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# The cosine method to Gardner equation and (2+1)- dimensional breaking soliton system

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**Abstract**

In this letter, we established a traveling wave solution by using cosine function algorithm for Gardner equation and (2+1)-dimensional breaking soliton system. The cosine method is used to obtain the exact solution.

*Key words:* Gardner equation; cosine function method; exact solution; (2+1)dimensional breaking soliton system

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## 1 introduction

Gardner equation known as the mixed $kdv-mkdv$ equation is very widely studied in various areas of physics that includes plasma physics, Fluid dynamic, Quantum Field Theory, solid state physics and other [3] the Gardner equation is solved by sin-cosine function method [2]. the breaking soliton system was used to describes the (2+1)-dimensional interaction of Riemann wave propagated along the y-axis with long wave propagated along the x- axis and it seems to have been investigated extensively where over lapping solutions have been derived. this system is solved to Generalized jacobi elliptic function method [4] and the $(G'/G)$-Expansion method [5].

## 2 The cosine-function method

Consider the nonlinear partial differential equation in the form

$$F(u, u_t, u_x, u_{xx}, u_{xxt}) = 0, \tag{1}$$

Where $u(x,t)$ is the solution of nonlinear partial differential equation Eq.(1). we use the

$$u(x,t) = f(\xi), \tag{2}$$

Where $\xi = x - ct$. This enables us to use the following changes:

$$\begin{aligned} &\frac{\partial}{\partial t}(\dots) = -c\frac{d}{d\xi}, \quad \frac{\partial}{\partial x}(\dots) = \frac{d}{d\xi}(\dots), \\ &\frac{\partial^2}{\partial x^2}(\dots) = \frac{d^2}{d\xi^2}(\dots), \ \dots \end{aligned} \tag{3}$$

Using Eq.(3) to transfer the nonlinear partial differential equation Eq.(**??**) to nonlinear ordinary differential equation

$$G(f, f', f'', f''', \dots) = 0. \tag{4}$$

The solution of Eq.(4) can be expressed in the form:

$$f(\xi) = \lambda \cos^{\beta}(\mu\xi), \quad |\xi| \leq \frac{\pi}{2\mu}, \tag{5}$$

Where $\lambda$, $\beta$ and $\mu$are unknown parameters which will be determined. Then we have:

$$\begin{aligned} f' = \tfrac{df(\xi)}{d\xi} &= \lambda\beta\mu \cos^{\beta-1}(\mu\xi)\sin(\mu\xi), \\ f'' = \tfrac{d^{f(\xi)}}{d\xi^2} &= -\lambda\beta\mu^2 \cos^{\beta}(\mu\xi) + \lambda\mu^2\beta(\beta-1)\cos^{\beta-2}(\mu\xi) \\ &\quad -\lambda\mu^2\beta(\beta-1)\cos^{\beta}(\mu\xi). \end{aligned} \tag{6}$$

Substituting Eq.(6) into the nonlinear ordinary differential equation Eq.(4) gives a trigonometric of terms. To determine the parameters first balancing the exponents of each pair of cosine to determine $\alpha$. Then we collect all terms with the same power in $\cos^{\beta}(\mu\xi)$ and put to zero their coefficients to get a system of algebraic equations among the unknown $\beta$, $\lambda$ and $\mu$. Now, the problem is reduced to a system of algebraic equations that can be solved to obtain the unknown parameters $\beta$, $\lambda$ and $\mu$. Hence, the solution considered in Eq.(5) is obtained [1].

## 3 Application

**Example 1**
**Gardner equation**

$$u_t - 6(u + \epsilon^2 u^2)u_x + u_{xxx} = 0. \tag{7}$$

By using the wave variable $\xi = x - ct$ and $u(x,t) = f(\xi)$ then equation becomes

$$-\frac{df(\xi)}{d\xi} - 6\left(f(\xi + \epsilon^2 f^2(\xi))\right)\frac{df(\xi)}{d\xi} + \frac{d^3 f(\xi)}{d\xi^3} = 0, \tag{8}$$

Intergrating Eq.(8)gives

$$-cf(\xi) - 6\frac{f^2(\xi)}{2} - 6\epsilon^2\frac{f^3(\xi)}{3} + \frac{d^2f(\xi)}{d\xi^2} = 0. \tag{9}$$

