Advances in Mathematical Finance
& Applications, 6(4), (2021), 757-767
DOI: 10.22034/AMFA.2019.1879646.1302

Published by IA University of
Arak, Iran
Homepage: www.amfa.iau-arak.ac.ir

Cost Malmquist Productivity Index in Non-Competitive Environment of Price in Data Envelopment Analysis and the Use of it in the Dealings of the Iranian Stock Exchange

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ARTICLE INFO

Article history:
Received 26 September 2019
Accepted 31 December 2019

Keywords:
Data Envelopment Analysis
Cost efficiency
Malmquist Productivity Index
non-competitive environment
Stock Exchange

ABSTRACT

The Malmquist Productivity Index (MPI) is a tool for analyzing the productivity. Considering its importance, different suggestions and studies have been offered on the MPI according to existing conditions of decision making units (DMUs) and the available data. The present research aimed to provide a Cost Malmquist Productivity Index (CMPI) in a non-competitive environment in which the price data changes from one under evaluation unit to another. Given the deficiency of Farrel's cost efficiency [1] and also the cost efficiency model presented by Tone [2], we presented CMPI in the presence of non-identical prices for various DMUs, Then, we evaluate a unit listed on the Iranian Stock Exchange by aforementioned CMPI.

1 Introduction

Data Envelopment Analysis (DEA) was introduced by Charnes et al. [3] as a powerful tool for measuring the relative efficiency of a set of Decision Making Units (DMUs). These DMUs receive the input x_i with the price of c_i and produce the output y_r with the price of p_r. DEA can evaluate efficiencies such as the cost efficiency (CE), revenue efficiency, profit efficiency and allocative efficiency (AE), etc., by the help of various mathematical models. The pricing information of inputs and outputs is available in some cases and should be considered in assessments, otherwise our estimates will not be complete in terms of prices. DMUs can be generally evaluated in competitive and non-competitive environments. Prices of all DMUs are equal in competitive environments, but they may have some or even more general differences in one or more indices, or can vary in all indices, and each input or output can have a separate price in non-competitive environments.

The productivity growth is a major source of the economic development; hence, the full understanding of determinants of the productivity is absolutely necessary; and the research on factors of changes in the productivity and its decomposition provides the valuable information for managers in both private and public sectors. The Malmquist Productivity Index (MPI) measures productivity changes of DMUs in different periods of time indicating that the efficiency of DMUs is improved, unaltered, or worsened. The MPI is usually computed as the product of the catch – up and frontier – shift. The MPI was first introduced by caves et al. [4], and then Nishimizu and Page [5] used the parametric programming to calculate the index. Grifell-Tatjé and Lovell [6] provided a generalization for the MPI. Afterwards, a lot of research was conducted on the growth and improvement of this productivity index, for instance, Althin [7] compared adjacent MPI to the base period MPI. Fuentes et al. [8] provided a parametric

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performance function method to estimate the MPI, and then Orea [9] offered a parametric decomposition of the centralized MPI. Lozano and Humphrey [10] studied a deficiency in the CMPI and measured the cost function efficiency in banking. Maniadakis and Thanassoulis [11] studied the CMPI in a competitive environment. Yu [12] investigated changes in the production capacity and the efficiency of input variable with a new decomposition of MPI. Emrouznejad et al. [13] studied the overall profit of MPI with the fuzzy interval data. Wang and Lan [14] measured the MPI as a new method based on two boundaries in the DEA. Tohidi et al. [15] introduced a global CMPI using the DEA. Tohidi and Razavyan [16] presented the circular MPI in the DEA. Kao and Hwang [17] studied the efficiency of several periods and the CMPI of two stages in production systems. Thanassoulis et al. [18] proposed the CMPI for evaluating the performance of groups; and Afsharian and Ahn [19] provided the overall MPI with a new approach to measurement of productivity changes over time. In this regard, Kao [20] provided the measurement and decomposition of MPI for parallel production systems. Walheer [21] studied the CMPI decomposition with shared inputs and exclusive outputs. According to the literature on the productivity analysis in the DEA, there are not many studies on the CMPI. The present study sought to introduce a new version of the CMPI, which was especially relevant to a situation where the price data of DMUs was not the same, and thus the index relate to a non-competitive environment.
This study has been structured as follows: In the second section, after a brief introduction of the DEA, the CMPI is introduced. The third section introduces the proposed method, and in the fourth section, the proposed method will be used for a numerical example of the Iranian Stock Exchange users and the final section presents the conclusion and suggestions.

