



# Measurement of Economics to Scale in Corporates of Tehran Stock Exchange

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## ARTICLE INFO

### Article history:

Received 09 June 2019

Accepted 04 September 2019

### Keywords:

Economics to scale

Nonlinear behavior of costs

Translog production function

## ABSTRACT

One of the most important effective factors in the economic growth is the increased efficiency of manufacturing sectors. Thus, it is necessary to review and measure the efficiency of business units from a variety of dimensions to plan the increase of efficiency in future courses. One of these dimensions is the economics to scale (ES) referring to the concept of enhanced earnings due to the increased manufacturing unit, which may be of an increasing, fixed or decreasing trend. ES can be measured through economic production functions. This paper has applied Translog production function to measure corporates' ES. Therefore, data of 105 manufacturing corporates have been collected during 2008-2017. With regard to linearity between independent variables, the elastic net regression method has been used. Results indicate that some industries are of increasing economics to scale trend and some have a decreasing ES. Results can be applied in order to plan and determine the appropriate production level in the management accounting of corporates

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## 1 Introduction

Investment is one of the most important tools for achieving economic growth in the country [1]. Economic growth can be affected by the increased production elements, efficiency level and productivity in the economic sectors. Traditional theories of growth primarily focus on the first element. In developing countries, lack of production elements especially capital may be considered as a basic constraint in the economic growth process, investment is strongly emphasized in the analysis of growth elements and productivity of desired elements has not been paid attention; it is an important factor in not achieving the suitable economic growth. In particular, proficiency and productivity of business units are the most important competitive elements. Business units reduce the expenses through increasing the proficiency and productivity levels and thereby, strengthening the competitive power. At owners' business units (shareholders) scale, they allocate their own financial sources to the units in order to obtain more returns and constitute a huge capital by the aggregation of smaller capitals while applying the sources optimally, achieving more returns and resulting in economic growth. The surplus return created in a business unit or group is caused by synergy [2]. Group dynamism is one of experimental psychological fields, which has been introduced after the world war II. When Levin as the founder of

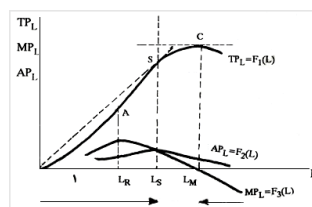
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this science described it with new features and content contrary to the previous ones, an opportunity was offered to present the issues of effective elements in group dynamism such as synergy in a group regarded as a key concept in the studies related to human sciences. Accordingly, the group has a structured integrity with different coordinates from total components. One of the most significant and interesting topics is the survey of synergy resulted from the formation of groups and forums; its recognition is the analytical base of lots of economic, psychological and sociological studies. Synergy has been taken from the Greek word "synergia" meaning joint work and cooperation in action. Synergy is the separate cooperation of several elements so that its general impact is more than the sum of effects independently. In a simpler definition, when two or more elements, flows or factors have mutual cooperation and interaction, an effect which is more than the effects of those elements separately will be created; then, the phenomenon "synergy" occurs [3]. In terms of economics and management studies, synergy is used to express the impact of group activities and increased efficiency of group work as compared to individual and independent work so that groups provide the capability of synergism and work more powerful than the members' individual power [4]. In other words, total sum is more than sum of members. Some believe that  $2+2=5$  or larger than 5. Therefore, it is expected that in business units along with various human and financial resources, the phenomenon "synergy" is formed and one of its aspects is the economics to scale and volume created by the elements inside and outside the organization. Wholesale purchase discounts, technology usage, reasonable financing, powerful management and marketing network are regarded as effective elements inside the organization and such factors as a chain of financial providers, educational system and financial service providers are outside the organization. Considering the above discussions, it can be stated that large business units are of increasing economics but inefficiency, lack of suitable sources combinations, and instability may lead to big expenses in every business unit. For instance, an expert is able to monitor 10 workers and each product can be manufactured in 5 h; if the number of workers increases to 15 people, the production hours will be increased for each product due to loss of ability to monitor the workers and the economics to scale will be decreased. Finally, this question is discussed "whether ES can be measured based on accounting information and at which level is ES of corporates in Tehran Stock Exchange ranked? Managers can provide each of these three functions with timely access to information by the accounting system recognition of cost behavior is one of the most important aspects of benefit analysis for managers. Study of cost behavior is not only important for academic researchers but also for ones who directly participate in activities related to their professional activities [5]. It is necessary to achieve the knowledge of cost behavior at all the cost levels and management accounting measures, especially controlling the costs, planning and making decisions. With the existence of enormous structures influencing the costs, the measures level is more likely to affect the cost amount. Measures level is expressed in a variety of ways like production unit ton, working hours, standard hours, sale, etc. Based upon the traditional theory, the cost behavior varies in relation to the measures level and the behavior prediction is done based on previous information [6]. Cost behavior refers to those adjusting the resources from a modern view and in order to understand the cost behavior, the decisions elements and consequences must be focused. Managers select the resources with respect to different constraints (e.g. demand, production technology, resources adjustment costs, corporate governance authority, debt contracts, and governance regulations), motivations (e.g. performance bonus, return objectives, and ownership) and biases (e.g. over trust). Lack of focus on traditional mechanic view concerning cost incentives and management decisions leads to develop a new and powerful method on cost be-

