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Finding a Better Time Estimation of a Trajectory

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ABSTRACT

The construction of a control is usually based on complete information on the coordinates of an object. In the absence of this information, the control can be constructed by solving two consecutive sub problems, namely, detecting a desired object and capturing the target object with complete information on its coordinates. In this paper we suppose that start-point for motions is variable and using the diagrams constructed in the plane of geometric objects of the operation zone of the manipulator, the problem of selecting a control under which a guaranteed search with a subsequent capture of a target object is carried out in a minimum time is solved.

1 Introduction

In control problems for an adaptive manipulator robot, the construction of a control is usually based on complete information on the coordinates of this object. In the absence of this information, the control can be constructed by solving two consecutive sub problems, detecting a desired object and capturing the target object with complete information on its coordinates. In this paper we suppose that start-point for motions is variable, namely in contrast to [1] we suppose that diameter of first circle of detection is variable and then we find optimum start-point for motions. So with generalization of [1] and using the diagrams constructed in the plane of geometric objects of the operation zone of the manipulator, the problem of selecting a control under which a guaranteed search with a subsequent capture of a target object is carried out in a minimum time is solved. Finally in end of paper we have solved an example and graph of its results are given.

2 Main Results

Suppose that we have a three-link manipulator operating in the Cartesian coordinate system whose links move relative to each other in three mutually perpendicular directions. The motion of this manipulator is controlled by electric drives consisting of separate excitation motors and reducers. We introduce the following assumptions [5]: (a) the reducers of the drives have large transmission ratios and (b) the electromagnetic time constants and characteristic times of putting the motors in the stationary rotation mode are much less than the time of working operation of the manipulator. Under these assumptions, motion control of the manipulator gripper $X(x_1, x_2, x_3)$ in each coordinate x_i is kinematics, and the planner and vertical motion of the manipulator gripper are described by the first, second and third equations of the system

$$\dot{x}_i = v_i; |v_i| < V_i; i = 1, 2, 3 \tag{2.1}$$

respectively,

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such that V_i is the maximum velocity of ith link. In accordance with the design of the manipulator in question, the operation zone of the gripper of the manipulator is a parallelepiped

$$D = \{(x_1, x_2, x_3); 0 < x_i \leq d; i = 1, 2, 3\} \tag{2.2}$$

and d is a constant. Assume that, in a certain rectangular bed

$$\hat{D}(a_1, a_2) = \{(x_1, x_2, 0); 0 < x_i \leq a_i; a_i \leq d; i = 1, 2\}$$

of the operation zone of manipulator (2), we have a point immobile object Y whose coordinates $y(t^0) = y^0$ are unknown.
We suppose that, in the course of motion, complete information on relations (1) and the parameters of the operation zone of manipulator (2), except the initial position $y^0 \in \hat{D}(a_1, a_2)$ of the object Y, is available to the controlled side. However, on the gripper X of the manipulator a sensor is mounted; with its use, on the rectangular bed $\hat{D}$, the mobile and varying circular information sensitivity domain $G(x(t))$, which allows one to gain more precise information on the coordinates of the location of the object Y in the case of its getting into this domain, $y \in G(x(t))$, is formed. We describe the domain $G(x)$ for any position $x \in D \subset R^3$ as follows:

$$G(x(t)) = G(\hat{x}(t), x_3(t), C) = \{\hat{\xi} \in R^2; |\hat{\xi} - \hat{x}| \leq \frac{h}{2} = Cx_3\} \tag{2.3}$$

such that $C = \tan(\alpha), |\alpha| < \frac{\pi}{2}, \hat{x} = (x_1, x_2) \in \hat{D}(a_1, a_2), 0 < x_3 < d, 0 < d$. Domain $G(x(t))$ in (3) represents a circle with the center at the point $\hat{x}(t) \in \hat{D}$ and the diameter $h = 2Cx_3, C = \tan(\alpha)$, where x_3 is the distance between the manipulator gripper and the rectangular bed $\hat{D}$ while C, a_1 and a_2 are numbers for which $h_{max} = 2Cmax(x_3); 0 < x_3 \leq d$ and the inequality $h_{max} < a_i \leq d, i = 1, 2$ is valid. In this paper we consider parameter C is constant.
Assume that the control of the manipulator gripper starts at the instant $t = t_0$ Suppose the control of the manipulator gripper starts at the instant $x^0 = (\hat{x}^0, x_3^0) \in D$ and ends at the instant $t = T$ when the following condition is satisfied

$$x(T) = x^1 \in D, x^1 = (\hat{x}^1, x_3^1), \hat{x}^1 = y, x_3^1 = 0 \tag{2.4}$$

