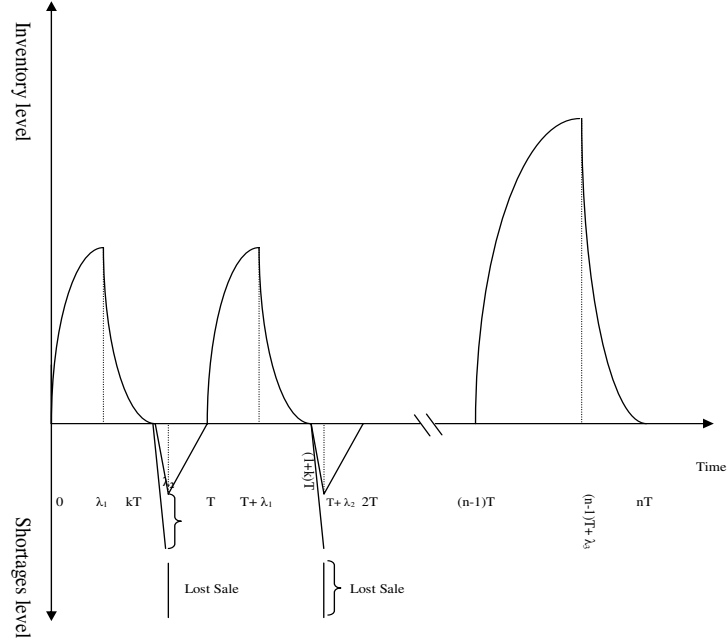
Figure 1. Graphical representation of the inventory system

In the last cycle, the inventory level is governed by the following differential equations ($I_i(t_i)$ denote the inventory level at any time $t_i$ in the $i - th$ part of the last cycle that $i = 1, 2$).

$$\frac{dl_1(t_1)}{dt_1} + \tau l_1(t_1) = p - D, \qquad 0 \leqslant t_1 \leqslant \lambda_3 \qquad (1)$$

$$\frac{dl_2(t_2)}{dt_2} + \tau l_2(t_2) = p - D, \qquad 0 \leqslant t_2 \leqslant kT - \lambda_3 \qquad (2)$$

Let $I_i(t_i)$ denote the inventory level at any time ti in the $(i - 2) - th$ part of the first to $(n - 1) - th$ cycles ($i = 3, 4, 5, 6$). The differential equations describing the inventory level at any time in the cycle are given as

$$\frac{dl_3(t_3)}{dt_3} + \tau l_3(t_3) = p - D, \qquad 0 \leqslant t_3 \leqslant \lambda_1 \qquad (3)$$

$$\frac{dl_4(t_4)}{dt_4} + \tau l_4(t_4) = -D, \qquad 0 \leqslant t_4 \leqslant kT - \lambda_1 \qquad (4)$$

$$\frac{dl_5(t_5)}{dt_5} = D\delta(\lambda_2 - kT - t_5), \qquad\qquad 0 \leqslant t_5 \leqslant \lambda_2 - kT \qquad (5)$$

$$\frac{dl_6(t_6)}{dt_6} = p - D, \qquad\qquad 0 \leqslant t_6 \leqslant T - \lambda_2 \qquad (6)$$

The boundary conditions are $I_1(0) = 0, I_2(T - \lambda_3) = 0$ $I_3(0) = 0,$ $I_4(kT - \lambda_1) = 0,$ $I_5(0) = 0,$ and $I_6(T - \lambda_2) = 0.$ Therefore:

$$I_1(t_1) = (P - D)(1 - e^{-tl_1})/\tau, \qquad 0 \leqslant t_1 \leqslant \lambda_3 \qquad (7)$$

$$I_2(t_2) = -D(1 - e^{t(T - \lambda_3 - t_2)})/\tau, \qquad 0 \leqslant t_2 \leqslant T - \lambda_3 \qquad (8)$$

$$I_3(t_3) = \frac{P - D}{\tau}(1 - e^{-t\tau_3}), \qquad 0 \leqslant t_3 \leqslant \lambda_1 \qquad (9)$$

$$I_4(t_4) = -D(1 - e^{t(kT - \lambda_1 - t_4)})/\tau, \qquad 0 \leqslant t_4 \leqslant kT - \lambda_1 \qquad (10)$$

$$I_5(t_5) = De^{-(\lambda_2 - kT)\alpha}(1 - e^{\alpha l_5})/\alpha, \qquad 0 \leqslant t_5 \leqslant \lambda_2 - kT \qquad (11)$$

$$I_6(t_6) = (P - D)(t_6 - T + \lambda_2), \qquad 0 \leqslant t_6 \leqslant T - \lambda_2 \qquad (12)$$

