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Imprecise Revenue Efficiency under Productivity Change

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ABSTRACT

Traditional data envelopment analysis (DEA) models evaluate the performance of decision-making units (DMUs) with the exact data and do not assume evaluation in the condition that the environment is uncertain. When some data are unknown, such as interval data, the DEA model is called imprecise DEA (IDEA). In this paper, we develop a new Malmquist productivity index(MPI) for dealing with interval data in DEA. Then, an approach for measuring the Malmquist productivity index using revenue efficiency is extended, too. The capabilities of the presented approach are shown by means of an example.

1 Introduction

Data envelopment analysis (DEA), occasionally called frontier analysis, was first put forward by Charnes et al. (1978). It is a performance measurement technique that, as we shall see, can be used for evaluating the relative efficiency of decision-making units (DMU's) in organizations (Tone et al. (2020)). Here a DMU is a distinct unit within an organization that has flexibility with respect to some of the decisions it makes, but not necessarily complete freedom with respect to these decisions. Revenue Efficiency (RE) evaluates the ability to produce current outputs at maximal revenue. in other words, the revenue performance of a firm that aims to make production using minimum inputs for the purpose of increasing profit. Ghiyasi (2017) presented an inverse DEA model based on cost and revenue efficiency in presence of price information. Nguyen et al. (2022) investigated the revenue efficiency differentials of the three airline business models in the U.S, using a two-stage data envelopment analysis approach.

Fare et al. [3] developed a DEA-based Malmquist productivity index which measures the productivity change over time. The Malmquist index was first suggested by Malmquist (1953) as

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a quantity index for use in the analysis of consumption of inputs, Fare et al. combined ideas on the measurement of efficiency from Farrell and the measurement of productivity from Caves et al.(1982) to construct a Malmquist productivity index has proven itself to be a good tool for measuring the productivity change of DMUs. Seyed Esmaeili et al. (2018) suggested a novel method to measure the productivity changes of hospitals over time in the presence of linguistic variables along with fuzzy data. Mombini et al. (2020) presented a global Malmquist index to evaluate the units under review in terms of economic efficiency, the units in terms of spending, production, revenue, and profit over several periods, and the rate of improvement or regression of each of these units.

Conventional DEA models deal with exact data, while in many real-world problems data are not exactly known and are stated in an imprecise one. Sharafi et al. (2015) discussed the new method for evaluation and ranking interval data with stochastic bounds. Poordavoodi et al. (2020) modified the interval data envelopment analysis models for QoS-aware Web service selection considering the uncertainty of QoS attributes in the presence of desirable and undesirable factors. Izadikhah et al. (2021) reformulated the conventional DEA models as an imprecise DEA problem and propose a novel method for evaluating the DMUs when the inputs and outputs are fuzzy and/or ordinal or vary in intervals. Mombini et al. (2022) investigated the sensitivity analysis in DEA and proposed an approach to determine the sustainability radius of the cost efficiency of units with interval data. Accordingly, the main aim of the current paper is to suggest a Malmquist productivity index using revenue efficiency for DMUs productivity evaluation with interval data.

2 Revenue Efficiency

Revenue Efficiency (RE) evaluates the ability to produce current outputs at maximal revenue. In this section, we assume that benefits are fixed and known, although they may possibly be different between the DMUs. In order to obtain a measure of revenue efficiency for DMUs with multiple inputs and outputs, the maximum revenue for the production of a DMU's current outputs with existing output benefits is obtained solving the following linear problem:

maximal revenue model

$$\begin{array}{lll} \max & \sum_{r=1}^{s} R_{ro} y_r^o & \\ s.t. & \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io} & i=1,...,m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} = y_r^o & r=1,...,s, \\ & \lambda_j \geq 0 & j=1,...,n, \\ & y_r^o \geq 0 & r=1,...,s. \end{array} \quad (1)$$

