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Research Paper

Measurement of Bitcoin Daily and Monthly Price Prediction Error Using Grey Model, Back Propagation Artificial Neural Network and Integrated model of Grey Neural Network

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ABSTRACT

One of the recent financial technologies is Block chain-based currency known as Cryptocurrency that these days because of their unique features has become quite popular. The first known Cryptocurrency in the world is Bitcoin, and since the cryptocurrencies market is a contemporary one, Bitcoin is currently considered as the pioneer of this market. Since the value of the previous Bitcoin prices data have a non-linear behaviour, this study aims at predicting Bitcoin price using Grey model, Back Propagation Artificial Neural Network and Integrated Model of Grey Neural Network. Then, the prediction's accuracy of these methods will be measured using MAPE and RMSE indices and also Bitcoin price data for a five-year period (2014-2018). The results had indicated that wen estimating Bitcoin daily prices, Back Propagation Artificial Neural Network model has the lowest absolute error rate (5.6%) compared to the Grey model and the integrated model. Additionally, for the monthly prediction of Bitcoin price, the integrated model, with the lowest absolute error rate (9%), has a better performance than the two other models.

1 Introduction

By looking at the global economy, it can be noticed that the level of transactions has crossed the limit of every country, and global trading is rapidly growing. Global trading requires the participation of intermediary institutions, and an increased number of these institutions may result in many obstacles, and it can also cause trading to be a very difficult and consuming endeavor. Nowadays, we have come to realize that recent technologies, such as cryptography and network computing in particular, have substantially affected the structure of global trading. One of the most significant of these is an emergent phenomenon known as cryptocurrency. The ecosystem of cryptocurrencies is in fact a private system for assisting the progress of trading among people without the presence of any central or intermediary institutions [31]. Bitcoin, which was first introduced in 2008 by a group of programmers known as Satoshi Nakamoto, was the first cryptocurrency. At first, Bitcoin was considered as an alternative to

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government fiat currency [10]. In a paper titled "A Peer-to-Peer Electronic Cash System", the developers of Bitcoin had implied that the governments were not eligible to create money [39]. Bitcoin's value on the very first days was less than even one cent, but with the introduction of block chain and all the other advantages brought by Bitcoin, many people became more interested in this form of cryptocurrency, and in December of 2017, Bitcoin's value reached up to 18,000 dollars.

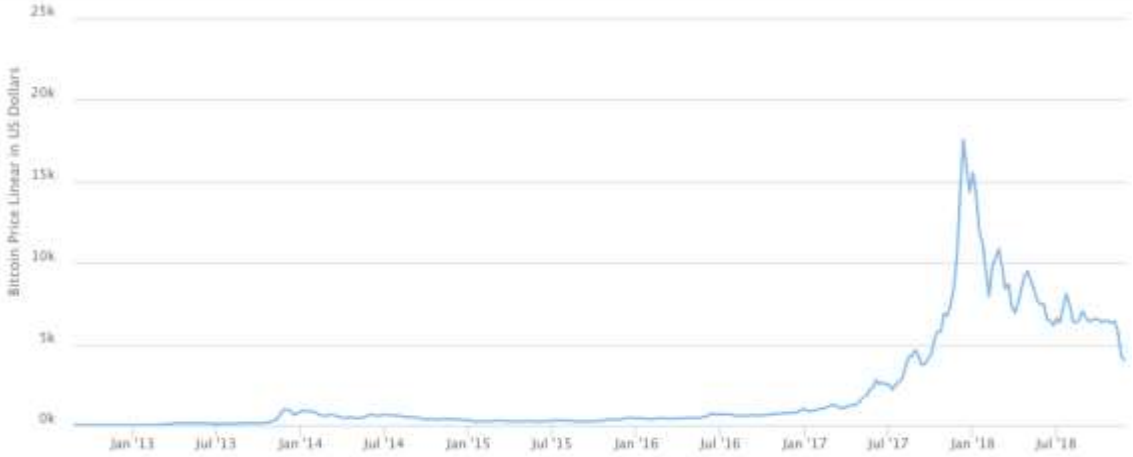

Fig.1: Bitcoins Price Chart (2013-2018)

Considering the numerous price changes of Bitcoin and other forms of cryptocurrencies, most people identify cryptocurrencies as some sort of financial asset, and in the recent financial literature, they are recognized as crypto assets. For that reason, the models that are used for estimating other sources of financial assets and commodities, such as stock, gold, oil, financial market indexes and etc., can also be useful for Bitcoins [25]. Bitcoin's price is quite important since it is recognized as some sort of financial asset that is globally accessed. Furthermore, Bitcoin has a great relationship with other kinds of cryptocurrencies. Therefore, an increase or a decrease in its price that can directly affect the cryptocurrency market is also highly remarkable [19]. Nevertheless, in comparison, countries that are quite competitive when it comes to electricity costs, consider Bitcoin as a very important component [6]. As for this, the electricity costs are far lower when compared to the global average. In comparison, this industry has the most return when it comes to the currencies' income, so proposing a model for estimating the Bitcoin's price is absolutely essential. In addition, the Central Bank of the Islamic Republic of Iran has emphasized that the increased use of cryptocurrencies has had potential effects on the monetary and currency policies of the country (The draft rules and regulations of the cryptocurrencies issues, [32]). Consequently, estimating a proper model in Iran is immensely recommended since this kind of cryptocurrency is applicable when getting around the sanctions or when it comes to expanding the cryptocurrencies industry because of the electricity's low costs which provide an amazing opportunity, in comparison to other countries, needed for earning the currencies' income.

