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A goal programming procedure for ranking decision making units in DEA

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Abstract

This research proposes a methodology for ranking decision making units by using a goal programming model. We suggest a two phases procedure. In phase 1, by using some DEA problems for each pair of units, we construct a pairwise comparison matrix. Then this matrix is utilized to rank the units via the goal programming model.

Key words: Data envelopment analysis; Pairwise comparison matrix; Goal programming; Ranking.
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1 Introduction

Data envelopment analysis (DEA), originally presented by Charnes et al. [6], is a well-known family of mathematical programming tools for assessing the relative efficiency of a set of comparable processing decision making units (DMUs). DEA successfully divides them into two categories: efficient DMUs and inefficient DMUs. The DMUs in the efficient category have identical efficiency scores. However, it is not appropriate to claim that they have the equivalent performance in actual practice. The efficient DMUs are not comparable among themselves in the CCR and other DEA models. One of the interesting research subjects is to discriminate between efficient DMUs. In the last decade, some DEA researchers initiated a new area called super-efficiency to rank the DEA efficient DMUs and developed various models [27]. Although the developed models are interesting and useful, in general, they have the drawbacks of lacking either stability or feasibility.

Several authors have proposed methods for ranking the best performers [2,15,29,36] and [35,45]. For a review of ranking methods, readers are refereed to Adler et al. [1]. In some cases, the models proposed by Andersen and Petersen [2] and Mehrabian et al. [29] can be infeasible. In addition to this difficulty, the Andersen and Petersen [2] model may be unstable because of extreme sensitivity to small variations in the data when some DMUs have relatively small values for some of their inputs. Jahanshahloo et al. [18] present a method for ranking extreme efficient decision making units in data envelopment analysis models with constant and variable returns to scale. In their method, they exploit the leave-one-out idea and l_1-norm, also, Jahanshahloo et al. [19] proposed a ranking system for extreme efficient DMUs based upon the omission of efficient DMUs from reference set of the inefficient DMUs. Li et al. [27] developed a super-efficiency model to overcome some deficiencies in the earlier models. Izadikhah [17] proposed a method for ranking decision making units with interval data by introducing two efficient and inefficient frontiers. Wang et al. [42] proposed a methodology for ranking decision making units. That methodology ranks DMUs by imposing an appropriate minimum weight restriction on all inputs and outputs, which is decided by a decision maker (DM) or an assessor in terms of the solutions to a series of linear programming (LP) models that are specially constructed to determine a maximin weight for each DEA efficient unit. Liu and Peng [28] proposed a methodology to determine one common set of weights for the performance indices of only DEA efficient DMUs. Then, these DMUs are ranked according to the efficiency score weighted by the common set of weights. For the decision maker, this ranking is based on the optimization of the groups efficiency. Jahanshahloo et al. [20] proposed two ranking methods. In the first method, an ideal line was defined and determined a common set of weights for efficient DMUs then a new efficiency score obtained and ranked them with it. In the second method, a special line was defined then compared all efficient DMUs with it and ranked them. Wang et al. [43] proposed a new methodology based on regression analysis to seek a common set of weights that are easy to estimate and can produce a full ranking for DMUs. Chen and Deng [8] proposed a new method for ranking units. Their method develop a new ranking system under the condition of variable returns to scale (VRS) based on a measure of cross-dependence efficiency, where the evaluation for an efficient DMU is dependent of the efficiency changes of all inefficient units due to its absence in the reference set, while the appraisal of inefficient DMUs depends on the influence of the exclusion of each efficient unit from the reference set. Recently, Rezai Balf et al. [31] proposed a method for ranking extreme efficient decision making units (DMUs). Their method uses L_∞(or Tchebychev) Norm, and it seems to have some superiority over other existing methods, because this method is able to remove the existing difficulties in some methods, such as Andersen and Petersen (AP) that it is sometimes infeasible. For more information about DEA see [21,24,14].