2 Background

2.1 DEA

Fig. 1 shows a schema of the concepts associated with Farrell [1]'s CE model. This is an example with two inputs (x_1, x_2) and one output (y) and input prices (c_1, c_2), assuming constant return to scale for PPS. ss' is the efficiency frontier and point P, within this set, indicates a DMU. The observed cost for DMU_P is shown by $C_P = x_{1P}c_{1P} + x_{2P}c_{2P}$. The straight line AA' that intersects P is *the cost line*, Point Q is the radial projection of P and PQ distance shows technical inefficiency.

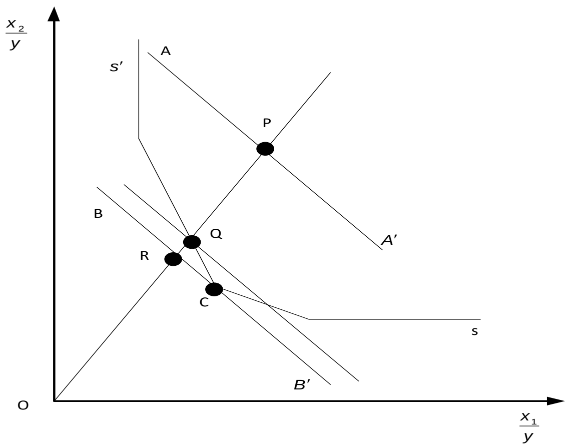

Fig. 1: Farrell's CE

The TE of DMU_P is the ratio of OQ/OP. If the cost line is shortened to the last point that is in contact with the efficient frontier, we will reach point C. It is a point producing y_P with the minimum possible cost. The reduced cost is originated from the advancement of allocation efficiency (AE) for DMU_P .

RQ distance indicate allocative inefficiency. Farrell [1] defined AE as the ratio of OR/OQ and the CE of DMU_P as follows:

$$CE = \frac{OR}{OP} = \frac{OQ}{OP} \times \frac{OR}{OQ} = TE \times AE$$

According to [1], the minimum cost required for producing y_o in the PPS $P = \{(x,y) | x \geq X\lambda, y \leq Y\lambda, \lambda \geq 0\}$ can be obtained using the following model:

$$\begin{aligned} cx^* = &\min_{x,\lambda} cx \\ s.t. \quad & x \geq X\lambda \\ & y_o \leq Y\lambda \\ & \lambda \geq 0 \end{aligned} \qquad (1)$$

Where $j = 1,...,n$ is the number of DMUs and $X_j = (x_{1j},...,x_{mj})^T$ and $Y_j = (y_{1j},...,y_{rj})^T$ are the input and output vectors of DMU_j, respectively. The value of Farrell's CE for DMU_o is determined as:

$$CE_o = \frac{\sum_{i=1}^{m} c_{io} x_{io}^*}{\sum_{i=1}^{m} c_{io} x_{io}}$$

Where x_{io}^* are the optimal solutions of model (1) and c_{io} is the price of the i'th input of DMU_o. Tone [2] discovered that something is wrong in Farrell's evaluation of CE in the presence of different prices (non-competitive environment). He encountered with this flaw by introducing a new CE value. Tone [2] discusses that if there are different DMUs with the same input and output and the price of a DMU is several times higher than another one, AE and CE may be the same in both DMUs. For example, suppose that DMU_A and DMU_B have the same inputs and outputs, i.e. $\theta_A^* = \theta_B^*$. Assume that the prices of DMU_A is twice as much as that of DMU_B, or $c_A = 2c_B$. Therefore, considering model (1) we have:

$$\begin{aligned} Min \quad & c_A x = (2c_B)x \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_i, \qquad i = 1,...,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rB}, \qquad r = 1,...,s \\ & \lambda_j \geq 0, \qquad j = 1,...,n \\ & x_i \geq 0, \qquad i = 1,...,m \end{aligned}$$

The above model gives the same optimal solution for both DMU_A and DMU_B, and $\theta_A^* = \theta_B^*$ Therefore, we have:

$$\gamma_A^* = \frac{c_A x_A^*}{c_A x_A} = \frac{2c_B x_B^*}{2c_B x_B} = \gamma_B^*$$