The objective of control of the gripper is to provide that condition (4) becomes valid in the smallest possible minimum time T.
We decompose the controlled motion of the gripper X into two stages, namely, the search and the subsequent steering to the desired point object Y. In this connection, as admissible functions, we take control functions of the form [4]

- If $t_0 \leq t \leq t^*$ then u=$u^0(x^0, t)$
- If $t^* \leq t \leq T$ then u=$u^1(x^*, t^*, x^1, t)$

(2.5)

Here x^0, x^*, x^1, t^* and T are parameters such that x^0, x^*, x^1 are members of D and $t_0 \leq t^* \leq T$. The stage of search and the stage of steering to the desired object correspond the controls u^0 and u^1, respectively. System (1) under the control of form (5) moves as follows. On the interval $[t_0, t^*]$ control of search u^0 is implemented, and then, after the instant of detection t^* (when the vector x^1 becomes known), on the interval $[t^*, T]$ the control u^1 , which steers the system from the detected point x^* to the point x^1 , is used. The question of the existence of the final instant $t^* \geq t_0$ is the main point in the search problem, and its solution depends on the method for controlling a gripper. If the projection of the trajectory of system (1) (under the control u^0) on the rectangular bed $\hat{D}$ is covering [3], then the time t^* at which the desired object is detected at the endpoint of this trajectory is called a guaranteed search time. The total time of search and steering involving the elapsed time to the detection and the remaining time $t^1 = T - t^*$ of driving to x^1 depends both on the control and the starting position of the gripper and on the geometric parameters of the operation zone of manipulation

$$T(D, x^0, u) = t^*(D, x^0, u^0) + t^1(x^*(x^0, u^0), u^1), D = D(a_1, a_2, a_3) \tag{2.6}$$

Assume that the first two coordinates $(x_1^0, x_2^0) = \hat{x}^0, 0 \leq x_1^0, x_2^0 \leq a_1, a_2$ of the initial position $x^0 = (\hat{x}^0, x_3^0) \in D$ and the parameter C are given. Then, the time of guaranteed search and steering (6) corresponds to each value of vertical coordinate $x_3^0, 0 \leq x_3^o \leq a_3$ and each admissible control $u = \{u^0, u^1\}$. Consider the following problem. It is required to find the minimum total time of guaranteed search and steering $T^*(a_1, a_2)$, the control $u^* = \{u^{0^*}, u^{1^*}\}$ and the initial diameter of the circle of detection $h_0^* = 2C(x_3^0)^*$ (or the initial coordinate $(x_3^0)^*$) that provide a minimum to functional

$$T^*(a_1, a_2) = \min_{0<h_0<h_{max}} \min_{u=\{u^0, u^1\}} T(a_1, a_2, x_3^0, u) \tag{2.7}$$

First, we describe two similar methods for controlling the motion of a gripper and then specify the conditions imposed on the parameters involved, under which the best (in terms of (7)) control is found. Consider two space broken lines L_1 and L_2 coming from the starting point $x_i^0 = (0, 0, x_3^{0(i)}), 0 < x_3^{0(i)} \leq a_3, i = 1, 2$ who's projections $\acute{L}_1$ and $\acute{L}_2$ on the rectangular bottom $\hat{D}$ in the plane ox_1x_2. Moving along the broken line L_i , the projection of the manipulator gripper scans the rectangle $\hat{D}$ along the broken line $\acute{L}_i$. Assume that the lengths of the sections of the broken lines that lie in the lateral sides of the rectangle decrease according to the arithmetic progression law with a constant $q_i, 0 < q_i < h_0^{(i)}$, where $h_0^{(i)} = 2Cx_3^{0(i)}, 0 < h_0^{(i)} \leq \min\{a_1, a_2\}$ is the initial diameter of the circle of detection. Then, the sum of diameters of the circles of detection on the lateral sides of the rectangle is expressed by the formula:

$$a_2 = (2h_0^{(1)} - (n_1 - 1)q_1)n_1/2; a_1 = (2h_0^{(2)} - (n_2 - 1)q_2)n_2/2 \tag{2.8}$$