Substituting Eq.(6) into Eq.(9) gives:

$$\begin{aligned} &-\lambda\cos^{\beta}(\mu\xi) - 3\lambda^2\cos^{2\beta}(\mu\xi) - 2\epsilon^2\lambda^3\cos^{3\beta}(\mu\xi) - \lambda\beta\mu^2\cos^{\beta}(\mu\xi) \\ &+ \lambda\beta(\beta-1)\mu^2\cos^{\beta}(\mu\xi) - \lambda\beta(\beta-1)\mu^2\cos^{\beta}(\mu\xi) = 0, \end{aligned} \tag{10}$$

By equating the exponents and the cofficients of each pair of the cosine function we obtain the following system of algebraic equations:

$$\begin{cases} -c\lambda - \lambda\beta\mu^2 - \lambda\beta(\beta-1)\mu^2 = 0, \\ -2\epsilon^2\lambda^3 + \lambda\beta(\beta-1)\mu^2 = 0. \end{cases} \tag{11}$$

$$3\beta = \beta - 2 \rightarrow \beta = 1.$$

By using maple for solving the system Eq(11):

$$\beta = 1-, \ \mu = \pm i\sqrt{c}, \ \lambda = \pm\frac{\sqrt{c}}{\epsilon} \tag{12}$$

Substituting Eq.(12) into Eq.(5) gives: (figure1)

$$\begin{aligned} u(x,t) &= \pm\frac{\sqrt{c}}{\cos}^{-1}(\pm i\sqrt{c}(x-ct)), \\ u(x,t) &= \pm\frac{\sqrt{c}}{\cosh}(\sqrt{c}(x-ct)). \end{aligned} \tag{13}$$

**Example 2**
**(2+1)-dimensional breaking soliton system**

$$\begin{aligned} &u_t + 4buv_x + 4bu_xv + bv_{xxy} = 0, \\ &v_x - u_y = 0. \end{aligned} \tag{14}$$

Suppose $u(x,y,t) = f(\xi)$ and $v(x,y,t) = g(\xi)$ and $\xi = x + ky - ct$ then

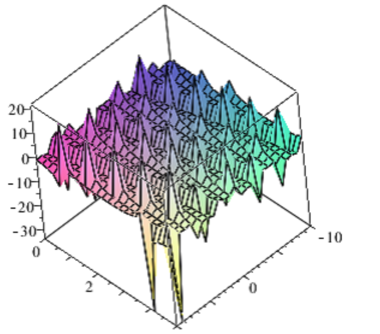



Fig. 1. shows the soliton solution for Gardner equation with increase time.

the (2+1)-dimensional soliton system becomes

$$-c\frac{df(\xi)}{d\xi}+4bf(\xi)\frac{dg(\xi)}{d\xi}+4b\frac{df(\xi)}{d\xi}g(\xi)+bk\frac{d^3f(\xi)}{d\xi^3}=0, \tag{15}$$

$$\frac{dg(\xi)}{d\xi}-k\frac{df(\xi)}{d\xi}=0, \tag{16}$$

Integrating Eq.(15), Eq.(16)gives:

$$-cf(\xi)+4bf(\xi)g(\xi)+bk\frac{d^2f(\xi)}{d\xi^2}=0, \tag{17}$$

$$g(\xi)-kf(\xi)=0, \tag{18}$$

From Eq.(18)

$$g(\xi)=kf(\xi)=0, \tag{19}$$

Substituting Eq.(19) into Eq.(17) gives:

$$-cf(\xi)+4bkf^2(\xi)+bk\frac{d^2f(\xi)}{d\xi^2}=0. \tag{20}$$

Substituting Eq.(6) into Eq.(20) gives:

$$\begin{aligned}&-c\lambda\cos^{\beta}(\mu\xi)+4bk\lambda^2\cos^{2\beta}(\mu\xi)+bk[-\lambda\beta\mu^2\cos^{\beta}(\mu\xi)\\&+\lambda\mu^2\beta(\beta-1)\cos^{\beta-2}(\mu\xi)\\&-\lambda\mu^2\beta(\beta-1)\cos^{\beta}(\mu\xi)\Big]=0,\end{aligned} \tag{21}$$