Many researchers (e.g., Belton and Vickers, [4]) highlight the relationship between DEA and Multi-Criteria Decision Analysis (MCDA): "Indeed in common with many approaches to multiple criteria analysis, DEA incorporates a process of assigning weights to criteria" (see also other references, Belton, [3]; Cook et al., [9,10]; Doyle and Green, [11]; Stewart, [39]). Ranking is very common in MCDA literature, especially when we have a discrete list of elements or alternatives with single or multiple criteria which we wish to evaluate and compare or select. Various approaches are suggested in the literature for fully ranking elements, ranging from the utility theory approach (see Keeney and Raiffa, [22]; Keeney, [23]; Sinuany-Stern and Mehrez, [37]; Fishburn, [12]), to the AHP developed by Saaty [33]. Sinuany-Stern et al. [38] present a two-stage model for fully ranking organizational units where each unit has multiple inputs and outputs. In the first stage, the Data Envelopment Analysis (DEA) is run for each pair of units separately. In the second stage, the pairwise evaluation matrix generated in the first stage is utilized to rank scale the units via the Analytical Hierarchical Process (AHP).

Interestingly, Charnes and Cooper have also had a significant impact on the development of Multiple Objective Linear Programming (MOLP) through the development of Goal Programming [7]. Since the 1970s, MOLP has become a popular approach for modifying and analyzing certain types of multiple criteria decision problems. The purpose of the current paper is ranking units by using goal programming method. Goal programming (GP) was originally proposed by Charnes and Cooper [7], and further development carried out by Lee [26], Ignizio [16], Tamiz [40], and Romero [32], among others [13,41,5]. It has been applied to many real-world problems in areas such as accounting, agriculture, economics, engineering, transportation, finance, government, international context, and marketing [34,25]. GP is an important technique for decision-makers (DMs) to consider simultaneously several objectives in finding a set of acceptable solutions. It can be said that GP has been, and still is, the most widely used technique for solving multi-criteria and multi-objective decision- making problems. Crisp comparison matrices lead to crisp weight vectors being generated. Accordingly, an interval comparison matrix should give an interval weight estimate. Therefore in Wang and Elhag [44], a goal programming (GP) method is proposed to obtain interval weights from an interval comparison matrix, which can be either consistent or inconsistent.

We suggest a multiple criteria decision making problem based on goal programming method for ranking decision making units in data envelopment analysis. We propose this method in two phases. First, we run DEA for every pair of units, two units at a time, ignoring the others. Then, from the results of the first phase, we create a pairwise comparison matrix to which we apply a goal programming procedure, which provides a full ranking scale of all the units. However, the obtained pairwise comparison matrix may be inconsistent. Since the pairwise comparison matrix is reciprocal in nature, its lower triangular judgments provide exactly the same information on the preferences of weights as its upper triangular judgments. So, the use of either the lower or upper triangular judgments should lead to the same priority rankings. By means of goal programming we generate weights from this inconsistent pairwise comparison matrix. These weights reflect the importance of each DMU and therefore we can rank these DMUs.

The current article proceeds as follows. In section 2, we present two concepts. First, we present the multiplier form of CCR model and then we present the goal programming method. In section 3, we develop our method for ranking DMUs in data envelopment analysis. Some examples are considered in section 4 which illustrate the proposed method. Also, the results of proposed method are compared with AP method and norm L_1 method. Conclusions are given in section 5.

2 Preliminaries

In this section, we briefly present some required concepts.

2.1 Data envelopment analysis

Consider n decision making units $DMU_j, \quad (j = 1, \ldots, n)$ which each DMU consumes inputs levels $x_{ij}, \quad i = 1, \ldots, m$ to produce output levels $y_{rj}, \quad r = 1, \ldots, s$. Suppose that $X_j = (x_{1j}, \ldots, x_{mj})$ and $Y_j = (y_{1j}, \ldots, y_{sj})$ are the vectors of inputs and outputs values respectively, for DMU_j, in which it has been assumed that $X_j \geq 0$ and $X_j \neq 0$, and $Y_j \geq 0$ and $Y_j \neq 0$. The relative efficiency score of the DMU_o is obtained from the following model which is called CCR multiplier model.

$$
\begin{aligned}
& e_o = max \sum_{r=1}^{s} u_r y_{ro} \\
& s.t. \\
& \qquad \sum_{i=1}^{m} v_i x_{io} = 1, \\
& \qquad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \; j = 1, \ldots, n, \\
& \qquad u_r, v_i \geq 0, \qquad\qquad r = 1, \ldots, s, \quad i = 1, \ldots, m
\end{aligned}
\tag{2.1}
$$

where $u_r, \quad (r = 1, \ldots, s)$ and $v_i, \quad (i = 1, \ldots, m)$ being the weight on output r and input i, respectively.