Bitcoin's price has had a non-linear trend over time, and also there are many assumptions considering Bitcoins that result in estimating a curve-fitting model with changing variables, loss of efficiency, and better performance of non-linear models for estimating the future Bitcoins' price [2]. Hence, the originality of this paper is the fact that three non-linear models, known as the Grey model, the Back Propagation of Neural Network model, and the Integrated model of Grey Neural Network, are used to estimate the Bitcoin's price in the short and long term to identify the model for better performance when estimating the daily and the monthly prices of Bitcoins.

2 Literature Review and Research Background

In this section, we review some important and related works.

2.1 Literature Review

The difference in investors' expectations is a key problem in capital market studies considered a risk [51]. Humans could not fulfill their needs on their own when living in primary societies. Therefore, based on humans' needs, the trading of commodities began to become prevalent, and the first form of money, known as commodity money, started finding its place. Issues related to commodity money caused human beings to use a specific kinds of commodities with better capabilities, so then they started using different kinds of metals, such as copper, iron, nickel, brass, silver, and gold. Silver and gold were preferred over other metals because of their superior features, and a twofold monetary system emerged which later on faced some problems. Therefore, countries turned to a monolithic monetary system. The problems associated with this system led to the emergence of paper money and credit money which substituted the metal monetary system. As for now, we have noticed that the electronic monetary system has found a social place, so the paper money system will be eliminated [47]. In fact, with the advancement of information technology, the electronic monetary system came to fore. In this system, money has a different form and is not a physical concept anymore. In fact, in this system, money is recognized by the numbers related to the computer and the networks. In other words, electronic or digital money, is a new mechanism instead of the paper money in banks, but in the recent years, a new form of monetary system has emerged that is totally different with all the previous systems and has a new unit of measurement, known as cryptocurrency [13].

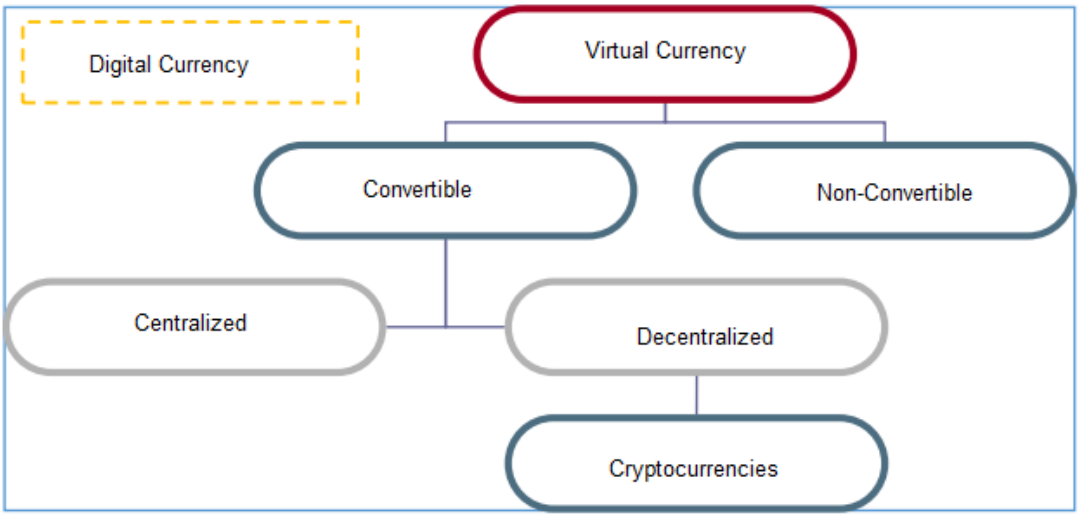

Fig. 1: Different Kinds of Virtual Currencies [31]

The idea of cryptocurrencies is based on the studies of David Cham and Stephan Brans in 1983. Other researchers such as Adam Black, has developed the idea of Hash, and then a researcher, named Wei Dai, suggested the protocol of cryptocurrency. Wei Dai is a specialist in the field of information technology that first proposed the concept of virtual currency as another name for cryptocurrency on his blog in 1998. His main purpose was to help with the financial problems and to create a monetary system without the use of intermediaries [21]. Of course, the theoretical foundations of these systems are based on the schools of Austrian economists and Hike (the winner of the noble prize in the economics field). Hike believes that an efficient currency is only created by the free rivalry of the private sectors. The

cryptocurrencies are considered as a subcategory of virtual currencies [42, 9], but his idea was only a theory till 2008. In the October of this year, a group of programmers known as Satoshi Nakamoto, suggested a cryptocurrency unit known as Bitcoin in their paper titled "A Peer-to-Peer Electronic Cash System". In 2009, Bitcoin was the first form of virtual money [46]. The United States congressional research service's report has recognized Bitcoin as a form of open source cryptocurrency. Germany's finance ministry has also considered Bitcoin as a recent financial tool which is similar to a private financial asset. Unlike the very first primary currencies, Bitcoin is not a centralized system, and all the transactions are done by the users of the payments' system of it [20]. Virtual currency is a general concept for different kinds of currencies. As you can see in Fig. 2, virtual currency is an intangible concept, therefore it is known as digital currency, and its difference is the fact that virtual currency has to relation with banking money and is a unique and recent form of currency which is also capable of solving mathematical algorithms. Sometimes, in scientific paper, digital money is used instead of virtual money because the word virtual has a negative concept when thinking of a real subject.