havior, which enables the researchers to open the black box of cost behavior and analyze a wide range of phenomena affecting the cost behavior through management decisions [7]. A question is presented "what is the source of economics to scale?" In accounting, the costs have been classified into variable, mixed, semi-variable, semi-fixed and fixed, which are generally classified into fixed and variable in analyses whereas the semi-variable and semi-fixed expenses have been of an increasing trend these days. Cost behavior refers to the costs of resources adjustment in the modern perspective. Adjustment costs are partial ad huge for variable and fixed resources, respectively. Though, lots of resources are put between these two ranges. For example, managers are able to hire or fire the skilled workers immediately but it requires imposing considerable adjustment costs like looking for, selecting and training new recruits and consent payment to the fired workers. Resources adjustment costs lead to dynamic impact on the relationship between activities and resources. According to traditional view, selecting the resources amount by the managers depends upon the same period activities, earlier resources amount (influencing adjustment costs in current period) and future expected activities (influencing future adjustment costs) as well as manager's motivations and behavioral biases [8]. Considering the aforementioned discussions, the synergy and economics to scale are resulted from fixed, semi-fixed and semi-variable expenses but in accounting, semi-variable and semi-fixed sections have not been regarded. For example, a large business unit uses wholesale purchase discounts. On the other hand, large business units involve the progressive rates due to the increased use of energy carriers. Also, it is expected that large units have less final financing costs than the small ones. Concerning other expenses (e.g. human resources and technology), it follows the same trend. Thus, complex business unit's conditions cannot be analyzed through linear relations and equations and nonlinear models should be used. Discussion of learning curve function and measurement model is a limited example of semi-variable expenses and applications of nonlinear models. In economy, based on the relationship between final production and average production, three production areas have been recognized. According to the fig .1, the production area 1 involves the production level 0 to the point S, the area 2 called the economic production area covers the area where final production is equal to average production to the area with zero final production (C to S points) and the rest is the area 3. Based on the definitions of the cost behavior in the area 1 from the points 0 to A, decreasing semi-variable and semi-fixed costs have led to the reduced expenses per unit; the severity of the decrease is the highest amount in the production capacity of business unit. From the points A to S, semi-fixed and semi-variable expenses have an ascending trend with a sloppy slope; as a result, final production has a decreasing trend but also, final production is more than the average production per unit. From the points S to C, semi-fixed and semi-variable expenses are of an ascending trend with slope more than the area 1 but it is still economic. After the point C due to the increasing expenses, final production is negative and average production income is decreasing per product.



**Fig.1.** Characteristics of Production Areas

In microeconomics theory, optimal production level refers to a level which uses economics to scale completely and minimizes the average cost. According to the definition of return to scale from the perspective of production function; constant return to scale (CRS) exists when the product changes in accordance with the changes in inputs. If the product increase is less than the input increase, there will be a decreasing return to scale (DRS) and if the product increase is more than the input increase, there will be an increasing return to scale (IRS). Economics to scale is a status when the production cost is decreased per product in addition to the increased production of an operation unit. Theoretically, the best activity amount of a production unit indicates the production volume with minimum average costs.

In this paper, to adapt the real circumstances, nonlinear production functions have been used to analyze and determine the return to scale in order to recognize the production area where a business unit operates and economics to scale. In the following, after reviewing the research background, it is investigated.

## 2 Research Background

Several experimental studies have been conducted using parametric and nonparametric methods in terms of production and return to scale as follows. Thompson [9] investigated the impact of work force, energy and capital on production using physical production function during 1951-2008 in America. Results indicated that energy is of a production capacity twice as much as work force. On the other hand, work force and overtime work are self-stretching but capacity for capital is weak. Mizobuchi [10] divided the work force productivity in three elements including technical advance, capital and return to scale and concluded that return to scale is an explanatory element of productivity gap between services and production sectors. In investigating the dynamism of productivity growth in Canadian Communication Industry, Gu and Lafrance [11] demonstrated that in addition to economics to scale and technical advance, the competitive process along with entry and exit of corporates to the market was important in specific periods. A study done by Chen [12] confirmed the evidence of increasing return to scale and relatively low technical advance in American manufacturing industries. In comparison with earlier experimental researches, a complete econometric analysis was used and more reliable results on RS estimates were presented. By Fleisher et al [13] the role of education in employee productivity has been studied in Chinese corporates. Researchers estimated the return rate of training costs by the means of employees' final production related to various educational levels based on production function. Thus, data of 338 business units were collected from 5 manufacturing industries during 1998-2000. Results indicated that education caused to increase final production by 30.1%. Diewert et al [14] computed the productivity growth and RS of 17 industries using indicator numbers in China. Results have shown that all the industrial sectors have increasing RS in China. In another research done by Diewert and Fox [15] RS, technical advances and exclusive profits have been estimated considering the existence of several inputs and products and the nonexistence of competition using Translog production function. Results indicated the increased RS and exclusive profits. Oliveira et al. [16] investigated the effect of increased RS and technology transfer on performance in Brazilian industries and estimated the coefficients of industrial production, workforce productivity, export rate and technology gap using the auto-regression model. Results have shown that Brazilian industry is of increasing RS. Sharma et al. [17] estimated the technical efficiency and total productivity of manufac-

turing elements among American states using random border function in 1997-2000. Based on research results, technical efficiency value was averagely high in the mentioned states and Alaska had the highest value. As well, changes in total productivity indicated that technical efficiency change contribution has been less than technological variations.