2.2 Goal Programming

Consider the following problem:

$$
\begin{aligned}
& max \; (f_1(x), \ldots, f_k(x)) \\
& s.t. \\
& \qquad x \in X
\end{aligned}
\tag{2.2}
$$

where $f_1, \ldots, f_k$ are objective functions and X is non-empty feasible region. Model (2.2) is called multiple objective programming. Goal programming is now an important area of multiple criteria optimization. The idea of goal programming is to establish a goal level of achievement for each criterion, therefore it is necessary for the decision maker to set goals for each objective that he/she wishes to obtain. A preferred solution is then defined as the one which minimizes the deviations from the set goals. Then GP can be formulated as the following achievement function.

$$
\begin{aligned}
& min \sum_{i=1}^{k}(d_i^+ + d_i^-) \\
& s.t. \\
& \qquad f_i(x) + d_i^- - d_i^+ = b_i,\ i = 1, \ldots, k, \\
& \qquad x \in X \\
& \qquad d_i^- d_i^+ = 0, \qquad\qquad i = 1, \ldots, k, \\
& \qquad d_i^-, d_i^+ \geq 0, \qquad\qquad i = 1, \ldots, k.
\end{aligned} \tag{2.3}
$$

The DMs for their goals set some acceptable aspiration levels, $b_i \ \ (i = 1, 2, \ldots, k)$, for these goals, and try to achieve a set of goals as closely as possible. The purpose of GP is to minimize the deviations between the achievement of goals, $f_i(x)$, and these acceptable aspiration levels, $b_i \ \ (i = 1, 2, \ldots, k)$. Also, d_i^+ and d_i^- are, respectively, over- and under-achievement of the ith goal.

3 Ranking decision making units by using goal programming

3.1 Some properties of the pairwise comparison matrix

First we briefly review some concepts about the pairwise comparison matrix.

Definition 3.1 *An $n \times n$ pairwise comparison matrix A is shown as follows:*

$$
A = \begin{pmatrix} 1 & a_{12} & \ldots & a_{1n} \\ a_{21} & 1 & \ldots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \ldots & 1 \end{pmatrix}.
$$

In the matrix A, the elements a_{ij} reflect the evaluation of unit i over unit j, (and also reflect the importance of criterion i over criterion j).

Definition 3.2 *Matrix A is* reciprocal *if $a_{ij} = \frac{1}{a_{ji}}, \ \ i, j = 1, \ldots, n$*

Definition 3.3 *Matrix A is* consistent *if* $a_{ij}a_{jk} = a_{ik}, \quad i,j,k = 1,\ldots,n$

If for some i, j, k, definition 3.3 does not hold, then A is said to be *inconsistent*.
One of the important purpose of construction of the pairwise comparison matrix is calculation the weight of each criterion. Hence we have the following remark from Saaty [33].

Remark 3.4 *If the pairwise comparison matrix A is reciprocal and consistent, then the importance weight $w_i, \quad i = 1,\ldots,n$, which reflects the relative importance given to unit i, is simply calculated as:*

$$w_i = \frac{a_{ij}}{\sum_{k=1}^{n} a_{kj}},\ i = 1,\ldots,n$$

3.2 The proposed Method

Assume that we have n decision making units which each unit consumes inputs levels $x_{ij}, \quad i = 1,\ldots,m$ to produce output levels $y_{rj}, \quad r = 1,\ldots,s$. Suppose that $X_j = (x_{1j},\ldots,x_{mj})$ and $Y_j = (y_{1j},\ldots,y_{sj})$ are the vectors of inputs and outputs values respectively, for unit j. For the purpose of ranking units we apply two following phases:

Phase 1 *Construction the pairwise comparison matrix.*
For any pair of units A and B, we perform the following DEA model as if only these two units exist [38].