The above figure indicates that virtual currency is divided into two categories known as Convertible and Non-Convertible which means is it possible to convert virtual money to real money? Virtual money is similar to the kind of money that can be won in computer games, so it cannot be converted to any form of banking money at all. This kind of money is usually centralized. On contrary, the convertible kind of virtual money can be converted into real money and vice versa, and it can be used to buy real and virtual products and services. This kind of virtual money is the most advanced type which indicates the maturation of virtual money [26]. This kind of money is divided into two groups known as Centralized and Decentralized. The centralized kind is a term that is used when the issuance and the control of currency is performed by another central institution (such as network-money). On the other hand, the decentralized kind is a term used when the issuance and the control of money is beyond the control and the management of the central institution, so it can be managed by all the participants of the network through cryptography, and Bitcoin and other form of cryptocurrencies are also considered to be part of this category [31]. When using Bitcoin, all the transactions are recorded in a public ledger. This ledger is absolutely transparent and can be accessed by all. Anybody in any situation can follow the Bitcoin's transactions using this method. This public ledger is called a Block Chain. The participants of every transaction cannot be followed in a public ledger, and they are only clarified by some combination of letters and number. The public ledger gets updated by all the members of Bitcoin (Nakamoto, [39]). In fact, the blocks are computer documents that record all the data related to the Bitcoin's network permanently. A block records some of the historical transactions of Bitcoins in a given period that has not been previously recorded.

Therefore, a block is like a page of a ledger. Every block records some part of previous block and the address to the next blocks, and this prevents from the editing or the elimination of a block. Therefore, a block is a permanent reservoir of all the documents that was once recorded and is no longer editable and cannot be eliminated. The new block needs to be confirmed before connecting to the other blocks. This confirmation used to be done in the traditional system of the central institution which used to verify your account at first to make sure that it had enough cash for the transaction. If there was enough cash in the account, the money was withdrawn and transferred to the recipient's account. However, in this system, you need to go through the Proof of Work option, which involves solving a series of complicated mathematical problems by the help of miners, to confirm the transactions [28]. The process of mining or creating a new Bitcoin is like the attachment of a math problem to every block. There are miners that are constantly processing and recording the transactions of the Bitcoin's network. They strive to complete the given block faster than other in order to receive a fee and also Bitcoins as a reward. Therefore,

in general, the process of creating and producing Bitcoin is like the given explanations. This process requires costly hardware and electricity [43]. Although there are different and contradictory views when considering Bitcoin, more research should be done. The draft rules and regulations of the cryptocurrencies issues written by the Central Bank of the Islamic Republic of Iran has emphasized that the increased use of cryptocurrencies has had potential effects on monetary and currency policies of the country.

2.2 Research Background

Burniske and Tatar [12] has provided creative guidelines for investing in Bitcoin and other forms of cryptocurrencies in their book titled "Crypto Assets" and have noted that Bitcoin and other forms of cryptocurrencies, such as Ethereum, Ripple, Litecoin, and etc. have most of the features of a financial asset and should be studied like other financial assets, such as stocks although Bitcoin can be used to buy real and virtual products and services [9]. However, researchers such as Yermack [49] has mentioned that Bitcoin's features are more compatible with an investor and is known as a risky financial asset. Some articles are written considering the fundamental factors that can affect Bitcoin's price. Bitcoin is a form of cryptocurrency, so determining the required factors for the advancement of this form of money is highly important [30]. All the studies that consider this concept have noted that the public interest of active investors in financial markets all the social networks' posts, such Twitter's, the number of times that the word Bitcoin was searched on the internet [24], the popularity of Bitcoin, the number of trades using this form of cryptocurrency and the related reports of popular news media [41], economic factors such as consumer's price and the USA's dollar index [50] are the general factor that can affect the Bitcoin's price.

Also, other studies that are discuss the significant relationship of Bitcoin with ounces of gold and other tradeable commodities in the exchange [6] and also the stock market indexes such as the Dow Jones and the S&P 500 index [17], and the results indicate that there is not a significant relationship between Bitcoins and the named factors. In general, most of the studies about this concept investigate the effective factors on Bitcoin, and none of them has ever presented a model for predicting the Bitcoin's price.

There are only a few studies in the world that are devoted to predicting the Bitcoin's price. In 2014, Shah and Zhang [44], used a regression analysis model for estimating the Bitcoin's price that helped them realize the return price of the Bitcoins. They proposed a useful trading strategy for Bitcoins by using this model which could achieve a higher return in comparison to the buy and hold strategy. Also, they asserted that the non-linear models are more efficient when it comes to predicting the Bitcoin's price. Nasr et al. [40] A special feature of economic forecasting compared to general economic modeling is that we can measure a model’s performance by comparing its forecasts to the outcomes when they become available. Madan et al., [35] has used logistic regression models, SVMs, and predicting accidental algorithms in an article titled Automated Bitcoin Transactions Using Machine Learning Algorithms, to forecast the Bitcoin's price.

They used the daily Bitcoin's price from 2010 to 2014 as the data for their model and forecasted the daily changes of Bitcoin's price, but they believed that other accurate models can be used. Therefore, they have suggested further research using the provided model for other financial assets. In the most recent study considering this issue titled Predicting Bitcoin's Price Using Neuro-Fuzzy techniques by Georg et al., [24] first a few articles for predicting the Bitcoin's price have been mentioned, and then using the neuro-fuzzy technique, they have advanced their neuro-fuzzy model. Lastly, they have provided a model for predicting the daily Bitcoin's price. Their suggestions can be used for further studies and also in assessing other model of the artificial neural network when predicting the Bitcoin's price. Alijani et al., [1] show a significant correlation between the price trend and return in the Bitcoin that

has been confirmed by various statistical methodology. Using statistical test stand reviewing trends and relationships between the variables, planning can be done to invest in it and its performance or inefficiency can be tested. The results of that research show a significant and positive relationship between the price and return of Bitcoin.