Deliktas et al [18] studied the total productivity changes using data envelopment analysis in public and private sectors of selected industries in Turkey in 1990-1998. Results have shown that although the increase of total productivity was low, the increase of technical productivity played a critical role in total productivity growth in both sectors. Piesse and Thirtle [19] investigated the efficiency level, technological changes and productivity in industrial and agricultural corporates in Hungary, measured the efficiency rate and examined the effective elements using random border and production functions in 1991-1998. According to the findings, extra costs were caused by mismanagement and substitution elasticity among production elements regarded as two important factors in relation to inefficiency. Also, impact of government subsidies payment to government departments led to inefficiency and positive impact of efficiency changes has been under the effect of technological regression so that total productivity has been reduced. Fingleton and McCombie [20] studied ES in a variety of industries using maximum likelihood method in 1979-1989 in Europe and stated that the European industries have increasing RS. Also, Basu and Fernald [21] used data of 34 American industries and suggested that although RS varies among different industries, all the industries averagely have constant or decreasing RS. ES-related researches have been conducted in Iran. Azamzadeh Shurki [22] addressed the impact of production function selection on the estimated values of structural parameters and the importance of correct production function selection. In order to estimate the production function of agriculture sector, Cobb-Douglas, Translog and Transcendental production functions were computed. In this study, impact of energy, capital and workforce inputs were studied on agricultural production in 1974-1979. Results indicated that Cobb-Douglas function was more suitable than two other functions and production elasticity of all the inputs were 0-1. Eslamloeiian and Ostadzad [23] reviewed the substitution elasticity between energy and other inputs using CES production function in Iran, designed a production function including such inputs a workforce, capital, energy and research costs for Iran's economic conditions and estimated continuous genetic algorithm numerically and nonlinearly. Results achieved from the substitution elasticity computations showed that as the workforce, capital and investment in R&D increased by 1%, energy saving was increased by 0.56, 0.59 and 0.46%, respectively. Furthermore, results given by final production computation indicated the increased final production of human force after the imposed war. In a similar study, they investigated various production functions such as GPF, CES, Cobb-Douglas, Translog, and Transcendental and generalized linear ones. These functions mostly are nonlinear and need a large sample volume. Moreover, in common econometrics, estimates are primarily done through minimizing RSS but it has lower efficiency as compared to absolute deviation value minimization. Results presented that Translog production function is more suitable for Iran. Also, 10% increase of energy and R&D expenses increased the production by 7.3 and 2.6%, respectively.

Lack of resources is one of major problems for the manufacturing corporates. Also the development of investment is one of the ways to solve economic problems. Due to resources Constraints, the importance of investment efficiency is emphasized. Particularly in a country like Iran under the current circumstances and despite sanctions, the importance of this issue becomes clearer [24]. Researches

focused on the increase of efficiency and production level at optimal level in order to enhance the return without attracting new resources. Most researches were done in a specific industrial level or used the collected data and reports of Central Banks; due to the aggregated information of corporates and reduction of sample size, the variables were aggregated and consequently, the results can only be interpreted at industry level and RS has not been measured separately for each corporate. In the following, using the data of corporates in stock exchange, ES will be measured at corporate level.

### 3 Research Methodology

In this research, production function has been used to measure ES, which is a completely physical concept and simply indicates the relationship between product and production costs. It is indicative of final product made by a combination of different elements. Studying various economic production functions, Translog function was selected to envelop the behavior of costs and to measure ES.

#### 3.1 Model of the Research

The function proposed by Christensen et al [25] estimates the interactions between variables in addition to the estimate of model parameters. Another reason to use it refers to nonlinear behavior of semi-fixed and semi-variable costs. It can be presented as follows (1).

$$Y=A.(x_1^{a1}).(x_2^{a2}).(x_1^{\frac{1}{2} \cdot \beta1 \cdot \ln x1}).(x_2^{\frac{1}{2} \cdot \beta2 \cdot \ln x2}).(x_1^{\frac{1}{2} \cdot \beta3 \cdot \ln x2}).(x_2^{\frac{1}{2} \cdot \beta4 \cdot \ln x1}) \quad (1)$$

To simplify the estimate of coefficients and analysis, its logarithm was used (2).

$$\ln Q = \ln \alpha_i + \sum_{i=1}^n \beta_i \ln x_i + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \gamma_{ij} \ln x_i \ln x_j + \frac{1}{2} \sum_{i=1}^n \gamma_{ii} (\ln x_i)^2 \quad (2)$$

After estimating the model and considering the impact of production elements on income, ES was computed by the equation (3).

$$E x_i = \frac{\partial Q}{\partial x_i} \times \frac{x_i}{Q} = \beta_i + \sum_{j=2}^n (\ln x_j) \quad (3)$$

Finally, by the elasticity sum of production elements, ES was estimated through the equation (4).

$$E_s = \sum_{i=1}^n E_i \quad (4)$$

#### 3.2 Model Variables

Translog function has had no constraints from the perspective of input variables. In economy, it is believed that production is a combination of work and capital. Christensen and Demski [26] in his book "Accounting Theory" introduced performance and ability of manager as the third factor. With the review of literature, direct materials as one of the most important elements have been added to the model. Thus, the variables are as follows:

1- Capital: the resources accessible for the business unit with the day value equal to the sum of stock day value and book value of debts.

2- Human force: the sum of wages in a financial period

3- Direct materials: costs of consumed materials in production in a financial period

4- Management ability: to measure the management ability using the method of Demerjian et al. [27]

In the mentioned model, the management ability is estimated through measuring the corporate efficiency, adding it in multivariate linear regression as a dependent variable and controlling the inherent features of corporate. In order to measure the corporate efficiency, data envelopment analysis model has been used.

$$\text{Max}_v \theta = \frac{\text{sales}}{v_1 \text{CGS} + v_2 \text{SG\&A} + v_3 \text{NetPPE} + v_4 \text{OpsLease} + v_5 \text{R\&D} + v_6 \text{Intan}} \quad (5)$$

The symbols used in the model are as follows:

Sales: the sale

CGS: the final cost of sold product

SG&A: the general, administrative, distribution and sale costs

Opslease: the operating lease costs

NetPPE: the fixed asset

R&D: the research and development

Intan: other intangible assets

The value estimated for the corporate efficiency is put in the range of 0-1 and the maximum value is 1; namely, as the value is smaller, the efficiency is lower. To control the impact of corporate inherent features in the above model, the corporate efficiency is separated into two sections including the corporate inherent features-based efficiency and management ability. Therefore, the corporate inherent features are controlled. Each controlling variable as the corporate inherent features can contribute the managers to make better decisions or on the contrary, they limit the management ability. Consequently, management ability can be estimated using the efficiency computed in the first stage and the below regression equation:

$$\text{Firm Efficiency}_i = \alpha + \beta_1(\text{TA})_i + \beta_2 \text{MSH}_i + \beta_3 \text{FCF}_i + \beta_4 \text{LN(AGE)}_i + \beta_5 \text{FCI}_i + \varepsilon_i \quad (6)$$

Where

Ln(TA): Corporate size equals natural log of sum of corporate assets

MSH: Corporate market share equals corporate sale to total industry sale ratio

FCF: Virtual variable; if it is positive, operational liquidity equals 1 and if it is negative, the liquidity will be 0.