The values of $\lambda_1, \lambda_2$ and $\lambda_5$ can be calculated with respect to $k$ and $T$, using the above equations. Solving $I_3(\lambda_1) = I_4(0)$ for $\lambda_1$ we have

$$\lambda_1 = Ln\frac{P - D(1 - e^{\tau kT})}{P}/\tau \qquad (13)$$

$\lambda_2$ can be calculated by solving $I_5(\lambda_2 - kT) = I_6(0)$

$$\lambda_2 = 1 - D(1 - k)]T/P \qquad (14)$$

Finally, solving $I_1(\lambda_3) = I_2(0)$ for $\lambda_3$ we have

$$\lambda_3 = Ln\frac{P - D(1 - e^{\tau T})}{P}/\tau \qquad (15)$$

The expected present value of the inventory costs are defined as: ECR (the replenishment costs), ECP (the purchasing costs), ECH (the carrying costs), ECS (the shortages costs, backordering and lost sale)

and ECD (the deterioration costs). The detailed analysis is given as follows.

## 3.1 The expected present value of ordering cost (ECR)

Assume CR as the ordering cost

$$CR = c_1 \left[ 1 + \sum_{i=0}^{n-1} e^{-R(jT+\lambda_2)} \right] \tag{16}$$

Consider

$$K(n) = \frac{1 - e^{-TnR}}{1 - e^{-RT}} \tag{17}$$

for simplifying. By replacing equation (14) in equation (16) and taking the expected value we have

$$ECR = c_1 E \left\{ 1 + k(n)e^{\frac{T[DR(1+k)+P^R]}{p}} \right\} \tag{18}$$

## 3.2 The expected present value of purchasing cost (ECP)

Let $ECP_1$ and $ECP_2$ as the expected present value of the purchase cost in the first to $(n-1) - th$ cycles and in the last cycle, respectively. The first purchase cost that is ordered at time zero equals to: $c_2 P \lambda_1$. Then, the next purchase will occur at time $\lambda_2$ and therefore, the first cycle purchase cost is

$$c_2 P \left[ \lambda_1 + (T - \lambda_2)e^{-\lambda_2 R} \right] \tag{19}$$

(19) The purchase cost for $j - th$ cycle, $(j = 2, 3, , n-1)$ is similar to the above equation with considering the discount factor, therefore, the total expected present of the purchase cost in the first $(n-1) - th$ cycles is

$$ECP_1 = c_2 PE \left\{ k(n_1) \left[ \frac{Ln[\frac{P-D(1-e^{\tau kT})}{p}]}{\tau} + [T - \frac{(P-D(1-k))T}{p}]e^{-[\frac{P-D(1-k)T}{p}]R} \right] \right\} \tag{20}$$

The production quantity in the last cycle will occur at time $(n-1)T$ and equals to $\lambda_3 P$. Therefore, the total expected present of the purchase cost in the last cycle will be

$$ECP_2 = c_2 P E \left[ Lu \frac{P - d(1 - e^{\tau T})}{P} e^{-(n-1)RT} \right] / \tau \qquad (21)$$

The total expected purchase cost over the time horizon would be

$$ECP = ECP_1 + ECP_2 \qquad (22)$$

## 3.3   Expected present value of holding cost (ECH)

Consider $ECH_1$ and $ECH_2$ as the expected present value of the holding cost during the first to $(n-1)-th$ cycles and the holding cost during the last cycle, respectively. In the first period, the holding costs for $j-th$ cycle is

$$CH_j = c_3 \left[ \int_0^{\lambda_1} l_3(t_3) e^{-Rt_3} \, dt_3 + \int_0^{kT - \lambda_1} l_4(t_4) e^{-Rt_4} \, dt_4 e^{-\lambda_1 R} \right] e^{-(j-1)RT},$$