3 Proposed Methodology

Empirical research can be categorized into two categories, consisting of experimental and descriptive, based on the method of data gathering. Also, the goal of this kind of research is practical. The statistical society of this kind of research is related to the daily Bitcoin's prices in a five years' period (2014-2018). As this research seeks a model for Bitcoin price prediction, the appropriate reference for bitcoin price data is first addressed, and the operational definitions of the modeling methods are described separately later in this section. Bitcoin's price data are gathered using a Bitcoin's price index at the Coindesk.com network site. Based on Dhyrberg [19] and Baur et al. [5] studies, the Coindesk.com network site is the best source for gathering Bitcoin's price data because the Bitcoin's price index of this network site is calculated using the average of Bitcoin's prices in four major cryptocurrencies markets, namely Bit stamp, Coin base, it Bit, and Bitfinex. Also, for calculating the Bitcoin's monthly prices, the weighted average of Bitcoin's daily prices is calculated in that month. The model used in this article are consisted of the Grey model, the Back Propagation of Neural Network model, and the Integrated model of Grey Neural Network which are described in this study.

3.1 Hypothesis

Considering theoretical foundations and the history of literature, Bitcoin has been known as a financial asset in the financial and economic literature [12], As it was mentioned Burniske and Tatar and have noted that cryptocurrencies have most of the features of a financial assets [49]. There are numerous articles that have considered this question "is Bitcoin a financial asset or a currency?", and they all have come up with the same answer which implies that Bitcoins have most of the features of a financial asset (Yermack, [49]; Glaser, [25]; Dhyrberg, [19]; Bouri et al., [6], Baur, [5]; Balcilar et al., [4]). Also, there are two studies in the cryptocurrencies" markets that discuss the inefficiency of this market (Urquhart, [48]; Nadarjah and Chu, [37]). Therefore, considering the low efficiency of cryptocurrencies' markets and other researchers' suggestions, the Bitcoin's price follows non-linear models while the regression and the linear models, cannot cover non-linear complicated relationships or of high levels [16]. Therefore, considering Bitcoin as a financial asset when providing a useful model for estimating its price is related to the group of studies that use non-linear models such as the artificial neural network, the Grey model, the Back Propagation of Neural Network model, and the Integrated model of Grey Neural Network when predicting the stocks' price, stock market indexes, other financial assets, firm's cash flows and etc. Harvey et al. [29] have created a neural network for the price changes of New York's stock exchange. Atsalakis and Valavanis [3] designed an integrated Neural Network model for predicting the index. Guresen et al. [27] used the neural artificial network model for predicting the NASDAQ index and stock's price with a high value.

Dash [14] compared the integrated Neural Network model with time series models, and the results indicated that the network had fewer errors. Also, there are studies that have investigated the efficiency of predicting using the Grey system. Deng [18] believes that the Grey theory also involves the fuzzy theory. In other words, the Grey theory can have a great performance in a fuzzy situation. In the Grey model, Sheng Chai et al. [45], compared the Grey model and the neural network as the required tools for predicting the stock index, and the result indicated the Grey model had fewer errors when predicting

the Taiwan's stock index. Naiming et al. [38] have compared the back propagation neural network and SVMs in a study of selected Shanghai's indexes. The results indicated that these models are considered to be better when predicting the Shanghai's index, and they have a more acceptable level of error. Jin Su [34] started predicting the cash flow of a bank using the Grey model and the neural network and indicated that the Grey model was more efficient when predicting the cash flows. Wang et al. [33] compared the neural network and the integrated method for predicting the stock index price. Based on the results of this study, the Grey Neural Network is more accurate when predicting the stock index price of the banking sector Therefore, the non-linear models used to predict the price of a financial asset can be tested for prediction of Bitcoin price. Therefore, the present paper hypotheses are as follows:

H_1: The integrated Grey neural network model outperforms the propagation artificial neural network model and the Grey model in predicting the daily price of Bitcoin.

H_2: The integrated Grey neural network model outperforms the propagation artificial neural network model and the Grey model in predicting the monthly price of Bitcoin.

3.2 Research Models

The operational definition of price prediction models, namely the Gray Model, the Back Propagation of Neural Network Model, and the Integrated Model of Gray Neural Network is discussed below, and the operational definition of the appropriate criterion for error measurement is also provided.

3.2.1 Grey Model

Grey theory was first proposed by Deng [18] in an international magazine named "System and Control Letter" titled "The Control Problems of Grey System". This theory quickly enhanced and was used in assessing, creating models, predicting, decision making, and control. This method can be used for creating models when there are unknown issues with limited data and incomplete information. In other words, this theory considers the kind of theories that have a limit, a specific range, and an uncertain definition. Therefore, Grey theory is also consisted of the fuzzy theory because the issues of the fuzzy theory have a clear definition and an uncertain domain, and its uncertainty can be described using the membership functions.

If Grey theory has a weak performance when studying problems with small samples and weak information, this theory will search for a realistic pattern based on the creating model methods and with the least amount of data available. Therefore, the Grey system theories consider issues that have a limit, a specific range, and an uncertain nature. Lastly, this theory can be used in a world of knowledge considering uncertain issues and somehow complicated matters. Therefore, it can be stated that this theory has a broader use comparing to statistics, the probabilities, and the fuzzy math that consider simple and uncertain issues, and it is considered as an enhanced form of the fuzzy theory (Malek and Dabaghi, [36]). Consider the time series below.

$$x^{(0)} = (x^{(0)}(1), x^{(0)}(2), \dots\dots, x^{(0)}(n)) \quad (1)$$

In this equation, $X^{(0)}(n)$ is a series of time series data in the time n. Therefore, the above equation can be rewritten:

$$x^{(1)} = (x^{(1)}(1), x^{(1)}(2), \dots\dots, x^{(1)}(n)) \quad (2)$$

In this equation, $X^{(1)}(k)$ can be calculated using the equation below

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i) \tag{3}$$

The new series generated, can be calculated based on the Accumulated Generating Operation (AGO). With the use of this operation, the non-negative, smooth, and discrete data can be transformed to a series. The differential equation below is named GM (1,1), which is in fact the Grey theory's main model.