Which is not acceptable, because in this case the price of DMU_B is 50% of the price of DMU_A. Tone argued that this problem roots in the production possibility set (PPS) constituted only by technical factors and lacks cost factors in its structure. To remove the problem, Tone [2] suggested a new PPS as $P_c = \{(\bar{x}, y) | \bar{x} \geq \bar{X}\lambda, y \leq Y\lambda, \lambda \geq 0\}$ where $\bar{X} = (\bar{x}_1, \dots, \bar{x}_n), \bar{x}_j = (c_{1j}x_{1j}, \dots, c_{mj}x_{mj})^T$ and $\bar{x}_{ij} = c_{ij}x_{ij}$. He evaluated the minimum cost of DMU_o using the following model:

$$\begin{aligned} e\bar{x}_o^* &= \min_{\bar{x},\lambda} e\bar{x} \\ s.t. \quad & \bar{x} \geq \bar{X}\lambda \\ & y_o \leq Y\lambda \\ & \lambda \geq 0 \end{aligned} \tag{2}$$

Finally, he proposed a new CE under different prices (non-competitive environment) as $\bar{\gamma}^* = \frac{e\bar{x}_o^*}{e\bar{x}_o}$.

2.2 Malmquist Productivity Index

MPI was introduced as a quantity index and presented as distance function ratios by Sten Malmquist [14] for analysing the consumption of production sources. He assumed that n DMUs use m inputs of x_i for producing s outputs of y_r in two time periods of t=1,2 and, PPS of time t as: $PPS^t = (X, Y)^t = \left\{(x, y) \middle| x \geq \sum_{j=1}^{n} \lambda_j x_j^t, 0 \leq y \leq \sum_{j=1}^{n} \lambda_j y_j^t, \lambda_j \geq 0\right\}$.

Notations of $(x_o, y_o)^1 = (x_o^1, y_o^1)$ and $(x_o, y_o)^2 = (x_o^2, y_o^2)$ are applied for DMU_o $(o \in \{1, \dots, n\})$ in the time periods of 1 and 2, respectively. The efficiency of the units of $(x_o, y_o)^1$ and $(x_o, y_o)^2$ are evaluated by the technological frontiers of 1 and 2 in Fig. 2.

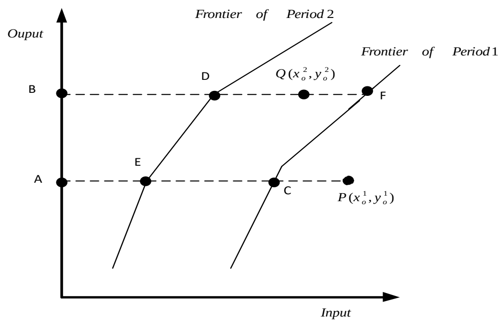

Fig. 2: Efficiency frontiers in two periods.

The catch – up is evaluated from periods 1 to 2 as follows:

$$\text{Catch-up} = \frac{\text{Efficiency of } (x_o, y_o)^2 \text{ with respect to priod 2 frontier}}{\text{Efficiency of } (x_o, y_o)^1 \text{ with respect to priod 1 frontier}} = \frac{\mathrm{BD}/\mathrm{BQ}}{\mathrm{AC}/\mathrm{AP}} = \frac{(AP)(BD)}{(AC)(BQ)} \tag{3}$$

and frontier – shift effect is as follows:

Frontier – shift for $(x_o, y_o)^1$:

$$\phi_1 = \frac{\text{Efficiency of } (x_o, y_o)^1 \text{ with respect to priod 1 frontier}}{\text{Efficiency of } (x_o, y_o)^1 \text{ with respect to priod 2 frontier}} = \frac{AC/AP}{AE/AP} = \frac{AC}{AE}$$

Frontier – shift for $(x_o, y_o)^1$:

$$\phi_2 = \frac{\text{Efficiency of } (x_o,y_o)^2 \text{ with respect to priod 1 frontier}}{\text{Efficiency of } (x_o,y_o)^2 \text{ with respect to priod 2 frontier}} = \frac{BF/BQ}{BD/BQ} = \frac{BF}{BD}$$

Frontier – shift for(x_o, y_o): $\phi = \sqrt{\phi_1 \phi_2} = \sqrt{\frac{AC}{AE} . \frac{BF}{BD}}$ (4)

MPI is computed as the product of the catch – up and frontier – shift. From (3), (4) we have:

$$MPI = \frac{AP}{BQ} \sqrt{\frac{BF}{AC} . \frac{BD}{AE}} \quad (5)$$

$MPI > 1$ indicates progress of productivity for DMU_o, $MPI = 1$ indicates the lack of change in productivity for DMU_o and $MPI < 1$ indicates the regress.