By equating the exponents and the coefficients of each pair of the cosine function we obtain the following system of algebraic equations:

$$\begin{cases} -c\lambda - bk\lambda\mu^2\beta - bk\lambda\mu^2\beta(\beta-1) = 0, \\ 4bk\lambda^2 + bk\lambda\mu^2\beta(\beta-1) = 0. \end{cases} \tag{22}$$
$$2\beta = \beta - 2 \to \beta = 2.$$

By using Maple for solving the system Eq.(22) we get:

$$\beta = -2,\ \mu = \pm\frac{i}{2}\sqrt{\frac{c}{bk}},\ \lambda = \frac{3}{8}\frac{c}{bk} \tag{23}$$

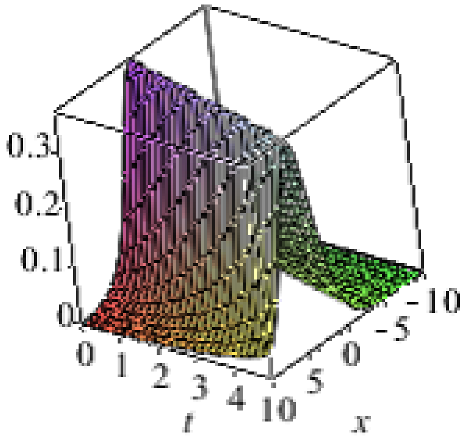
Then by substituting Eq.(23) into Eq.(5) then, the exact soliton solutions of the (2+1)-dimensional breaking soliton system can be written in the form: (figure 2, 3)

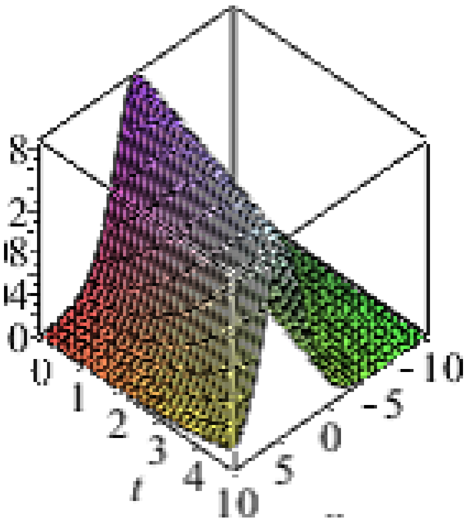
$$u(x,y,t) = \frac{3}{8}\frac{c}{bk}\cos^{-2}\left(\pm\frac{i}{2}\sqrt{\frac{c}{bk}}(x+ky-ct)\right),$$
$$u(x,y,t) = \frac{3}{8}\frac{c}{bk}\sec h^2\left(\frac{1}{2}\sqrt{\frac{c}{bk}}(x+ky-ct)\right). \tag{24}$$
$$v(x,y,t) = \frac{3}{8}\frac{c}{b}\sec h^2\left(\frac{1}{2}\sqrt{\frac{c}{bk}}(x+ky-ct)\right). \tag{25}$$

Figures 3 and 3 show soliton solutions $u$, $v$ of the (2+1)-dimensional breaking soliton
system at $y = 0$ to $b = 1$, $k = 2$

## 4 Conclusion

In the letter, the cosine function method has been successfully applied to find the solution for two nonlinear partial differential equations such as Gardner and (2+1)-dimensional breaking soliton system. The cosine function method is used to find a new exact solution.




## References

[1] A. H. A. Ali, A. A. Soliman, et. al., soliton solution for nonlinear partial differential equations by cosine-function method. Physics letters A 368 (2007) 299-304.

[2] Anwar jaafar mohamad et. al., The sine-cosine function method for the

exact solutions of nonlinear partial differential equations.

[3] Biswas, A. soliton pertur bation theory for the Garnner equation, Adv studies theor phys.vol 16, no .2, (2008) pp.787-794.

[4] Ibrahim Enam Inan Generalized jacobi Elliptic function method for traveling wave solutions of(2+1)-dimensional breaking soliton equation. cankaya university journal of science and Engineering volume 7 (2010), no,1,39-50.

[5] Yuanming chen, songhua ,Non- Traveling wave solutions for the (2+1)-Dimensional breaking soliton system, college of sciences, zhejiang lishui university, lishui, china ,2012 ,3,813-818