$$\frac{d\,x^{(1)}}{dt} + a\,x^{(1)} = u \tag{4}$$

In this equation, an and u are the parameters of the differential equation calculated using the least squares method:

$$\alpha = \begin{bmatrix} a \\ u \end{bmatrix} = (B^T B)^{-1} B^T Y_N \tag{5}$$

In this estimation, B and Y_N, can be calculated using the method below.

$$Y_N = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ . \\ . \\ . \\ x^{(0)}(n) \end{bmatrix} \qquad B = \begin{bmatrix} -0.5(x^{(1)}(2) + x^{(1)}(1)) & 1 \\ -0.5(x^{(1)}(3) + x^{(1)}(2)) & 1 \\ . & . \\ . & . \\ -0.5(x^{(1)}(n) + x^{(1)}(n-1)) & 1 \end{bmatrix} \tag{6}$$

The answer of this differential model is Grey's predicting model for $x^{(1)}$.

$$\hat{x}^{(1)}(k+1) = \left(x^{(0)}(1) - \frac{u}{a}\right) e^{-ak} + \frac{u}{a} \tag{7}$$

Therefore, Grey's model for predicting $x^{(0)}$ can be calculated using the model below.

$$\hat{x}^{(0)}(k+1) = \left(x^{(0)}(1) - \frac{u}{a}\right)(1 - e^{a}) e^{-ak} \tag{8}$$

Using the Regressive Reduction method, the below equation can be achieved.

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \tag{9}$$

The parameters an and u in Grey's model named GM (1,1), can be calculated using the data of predictions. There is an entry series in GM (1,1), and the predicting operation generates another series. There is other model based on this method as well that has multiple entries and predictions. The conditions and the essentials for the data used in Grey's model are very important. The coefficient of a should not equal to zero (There is another solution if it equals to zero), and the data should be smooth (they should not change so often) and non-negative in this equation (Chua and Huang, [11]). Also, the conditions of the smoothness of data and their stepwise should be followed and confirmed. Smoothness and stepwise are defined by ρ and σ relatively. The equations used for calculating these two factors are provided below.

$$\rho(k) = \frac{x(k)}{\sum_{i=1}^{k-1} x(i)} \tag{10}$$

$$\sigma(k) = \frac{x(k)}{x(k-1)}; k = 2,3,\dots,n \tag{11}$$

In fact, if these two ratios have the below conditions, Grey's model will be more accurate and efficient.

$$\frac{\rho(K+1)}{\rho(K)} < 1; K = 2,3,\dots,n-1 \tag{12}$$

$$\rho(K)\epsilon[0,\varepsilon]; k = 3,4,\dots,n \tag{13}$$

$$\varepsilon < 0.5$$
$$\sigma(k+1)\epsilon[1,1+\varepsilon] \tag{14}$$

3.2.2 The Back Propagation of Neural Network Model

The neural network is an efficient toll when estimating complicated and non-linear functions. Different regression models search for a linear or an estimation curve in a way that the distance between the points of the given line or the curve from the actual values equals to the least value possible. It should be noted that the financial variable, such as price, cash flows, returns, and etc. follow very complicated non-linear relations in a way that the estimation curve may equal to a very noticeable number. On the other hand, the regression models may have errors, and there are different assumptions. Therefore, the regression models with too many changes in their values used for estimating the curve may lose their efficiency. One of the most important uses of neural network is estimating the curves based on the previous information. In fact, the neural network when integrated with learning operations can decrease the estimation errors of the curve. Of course, the neural network has other capabilities, such as categorizing, extracting the feature, associative memory, compacting, optimization, control, and identifying, but it can be stated that its basis of work is the same regarding all of these capabilities.

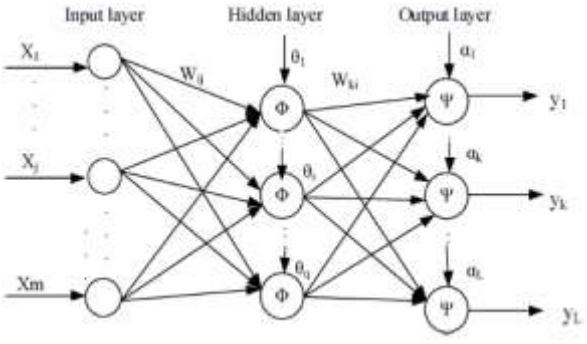

Fig.3: Neural Network's Layers

x_i: The entry values of the neural network
w_{ij}: The i[th] entry weight values to neuron j
θ_i: The i[th] neuron's threshold

ϕ: The transformative function of the hidden layer

w_{ki}: The weight entry values from k[th] neuron of the hidden layer to the i[th] neuron of the exit layer

y_i: The exit value of i[th] neuron of the neural network's exit layer

The Back Propagation of Neural Network model can be briefly described using the steps below.

1- Determining the entry series and giving values to the nodes' weights and threshold of each node
2- Calculating the exit value of the neural network
3- Calculating the exit value of the entry layer using the transferring neural function and the transferring weights
4- The exit value of the entry level is defined as O_i^I , and it equals to the first entry series

$$O_i^I = x_i \tag{15}$$

Calculating the exit value of the hidden layer using the neural transferring function and the transferring weights can be done using the equation below.

$$\sigma_k^H = O_k^H(1 - O_k^H)\sum_{j=1}^{m} \sigma_j^o . w_{jk}^{OH} \tag{16}$$

$$net_k^H = \sum_{i=1}^{n} w_{Ki}^{HI}\, O_i^I, k = 1,2, \dots, l \tag{17}$$

$$O_k^H = f\left(net_k^H - O_k^H\right) \tag{18}$$

Calculating the exit value of the exit layer using the neural transferring function and the transferring weights can be done using the equations below.

$$net_k^H = \sum_{i=1}^{n} w_{jk}^{OH}\, O_k^H, j = 1,2, \dots, m \tag{19}$$

$$y_j = f\left(net_j^O - \theta_j^O\right) \tag{20}$$

In the above equation, $\boldsymbol{y_j}$is considered as the exit level of the neural network.