Age: Natural log of corporate activity age

FCI: A virtual variable; it equals 1 for the export corporates, otherwise, it is 0.

$\varepsilon_i$ : Remaining amount of regression equation indicating the corporate management ability.

## 4 Research Data

Data have been obtained from a manufacturing corporate. Corporates of Tehran Stock Exchange have been classified based upon production type, products and activity in industries. Thus, to choose the sample, a classifying sampling method has been used. Because of few active corporates in some industries, homogenous industries were combined so that total nine industries were classified. The research sample included 105 corporates which were selected randomly research period in 10 years from 2008 to 2017. The required data were extracted from the audited financial statements and the financial attachments through Codal site and Rahavarnovin database. Using data envelopment analysis and regression models, the management ability variable has been measured. Data envelopment analysis (DEA) is a technique to assess the relative performance of decision making units (DMUs). Return scores DEA (y) are given in the interval (0,1). Regression model selection is important for the second stage of DEA. Generally, in previous researches, an OLS-based linear regression model was used to test the model but the linear regression was not suitable for the analysis giving no correct results. Osmerla et al. stated that the results of DEA presented the efficiency value as (0-1) which is boundary. In such conditions that the boundary variable as a dependent one at the interval (0-1) is put in the model, the model relation will not be linear due to the small amount between the year-corporates and fractional regression must be used. In fractional regression, the model is estimated by QML and there is no need to review classic linear regression hypotheses. Among the common software concerning econometrics, Stata 14 is suitable and was used. Therefore, after the model estimate, the remaining values were extracted as management ability.

Because of structural differences of industries, the impact of industry has to be considered. Generally, the industry type as a virtual variable is added to the model. In regression models, constant amount indicates width from origin and variables coefficient indicates regression line slope. Thus, it is probable that industry type influences the width from origin or regression line slope. If industry type as a virtual variable enters the model without any interactions with independent variables, the model estimate results regard the significance of industry type effect on differences of width from origin. In this paper, ES has been measured according to regression line slope. Industry type impact has to enter the model with an interaction to independent variables so that number of model variables will increase to 112 (8\*12); in addition to the problem in results interpretation, it is possible that model coefficients cannot be reliable. For this reason, model estimate should be done separately for each industry. Homogeneous industries are combined and industries with less active corporates are categorized together. Final model is in the following linear regression form.

$$\begin{aligned} \text{Lnsale} = & \beta_0 + \beta_1 \text{Indm}_{it} + \beta_2 \text{Inwage}_{it} + \beta_3 \text{Inmv}_{it} + \beta_4 \text{Inmang}_{it} + \frac{1}{2} \beta_5 (\text{Indm}_{it})^2 + \frac{1}{2} \beta_6 (\text{Inwage}_{it})^2 + \frac{1}{2} \beta_7 (\text{Inmv}_{it})^2 + \frac{1}{2} \beta_8 (\text{Inmang}_{it})^2 \\ & + \beta_9 \text{Indm}_{it} * \text{Inwage}_{it} + \beta_{10} \text{Indm}_{it} * \text{Inmv}_{it} + \beta_{11} \text{Inmang}_{it} * \text{Indm}_{it} + \\ & \beta_{12} \text{Inwage}_{it} * \text{Inmv}_{it} + \beta_{13} \text{Inmang}_{it} * \text{Inwage}_{it} + \beta_{14} \text{Inmang}_{it} * \text{Inmv}_{it} + \varepsilon_{it} \end{aligned} \quad (7)$$

Variables are:

Insale; Natural log of sale

Indm: Natural log of direct materials

Inwage: Natural log of wages

Inmv: Natural log of corporate value

Inmang: Natural log of management ability



## 4.1 Model Test

To analyze primary data, descriptive statistics of research variables have been presented in Table 1 indicating the amounts of average, mean, maximum, minimum and standard deviation.

**Table 1:** Descriptive statistics of research variables

variables	N	Ave.	S. D	min	max
LnTa	1,050	13.903	1.534	10.816	19.133
MSH	1,050	0.062	0.089	0.004	0.559
lnAGE	1,050	3.52	0.409	1.792	4.159
EFE	1,050	0.807	0.255	0.003	1
mange	1,050	0	0.314	-0.879	0.887
lnsale	1,050	15.539	1.018	10.112	19.724
C.G.S	1,050	15.344	1.925	9.711	19.679
SGA	1,050	12.431	1.766	8.073	16.594
PPE	1,050	1819541	6836014	3238	67800000
RD	1,050	1132.201	6278.08	0	89628
OPL	1,050	7103.367	55124.42	0	743849
lnwage	1,050	11.763	1.319	8.502	16.86
lnm	1,050	12.711	1.84	7.077	19.673
lnmv	1,050	14.21	1.55	10.735	19.268

Source: Research Results.

One of regression hypotheses is lack of linear relationship between explanatory model variables using OLS method. Correlation coefficient was computed and results have been presented in Table 2.