3 Proposed Cost Malmquist Productivity Index

Suppose that *n* DMUs use *m* input x_i for producing *s* output of y_r. There is available information about these *n* DMUs during t and t+1 periods. $DMU_o^t = (x_{io}^t, y_{ro}^t)$ for $i = 1,...,m$ and $r = 1,...,s$ represents coordinates of under-evaluation unit at time t; and it is similar to t+1. Division of quantities associated with a quantity is usually used to express the variation of that quantity in two periods of time. Percentages of progress and regress, the increase or decrease, etc. can be thus determined. The present paper aimed to estimate the productivity status of DMUs in terms of progress or regress, and its factors in the presence of pricing data of inputs. Obviously, changes in the efficiency can be considered as a criterion for changing the productivity in the normal state. $\frac{\mathrm{TE}_o^{t+1}}{\mathrm{TE}_o^t}$ fraction can be used in this regard. If we have the pricing data of inputs, we can use $\frac{\mathrm{CE}_o^{t+1}}{\mathrm{CE}_o^t}$ fraction in which CE_o^t and CE_o^{t+1} respectively represent the CE of DMU_o during t and t+1. We assume that the price data changes from one DMU to another, but it also changes from one period to another. We take into account the efficiency of $\frac{\mathrm{CE}_o^{t+1}}{\mathrm{CE}_o^t}$ and seek to evaluate each of them in terms of t like a research by Tone [2]. To this end, we consider technical, cost and allocative efficiency which is obtained from the second frontier. Obviously, the denominator can be calculated like in [2], that is CE_o^t ,because everything is related to *t* period. Now, we assume that the numerator,CE_o^{t+1}, is intended, that is, prices of DMU_o during t+1 with its input and output values during t+1 compared to cost and technological frontiers of t. We first consider values of DMU_o^{t+1} (i.e. regardless of prices). In this case, DMU_o^{t+1} has three modes in terms of t frontier: inside, on and outside the frontier. Nevertheless, the technical efficiency value is less than, equal to or greater than 1; hence, a projection point can be obtained. At this stage, we can evaluate the progress in the technology or efficiency like the normal state of MPI because we can examine $\frac{TE_o^{t+1}}{TE_o^t}$ fraction. If its value is less than 1, then it is technically regressed, but if it is more than 1, then it is technically progressed. Now, according to Tone and Tsutsui [22], corresponding to the PPS^t, we consider $\overline{PPS}^t$ that is defined as follows.

$$\overline{PPS}^t = \left(\tilde{X},\bar{Y}\right)^t = \left\{\left(\tilde{x},\bar{y}\right) \middle| \tilde{x} \ge \sum_{j=1}^{n} \lambda_j \tilde{x}_j^t, 0 \le \bar{y} \le \sum_{j=1}^{n} \lambda_j \bar{y}_j^t, \lambda_j \ge 0\right\} \quad (6)$$

Where $\tilde{X}^t = (\tilde{x}_1^t,...,\tilde{x}_n^t)$, $\tilde{x}_j^t = (c_{1j}^t x_{1j}^t,...,c_{mj}^t x_{mj}^t)^T$ and $C^t = (c_1^t,...,c_m^t)$ are the input prices at time period t. This PPS is developed using the projection points of the DMUs at period t and the data at period t. Therefore, it can be said that all the corresponding technical failures of the DMUs have already been eliminated. The inputs observed in this PPS is the type of costs associated with each of the inputs of the DMUs. Now, we consider the point of DMU_o^{t+1} with respect to the frontier t where $(x_o^{*t+1}, y_o^{*t+1})_t$ (i.e. the projection of point (x_o, y_o) at period t + 1with respect to the frontier t). By taking into account the prices related to (x_o^{t+1}, y_o^{t+1}), i.e. C_o^{t+1}, we reach the point $(\tilde{x}_o^{t+1}, \bar{y}_o^{t+1})$ where $\tilde{x}_o^{t+1} = c_i^{t+1} x_{io}^{t+1}$.