$$j = 1, 2, ..., n-1 \quad (23)$$

After some calculations and taking the expected value we have

$$ECH_1 = c_3 E \left\{ K(n-1) \left[ \frac{(P-D) \left\{ e^{-\lambda_1 R} \left[ -R(1 - e^{-\tau \lambda_1}) - \tau \right] + \tau \right\} + D \left[ -(R+\tau) e^{-\lambda_1 R} + \tau e^{-kTR} + R e^{kT} e^{-\lambda_1(R+\tau)} \right]}{\tau R(R+\tau)} \right] \right\}$$

$$(24)$$

For the last cycle, holding cost will be

$$CH_n = c_3 \left[ \int_0^{\lambda_3} I_1(t_1) e^{-Rt_1} \, dt_1 e^{-R(n-1)T} + \int_0^{T - \lambda_3} I_2(t_2) e^{-Rt_2} \, dt_2 e^{-R((n-1)T + \lambda_3)} \right] \qquad (25)$$

After some complex calculations and taking the expected value we have

$$
ECH_2 = -c_3 E \left\{ e^{-R[\lambda_3+(n-1)T]} \left[ \begin{array}{c} \dfrac{(P-D)\left[R(1-e^{-\tau\lambda_3})+\tau(1-e^{\lambda_3 R})\right]}{-\tau R(R+\tau)} + \\[2mm] \dfrac{D\left[\tau e^{-R(T-\lambda_3)} - R(1-e^{\tau(T-\lambda_3)})-\tau\right]}{-\tau R(R+\tau)} \end{array} \right] \right\} \qquad (26)
$$

So, the total expected present value of the holding costs over the time horizon is

$$
ECH = ECH_1 + ECH_2 \qquad (27)
$$

## 3.4    The expected present value of shortages cost (ECS)

ECS shows the expected present value of the shortages cost, including backorder and lost sales, during the first to $(n-1)-th$ cycles. Shortages are not allowed in the last cycle. Therefore

$$
ECS = E\left\{ K(n-1) \left[ \begin{array}{l} \int_0^{\lambda_2-kT}\left[c_4 e^{-Rt_5}\sigma(t_5)+c_5(1-\sigma(t_5))e^{-(\lambda_2-kT)R}\right]-I_5(t_5)]dt_5\, e^{-kTR} \\[2mm] +\int_0^{T-\lambda_2}\left[c_4 e^{-Rt_6}\sigma(t_6)+c_5(1-\sigma(t_6))e^{-(T-\lambda_2)R}\right]-I_6(t_6)]dt_6\, e^{-\lambda_2 R} \end{array} \right] \right\}
$$

$$(28)$$

## 3.5 The expected present value of deteriorating cost (ECD)

Denote $DI_1$ the quantity of inventory items which have been deteriorated per cycle in the first to the $(n-1)-th$ cycles

$$
DI_1 = \tau\left[\int_0^{\lambda_1} I_3(t_3)dt_3 + \int_0^{kT-\lambda_1} I_4(t_4)dt_4\right]
$$
$$
= \dfrac{(P-D)(\tau\lambda_1 -1-e^{-\tau\lambda_1})-D(1+\tau(kT-\lambda_1)-e^{\tau(kT-\lambda_1)})}{\tau} \qquad (29)
$$

Now, assume $ECD_1$ as the expected present value of the deterioration cost during the first to the $(n-1)-th$ cycles. Also, $ECD_2$ is defined the expected present value of the deterioration cost during the last cycle. $ECD_1$ after taking the expected value will be

$$ECD_1 = \frac{c_6}{\tau} E\left\{K(n-1)\left[(P-D)\left(\tau\lambda_1 - 1 - e^{-\tau\lambda_1}\right) - D\left(1 + \tau(kT - \lambda_1) - e^{\tau(kT-\lambda_1)}\right)e^{-\lambda_1 TR}\right]\right\} \quad (30)$$

For the last cycle, deterioration cost will be

$$ECD_2 = \frac{c_6}{\tau} E\left\{\left[(P-D)\left(\tau\lambda_3 - 1 + e^{-\tau\lambda_3}\right) - D\left(1 + (T-\lambda_3)\tau - e^{\tau(T-\lambda_3)}\right)e^{-\lambda_3 R}\right]e^{-(n-1)RT}\right\} \quad (31)$$