**Table 2:** Correlation Coefficient

Correlation	lnm	lnwage	lnmv	lnmang	lnm <sup>2</sup>	lnwage <sup>2</sup>	lnmv <sup>2</sup>
lnm	1						
lnwage	0.74	1					
lnmv	0.75	0.9	1				
lnmang	0.11	0.05	0.01	1			
lnm <sup>2</sup>	0.99	0.75	0.77	0.1	1		
lnwage <sup>2</sup>	0.74	1	0.9	0.04	0.75	1	
lnmv <sup>2</sup>	0.75	0.9	1	0	0.77	0.91	1
lnmang <sup>2</sup>	-0.09	-0.08	-0.05	-0.77	-0.09	-0.07	-0.05
lnm*lnmv	0.95	0.86	0.91	0.06	0.96	0.86	0.92
lnm*lnwage	0.94	0.91	0.87	0.08	0.95	0.92	0.88
lnm*lnmang	0.08	0.03	-0.01	0.99	0.07	0.02	-0.02
lnmv*lnwage	0.09	0.03	-0.02	0.99	0.08	0.02	-0.02
lnmv*lnmang	0.09	0.02	-0.02	0.99	0.08	0.01	-0.02
lnwage*lnmang	0.76	0.97	0.97	0.03	0.78	0.98	0.97
Correlation	lnmang <sup>2</sup>						

**Table 2:** Continue

Correlation	Indm	lnwage	lnmv	lnmang	Indm^2	lnwage^2	lnmv^2
		Indm* lnmv	Indm* lnwage	Indm* lnmang	lnmv* lnwage	lnmv* lnmang	lnwage* lnmang
lnmang^2	1						
Indm*lnmv	-0.08	1					
Indm*lnwage	-0.09	0.98	1				
Indm*lnmang	-0.75	0.04	0.05	1			
lnmv*lnwage	-0.76	0.04	0.06	0.99	1		
lnmv*lnmang	-0.75	0.04	0.05	1	1	1	
lnwage*lnmang	-0.06	0.91	0.92	0	0	-0.01	1

Source: Research Results.

Correlation coefficient table indicates a linear relationship between some independent variables, which is reasonable since second power of each variable and their interaction are considered in the model, Lasso model cannot be estimated with respect to intense relationships between variables. In general, variables are deleted and logarithms are given in the model to solve the linearity problem but none of common methods works because all the variables are related to production function. Studying the statistics articles, and new methods to remove the linearity problem such as ridge, Lasso and elastic net regression, it was concluded that old least square regression and its weaknesses are explained followed by a new regression [28].

#### 4.1.1 Ordinary Least Square Method (OLS)

In this method, model coefficients are estimated in a way that the sum of remaining least square is as below:

$$RSS(\beta) = \sum_{i=1}^m (y_i - p_i^T \beta)^2 \tag{8}$$

Given that RSS is a quadratic function in the mentioned function, there will be always a least value.

To select the best amounts of  $\beta$ , the equation (8) is rewritten in a matrix form:

$$RSS(\beta) = (y - P\beta)^T (y - P\beta) \tag{9}$$

In this equation, P is the matrix of  $m \times n$  where m is number of program implementation and n is number of determiners. To estimate the least RSS, the equation (9) is driven as follows:

$$P^T (y - P\beta) = 0 \tag{10}$$

If  $P^T P$  is reversible, a unique solution will be achieved for the vector of coefficients as follows:

$$\beta = (P^T P)^{-1} P^T y \tag{11}$$

It can be shown that the mean square error of OLS that measures the square of estimated regression coefficients will be computed as follows:

$$E[(\hat{\beta} - \beta)^T (\hat{\beta} - \beta)] = \sigma^2 \sum_{j=1}^n \lambda_j^{-1} \tag{12}$$

$\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_p$  is the specific values and  $\sigma^2$  is the variance of estimate coefficients. Due to the existence of high multivariate linearity between the determiners, specific values are small and as a result, mean square error will be large. Second problem is that as the model is more complicated (number of determiners is more), variance of coefficients is increased leading to the increased inherent error of prediction. Third problem is the existence of linear independence condition between the matrix rows and columns in terms of P values. Linear dependence creates when at least one row or column is a combination of one or few other rows or columns. In other words, there is a vector of non-zero coefficients producing 0 if multiplied by matrix vector. Thus, the dependence causes an irreversible matrix.

#### 4.1.2 Ridge Regression

To solve the OLS problem, little bias (error) is considered in the estimate of  $\beta$  leading to the inherent reduction of variance. To achieve this goal, an error-variance balance should be regarded. On the other hand, since no constraints are imposed on coefficients in OLS, it may grow considerably. To control the coefficients amounts tended to infinity because determiners approach 0 and balance error-variance which is created by correlation of coefficients, such regularization methods as ridge regression can be used. To control the growth of coefficients, ridge regression imposes a constraint on total squared residue maximization.

$$\text{minimize } \sum_{i=1}^n (y_i - \beta^T p_i)^2 \quad \text{s.t.} \quad \sum_{j=1}^n \beta_j^2 \leq t \quad (13)$$

The equation (13) is rewritten in a matrix as follows:

$$\text{minimize } (y - P\beta)^T (y - P\beta) \quad \text{s.t.} \quad \sum_{j=1}^n \beta_j^2 \leq t \quad (14)$$

It is assumed that P has been normalized (mean 0 and variance 1) and y is co-centralized. So, the determiner  $P_j$  will be converted to:

$$r_{ij} = \frac{(p_{ij} - \bar{p}_j)}{\sqrt{\sum (p_{ij} - \bar{p}_j)^2}} \quad (15)$$