This point has three modes with respect to the PPS of $\bar{T}^t$; Inside, outside and on the frontier. By measuring the efficiency (radial, non-radial and SBM) with respect to that frontier, we reach a point on the frontier. The position of the point relative to $(\tilde{x}_{io}^{t+1}, y_o^{t+1})$ clearly depends on how prices change because we have already taken into account the inefficiency, efficiency, or super efficiency derived from the physical input and output quantities, and therefore, any progress or regress of productivity is due to a change in prices. We call this new projection point $(\tilde{x}_o^{*t+1}, \tilde{y}_o^{*t+1})$ We also call the radial efficiency that is obtained from this point (ρ^*). But we have not yet calculated the CE $(\tilde{x}_o^{t+1}, \bar{y}_o^{t+1})$ compared to the frontier at period t. Similar to the model by Tone and Tsutsui [22], we measure the CE of a point with a minimum cost in the PPS at period t, and, at least, with the output value of y_o^{t+1}, using the following model.

$$\begin{aligned} & Min\ 1.x \\ & s.t.\ \sum_{j=1}^{n} \lambda_j \tilde{x}_j^t \le x \\ & \qquad \sum_{j=1}^{n} \lambda_j y_j^t \ge y_o^{t+1} \\ & \qquad \lambda_j \ge 0 \end{aligned} \quad (7)$$

Assuming that the optimal solution of the above model is x^{**t}, the CE of $(\tilde{x}_o^{t+1}, \bar{y}_o^{t+1})$ is measured $\frac{1x^{**t}}{1\tilde{x}_o^{t+1}}$. We consider the quotient of $\frac{1x^{**t}}{1\tilde{\tilde{x}}_o^{*t+1}}$ to be the result of allocative efficiency, inefficiency, or super efficiency. Therefore, it can be concluded that CE_o^{t+1} with respect to the cost frontier of period t can be divided into three components as in the study by Tone and Tsutsui [22]. The same applies to CE_o^t. So:

$$CMPI_o^t = (\frac{CE_o^{t+1}}{CE_o^t}) = \frac{(TE_o^{t+1})_t}{(TE_o^t)_t} . \frac{(DTE_o^{t+1})_t}{(DTE_o^t)_t} . \frac{(CAE_o^{t+1})_t}{(CAE_o^t)_t} \quad (8)$$

DTE_o^t is the price efficiency or cost technical efficiency of DMU_o with the data relative to the frontier of t and at period t. Similarly, we can measure the same efficiency with respect to the cost frontier of period t + 1 which is indicated by $CMPI_o^{t+1}$. Based on the traditional MPI, using $CMPI_o^t$ and $CMPI_o^{t+1}$, we define CMPI by their geometric mean, i.e., $CMPI_o = \sqrt{CMPI_o^t . CMPI_o^{t+1}}$.

$CMPI_o > 1$: indicates progress in total factor Productivity of DMU_o from period t to t+1.

$CMPI_o = 1$: indicates quo in total factor Productivity of DMU_o from period t to t+1.

$CMPI_o < 1$: indicates deterioration in total factor Productivity of DMU_o from period t to t+1.

The traditional MPI decomposed is as follows:

$$MPI = \left[\frac{TE^t(x^{t+1}, y^{t+1})}{TE^t(x^t, y^t)} \times \frac{TE^{t+1}(x^{t+1}, y^{t+1})}{TE^{t+1}(x^t, y^t)}\right]^{1/2} = \\ = \frac{TE^{t+1}(x^{t+1}, y^{t+1})}{TE^t(x^t, y^t)}\left[\frac{TE^t(x^{t+1}, y^{t+1})}{TE^{t+1}(x^{t+1}, y^{t+1})} \times \frac{TE^t(x^t, y^t)}{TE^{t+1}(x^t, y^t)}\right] \tag{9}$$