Therefore, the expected present value of the deterioration cost over the time horizon is

$$ECD = ECD_1 + ECD_2 \qquad (32)$$

Considering the above mentioned analysis, the expected present value of the total system costs over the time horizon for a given value of H, is as follow

$$ETC(n,k) = ECR + ECP + ECH + ECS + ECD \qquad (33)$$

Note that the time horizon H has a $p.d.f. f(h)$. So, the present value of expected total cost from n complete cycles, $ETVC(n,k)$, is given by

$$ETVC(n,k) = \sum_{n=0}^{\infty} \int_{nT}^{(n+1)T} ETC(n,k)f(h)dh \qquad (34)$$

## 4.    The solution procedure

The problem is to determine $n^*, k^*$ and $ETVC(n^*, k^*)$ where $k$ is a continuous variable $(0 < k < 1)$, and $n$ is a discrete variable. Therefore, for any given $n$, the necessary condition for the minimum of $ETVC(n,k)$ is

$$\frac{dETVC(n,k)}{dk} = 0 \qquad (35)$$

For a given value of $n$, derive $k^*$ from Equation (37). By substituting $(n, k^*)$ into equation (36), $ETVC(n, k^*)$ is derived. Then, $n$ increase by the increment of one continually and $ETVC(n, k^*)$ calculate again. We repeat this algorithm until finding the minimum value of $ETVC(n, k^*)$.

The $(n^*, k^*)$ and $ETVC(n^*, k^*)$ values constitute the optimal solution and satisfy the following conditions

$$\Delta ETVD(n^* - 1, k^*)\langle 0 \langle \Delta ETVC(n^*, k^*) \tag{36}$$

Where

$$\Delta ETVC(n^*, k^*) = ETVC(n^* + 1, k^*) - ETVC(n^*, k^*) \tag{37}$$

To ensure convexity of the objective function, the derived values of $(n^*, k^*)$ must satisfy the following sufficient condition

$$\frac{d^2 ETVC(n, k)}{dk^2} \geqslant 0 \tag{38}$$

## 5.   Numerical Example

The following numerical example is illustrated the model to obtain the optimal replenishment and shortages policy.

Let $P = 10000 units/year$, r=\$ $0.2/\$ /year$, $\tau = 0.05/unit/year$, $\delta(t) = e^{-0.5t}$, $D = 6000 units/year$, $c_1 = \$ 150/order$; $c_2 = \$ 20/unit$; $c_3 = \$ 5/unit/year$; $c_4 = \$ 5/unit/year$; $c_5 = \$ 15/unit$ and $c_6 = \$ 30/unit$.

The inflation rate is stochastic with Uniform distribution:$i \sim U(\$ 0.08/\$ /year, \$0.15/\$ /year)$. Also, the time horizon has Normal distribution with mean of 10years:$H \sim N(10, 1.5^2)$.

This problem has been solved via a numerical method, which has been explained in the previous section and the results are illustrated in Table 1. It can be seen that the minimum expected cost is 290023.2\$ for $n^* = 31$ and $k^* = 0.613$ (The shortages occur after elapsing 61.3% of the cycle time). '[Insert Table 1 about here]'

**Table 1.** The optimal solution of $ETVC(n, k)$

| n | k | ETVC (n,k) | N | k | ETVC (n,k) |
|---|---|---|---|---|---|
| 5 | 0.623 | 321582.4 | 32 | 0.613 | 290892.2 |
| 10 | 0.621 | 314464.2 | 35 | 0.612 | 293167.7 |
| 20 | 0.617 | 301458.7 | 40 | 0.610 | 297256.4 |
| 30 | 0.614 | 290732.5 | 50 | 0.606 | 308264.6 |
| 31 | **0.613**[*] | **290023.2**[*] | 60 | 0.602 | 321954.3 |

## 6.    Conclusion

In this paper, an inflationary inventory model has been presented with shortages over a stochastic time horizon and shortages are partially backlogged. Moreover, the demand is a function of the inflation rate.

It most of the last papers inflation rate, usually, has been assumed constant over the time horizon. However, many factors may also affect the rate of inflation. Therefore, the stochastic inflation has been assumed in this paper. The numerical example has been given to illustrate the theoretical results the study has been conducted under the Discounted Cash Flow approach.