In the above equation,  $r_{ij}$  indicates the correlation between the amounts of determiners  $i$ th and  $j$ th in different implementations.  $r_{ij}$  is an element of correlation matrix among determiners. Elements of row  $k$ th or  $R_k$  are centralized in the matrix and scaled in a way to have the length unit. Therefore, correlation matrix is involved instead of primary matrix of determiners here. Ridge regression can be rewritten as PRSS:

$$PRSS(\beta)_{\ell_2} = \sum_{i=1}^m (y_i - R_i^T \beta)^2 + \lambda \sum_{j=1}^p \beta_j^2 = (y - R\beta)^T (y - R\beta) + \lambda \|\beta\|_2^2 \quad (16)$$

Solving the equation (16) led to the least mean prediction error as compared to the sum of least squares. After deriving the equation (16) in relation to  $\beta$ , the below equation is achieved:

$$\frac{\partial PRSS(\beta)_{\ell_2}}{\partial \beta} = -2R^T (y - R\beta) + 2\lambda\beta \quad (17)$$

$$\hat{\beta}_\lambda^{ridge} = (R^T R + \lambda I_p)^{-1} R^T y$$

As it has been shown in the equation (17), ridge regression changed the equation as follows:

$$\begin{aligned} (1 + \lambda)\beta_1^{ridge} + r_{1,2}\beta_2^{ridge} + \dots + r_{1,n}\beta_n^{ridge} &= r_{1y} \\ r_{2,1}\beta_1^{ridge} + (1 + \lambda)\beta_2^{ridge} + \dots + r_{2,n}\beta_n^{ridge} &= r_{2y} \\ \dots & \\ r_{n,1}\beta_1^{ridge} + r_{n,2}\beta_2^{ridge} + \dots + (1 + \lambda)\beta_n^{ridge} &= r_{ny} \end{aligned} \quad (18)$$

In the equation,  $r_{ij}$  is the correlation between  $i$ th and  $j$ th determiners and  $r_{1y}$  is the correlation between the determiner  $i$ th and the termination status. The solution (18)  $\beta_1^{ridge}, \beta_2^{ridge}, \dots, \beta_n^{ridge}$ , is a combination of ridge estimation coefficients indicating each determiner effect on the termination status. Estimate is done by  $\lambda$  which is a regularization parameter called shrinking parameter with a series of capabilities. First, the existence of  $\lambda$  in the equation (17) makes the problem non-singular even if  $R^T R$  is irreversible. This parameter also controls the amount of coefficients and regulator. With the decreased  $\lambda$ , solutions tend to OLS methods. On the other hand, the increased  $\lambda$  leads to give 0 for all the coefficients. To select  $\lambda$ , different ways have been suggested and the best one has been cross validation. In this method, a value is selected for  $\lambda$  and the mean square error is minimized.

Advantage of choosing the features using ridge regression is that a constant estimate is provided, which can be useful when there is a lot of dependence among prediction variables and after making a ridge equation with fitted determiners data, most determiners are omitted: determiners with small but constant coefficients and those with non-constant ones. Remaining determiners are used to make a new ridge regression. This process continues till suitable number of determiners with high coefficients is chosen. But the main problem of ridge regression is that the step-to-step deletion can cause the deletion if some useful determiners which have been given small coefficients due to be dependent on others because it is not of a feature selection nature. In fact, the real problem is that all determiners are given a coefficient; if there are lots of determiners, model will be large and uninterpretable. Consequently, ridge regression does not make the coefficients zero for the extra determiners or those without prediction value.

### 4.1.3 Lasso Method

It is another shrinking strategy having suitable features of Ridge regression and attributes subset selection. In this method, such interpretative models as attribute selection method with Ridge regression stability are produced. Lasso tries to allocate high coefficients to determiners related to error. The main difference with ridge method is the penalty parameter. Penalty parameter  $\ell_2$  replaces  $(\sum_1^n |\beta_j^2|)$  in Ridge method and  $\ell_1$  replaces  $(\sum_1^n |\beta_j|)$  in Lasso method. Lasso penalty produces nonlinear solutions in the relation between determiners and termination status  $y_i$  and thus, quadratic linear programming log can be used to estimate the coefficients. Lasso aims to minimize sum of remaining squares:

$$\min imize (y - P\beta)^T (y - P\beta) \text{ s.t. } \sum_{j=1}^n \beta_j \leq s \quad (19)$$

The equation (20) is the loss function:

$$PRSS(\beta)_{\ell_1} = \sum_{i=1}^m (y_i - R_i^T \beta)^2 + \lambda \sum_{j=1}^p |\beta_j| = (y - R\beta)^T (y - R\beta) + \lambda \|\beta\|_1 \quad (20)$$

Lasso has the regularization parameter  $\lambda$  which determines the regularization amount. When a suitable amount is chosen, some solutions will be obtained that make zero the coefficient of most determiners with no relation to program termination status. Therefore, Lasso is able to select features well. Of course, if a suitable amount is not selected, most coefficients may be deleted mistakenly leading to develop an unsuitable model.

However, Lasso method has some faults: first, there are lots of mutual correlations if there is a group of variables (determiners). It only selects a variable (determiner) voluntarily out of other variables and makes the rest zero. This approach causes a serious problem in the error positioning process since an error may be made due to an incorrect relation to some determiners and all these determiners make an error together. In such circumstances, by deleting a determiner, the main cause of an error is omitted. Thus, finding a group of high correlation determiners is important. This group as the error detectors is recognized as error predictors. In addition, if number of attributes (p) is more than number of observations (n), Lasso considers the maximum n, which is not appropriate for many applications. Particularly, in error positioning process, since number of determiners may be more than number of successful and unsuccessful implementations, Lasso method creates a weak model [30].