In the model above, R_{ro} is the benefit of output r for the DMU_O under assessment. y_r^o is a variable that , at the optimal solution , gives the amount of output r to be used by DMU_O in order to produce the current outputs at maximal revenue. Note that, this model assumes that the output benefits at each DMU $(R_{ro}, r = 1, ..., s)$ are fixed and known, although they can differ between DMUs. Revenue efficiency is then obtained as the ratio of maximum revenue with specefic benefits (the optimal solution to model (1)) to the observed revenue at DMU_O , as follows:

$$RevenueEfficiency_o = \frac{\sum_{r=1}^{s} R_{ro} y_r^{o^*}}{\sum_{r=1}^{s} R_{ro} y_{ro}} \quad (2)$$

3 Revenue Efficiency With Interval Data

Assume that,We have n DMUs with interval inputs and outputs, and Rrj is benefits output r of DM Uj which Rrj ∈ [Rmin, Rmax]. In order to obtain a measure of revenue efficiency which is an interval for this kind of data the following are suggested:

$$\begin{array}{lll} \max & \sum_{r=1}^{s} R_{ro}^{max} y_r^o & \\ s.t. & \sum_{j=1, j\neq o}^{n} \lambda_j x_{ij}^l + \lambda_o x_{io}^u \leq x_{io}^u & i=1,...,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj}^u = y_r^o & r=1,...,s, \\ & \lambda_j \geq 0 & j=1,...,n, \\ & y_r^o \geq 0 & r=1,...,s. \end{array} \quad (3)$$

$$\begin{array}{lll} \max & \sum_{r=1}^{s} R_{ro}^{min} y_r^o & \\ s.t. & \sum_{j=1,j\neq o}^{n} \lambda_j x_{ij}^u + \lambda_o x_{io}^l \leq x_{io}^l & i=1,...,m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj}^l = y_r^o & r=1,...,s, \\ & \lambda_j \geq 0 & j=1,...,n, \\ & y_r^o \geq 0 & r=1,...,s. \end{array} \quad (4)$$

suppose $yro*$ and $yo*$, are the optimal solutions for (3),(4)respectively. The following revenues for *DMUo* may be calculated as follows:

$$\bar{R} = \frac{\sum_{r=1}^{s} R_{ro}^{max} \bar{y}_r^{o^*}}{\sum_{r=1}^{s} R_{ro}^{min} y_{ro}} \quad (5)$$

$$\underline{R} = \frac{\sum_{r=1}^{s} R_{ro}^{min} \underline{y}_r^{o^*}}{\sum_{r=1}^{s} R_{ro}^{max} y_{ro}} \quad (6)$$

Theorem 1: Any $R \in [\underline{R}, \bar{R}]$,can be treated as revenue efficiency for DMU_o.

4 Malmquist productivity index

Malmquist productivity Index is defined with assimilation efficiency changes of each unit and technology changes. MPI can be calculated via several functions, such as distance function:

$$D(X_o, Y_o) = \inf\{\theta : (\theta X_o, Y_o) \in PPS\} \quad (7)$$

This equation shows in special conditions,only the efficiency frontier change at time t+1 related to t;that could not be a suitable criterion to calculate the technology change. This distance function does not define the ineffi- ciency values.The efficiency frontier will be specified for each DMU with DEA.Production function is hypothesized instant t and t+1. Calculation of the MPI needs to four linear programming problems as below:

$o \in Q = \{1, ..., n\}$

$$D_o^t(X_o^t,Y_o^t) = \begin{array}{lll} \min & \theta & \\ s.t. & \sum_{j=1}^{n}\lambda_j x_{ij}^t \leq \theta x_{io}^t & i=1,...,m, \\ & \sum_{j=1}^{n}\lambda_j y_{rj}^t \geq y_{ro}^t & r=1,...,s, \\ & \lambda_j \geq 0 & j=1,...,n. \end{array} \quad (8)$$