# References

[1] aBalkhi, Z. T. (2004), On the Optimality of Inventory Models with Deteriorating Items for Demand and On-Hand Inventory Dependent Production Rate, IMA Journal Management Mathematics, 15, 67-86.

[2] bBalkhi, Z. T. (2004), An Optimal Solution of a General Lot Size Inventory Model with Deteriorated and Imperfect Products, Taking into Account Inflation and Time Value of Money, International Journal of Systems Science, 35, 87-96.

[3] Buzacott, J. A. (1975), Economic Order Quantities with Inflation, Operational Research Quarterly, 26, 553-558.

[4] Chang, C. T. (2004), An EOQ Model with Deteriorating Items under Inflation when Supplier Credits Linked to Order Quantity, International Journal of Production Economics, 88, 307-316.

[5] Chen, J. M. (1998), An Inventory Model for Deteriorating Items with Time-Proportional Demand and Shortages under Inflation and Time Discounting, International Journal of Production Economics, 55, 21-30.

[6] Chen, J. M. and Lin, C. S. (2002), An Optimal Replenishment Model for Inventory Items with Normally Distributed Deterioration, Production Planning and Control, 13, 470-480.

[7] Chern, M. S., Yang, H. L., and Teng, J. T. (2008), Partial backlogging inventory lot-size models for deteriorating items with fluctuating demand under inflation, European Journal of Operational Research, 191, 127-141.

[8] Chung, K. J. (2003), An Algorithm for an Inventory Model with Inventory-Level-Dependent Demand Rate, Computers and Operations Research, 30, 1311-1317.

[9] Chung, K. J. and Tsai, S. F. (2001), Inventory Systems for Deteriorating Items with Shortage and a Linear Trend in Demand-Taking Account of Time Value, Computers and Operations Research, 28, 915-934.

[10] Datta, T. K. and Pal, A. K. (1991), Effects of Inflation and Time-Value of Money on an Inventory Model with Linear Time-Dependent Demand Rate and Shortages, European Journal of operational Research, 52, 326-333.

[11] Dey, J. K., Mondal, S. K., and Maiti, M. (2008), Two storage inventory problem with dynamic demand and interval valued lead-time over finite time horizon under inflation and time-value of money, European Journal of Operational Research, 185, 170-194.

[12] Goyal, S. K. and Giri, B. C. (2001), Recent Trends in Modeling of Deteriorating Inventory, European Journal of Operational Research, 134, 1-16.

[13] Hadley, G. A. (1964), Comparison of Order Quantities Computed Using the Average Annual Cost and the Discounted cost, Management Science, 10, 472-467.

[14] Hariga, M. A. (1995), Effects of Inflation and Time-Value of Money on an Inventory Model with Time-Dependent Demand Rate and Shortages, European Journal of operational Research, 81, 512-520.

[15] Hariga, M. A. and Ben-Daya, M. (1996), Optimal Time Varying Lot-Sizing Models under Inflationary Conditions, European Journal of operational Research, 89, 313-325.

[16] Hsieh,T. P. and Dye, C. Y. (2010), Pricing and lot-sizing policies for deteriorating items with partial backlogging under inflation, Expert Systems with Applications, In Press.

[17] Horowitz, I. (2000), EOQ and Inflation Uncertainty, International Journal of Production Economics, 65, 217-224.

[18] Hou, K. L. (2006), An Inventory Model for Deteriorating Items with Stock-Dependent Consumption Rate and Shortages under Inflation and Time Discounting, European Journal of Operational Research, 168, 463-474.

[19] Hou, K.L. and Lin, L. C. (2004), Optimal Inventory Model with Stock-Dependent Selling Rate under Maximal Total Present Value of Profits, Proc Fourth IASTED International Conference on Modeling, Simulation, and Optimization, 7-12.

[20] Jaggi, C. K., Aggarwal, K. K., and Goel, S. K. (2006), Optimal order policy for deteriorating items with inflation induced demand, International Journal of Production Economics, 103, 707-714.

[21] Liao, H. C. and Chen, Y. K. (2003), Optimal Payment Time for Retailer's Inventory System, International Journal of Systems Science, 34, 245-253.

[22] Lo, S. T., Wee, H. M., and Huang, W. C. (2007), An integrated production-inventory model with imperfect production processes and Weibull distribution deterioration under inflation, International Journal of Production Economics, 106, 248-260.