### 4.1.4 Elastic Net

With respect to the mentioned attributes for Ridge regression and Lasso method, a method is needed to debug the program in a way that it combines strengths of two desired methods and remove the weaknesses. It is proposed by Zu and Histi, which is called elastic net. Elastic net is another shrinking method with grouping impact which is able to develop such groups as high correlation attributes. This advantage results in a really appropriate method to select error predictors. Parameters  $\beta$  in elastic net is estimated as follows:

$$\hat{\beta}^{elastic.net} = arg \min_{\beta} \sum_{i=1}^m (y_i - \beta_0 - \sum_{j=1}^n p_{ij} \beta_j)^2 \tag{21}$$

$$subject\ to\ \lambda \sum_{j=1}^n \beta_j^2 + (1 - \lambda) \sum_{j=1}^n \beta_j \leq r$$

In the equation (21), r is the regularization parameter depending on attributes existing in the data set. Elastic net imposes two penalties on coefficients.  $\lambda$  is in interval [0,1) in penalty function. If  $\lambda=0$ , the equation (21) is converted to Lasso equation and if  $\lambda=1$ , the equation will be ridge regression. Thus, with a suitable selection of  $\lambda$  between 0 and 1, the equation will have attributes of Ridge and Lasso methods. Like Ridge and Lasso methods, matrix form of elastic net is as follows:

$$L(\lambda_1, \lambda_2, \beta) = |y - P\beta|^2 + \lambda_2 |\beta|^2 + \lambda_1 |\beta|_1 \tag{22}$$

Various methods exist to choose these parameters. If there are only learning data, the cross validation will be a suitable strategy to estimate the prediction error and compare a variety of models. For  $\lambda$  in elastic net, such amounts as 0.001, 0.05, 0.1, ..., 0.95 have been tested and according to the attributes and size of desired program, numbers are considered for r. So, amounts are selected for parameters to reduce the error of incorrect classification [31]. As it has been explained, elastic net regression method has been used to implement the research model in software Stata.

#### 4.1.5 Results of Model Estimate Using Elastic Net Regression

Model estimate at error level of 5% has been presented in Table 3. Note that the coefficient is considered 0 for insignificant variables.

**Table 3:** Model estimation by elastic net method

Industry	Metals	Chemical	Medicine	Cement	sugar Food and	and Mineral tile	car and parts	Machine	others
Indm	0.517	-0.106	0	1.595	0.084	0.077	0.845	0.873	0.559
Inwage	0.221	0	0.107	0.311	0.132	0.071	0.001	0	0.286
Inmv	0	0.2	0.579	0	0.109	0.062	0.051	0	0.195
Inmang	0	0.33	0	0	0.03	0.024	0	0.004	0
(Indm^2)	0	0.035	0.028	-0.169	0.007	0.007	0	0	0
(Inwage^2)	0	0.007	0	0	0.012	0.006	0	0	0
(Inmv^2)	0.033	0.024	0	0	0.008	0.004	0.004	0	0
(Inmang^2)	-0.14	-0.346	-0.167	0	-0.078	0.141	0	0	0
Indm*Inwage	0.008	0	0	0	0.005	0.004	0	0	0
Indm*Inmv	0	0	0	0.033	0.004	0.004	0	0	0
Indm*Inmang	0	0	0	0	0.01	0.006	0	0	0
Inwage*Inmv	0.01	0	0	0.839	0.001	0.004	0	0	0
Inwage*Inmang	0	0	0	-0.751	0.002	0.002	0	0	0
Inmang*Inmv	0	0	0	0	0.002	0.002	0	0.003	0
d1	0.331	0.137	0.147	-0.212	-0.045	0.138	0	-0.056	0
d2	0.37	0.199	0.272	-0.016	0.012	0.089	0	0	0

**Table 3:** Continue

Industry	Metals	Chemical	Medicine	Cement	Food and sugar	Mineral and tile	car and parts	Machine	others
d3	0.268	0.101	0.264	0.11	-0.037	0.113	0	0.056	0
d4	0.228	0.064	0.103	0.151	0.003	0.058	0	0	0
d5	0.089	0.083	0.168	0.193	0.006	-0.049	0	0	0
d6	0.214	0.136	0.191	0.275	0.039	0.081	0	0	0
d7	0.033	0	0	0.174	0.008	0.068	0	0	0
d8	0.101	0	0	0.275	0.022	0.132	0	0	0
d9	0	-0.157	-0.048	0	-0.019	0.057	0	0	0
_cons	2.011	6.627	1.604	-2.822	5.226	7.29	1.514	2.183	0.267
R-squared	0.978	0.979	0.972	0.896	0.918	0.836	0.989	0.972	0.939
lambda	0.002	0.003	0.004	0	0.286	0.572	0.029	0.024	0.048

Source: Research Results.

### 4.2 Role of Production Elements in Income

In economy, the term "elasticity" of production elements indicates the increased impact of production element on the increase of income. To estimate it, production to element ratio has been derived and after the computation of elasticity, elasticity of production elements is summed up to give ES. For example, concerning significant variables in the first year for metals industry, final production function is as follows:

$$\ln \text{sale}_t = 0.5167 \cdot \ln \text{dm} + 0.2210 \cdot \ln \text{wage} + \frac{1}{2} \cdot 0.0331 \cdot (\ln \text{MV})^2 - \frac{1}{2} \cdot 0.1398 \cdot (\ln \text{mang})^2 - 0.0078 \cdot \ln \text{dm} \cdot \ln \text{wage} + 0.010 \cdot \ln \text{wage} \cdot \ln \text{MV} + 2.0107 + 0.3312 \tag{23}$$

The above result will be written as a nonlinear function:

$$\text{Sale} = 2.3419 \cdot \text{DM}^{0.5167} \cdot \text{wage}^{0.2210} \cdot \text{MV}^{0.1655} \cdot \ln \text{mv} \cdot \text{mang}^{-0.0699} \cdot \text{DM}^{0.0078} \cdot \ln \text{wage} \cdot \text{wage}^{0.010} \cdot \ln \text{mv} \tag{24}$$