In the model above ,*xt io*is the *ith* input and *yro t* is the *rth* output of *DMUo* at time t,the value of efficiency(*Dot*(*Xot, Yot*) = *θ*∗) shows that how much can be decrease inputs of *DMUo* to production that output.
If in the previous model,we put t+1 instead t ,then CCR problem (6) is calculated at time t+1 and is equal *Dt*+1(*Xot*+1, *Yot*+1) and is the technical efficiency for *DMUo* at time t+1.The value of *Dt*(*Xot*+1, *Yot*+1) for *DMUo*,is the distance of *DMUo* at t+1 with the frontier of time t,calculated by below problem:

$$D^t(X_o^{t+1},Y_o^{t+1}) = \begin{array}{lll} \min & \theta & \\ s.t. & \sum_{j=1}^{n}\lambda_j x_{ij}^t \leq \theta x_{io}^{t+1} & i=1,...,m, \\ & \sum_{j=1}^{n}\lambda_j y_{rj}^t \geq y_{ro}^{t+1} & r=1,...,s, \\ & \lambda_j \geq 0 & j=1,...,n. \end{array} \quad (9)$$

The same models *Dt+1*(*Xot, Yot*) and *Dt+1*(*Xot+1, Yot+1*) are calculated. Fare hypotheses *Dot+1*(*Xot+1, Yot+1*) and *Dot*(*Xot, Yot*) must be equal to 1 to be efficient. Therefore he defined relative efficiency change as:

$$TEC_o = \frac{D_o^{t+1}(X_o^{t+1},Y_o^{t+1})}{D_o^t(X_o^t,Y_o^t)} \quad (10)$$

He described one geometric compotation to determine technology change between t and t+1:

$$FS_o = \sqrt{\frac{D_o^t(X_o^{t+1},Y_o^{t+1})}{D_o^{t+1}(X_o^{t+1},Y_o^{t+1})}\cdot\frac{D_o^t(X_o^t,Y_o^t)}{D_o^{t+1}(X_o^t,Y_o^t)}} \quad (11)$$

MPI will be evaluated from multiplication efficiency change and technology change for each input oriented *DMUo* at time t and t+1:

$$M_o = \frac{D_o^{t+1}(X_o^{t+1},Y_o^{t+1})}{D_o^t(X_o^t,Y_o^t)} \cdot \sqrt{\frac{D_o^t(X_o^{t+1},Y_o^{t+1})}{D_o^{t+1}(X_o^{t+1},Y_o^{t+1})} \cdot \frac{D_o^t(X_o^t,Y_o^t)}{D_o^{t+1}(X_o^t,Y_o^t)}} \quad (12)$$

The simple form of relation (10)is:

$$M_o = \sqrt{\frac{D_o^t(X_o^{t+1},Y_o^{t+1})}{D_o^t(X_o^t,Y_o^t)} \cdot \frac{D_o^{t+1}(X_o^{t+1},Y_o^{t+1})}{D_o^{t+1}(X_o^t,Y_o^t)}} \quad (13)$$

So,we have three conditions:

1.$M_o > 1$,increase productivity and observe progress.

2.$M_o < 1$,decrease productivity and observe regress.

3.$M_o = 1$,no change in productivity at time t+1 in comparison to t.

5 Revenue Malmquist productivity index

5.1 Background

Consider that in time period t, producers are using inputs $x^t \in R^m$, to produce outputs $y^t \in R^s$. Define now the production technology of period t in terms of the output offer set, which is:

$L^t(X^t) = \{Y^t : Y^t$ can produce $X^t\}$ (14)

$L^t(X^t)$ contains all output vectors,which can be produce from X^t. Assume that $L^t(X^t)$ is non-empty, closed, convex, bounded and satisfies strong dis- posability of inputs and outputs.

$$\text{Isoq}L^t(X^t) = \{Y^t : Y^t \in L^t(X^t), \lambda Y^t \notin L^t(X^t) for \lambda > 1\} \quad (15)$$

$L^t(X^t)$ is bounded from below by the output isoquant, that is:

Isoq$L^t(X^t)$defines a boundary (frontier) to the output offer set in the sense that any radial expansion of output vectors that lie on the frontier is not possible within $L^t(X^t)$.

$$D_o^t(X^t,Y^t) = \sup\{\varphi : (\varphi Y^t) \in L^t(X^t), \varphi > 0\} \quad (16)$$

Alternatively, with reference to the output offer set, define the technology of production in terms of the output distance function as:

where the subscript o denotes output orientation.