Where n_i is an integer that specifies the number of described sections or steps of scanning. In contrast to [2], where the case of "slow" search is considered, we investigate the case of "fast" search [3], when the time $\frac{h_0^{(1)}}{2V_2}(\frac{h_0^{(2)}}{2V_1})$ of planar movement of the center of the circle of detection with a maximum speed $v = V_2(v = V_1)$ over the distance equal to the radius of the circle $\frac{h_0^1}{2}(\frac{h_0^1}{2})$is less than the time of the vertical movement $\frac{x_3^{0(1)}}{V_3}(\frac{x_3^{0(2)}}{V_3})$ where $x_3^{0(i)} = \frac{C^{-1}h_0^i}{2}$. Then, the controls under which the gripper X moves along the trajectories $L_i(i = 1, 2)$ are given as follows:

- If $t_0 \leq t \leq t_1$ then $u_{L_1} = \{v_1 = 0, v_2 = V_2, v_3 = 0\}$
- If $t_{4r+1} \leq t \leq t_{4r+2}; r = 0, 1, 2, ..., [\frac{n_1+1}{2}]$ then $u_{L_1} = \{v_1 = V_1, v_2 = 0, v_3 = 0\}$
- If $t_{4r+3} \leq t \leq t_{4r+4}; r = 0, 1, 2, ..., [\frac{n_1+1}{2}]$ then $u_{L_1} = \{v_1 = -V_1, v_2 = 0, v_3 = 0\}$
- If $t_{2r+2} \leq t \leq t_{2r+3}, r = 0, 1, 2, ..., n_1 - 1$ then $u_{L_1} = \{v_2 = (\frac{2h_0^{(1)} - (2r+1)q_1}{q_1})(CV_3), v_1 = 0, v_3 = V_3\}$
- In steering step $u_{L_1} = \{v_1 = 0, v_2 = CV_3, v_3 = V_3\}$

(2.9)

- If $t_0 \leq t \leq t_1$ then $u_{L_2} = \{v_1 = V_1, v_2 = 0, v_3 = 0\}$
- If $t_{4r+1} \leq t \leq t_{4r+2}; r = 0, 1, 2, ..., [\frac{n_2+1}{2}]$ then $u_{L_2} = \{v_1 = 0, v_2 = V_2, v_3 = 0\}$
- If $t_{4r+3} \leq t \leq t_{4r+4}; r = 0, 1, 2, ..., [\frac{n_2+1}{2}]$ then $u_{L_2} = \{v_1 = 0, v_2 = -V_2, v_3 = 0\}$
- If $t_{2r+2} \leq t \leq t_{2r+3}, r = 0, 1, 2, ..., n_2 - 1$ then $u_{L_2} = \{v_1 = (\frac{2h_0^{(2)} - (2r+1)q_2}{q_2})(CV_3), v_2 = 0, v_3 = V_3\}$
- In steering step $u_{L_2} = \{v_1 = CV_3, v_2 = 0, v_3 = V_3\}$

(2.10)