The above model is derived based on the elements:

$$\begin{aligned} E_{\text{DM}} &= 0.5167 - 0.0078 \cdot \ln \text{wage} \\ E_{\text{wage}} &= 0.2210 - 0.0078 \cdot \ln \text{DM} + 0.010 \cdot \ln \text{mv} \\ E_{\text{mv}} &= 0.0331 \cdot \ln \text{mv} + 0.010 \cdot \ln \text{wage} \\ E_{\text{mang}} &= -0.1398 \cdot \ln \text{mang} \end{aligned} \tag{25}$$

Simply, putting the amounts for each year-corporate can compute the elasticity. For example, average values of variables in metals industry have entered the elasticity model and ES has been given

$$\begin{aligned} E_{\text{DM}} &= 0.4191 \\ E_{\text{wage}} &= 0.1170 \\ E_{\text{mv}} &= 0.4887 \end{aligned} \tag{26}$$

$$E_{\text{mang}} = 0.1435$$

$$E_s = \sum_{i=1}^4 E_i = 1.168468$$

For other industries, elasticity and ES were estimated and maximum and minimum results have been presented in Table 4.

**Table 4:** The minimum and maximum economics on a scale (ES)

industry	Min			max		
	firm	Year	EOS	firm	year	EOS
Metals	jaam darou	2009	1.006	SADID PIPE & EQUIPMENT Co	2010	1.383
Chemical	Loabiran	2007	1.092	Loabiran	2016	1.971
Medicine	Osve Pharmaceutical Co	2007	0.921	Razak Laboratory	2016	1.424
Cement	Doroud cement co	2016	0.241	khazar sement	2008	1.706
Food and sugar	Shoko Pars	2010	1.047	pak dairy	2016	1.437
Mineral and tile	Iranl glass wool	2007	0.707	Qazvin glass	2015	0.997
car and parts	Irka Part	2007	0.941	Iran khodro	2016	0.967
Machine	Firouz Engineering	2007	0.915	taraktorsazi Iran	2016	0.931
others			1.04			1.04

Source: Research Results.

## 5 Results

Considering the increased competition among business units, determining a level of production will have the most ES. In current research, using economic production function and accounting variables, ES has been separately computed and results showed that metals, chemical, food and sugar industries have an increasing ES because the minimum amount is more than 1. For pharmacy and cement industries, the minimum amount is smaller than 1 and the maximum amount is larger than 1. Therefore, RS is increasing for some year-corporates and it is decreasing for others. For automobile, machinery, minerals and tiles industries, the most amount is less than 1 indicating the decreasing RS. For other industries, due to insignificant variables, Translog function changed to Cobb-Douglas function and thus, RS was obtained as 1.040 since samples were combined with different structures from different industries.

## 6 Conclusion and Suggestions

Human beings always try to achieve the most profit from the resources so that it is necessary to evaluate the performance of economic activities. One of important performance aspects is economies of scale (ES) which determines how much the income increases as a result of increased production costs. According to structural differences in a variety of industries, measuring economies of scale has been done separately concerning each industry. Results indicated that all the corporates in metals, chemical materials and food industries have the increasing economies of scale. Results are in conformity with those reported by Thompson [9], Chen [12], Diewert et al. [14], Oliveira et al. [16], Fingleton and



McCombie[20], and Eslamloei and Ostadzad [23]. Automobile, machinery, minerals and tiles industries were of the decreasing economies of scale. These results are confirmed by the findings reported by Piesse and Thirtle[19] and Azamzadeh Shurki et al[22]. Some pharmaceutical and cement industries have the increasing economies of scale and few corporates had a decreasing ES during the research period. Results can be interpreted in a manner that the competition or being monopoly in the industry and being investor or user are two effective elements in ES. For example, if the competition level is high among the corporates, they try to plan and use the resources optimally in order to survive in the market competition; consequently, their ES will be increasing. Also, if the corporate is an investor and exploits the capacities completely due to high constant costs, they will have an increasing ES.

For most year-corporates, such variables as materials, wages and market value of resources are significant showing a nonlinear relationship between elements and income. These results confirm the discussions on the importance of semi-fixed and semi-variables costs. Results concerning management ability were not significant for most industries and some had positive impact and others were negative. So, the results were different but with some constraints. Because the measurement of management ability used DEA and business unit had the best performance selected as the efficient unit after the efficiency determination and other unit's efficiency was assessed so that efficiency was relatively measured, not absolutely. In planning for production, a linear model is generally used; in linear models, the increased production amount to nominal capacity will be of ES but research findings suggest that nonlinear models like Translog function are used in planning. Also, it is necessary to separate the combined, semi-variable and semi-fixed variables precisely and training the accountants in terms of production function and analysis is recommended. Another constraint of current research is the application of corporate market value in addition to management ability measurement. Because of severe inflation in some research periods, using book value of assets and making conclusions accordingly are not valid. Therefore, book value of corporate as the corporate financial resources was used to control the inflation impact. Though, other elements influencing the book value limited the research results except for economic factors. In the corporates, a linear model like variable-based costing is used for planning and making decisions on production. In the linear model, the increased production rate is of ES. Research findings indicated that due to nonlinear behavior of costs, optimum production rate should be regarded with respect to ES and necessarily, the increase of production will not lead to the increased income and profit. Thus, the corporate managers are recommended to apply nonlinear models and functions like translog in order to determine the optimum production level concerning the planning and decision making processes. In operational auditing, ES is investigated as a criterion to evaluate the business performance. Generally, auditors measure it through financial ratios but financial ratios of performance are relative and one-dimensional. Consequently, they have some constraints. Therefore, operational auditors and organizational ones are suggested to compute ES based on accounting data and production functions. According to lack of training in terms of production functions for the accountants, it is proposed that the educational producers should produce training programs on industrial and management accounting, and production functions discussions and their analysis for the accounting students.

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