We can write us decompose in two forms. A form in terms of the CE itself, and similar to the traditional MPI, in this case, it is decomposed in a phrase like (9) and we have only the CE instead of technical efficiency. The first sentence is DOE's name, change in overall efficiency and the second sentence is the DCT, which represent changes in cost technology. Therefore, we can decompose the above decomposition in terms of the technical, price, and allocative efficiencies. Therefore, we can write the CMPI as follows:

$$\begin{aligned} CMPI_o &= \sqrt{\left(\frac{CE_o^{t+1}}{CE_o^t}\right)_t \left(\frac{CE_o^{t+1}}{CE_o^t}\right)_{t+1}} = \\ &= \sqrt{\frac{(TE_o^{t+1})_t}{(TE_o^t)_t} \times \frac{(DTE_o^{t+1})_t}{(DTE_o^t)_t} \times \frac{(CAE_o^{t+1})_t}{(CAE_o^t)_t} \times \frac{(TE_o^{t+1})_{t+1}}{(TE_o^t)_{t+1}} \times \frac{(DTE_o^{t+1})_{t+1}}{(DTE_o^t)_{t+1}} \times \frac{(CAE_o^{t+1})_{t+1}}{(CAE_o^t)_{t+1}}} \\ &= \sqrt{\frac{(TE_o^{t+1})_t}{(TE_o^t)_t} \times \frac{(TE_o^{t+1})_{t+1}}{(TE_o^t)_{t+1}}} \times \sqrt{\frac{(DTE_o^{t+1})_t}{(DTE_o^t)_t} \times \frac{(DTE_o^{t+1})_{t+1}}{(DTE_o^t)_{t+1}}} \times \sqrt{\frac{(CAE_o^{t+1})_t}{(CAE_o^t)_t} \times \frac{(CAE_o^{t+1})_{t+1}}{(CAE_o^t)_{t+1}}} \\ &= \frac{TE_o^{t+1}(x^{t+1}, y^{t+1})}{TE_o^t(x^t, y^t)} \times \sqrt{\frac{TE_o^t(x^{t+1}, y^{t+1})}{TE_o^{t+1}(x^{t+1}, y^{t+1})} \times \frac{TE_o^t(x^t, y^t)}{TE_o^{t+1}(x^t, y^t)}} \\ &\times \frac{DTE_o^{t+1}(x^{t+1}, y^{t+1})}{DTE_o^t(x^t, y^t)} \times \sqrt{\frac{DTE_o^t(x^{t+1}, y^{t+1})}{DTE_o^{t+1}(x^{t+1}, y^{t+1})} \times \frac{DTE_o^t(x^t, y^t)}{DTE_o^{t+1}(x^t, y^t)}} \\ &\times \frac{CAE_o^{t+1}(x^{t+1}, y^{t+1})}{CAE_o^t(x^t, y^t)} \times \sqrt{\frac{CAE_o^t(x^{t+1}, y^{t+1})}{CAE_o^{t+1}(x^{t+1}, y^{t+1})} \times \frac{CAE_o^t(x^t, y^t)}{CAE_o^{t+1}(x^t, y^t)}} \end{aligned} \tag{10}$$

TE_o^t indicates the technical efficiency for DMU_o in the period t also DTE_o^t indicates the price efficiency for DMU_o in the period t and CAE_o^t indicates the AE for DMU_o in the period t. Similarly, we also have the efficiencies for the period t + 1.

4 An Empirical Example

In this section, for better understanding and most of the literature mentioned in the pre – section, we

study part of the transactions of one of the Iranian Stock Exchange users in two consecutive periods (We consider a basket including 5 shopping and sales, each of which is considered a DMU). We measure the CMPI for these 5 DMU to determine where each of the five deals has been developed during periods of time in terms of CE, or which of the transactions have retreated or which was unchanged. These units are listed as follows: DMU 1; Symbol Saderat Bank's stakes, DMU 2; Symbol Maskan Shomal Shargh's stakes, DMU 3; Symbol Keshtirany darya khazar's stakes, DMU 4; Symbol Zob Ahan Isfehan's stakes, DMU 5; Symbol Tejarat Bank's stakes.
As mentioned before, between the two periods of t = August 2019 and t + 1= September 2019.That, the number of shares purchased by the user as input and the number of shares sold by the user as the output is introduced. Table 1 shows the data about the input and output and the input costs of these units at the time period t, and Table 2 shows the same data in the period t+1.