$$\begin{aligned} e_{AA} = max & \sum_{r=1}^{s} u_r y_{rA} \\ s.t. & \\ & \sum_{i=1}^{m} v_i x_{iA} = 1, \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0,\ j = A, B, \\ & u_r, v_i \ge 0, \qquad r = 1,\ldots,s, \quad i = 1,\ldots,m. \end{aligned} \tag{3.1}$$

In order to cross evaluate unit B, using the optimal weights of unit A, we calculate $e_{BA} = \sum_{r=1}^{s} u_r y_{rB} / \sum_{i=1}^{m} v_i x_{iB}$. Since there are only two units in Problem (3.1), it is simple to show that $e_{BA} = e_{BB}$ and $e_{AB} = e_{AA}$, thus the evaluation of A over B is e_{AA}/e_{BB}. In order to cross evaluate unit B using the optimal weights of unit A, when $u_r \geq \varepsilon, v_i \geq \varepsilon$, we may have more than one optimal solution for the optimal weight; thus, given the optimal solution for unit A, e_{AA}, we solve the following problem according to Oral et al. [30] to guarantee the best cross evaluation for unit B:

$$\begin{aligned} & e_{BA} = max \sum_{r=1}^{s} u_r y_{rB} \\ & s.t. \\ & \qquad \sum_{i=1}^{m} v_i x_{iB} = 1, \\ & \qquad \sum_{r=1}^{s} u_r y_{rj} - e_{AA} \sum_{i=1}^{m} v_i x_{ij} \leq 0, \; j = A, B, \\ & \qquad u_r, v_i \geq \varepsilon, \qquad\qquad r = 1, \ldots, s, \quad i = 1, \ldots, m. \end{aligned} \tag{3.2}$$

Actually, e_{BA} is the optimal cross evaluation of unit B (See [38]). Symmetrically, e_{BB} and e_{AB} are calculated. Finally, based on these results, we construct the pairwise comparison matrix from the results of the paired DEA described above, so that for every pair of units j and k:

$$a_{jk} = \frac{e_{jj}+e_{jk}}{e_{kk}+e_{kj}} \; and \; a_{jj} = 1.$$

Therefore we construct matrix A as pairwise comparison matrix. The matrix A is $n \times n$. This matrix is reciprocal, since $a_{jk} = \frac{1}{a_{kj}}$. This matrix has not been evaluated subjectively by a decision maker, rather, it is an objective evaluation (with direct comparisons), calculated from the DEA pairwise runs, which provide cross evaluation, thus allowing each unit to receive its most favorable evaluation relative to any other unit.

Phase 2 *Ranking using goal programming*

The pairwise comparison matrix A, generated in the first phase, is often inconsistent (see examples 4.1 and 4.2). Therefore the relation

$$w_i = \frac{a_{ij}}{\sum_{k=1}^{n} a_{kj}},\ i = 1, \ldots, n$$

is no longer holds. We must obtain importance weights $w_i,\ \ i = 1, \ldots, n$ such that $a_{ij} = \frac{w_i}{w_j}$ or equivalently $a_{ij}w_j - w_i = 0$. Hence, we introduce deviation variables p_{ij} and q_{ij}, which leads to:

$$a_{ij}w_j - w_i + p_{ij} - q_{ij} = 0,\ i, j = 1, \ldots, n \tag{3.3}$$

where p_{ij} and q_{ij} are both nonnegative real numbers, but can't be positive at the same time, i.e. $p_{ij}q_{ij} = 0$. Since A is reciprocal, then the use of upper or lower triangular components of matrix A would lead to same priorities and ranking. Hence we apply the goal programming method. It is desirable that the deviation variables p_{ij} and q_{ij} are kept to be small as possible, which leads to the following goal programming model:

$$\begin{aligned}
& d^* = min \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (p_{ij} + q_{ij}) \\
& s.t. \\
& \qquad a_{ij}w_j - w_i + p_{ij} - q_{ij} = 0,\ i = 1, \ldots, n-1, \quad j = i+1, \ldots, n \\
& \qquad \sum_{i=1}^{n} w_i = 1, \\
& \qquad w_i, p_{ij}, q_{ij} \geq 0, \qquad\qquad for\ \ all\ \ i\ \ and\ \ j.
\end{aligned} \tag{3.4}$$

It can be seen that, in the proposed model we neglected the constraint $p_{ij}q_{ij} = 0$, since the structure of linear programming results that in optimality at least one of the p_{ij} or q_{ij} is zero. The goal programming model (3.4) considers only the upper triangular judgments of comparison matrices when generating weights because no new information is embodied in the lower triangular judgments.
By solving model (3.4), the optimal weight vector $W = (w_1, \ldots, w_n)$ is obtained. We assign the rank 1 to the unit with the maximal value of w_j, etc., in a decreasing order of w_j.