Calculating the errors of the estimated model can be done using the difference between the exit value calculated and the real values and statistical parameters such as the least squared of the error sentences.

$$E = \frac{1}{2}\sum_{j=1}^{m}\left(d_j - y_j\right)^2 \tag{21}$$

If the error's value is less than the expected value, go back to step number 5, and if this does not happen, go to step number 4.

Adjusting the neural network's weights

The weight between the hidden and the exit layer can be adjusted as below.

$$w^{-OH}_{jk} = w_{jk}^{OH} + \Delta w_{jk}^{OH} \tag{22}$$

$$\Delta w_{jk}^{OH} = \text{ɲ}\sigma_j^o . O_k^H \tag{23}$$

$$\sigma_j^O = \left(d_j - y_j\right).y_j(1 - y_j) \tag{24}$$

The weight between the entry layer and the hidden layer can be adjusted by the equations below.

$$\Delta w_{ki}^{HI} = \text{ɲ}\sigma_k^H . O_k^I \tag{25}$$

3.2.3 The Integrated Model of Grey Neural Network

In this model, the main foundation is that the expected values can be calculated using the Grey model GM (1,1). Then, these values can be used in the neural network as the entry values, and here the expected values are as the same real values. After adjusting the weights and the values of the thresholds, the expected values of the Grey model will be closer to the real values. In fact, the neural network is the process of improving and decreasing the predicting errors, and this adjusted learnt neural network can be used for adjusting the next values of the Grey's predicting model. These processes can be done on standardized data, and they can have their main values after predicting.

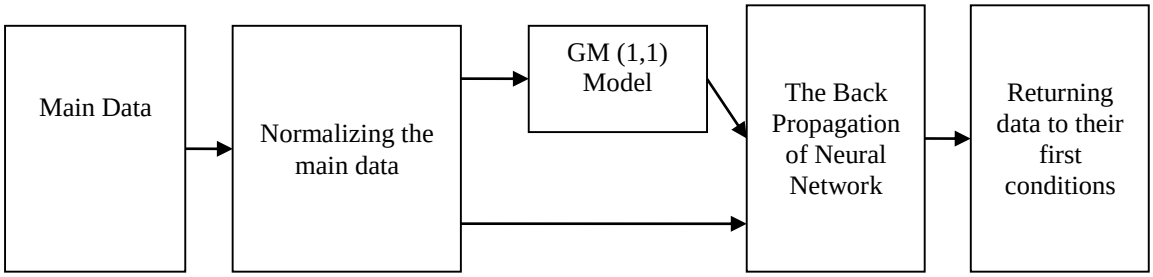

Fig.4: Grey-Neural Network processes

For achieving the final values of the predictions in the integrated model of Grey Neural Network, the steps below should be taken. The P series are the values of the first series of predictions, defined as $\boldsymbol{x}^{(\mathbf{0})}$which has n-1 components. Another series, known as the T series, is generated using the P series.

$$T = (x^{(0)}(2), x^{(0)}(3), \dots\dots, x^{(0)}(n)) \tag{26}$$

The entry vector used in the Back Propagation of Neural Network is P, and the target vector for the neural network is T. Then, the Back Propagation of Neural Network is taught using these two vectors to define the weight and the threshold values. A part of the data is used for testing purposes. Then, the expected values of the Grey's model are used in the Back Propagation of Neural Network, and the exit values will be compared to the real values in order to calculate the error values. In this study, the integrated method of the neural network has multiple entry and exit points. In a neural network with a structure consisting of multiple entries and exists, when the P vector is achieved, both P and T vectors will be used in the neural network. Therefore, the number of the entries and the exits should equal to the number when testing the integrated model.

3.2.4 Measuring the Errors of the Predicting Model

When analyzing a predicting mode or when choosing the best model among different model, an index is needed to make the necessary decision for accepting or denying the estimating model. In addition, there is uncertainty in all of predictions. The predictions' errors are caused by the fact that one or multiple components of the time series predicting components, like the trend component, are not considered to be seasonal or periodically, or another reason is that they have irregular fluctuations. The sum of all the errors in a predicting method, with n being the total of time periods, can be calculated using the equation below.

$$SE = \sum_{t=1}^{n} [y_t - \hat{y}_t] \tag{27}$$

When the predicting errors are accidental, some of the errors are positive, and the rest of them are negative, so the effect will be neutral, and calculating the errors will face some problems. The absolute values of the predicting error can be calculated using the equation below.

$$|e_t| = |\, y_t - \hat{y}_t| \tag{28}$$

Now, for calculating the errors in the time period of n, the mean absolute deviation (MAD) can be calculated using the equation below.

$$MAD = \frac{\sum_{t=1}^{n} |y_t - \hat{y}_t|}{n} \tag{29}$$

This error can be calculated using another method, knows as the mean square error (MSE) as well using the equation below.

$$MSE = \frac{\sum_{t=1}^{n} [y_t - \hat{y}_t]^2}{n} \tag{30}$$

In this method, the root mean square error is calculated, using the equation below, because the squares of the errors will generate a noticeable value.

$$RMSE = \sqrt{\frac{\sum_{t=1}^{n} [y_t - \hat{y}_t]^2}{n}} \tag{31}$$

Sometimes, calculating the percentage of the prediction errors is better than the real values. Therefore, the mean absolute percentage error is an important parameter for calculating the percentage of predictions. This parameter can be calculated using the equation below.

$$MAPE = \frac{\sum_{t=1}^{n} |\frac{y_t - \hat{y}_t}{y_t}|}{n} \tag{32}$$

In the following section, the described parameters will be used for calculating the prediction errors of the article's model to estimate the Bitcoin's price.