$D^t_o(X^t, Y^t)$ *in (16) is the highest possible demand,which can be multiplied with* Y^t *remains in* $L^t(X^t)$. *If* $D^t_o(x^t, y^t) > 1$,*then* $Y^t \in intL^t(X^t)$ *and if* $D^t_o(X^t, Y^t) = 1$ *then* $Y^t \in IsoqL^t(X^t)$.$D^t_o(x^t, Y^t)$ *is similar with the definition of technical efficiency in output oriented:*

$$TE^t(x^t, y^t) = \max\{\phi : \phi Y^t \in L^t(X^t), \phi > 0\} \qquad (17)$$

When output prices,$W^t \in R^S$, are available, the revenue function is de- fined:

$$R^t(X^t, W^t) = \max\{W^t Y^t : Y^t \in L^t(X^t), W^t > 0\} \qquad (18)$$

$R^t(X^t, W^t)$ is the maximum revenue of producing output Y^t . Frontier of this set is:

$$IsoqR^t(X^t, W^t) = \{Y^t : W^t Y^t = R^t(X^t, W^t)\} \qquad (19)$$

This boundary contains the output vectors that can have the maximum revenue with their price W^t.Therfore technical efficiency and distance function have the same definition.

$$OE^t_o(X^t, Y^t, W^t) = \frac{W^t Y^t}{R^t(X^t, W^t)} \qquad (20)$$

Overall(or revenue)efficiency defines:

because technical efficiency is less than overall efficiency for each unit ,then:

$$TE^t_o(X^t, Y^t) \leq OE^t_o(X^t, Y^t, W^t) \qquad (21)$$

According to technical efficiency is the same as distance function:

$$D^t_o(X^t, Y^t) \leq \frac{W^t Y^t}{R^t(X^t, W^t)} \qquad (22)$$

Allocative efficiency defines as follows:

$$AE^t_o(X^t, Y^t, W^t) = \frac{W^t Y^t}{D^t_o(X^t, Y^t) R^t(X^t, W^t)} \qquad (23)$$

Assume two time periods t and t+1 respectively and define in each one of them technology and production. Taking time period t as the reference period, the output oriented Malmquist index (OM) is:

$$OM^t = \frac{D_o^t(X^t,Y^t)}{D_o^t(X^{t+1},Y^{t+1})} \quad (24)$$

In a similar fashion, with reference to period t+1, one may define the fol- lowing index:

$$OM^{t+1} = \frac{D_o^{t+1}(X^t,Y^t)}{D_o^{t+1}(X^{t+1},Y^{t+1})} \quad (25)$$

Malmquist productivity index(OM)is a geometric component of (24) and (25):

$$OM = \sqrt{\frac{D_o^t(X^t,Y^t)}{D_o^t(X^{t+1},Y^{t+1})}\frac{D_o^{t+1}(X^t,Y^t)}{D_o^{t+1}(X^{t+1},Y^{t+1})}} \quad (26)$$

OM is Malmquist productivity index and has inverse relative with M_O defi- nition previous section.Three conditions are existed:

1.OM > 1, observe progress.

2.OM < 1, observe regress.

3.OM $= 1$,do not observe any change in productivity .

5.2 Revenue Malmquist productivity index

By using allocative and technical efficiency, output's price productivity changes are determined.To take care of (22) to (24) revenue Malmquist productivity index(RM) is calculated as:

$$RM^t = [\frac{W^tY^t/R^t(X^t,W^t)}{W^tY^{t+1}/R^t(X^{t+1},W^t)}] \quad (27)$$

$$RM^{t+1} = [\frac{W^{t+1}Y^t/R^{t+1}(X^t,W^t)}{W^{t+1}Y^{t+1}/R^{t+1}(X^{t+1},W^{t+1})}] \quad (28)$$

$$RM = [\frac{W^tY^t/R^t(X^t,W^t)}{W^tY^{t+1}/R^t(X^{t+1},W^t)}\frac{W^{t+1}Y^t/R^{t+1}(X^t,W^{t+1})}{W^{t+1}Y^{t+1}/R^{t+1}(X^{t+1},W^{t+1})}]^{\frac{1}{2}} \quad (29)$$