Table 1: Data for Period t (August 2019)

DMU	Input (x^t)	Cost (c^t)	Output (y^t)	$\bar{x}^t$
1	5000	445	3000	2225000
2	300	1683	200	504900
3	270	15798	70	4265460
4	6000	1595	6000	9570000
5	12000	421	2000	505200

Table 2: Data for Period t + 1 (September 2019)

DMU	Input (x^{t+1})	Cost (c^{t+1})	Output (y^{t+1})	$\bar{x}^{t+1}$
1	16000	413	6000	6608000
2	1400	2163	1400	3028200
3	90	17941	80	1614690
4	12000	1615	2000	19380000
5	35000	408	5000	1428000

We calculated the technical efficiency of DMUs with the CCR model in the PPS P relative to their own frontiers and the other period frontiers, and we demonstrated the results in Table 3.

Table 3: Technical Efficiency of DMUs in PPS P.

DMU	$TE^t(x^t, y^t)$	$TE^{t+1}(x^{t+1}, y^{t+1})$	$TE^{t+1}(x^t, y^t)$	$TE^t(x^{t+1}, y^{t+1})$
1	0.60	0.37	0.60	0.40
2	0.67	1	0.57	1
3	0.26	0.89	0.60	0.90
4	1	0.17	1	0.20
5	0.17	0.14	0.20	0.15

Now the coordinates of the projection points are captured for inefficient DMUs and we multiply their coordinates at the DMUs cost and this time, we calculated the price efficiency of DMUs in the PPS P_c relative to their own frontiers and the other period frontiers, and the result is shown in Table 4.
In this part, we project units on the P_c frontier and calculate cost of the DMUs and finally, we calculate the AE, which equals a ratio of the lowest cost composition to the cost combined of each DMU, in the PPS P_c relative to their own frontiers and the other period frontiers, and the result is shown in Table 5.

Table 4: Price Efficiency of DMUs in PPS P_c .

DMU	$DTE^t(x^t,y^t)$	$DTE^{t+1}(x^{t+1},y^{t+1})$	$DTE^{t+1}(x^t,y^t)$	$DTE^t(x^{t+1},y^{t+1})$
1	0.88	0.94	0.84	0.97
2	0.97	0.90	0.93	0.94
3	1	0.91	0.96	0.95
4	0.88	1	0.84	1.04
5	0.89	0.92	0.85	0.96

Table 5: AE of DMUs in PPS P_c .

DMU	$CAE^t(x^t,y^t)$	$CAE^{t+1}(x^{t+1},y^{t+1})$	$CAE^{t+1}(x^t,y^t)$	$CAE^t(x^{t+1},y^{t+1})$
1	0.27	0.56	1.11	0.14
2	1	0.47	4.01	0.11
3	0.29	1	1.18	0.24
4	0.03	0.40	0.15	1.01
5	0.43	0.69	1.74	0.17

Finally, we calculate the CE of DMUs that is multiplication of the technical, price and cost-allocation efficiencies, in the PPS P_c relative to their own frontiers and the other period frontiers, and the results are shown in Table 6. According to (9), the CMPI measure is calculated for 6 units. The results are shown in last column of Table 6.

Table 6: CE of DMUs in PPS P_c .

DMU	$CE^t(x^t,y^t)$	$CE^{t+1}(x^{t+1},y^{t+1})$	$CE^{t+1}(x^t,y^t)$	$CE^t(x^{t+1},y^{t+1})$	$CMPI$
1	0.01	0.07	0.1	0.14	0.32
2	1	1	0.96	0.11	0.31
3	1	1	0.96	0.24	2.54
4	1	1	0.96	1.01	1.85
5	1	1	0.96	0.17	0.33

Units like DMU_3, DMU_4 whose CMPI measure is greater 1 during in the two periods of time, they have improved in terms of productivity. But DMU_1, DMU_2, DMU_5, DMU_6 has productivity deterioration in the tow period of time. The results shows that the desired user did not perform well on the transactions related to units 1, 2, and 5, and the productivity index of those units has retreated in September compared to August 2019. But the productivity index for two units is 3 and 4 more than 1 and that means that transactions in these units have been more productive in September compared to August 2019.

5 Conclusion

In this study, a CMPI was presented in a non-competitive environment, in which price data undergo a change from one evaluation unit to another. Given the deficiency of Farrel's CE [1] and also the CE model presented by Tone [2], and by taking advantage of the idea of changing the productivity of DMUs at different time periods, we presented CMPI in the presence of non-identical prices for various DMUs, then tested our model on data extracted from Iranian Stock Exchange Users. For future studies, we can suggest the use of fuzzy data or imprecise data for the method presented in this study. We can also examine the congestion issue for pricing data.

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