4 Research Analysis and Finding

4.1 Creating a Proper Model for Predicting the Daily Price of Bitcoin

In this part, the neural network, the Back Propagation of Neural Network, and the integrated model of Grey Neural Networks, the last 15 days of Bitcoin prices are estimated and compared to the real Bitcoin's price. By calculating the error parameters, like RMSE and MAPE, the best model to predicting the daily

price of Bitcoin will be chosen. The daily data of Bitcoin's prices from the beginning of 2014 to the end of 2018 for the neural network, the Back Propagation of Neural Network, and the integrated model of Grey Neural Networks is used. The entry data to the neural artificial network is categorized to three sections: A) The Training Set, B) The Validation Set, and C) The Test Set (which is the final 182 days that approximately equals to 10 percent of the data), and also there are three hidden layers. In addition, when the number of the hidden layers and the neurons are not enough in every layer, the network cannot come up with an efficient response, and if the number is more than the amount needed, the network is going to be unstable. The Integrated model of Grey Neural Network uses 50 percent, the training set uses 40 percent, and the testing uses 10 percent of the data. Also, the Grey model is GM (1,1). The table below shows the real data of Bitcoin and the estimated data for the last 15 days of the 2018 using the three mentioned models.

Table 1: The Real and the Estimated Bitcoin's Daily Price Using the Neural Network, the Back Propagation of Neural Network, and the Integrated Model of Grey Neural Networks

Period	**Day 1**	**Day 2**	**Day 3**	**Day 4**	**Day 5**
Bitcoin's real price	3548	3716	3737	4138	3899
Neural network estimated price	3551	3560	3565	3583	3607
Grey's model estimated price	21270	21313	21355	21398	21441
the Integrated model of Grey Neural Network estimated price	3805	3761	3712	3610	3574
Period	**Day 6**	**Day 7**	**Day 8**	**Day 9**	**Day 10**
Bitcoin's real price	4045	4008	4082	3835	3849
Neural network estimated price	3624	3647	3653	3660	3669
Grey's model estimated price	21484	21527	21570	21613	21656
the Integrated model of Grey Neural Network estimated price	3503	3441	3442	3360	3357
Period	**Day 11**	**Day 12**	**Day 13**	**Day 14**	**Day 15**
Bitcoin's real price	3646	3948	3797	3896	3747
Neural network estimated price	3687	3701	3704	3701	3709
Grey's model estimated price	21699	21743	21786	21830	21873
the Integrated model of Grey Neural Network estimated price	3358	3364	3371	3354	3324

Based on the above table, the indexed used for calculating the errors of the artificial neural network, the Grey model, and the Integrated model of Grey Neural Network for predicting the Bitcoin's daily price compared to its real values have been calculated that is available in the table below.

Table 2: The parameters of Bitcoin's Daily Price Using the Neural Network, the Back Propagation of Neural Network, and the Integrated Model of Grey Neural Networks

The error's parameter	**MAPE**	**RMSE**	**MSE**
The values of the neural network	5.6%	272	74141
The values of Grey model	10.5%	449	201450
The values of the Integrated model of Grey Neural Network	460%	17712	313743317

Based on Table 2, the value of the average calculated by Back Propagation of Neural Network model equals to 5.6 percent and the square root of the errors equals to 272. Also, the absolute average of the errors' percentage in Grey's model and in the integrated model of Grey Neural Network relatively equal to 460 percent and 10.5 percent, and this indicates a higher error, in comparison to the neural network. Since the RMSE and the MAPE parameters of Back Propagation of Neural Network model are less than the value of these parameters in the Grey model and the integrated model of Grey Neural Network, the artificial neural network has a better performance when estimating the Bitcoin's daily prices. Also, the

Grey model has a very high absolute average of the errors, so it is not a proper model for estimating the Bitcoin's daily price.

Fig.5: The Predicting Chart for the Prices of the Neural Network Model and the Integrated Model of Grey Neural Network in Comparison to Bitcoin's Real Price

Fig. 5 idicates that the Back Propagation of Neural Network model has a better performance when estimating the Bitcoin's daily prices. Therefore, the research first hypothesis is rejected.

4.2 Creating Proper Models for Predicting the Monthly Prices of Bitcoin

When choosing the best model for predicting Bitcoin's monthly prices, daily price data of Bitcoin is gathered from January of 2014 to June of 2018. Also, the weighted average is calculated to find the values of monthly prices. The Back Propagation of Neural Network uses the error parameters, RMSE and MAPE to choose the best model for estimating Bitcoin's monthly price with the least amount of error when compared to the other models. When creating the Back Propagation of Neural Network model, two hidden layers consisting of 50 percent of the data for the training set, 40 percent of data for the validation test, and 10 percent of the remaining data for the testing set. Also, the Grey model is GM (1,1). In the Integrated model of Grey Neural Network, 50 percent of the data is used for training set, 40 percent is used for the validation set, and the remaining 10 percent is used for the test set. The table below shows Bitcoin's real data and the estimated data for the last 5 months of 2018, using the neural network, the Back Propagation of Neural Network, and the integrated model of Grey Neural Networks.

Table 3: The Real and the Estimated Bitcoin's Daily Price Using the Neural Network, the Back Propagation of Neural Network, and the integrated Model of Grey Neural Networks

Period	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6
Bitcoin's real price	7368	6632	6591	9480	4788	3715
Neural network estimated price	4513	5842	7163	8903	7687	6685
Grey's model estimated price	1839	1929	2024	2123	2275	2366
the Integrated model of Grey estimated price Neural Network	6712	7264	6654	6615	4797	3722

Based on Table 3, the measuring indexes of the errors from the neural network, the Back Propagation of Neural Network, and the integrated model of Grey Neural Networks, are calculated in the table below for predicting the monthly price of Bitcoin in comparison to its real price.