where $W^tY^t = \sum_{n=1}^{N} w_n^t y_n^t$ denotes the nth output and Rt(Xt, W t) is the maximum revenue which is calculated in (18).OM index discusses outputs quantity and RM index discusses outputs revenue. $W^tY^t/R^t(X^t,W^t)$ is the revenue efficiency to product Y^t at time period t with output price W^t.This fraction compares revenue of output Y^t and the maximum product revenue and its value is not less than 1. Value 1 means this output has the maximum revenue and value greater than 1 means this output can be decreased.This fraction is exactly overall efficiency as defined in (20).Therfore with using overall efficiency and OM and RM can be provided.RM is the value that shows which output's part can increase arrive revenue frontier.(Using constant return to scale is not necessary,but it is only for clear and distinction bench mark of revenue frontier).

Similarity OM index ,for RM can say:

1.RM $>$ 1,observe progress and decrease productivity.

2.RM $>$ 1,observe regress and increase productivity.

3.RM $>$ 1,no change in productivity.

6 Malmquist Productivity Index with Interval Data

In the section (4),we defined Malmquist productivity index.With interval data the Malmquist productivity index may be defined as follows.Consider four models in the following form,so the M.P.I is an interval which is defined.

$$\begin{array}{llll} \underline{\theta}_t^t = & \min & \theta & \\ & s.t. & \sum_{j=1}^{n} \lambda_j x_{ij}^{tl} \leq \theta x_{io}^{tu} & i = 1, ..., m, \\ & & \sum_{j=1}^{n} \lambda_j y_{rj}^{tu} \geq y_{ro}^{tl} & r = 1, ..., s, \\ & & \lambda_j \geq 0 & j = 1, ..., n. \end{array} \tag{30}$$

$$\begin{array}{llll} \overline{\theta}_t^{t+1} = & \min & \theta & \\ & s.t. & \sum_{j=1}^{n} \lambda_j x_{ij}^{tu} \leq \theta x_{io}^{t+1l} & i = 1, ..., m, \\ & & \sum_{j=1}^{n} \lambda_j y_{rj}^{tl} \geq y_{ro}^{t+1u} & r = 1, ..., s, \\ & & \lambda_j \geq 0 & j = 1, ..., n. \end{array} \tag{31}$$

$$\begin{array}{llll} \overline{\theta}_{t+1}^{t+1} = & \min & \theta & \\ & s.t. & \sum_{j=1}^{n} \lambda_j x_{ij}^{t+1u} \leq \theta x_{io}^{t+1l} & i = 1, ..., m, \\ & & \sum_{j=1}^{n} \lambda_j y_{rj}^{t+1l} \geq y_{ro}^{t+1u} & r = 1, ..., s, \\ & & \lambda_j \geq 0 & j = 1, ..., n. \end{array} \tag{32}$$

$$\underline{\theta}^{t}_{t+1} = \begin{array}{lll} \min & \theta & \\ s.t. & \sum_{j=1}^{n} \lambda_j x_{ij}^{t+1l} \leq \theta x_{io}^{tu} & i=1,...,m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj}^{t+1u} \geq y_{ro}^{tl} & r=1,...,s, \\ & \lambda_j \geq 0 & j=1,...,n. \end{array} \quad (33)$$

$$\overline{M} = [\frac{\overline{\theta}^{t+1}_{t}}{\underline{\theta}^{t}_{t}} \times \frac{\overline{\theta}^{t+1}_{t+1}}{\underline{\theta}^{t}_{t+1}}]^{\frac{1}{2}} \quad (34)$$

Therefore from the above models, we have:

The values *θtt, θt t+1, θt t+1* and *θt t+1 +1* can be evaluated, too. So, we obtain:

$$\underline{M} = [\frac{\underline{\theta}^{t+1}_{t}}{\overline{\theta}^{t}_{t}} \times \frac{\underline{\theta}^{t+1}_{t+1}}{\overline{\theta}^{t}_{t+1}}]^{\frac{1}{2}} \quad (35)$$

Theorem 2: By using of previous models $M \in [\underline{M}, \overline{M}]$, that M is the Malmquist productivity index.

proof: Proof is evident.