Theorem 3.5 *The vector of weights generated by upper triangular components of pairwise comparison matrix A are same as the vector of weights generated by lower triangular components.*

Proof. Since the pairwise comparison matrix A is reciprocal, the proof is evident.

Theorem 3.6 *Model (3.4) is always feasible.*

Proof. Consider the vector $\widetilde{W} = (\widetilde{w}_1, \ldots, \widetilde{w}_n)$ which has the following conditions:

$$\begin{cases} \sum_{i=1}^{n} \widetilde{w}_i = 1, \\ \widetilde{w}_i \geq 0. \end{cases}$$

Then we define $\widetilde{p}_{ij}$ and $\widetilde{q}_{ij}$ as follows:

$$\begin{cases} \widetilde{p}_{ij} = \max\{-(a_{ij}w_j - w_i), 0\}, \\ \widetilde{q}_{ij} = \max\{(a_{ij}w_j - w_i), 0\}. \end{cases}$$

It is clear that $(\widetilde{W}, \widetilde{p}_{ij}, \widetilde{q}_{ij})$ is a feasible solution for model (3.4).

Theorem 3.7 *The pairwise comparison matrix A is consistent if and only if $d^* = 0$.*

Proof. Let us first prove that, if $d^* = 0$ then matrix A is consistent.
Since $d^* = 0$ we have $p_{ij} = q_{ij} = 0$. Therefore $a_{ij}w_j - w_i = 0$ and hence $a_{ij} = \frac{w_i}{w_j}$. This gives $a_{ij}a_{jk} = a_{ik}$, and we conclude that matrix A is consistent.

Conversely, suppose that matrix A is consistent. That is

$$a_{ij}a_{jk} = a_{ik}, \qquad i, j, k = 1, \ldots, n.$$

Now, if we define

$$\begin{cases} \overline{w}_j = \frac{a_{jk}}{\sum_{t=1}^{n} a_{tk}}, \; j = 1, \ldots, n \\ \overline{p}_{ij} = \overline{q}_{ij} = 0, \end{cases}$$

then it is easy to check that $(\overline{W}, \overline{p}_{ij}, \overline{q}_{ij})$ is feasible for model (3.4). Since model (3.4) has minimization form, we conclude that $d^* = 0$.

4 Illustrating examples

In this section we present two illustrating examples showing that the new method can rank all DMUs.

Example 4.1 *The case of single input and output.*

Consider four DMUs with a single input and single output. The data are shown in Table 1.

Table 1. The data set of example 4.1.

DMU	Input	Output
1	1	3
2	2.1	5.2
3	4.5	8.2
4	3.2	9.6

By using formula (3.2) and (3.3) we can construct the pairwise comparison matrix A as follows:

$$A = \begin{pmatrix} 1 & 1.09565 & 1.24424 & 1 \\ 0.91270 & 1 & 1.15214 & 0.91269 \\ 0.80370 & 0.86795 & 1 & 0.80370 \\ 1 & 1.09565 & 1.24424 & 1 \end{pmatrix}.$$