[23] Maity, A. K. (2010), One Machine Multiple-Product Problem with Production-Inventory System under Fuzzy Inequality Constraint, Applied Soft Computing, In Press.

[24] Maity, K. and Maiti, K. (2008), A numerical approach to a multi-objective optimal inventory control problem for deteriorating multi-items under fuzzy inflation and discounting, Computers & Mathematics with Applications, 55, 1794-1807.

[25] Maiti, A. K., Maiti, M. K., and Maiti, M. (2006), Two storage inventory model with random planning horizon, Applied Mathematics and Computation, 183, 1084-1097.

[26] Mirzazadeh, A. and Sarfaraz, A. R. (1997), Constrained Multiple Items Optimal Order Policy under Stochastic Inflationary Conditions, Proc second Annual International Conference on Industrial Engineering Application and Practice, USA, San Diego, 725-730.

[27] Mirzazadeh, A. (2008), Economic order interval under variable inflationary conditions. Hamburg International Conference of Logistics (HICL2008), Hamburg, Germany.

[28] Mirzazadeh, A. (2007), Effects of uncertain inflationary conditions on inventory models using the average annual cost and the discounted cost. Eighth International Conference on Operations & Quantitative Management (ICOQM-8), Bangkok, 17-20.

[29] Misra, R. B. (1979), A Note on Optimal Inventory Management under Inflation, Naval Research Logistics Quarterly, 26, 161-165.

[30] Moon, I., Giri, B. C., and Ko, B. (2005), Economic Order Quantity Models for Ameliorating/Deteriorating Items under Inflation and Time Discounting, European Journal of Operational Research, 162, 773-785.

[31] Sana, S. S. (2010), Demand influenced by enterprises' initiatives-A multi-item EOQ model of deteriorating and ameliorating items, Mathematical and Computer Modelling, 52, 284-302.

[32] Sarkar, B. and Moon, I. (2011), An EPQ model with inflation in an imperfect production system, Applied Mathematics and Computation, 217, 6159-616.

[33] Sarker, B. R., Jamal, A. M. M., and Wang, S. (2000), Supply Chain Models for Perishable Products under Inflation and Permissible Delay in Payments, Computers and Operations Research, 27, 59-75.

[34] Sarker, B. R. and Pan, H. (1994), Effects of Inflation and the Time Value of Money on Order Quantity and Allowable Shortage, International Journal of Production Economics, 34, 65-72.

[35] Su, C. T., Tong, L. I., and Liao, H. C. (1996), An Inventory Model under Inflation for Stock Dependent Demand Rate and Exponential Decay, Operations Research, 33, 71-82.

[36] Vrat, P. and Padmanabhan, G. (1990), An Inventory Model under Inflation for Stock Dependent Consumption Rate Items, Engineering Costs and Production Economics, 19, 379-383.

[37] Wee, H. M. and Law, S. T. (1999), Economic Production Lot Size for Deteriorating Items Taking Account of the Time Value of Money, Computers and Operations Research, 26, 545-558.

[38] Wee, H. M. and Law, S. T. (2001), Replenishment and Pricing Policy for Deteriorating Items Taking into Account the Time-Value of Money, International Journal of Production Economics, 71, 213-220.

[39] Yang, H. L. (2004), Two-Warehouse Inventory Models for Deteriorating Items with Shortages under Inflation, European Journal of Operational Research, 157, 344-356.

[40] Yang, H. L. (2006), Two-warehouse partial backlogging inventory models for deteriorating items under inflation, International Journal of Production Economics, 103, 362-370.

[41] Yang, H. L., Teng, J. T., and Chern, M. S. (2010), An inventory model under inflation for deteriorating items with stock-dependent consumption rate and partial backlogging shortages, International Journal of Production Economics, 123, 8-19.

[42] Yang, H. L., Teng, J. T., and Chern, M. S. (2001), Deterministic Inventory Lot-Size Models under Inflation with Shortages and Deterioration for Fluctuating Demand, Naval Research Logistics, 48, 144-158.

[43] Uthayakumar, R. and Geetha, K. V. (2009), Replenishment policy for single item inventory model with money inflation, Opsearch, 46, 345-357.