7 Revenue Malmquist with Interval Data

In the above mentioned model,it has been assumed that R_{rj} were fixed.If R_{rj} lies in the interval,the following models may be used to evaluate the M.P.I.

$$\begin{array}{lll} \max & \sum_{r=1}^{s} R_{ro}^{max} y_r^o & \quad (36) \\ s.t. & \sum_{j=1, j\neq o}^{n} \lambda_j x_{ij}^{t+1l} + \lambda_o x_{io}^{t+1u} \leq x_{io}^{t+1u} & i=1,...,m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj}^{t+1u} = y_r^o & r=1,...,s, \\ & \lambda_j \geq 0 & j=1,...,n, \\ & y_r^o \geq 0 & r=1,...,s. \end{array}$$

$$\begin{array}{lll} \max & \sum_{r=1}^{s} R_{ro}^{max} y_r^o & \quad (37) \\ s.t. & \sum_{j=1}^{n} \lambda_j x_{ij}^{tl} \leq x_{io}^{t+1u} & i=1,...,m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj}^{tu} = y_r^o & r=1,...,s, \\ & \lambda_j \geq 0 & j=1,...,n, \\ & y_r^o \geq 0 & r=1,...,s. \end{array}$$

$$\begin{array}{lll} \max & \sum_{r=1}^{s} R_{ro}^{min} y_r^o & \\ s.t. & \sum_{j=1, j\neq o}^{n} \lambda_j x_{ij}^{tu} + \lambda_o x_{io}^{tl} \leq x_{io}^{tl} & i=1,...,m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj}^{tl} = y_r^o & r=1,...,s, \\ & \lambda_j \geq 0 & j=1,...,n, \\ & y_r^o \geq 0 & r=1,...,s. \end{array} \quad (38)$$

$$\begin{array}{lll} \max & \sum_{r=1}^{s} R_{ro}^{min} y_r^o & \\ s.t. & \sum_{j=1}^{n} \lambda_j x_{ij}^{t+1u} \leq x_{io}^{tl} & i=1,...,m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj}^{t+1l} = y_r^o & r=1,...,s, \\ & \lambda_j \geq 0 & j=1,...,n, \\ & y_r^o \geq 0 & r=1,...,s. \end{array} \quad (39)$$

The following revenues from models (36),(37),(38), and(39) may be calculated as follows, respectively:

$$\bar{R}_{t+1}^{t+1} = \frac{\sum_{r=1}^{s} R_{ro}^{max} y_r^{o*}}{\sum_{r=1}^{s} R_{ro}^{min} y_{ro}}, \qquad \bar{R}_{t}^{t+1} = \frac{\sum_{r=1}^{s} R_{ro}^{max} y_r^{o*}}{\sum_{r=1}^{s} R_{ro}^{min} y_{ro}},$$

$$\underline{R}_{t}^{t} = \frac{\sum_{r=1}^{s} R_{ro}^{min} \underline{y}_r^{o*}}{\sum_{r=1}^{s} R_{ro}^{max} y_{ro}}, \qquad \underline{R}_{t+1}^{t} = \frac{\sum_{r=1}^{s} R_{ro}^{min} \underline{y}_r^{o*}}{\sum_{r=1}^{s} R_{ro}^{max} y_{ro}}.$$

Therefore, we have from the above models M_R as follows:

$$\overline{M}_R = [\frac{\overline{R}_t^{t+1}}{\overline{R}_t^t} \times \frac{\overline{R}_{t+1}^{t+1}}{\overline{R}_{t+1}^t}]^{\frac{1}{2}} \quad (40)$$

In a similar fashion, we can obtain $\underline{M}_R$ with using previous models as follows:

$$\underline{M}_R = [\frac{\underline{R}_t^{t+1}}{\overline{R}_t^t} \times \frac{\underline{R}_{t+1}^{t+1}}{\overline{R}_{t+1}^t}]^{\frac{1}{2}} \quad (41)$$

Theorem 3): Any $M_R \in [\underline{M}_R, \overline{M}_R]$, that M_R is Revenue Malmquist pro- ductivity index.