We can easily check that the pairwise comparison matrix A is reciprocal but it is inconsistent. Now, for ranking DMUs we apply a goal programming model (3.4) to matrix A. Therefore we must solve the following goal programming model

$$
\begin{aligned}
d^* = min\ & p_{12} + q_{12} + p_{13} + q_{13} + p_{14} + q_{14} + p_{23} + q_{23} + p_{24} + q_{24} + p_{34} + q_{34} \\
s.t.\ & \\
& 1.09565 w_2 - w_1 + p_{12} - q_{12} = 0, \\
& 1.24424 w_3 - w_1 + p_{13} - q_{13} = 0, \\
& w_4 - w_1 + p_{14} - p_{14} = 0, \\
& 1.15214 w_3 - w_2 + p_{23} - q_{23} = 0, \\
& 0.91269 w_4 - w_2 + p_{24} - q_{24} = 0, \\
& 0.80370 w_4 - w_3 + p_{34} - q_{34} = 0, \\
& w_1 + w_2 + w_3 + w_4 = 1, \\
& w_i, p_{ij}, q_{ij} \geq 0, \quad 1 \leq i, j \geq n.
\end{aligned} \tag{4.1}
$$

By solving model (4.1), we obtain the optimal vector $W = (w_1, \ldots, w_n)$. We assign the rank 1 to the unit with the maximal value of w_j, etc., in a decreasing order of w_j. The result is shown in Table 2.

Table 2. The result of proposed method for example 4.1.

DMU	The proposed score	Proposed Ranking	Ap Score
1	w_1=0.269077334873812356	1	1.0000000
2	w_2=0.246	3	0.8253968
3	w_3=0.216	4	0.6074074
4	w_4=0.269077334873812300	2	1.0000000

In Table 2, we also present the AP result of these DMUs. We can see that, the AP method can't rank these DMUs, but the proposed method presents a fully ranking.

The optimal objective of model (4.1) is $d^* = 0.00357$, which shows that the pairwise comparison matrix A is inconsistent by Theorem 3.7.

Example 4.2 *The case of multiple inputs and outputs.*

Consider eight DMUs with two inputs and two outputs as defined by Table 3.

Table 3. The data set of example 4.2.

DMU	Input 1	Input 2	Output 1	Output 2
1	20	12	60	36
2	10	15	30	45
3	15	12	30	36
4	5	70	15	80
5	3	9	3	9
6	9	18	1	18
7	63	19	8	19
8	22	73	1	3

First, we construct the pairwise comparison matrix A by using formula (3.2) and (3.3). Matrix A is $n \times n$ and is reciprocal. Now, for ranking DMUs we apply a goal programming model (3.4) to matrix A. The result of the goal programming model is shown in Table 4.

Table 4. The result of proposed method for example 4.2.

DMU	The proposed score	Proposed Ranking
1	w_1=0.1392200931	4
2	w_2=0.1438343458	1
3	w_3=0.1401384993	2
4	w_4=0.1392200926	5
5	w_5=0.1392200946	3
6	w_6=0.1284378144	6
7	w_7=0.0958640004	7
8	w_8=0.0740650956	8

By solving model (4.1), we obtain the optimal vector $W = (w_1, \ldots, w_n)$. We assign the rank 1 to the unit with the maximal value of w_j, etc., in a decreasing order of w_j. Clearly , the proposed method presents a fully ranking. We can see that the unit 2 is the best and the unit 8 is the worst unit. The optimal objective of the goal programming model is $d^* = 0.257758$, which shows that the pairwise comparison matrix A is inconsistent by Theorem 3.7.

Example 4.3 ***Case Study:*** *Ranking of Japanese companies.*

We apply our approach by a data set consists of 10 largest Japanese companies in 1999 (see Table 5). Ten Japanese companies are evaluated in term of three inputs and one output. The DEA inputs are asset, equity and number of employees and the DEA output is revenue. Table 5 shows the input and output data of the ten companies.

Table 5. The data for ten Japanese companies.

DMU	Company	Asset	Equity	Employee	Revenue
1	Marubeni Corp.	49742.9	2704.3	5844	91361.7
2	Mitsubishi Corp.	67553.2	7253.2	36000	104456.3
3	Nippon Telegraph and Tel.	133008.8	47467.1	138150	74323.4
4	Hitachi Ltd.	73917	21914.2	328351	60937.9
5	Itochu Corp.	51432.5	2333.8	5775	106184.1
6	Sumitomo Corp.	41168.4	4351.5	30700	86921
7	Honda Motor	38455.8	13473.8	112200	47597.9
8	Fujitsu Ltd.	39052.2	8901.6	188000	40050.3
9	Nissan Motor	52842.1	9583.6	39467	50263.5
10	Japan Tobacco	17023.6	10816.6	31000	29612.2

By using relations (3.2) and (3.3), we calculate the pair-wise comparison matrix. The results are shown in Table 6. In Table 6, only the upper triangular elements are shown.