In (9) and (10), the instants of time $t_j^{(i)}, t_{j+1}^{(i)}, t_{j+2}^{(i)}, i = 1, 2$ are found from motion equation (1) by substituting into them controls (9) and (10) and the subsequent integration under boundary conditions that represent the coordinates of the vertices $L_i^j, L_i^{j+1}, L_i^{j+2}, (i = 1, 2)$.
The gripper X under controls (9) and (10) moves as follows. On the leg $L_1^{(0)} L_1^{(1)} (L_2^{(0)} L_2^{(1)})$ the planer motion of the gripper X is carried out with a maximum speed $V_2(V_1)$ and on the leg $L_1^{(1)} L_1^{(2)} (L_2^{(1)} L_2^{(2)})$ the planer motion of the gripper X is carried out with a maximum speed $V_1(V_2)$ and a fixed diameter of detection $h_0^{(1)} = 2Cx_3^{0(1)} (h_0^{(2)} = 2Cx_3^{0(2)})$ i.e. for $v_3 = 0$. Starting from the vertex $L_1^{(2)}(a_1, \frac{h_0^{(1)}}{2}, x_3^{0(1)})(L_2^{(2)}(\frac{h_0^{(2)}}{2}, a_2, x_3^{0(2)}))$ the object X moves over the coordinates $x_2(x_1$ and x_3 simultaneously. At time $t = \frac{q_i}{2CV_3}, (x_3^{0(i)} - x_3^{1(i)} = \frac{q_i}{2C}), i = 1, 2$ the gripper X moves from the position $L_1^{(2)}(L_1^{(2)})$ to the position $L_1^{(3)}(a_1, h_0^{(1)} + \frac{h_0^{(1)} - q_1}{2}, x_3^{1(1)}) L_2^{(3)}(h_0^{(2)} + \frac{h_0^{(2)} - q_2}{2}, a_2, x_3^{1(2)})$ while the center of the circle of detection moves from the point $(a_1, \frac{h_0^{(1)}}{2})((\frac{h_0^{(2)}}{2}), a_2)$ to the position $(a_1, h_0^{(1)} + \frac{(h_0^{(1)} - q_1)}{2})((h_0^{(2)} + \frac{(h_0^{(2)} - q_2)}{2}, a_2))$ at the speed $v_2 = \frac{(2h_0^{(1)} - q_1)CV_3}{q_1}(v_1 = \frac{(2h_0^{(2)} - q_2)CV_3}{q_2})$. In accordance with the described method, the motion along the final leg of the trajectory allows steering the gripper to the endpoint of the trajectory $L_i, i = 1, 2$. The projections $\acute{L}_i, i = 1, 2$ of the trajectories $L_i, i = 1, 2$ cover the rectangle bottom $\hat{D}$; therefore, the motion of the gripper along the trajectories $L_i, i = 1, 2$ under the above described controls allows detecting a desired object located in $\hat{D}$ in a finite time [3]. Investigations made analogously to [3] have demonstrated that one of the trajectories $\acute{L}_i, i = 1, 2$ depending on the geometric parameters of $\hat{D}$, has a minimum length among the set of trajectories that cover the rectangular domain with a given accuracy, which are constructed on the basis of the two initial trajectories $\acute{L}_i, i = 1, 2$. The total time of controlling the motion of the gripper X along the trajectories $L_i, i = 1, 2$ is computed as follows:

$$T_{L_1}(a_1, a_2, h_0^{(1)}, q_1) = t_{L_1}^* + t_{L_1}^1 \quad (2.11)$$

such that:

- $t_{L_1}^*(a_1, a_2, h_0^{(1)}, q_1) = \frac{h_0^{(1)}}{2V_2} + \frac{n_1 a_1}{V_1} + \frac{(n_1 - 1)q_1}{2CV_3}$
- $t_{L_1}^1(h_0^{(1)}, q_1) = \frac{h_n^{(1)}}{2CV_3} = \frac{h_0^{(1)} - (n_1 - 1)q_1}{2CV_3}$
- $T_{L_1}(a_1, a_2, h_0^{(1)}, n_1) = \frac{h_0^{(1)}}{2}(\frac{V_2 + CV_3}{CV_2 V_3}) + \frac{n_1 a_1}{V_1}$

And

$$T_{L_2}(a_1, a_2, h_0^{(2)}, q_2) = t_{L_2}^* + t_{L_2}^1 \quad (2.12)$$

such that:

- $t^*_{L_2}(a_1, a_2, h_0^{(2)}, q_2) = \frac{h_0^{(2)}}{2V_1} + \frac{n_2 a_2}{V_2} + \frac{(n_2-1)q_2}{2CV_3}$
- $t^1_{L_2}(h_0^{(2)}, q_2) = \frac{h_n^{(2)}}{2CV_3} = \frac{h_0^{(2)}-(n_2-1)q_2}{2CV_3}$
- $T_{L_2}(a_1, a_2, h_0^{(2)}, n_2) = \frac{h_0^{(2)}}{2}(\frac{V_1+CV_3}{CV_1V_3}) + \frac{n_2 a_2}{V_2}$