Table 4: The parameters of Bitcoin's Monthly Price Using the Neural Network, the Back Propagation of Neural Network, and the Integrated Model of Grey Neural Networks

The error's parameter	MAPE	RMSE	MSE
The values of the neural network	34%	2018	4443390
The values of Grey model	63%	4757	2634326
The values of the Integrated model of Grey Neural Network	9%	1227	1507014

Based on Table 4, the integrated model of Grey Neural Network has the least amount of absolute average of the errors percent that equals to less than 10 percent, so this model has a better performance for estimating the Bitcoin's monthly price compares to Grey model and the neural network. The square root of squares of the errors of the neural network and Grey model equal to 2018 and 4757 relatively, so the performance of the neural network is better than Grey model, but the value of MAPE equal to 34 percent, so just like Grey model, it is not considered a proper model for estimating Bitcoin's monthly price.

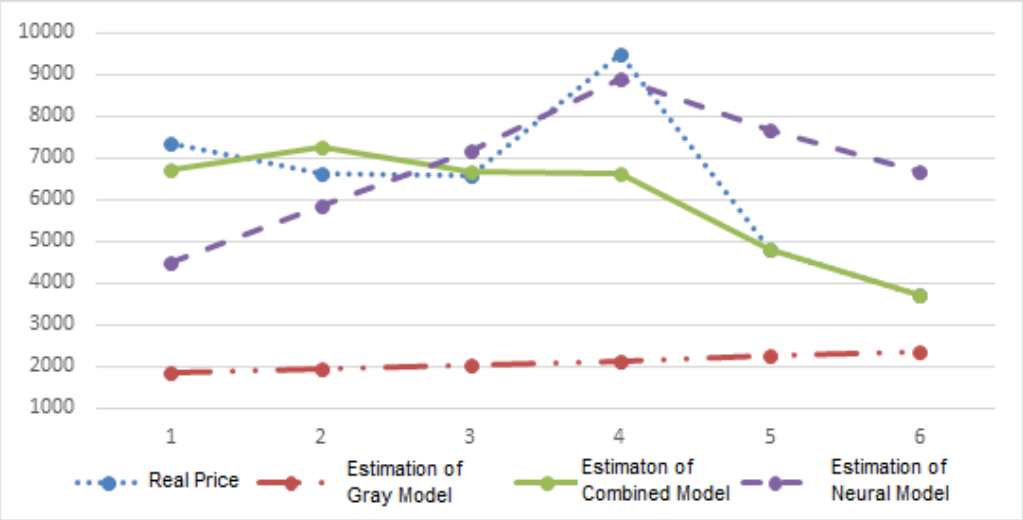

Fig. 6: The Predicting Chart for the Prices of the Neural Network Model and the Integrated Model of Grey Neural Network in Comparison to Bitcoin's Real Monthly Price

As it is indicated in Fig. 6, the integrated model of Grey Neural Network has the best performance when estimating the Bitcoin's monthly price since the Bitcoin's price is approximated with a very small error in the fifth and the sixth month. Grey model hast the highest error when estimating the Bitcoin's monthly price, and it is suggested not to use this model for estimating the daily and also the monthly prices by itself. Therefore, the research second hypothesis is approved.

5 Discussion and Conclusion

The financial crisis in 2008 indicates that just like the economists' point of view, the monetary and the banking system is relied on the dollar value, and there are some problems with this fact. The analysis show that the mechanism of creating money and other inflationary actions of the central banks usually prefers short term benefits of the governments over rationality and long term consistency and can create a new financial system using technology. Bitcoin and other forms of cryptocurrencies are examples of

creations that have emerged by the advancement of the internet, and they are actually considered as an economical idea which has a computer software structure. If people were not interested in this concept, these financial assets would have been forgotten in the very first years. Advantages such as the optional way of transferring money, non-terminable transaction, accelerating the transaction, decreasing the expenses of out of the border transactions, and elimination of intermediaries are some of these technological aspects.

Bitcoin is a financial asset, and since it is globally accessed, its price is highly important. In addition, Bitcoin has a close relation to other cryptocurrencies, and a decrease or an increase in its price directly affects the cryptocurrencies market. On the other hand, the cost of electricity in Iran is much lower compared to the global average. This industry is one of the industries with the highest return and currency income in Iran, so providing a proper model for estimating Bitcoin's price is highly important. Besides, the Islamic Republic Central Banks has emphasized that the extended use of cryptocurrencies has noticeable effects on monetary and currency related policies of the country. This study uses three models, namely the neural network, the Back Propagation of Neural Network, and the integrated model of Grey Neural Networks, for estimating the Bitcoin's daily and monthly price. The results indicated that when estimating the Bitcoin's daily price, the neural network model has the least MAPE and RMSE. Also, based on these results and studies such as Silva Filho et al. [3], Madan et al. [35], and Shah and Zhang [44], the non-linear models are more suitable when estimating Bitcoin's price.

Since Bitcoin and other forms of cryptocurrencies are considered to be the latest financial tools globally, further empirical research is needed for predicting the price of the existing cryptocurrencies by modeling other non-linear models, as well as to study and come up with new ideas on the application areas of these tools to finance ideas by concluding smart contracts and releasing the relevant token. Therefore, it is suggested that researchers use other meta-heuristic algorithms such as ant colony optimization algorithm (ACO), and collective behavior of starling birds, and improve models using genetic algorithm. Additionally, it is suggested that policy makers make decision about the entry of this tool in the money and capital markets considering the entry of Bitcoin and other cryptocurrencies to other financial markets as well as the potential of this tool to circumvent sanctions. It is also recommended as a practical suggestion to financial institutions and the investors in the field of cryptocurrency and bitcoin mining that use the Neural Network Model for their daily time frame and utilize the model of Gray Neural Network for their monthly time frame to predict the bitcoin price for in terms of profitability.

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