Table 6. The obtained pair-wise comparison matrix for Japanese companies.

	Marubeni Corp.	Mitsubishi Corp.	Nippon Telegraph and Tel.	Hitachi Ltd.	Itochu Corp.	Sumitomo Corp.	Honda Motor	Fujitsu Ltd.	Nissan Motor	Japan Tobacco
Marubeni Corp.	1	1.0850485	1.5340460	1.3818415	0.9448137	1	1.1953904	1.284262	1.3177961	1.0273740
Mitsubishi Corp.	–	1	1.4702415	1.3067409	0.8750818	1	1.1122737	1.2042129	1.2392456	1
Nippon Telegraph and Tel.	–	–	1	1	0.6153070	0.6322331	1	1	0.7935435	0.7857989
Hitachi Ltd.	–	–	–	1	0.6989640	0.6946717	0.8929622	0.9355307	0.9321312	1
Itochu Corp.	–	–	–	–	1	1	1.2509355	1.3370744	1.36935	1.0856445
Sumitomo Corp.	–	–	–	–	–	1	1.2612874	1.3468555	1.3788354	1.0967049
Honda Motor	–	–	–	–	–	–	1	1	1	1
Fujitsu Ltd.	–	–	–	–	–	–	–	1	1	1
Nissan Motor	–	–	–	–	–	–	–	–	1	1
Japan Tobacco	–	–	–	–	–	–	–	–	–	1

By using the goal programming method, we can rank the DMUs. The result are shown in Table 7. Also, in Table 7, we can see the ranking results of AP method and L_1-norm method. Clearly, the results of proposed method and other methods are close.

Table 7. The final results of proposed method and other methods for ranking Japanese companies.

Company	Proposed Method		CCR efficiency	AP Method		Norm L_1 Method	
	Score	Rank	Score	Score	Rank	Score	Rank
Marubeni Corp.	0.1169779	3	0.8894680560	0.8894680560	3	0.8894680560	3
Mitsubishi Corp.	0.1091745	4	0.7392753542	0.7392753542	5	0.7392753542	5
Nippon Telegraph and Tel.	0.0771999	10	0.2646576597	0.2646576597	10	0.2646576597	10
Hitachi Ltd.	0.08465365	9	0.3904652830	0.3904652830	9	0.3904652830	9
Itochu Corp.	0.1215543	2	1.0000000000	1.3467486565	1	8.2069863E+3	1
Sumitomo Corp.	0.1221068	1	1.0000000000	1.0226778975	2	9.3361275E+2	2
Honda Motor	0.09066065	6	0.5862261891	0.5862261891	6	0.5862261891	6
Fujitsu Ltd.	0.09066065	6	0.4857351624	0.4857351624	7	0.4857351624	7
Nissan Motor	0.08876785	8	0.4505177769	0.4505177769	8	0.4505177769	8
Japan Tobacco	0.09824378	5	0.8238697216	0.8238697216	4	0.8238697216	4

5 Conclusion

In summary, this paper proposed an integrated methodology for ranking decision making units by integration of DEA and goal programming model. The integrated model can be used for ranking decision making units by considering multi objective criteria. For the purpose of ranking units, we suggested a two phases procedure. In phase 1, by using some DEA problems for each pair of units, we constructed a pairwise comparison matrix. Then this matrix was utilized to rank the units via the goal programming model. The advantage of this hybrid method is that all preferences are derived mathematically from the input/output data, by using pairwise DEA models. Thus, there is no subjective evaluation, and the proposed model is always feasible. Also this method is able to rank all DMUs (See examples 4.1, 4.2 and 4.3). However this method doesn't work in the case of variable return to scale, therefore researchers can extend the proposed method to work in this case. Also the ranking by the proposed method is dependent to the situation of inefficient DMUs in PPS, therefore the inefficient DMUs are involved in the proposed method and play important roll. Also in order to evaluate and ranking DMUs, researchers can combine DEA models with other MCDM methods.

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