8 Conclusion

This paper explored the assessment of productivity index with interval data and using revenue efficiency. The applicability of the methods in the bank branch seems to be very useful and consistent ith common sense of the board of directory and managerial port of view.

References

[1] Berg,S.A.,Forsund,F.R.,Jansen,E.S., *Malmquist indices of produc- tivity growth during the deregulation of Norwegian banking,1980-89*. Scandinavian Journal of economics(supplement), 1992, 211-228.

[2] Despotis,D.K., Smirlis,Y.G., *Data envelopment analysis with imprecise data*. European Journal of Operational Research. 2002, 140, 24-36.

[3] Malmquist,S., Index numbers and indifference surfaces. Trabajos de Estadistica. 1953, 4,209-242.

[4] Robert M.Therall, *Measures in DEA with an Application to the Malmquist Index*. Journal of Productivity Analysis, 2000, 13,125-137. Thanassoulis,E.,Maniadakis,N., *A cost Malmquist productivity index*. European Journal of Operational Research. 2004, 154,396-409.

[5] Charnes; W. W. Cooper; E. Rhodes; *Measuring the efficiency of decision making units*. European Journal of Operational Research 1978, 2, 429-444, 10.1016/0377-2217(78)90138-8.

[6] Tone, K., Toloo, M., & Izadikhah, M, *A modified slacks-based measure of efficiency in data envelopment analysis*. European Journal of Operational Research, 2020, 287(2).

[7] Fare,R.,Grosskopf,Norris M.,Zhang,Z., Productivity growth,technical progress,and efficiency change in industrialized counties.American Economic Review.1994, 84,66-83.

[8] Ghiyasi, M., Inverse DEA based on cost and revenue efficiency, Computers & Industrial Engineering, 2017, 114, 258-263

[9] Nguyen, M-A., Yu, M-M., Lirn, T-C., Revenue efficiency across airline business models: A bootstrap non-convex meta-frontier approach, Transport Policy, 2022, 117, 108-117

[10] Fatemeh Sadat Seyed Esmaeili, Pejman Peykani, Mohsen Rostamy-Malkhalifeh, Farhad Lotfi. Measuring Productivity Changes of Hospitals in Tehran: The Fuzzy Malmquist Productivity Index. International Journal of Hospital Research, Iran University of Medical Sciences, 2018. ⟨hal-02401184v2⟩

[11] Mombini, E., Rostamy-Malkhalifeh, M., Saraj, M., Zahraei, M., & Tayebi Khorami, R. Global malmquist index for measuring the economic productivity changes. Measurement and Control, 2020, 53(7–8), 1278–1285. https://doi.org/10.1177/0020294020923096

[12] Poordavoodi, A., Moazami Goudarzi, M.R., Haj Seyyed Javadi, H., Rahmani, A.M., Izadikhah, M., Toward a More Accurate Web Service Selection Using Modified Interval DEA Models with Undesirable Outputs, Computer Modeling in Engineering & Sciences, 2020, 123(2), P. 525-570, Doi: 10.32604/cmes.2020.08854

[13] Hamid Sharafi, Mohsen Rostamy-Malkhalifeh, Alireza Salehi, Mohammad Izadikhah, Efficiency Evaluation and Ranking DMUs in the Presence of Interval Data with Stochastic Bounds, International Journal of Data Envelopment Analysis, 2015, 3(1), 617-626

[14] Izadikhah, M., Roostaee, R., & Emrouznejad, A., Fuzzy Data Envelopment Analysis with Ordinal and Interval Data. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 2021, *29*(3), 385-410.

https://doi.org/10.1142/s0218488521500173

[15] Mombini, E., Rostamy-Malkhalifeh, M., Saraj, M. The sustainability radius of the cost efficiency in Interval Data Envelopment Analysis: A case study from Tehran Stocks. *Advances in Mathematical Finance and Applications*, 2022; 7(2): 279-291. doi: 10.22034/amfa.2021.1917327.1528