In (11), ((12)), $t^*_{L_1}(t^*_{L_2})$ is the time of "scanning" the rectangle $\hat{D}$ in the motion of X along the trajectory of length:

$$d^*_{L_1}(a_1, a_2, h_0^{(1)}, q_1) = \frac{h_0^{(1)}}{2} + \frac{(n_1-1)(2h_0^{(1)} - (n_1-1)q_1)}{2} + n_1 a_1$$

$$(d^*_{L_2}(a_1, a_2, h_0^{(2)}, q_2) = \frac{h_0^{(2)}}{2} + \frac{(n_2-1)(2h_0^{(2)} - (n_2-1)q_2)}{2} + n_2 a_2)$$

While $t^1_{L_1}(t^1_{L_2}$ is the time of steering to the detected point along the final leg.

2.1 OPTIMAL CHOICE

With use [1] we write problem (7) in the following form:

$$T^*(a_1, a_2) = \min_{0<h_0^i<h_{max}} \min_{n_i \in Z} \{T_{L_1}(a_1, a_2, h_0^{(1)}, n_1), T_{L_2}(a_1, a_2, h_0^{(2)}, n_2)\}; h_{max} \le a_1, a_2 \le d \tag{2.13}$$

At first we consider T_{L_1}. Assume the problem is solved, so $q_1 = \frac{2Ch_0^{(1)}V_3}{V_2+CV_3}$ [1], and $q_1, h_0^{(1)}$ and $n_1 \in Z$ satisfy in following equality

$$a_2 = \frac{n_1}{2}(2h_0^{(1)} - (n_1-1)q_1) \tag{2.14}$$

Therefore the equation

$$n_1^2 q_1 - n_1(2h_0^{(1)} + q_1) + 2a_2 = 0 \tag{2.15}$$

has a real positive root (in fact integer positive root) with respect n_1 then $\triangle = (2h_0^{(1)} + q_1)^2 - 8a_2q_1 > 0$ and after simple rearrangements, we obtain

$$2(h_0^{(1)})^2 V_2^2 + 16Ch_0^{(1)}V_3(V_2+CV_3)(h_0^{(1)} - a_2) > 0 \tag{2.16}$$

In other hand for solution (15) we have:

$$n_1 = \frac{(2h_0^{(1)} + q_1) \pm \sqrt{\triangle}}{2q_1} = \frac{2h_0^{(1)}(V_2+2CV_3) \pm \sqrt{4h_0^{(1)}V_2^2 + 16CV_3h_0^{(1)}(V_2+CV_3)(h_0^{(1)} - a_2)}}{4Ch_0^{(1)}V_3}$$

Then

$$n_1 = \frac{V_2+2CV_3}{2CV_3} \pm \sqrt{(\frac{V_2+2CV_3}{2CV_3})^2 - \frac{1}{h_0^{(1)}}(\frac{a_2(1+CV_3)}{CV_3})} \tag{2.17}$$

Using (16), so $(\frac{V_2+2CV_3}{2CV_3})^2 - \frac{1}{h_0^{(1)}}(\frac{a_2(1+CV_3)}{CV_3}) > 0$, if let $\beta = \frac{V_2+2CV_3}{2CV_3}$ and $\gamma = \frac{a_2(1+CV_3)}{CV_3}$ then $\beta^2 - \frac{1}{h_0^{(1)}}\gamma > 0$ or

$$h_0^{(1)} > \frac{\gamma}{\beta^2} \tag{2.18}$$

Now using (9) and (15), therefore

$$T_{L_1}(h_0^{(1)}) = \frac{h_0^{(1)}}{2}(\frac{V_2+CV_3}{CV_2V_3}) + \frac{a_1}{V_1}(\beta \pm \sqrt{\beta^2 - \frac{\gamma}{h_0^{(1)}}}) \tag{2.19}$$

At first we continue with minus sign, namely

$$T_{L_1}(h_0^{(1)}) = \frac{h_0^{(1)}}{2}(\frac{V_2+CV_3}{CV_2V_3}) + \frac{a_1}{V_1}(\beta - \sqrt{\beta^2 - \frac{\gamma}{h_0^{(1)}}})$$

and for simplicity we use h instead $h_0^{(1)}$ and $\alpha = \frac{V_2+CV_3}{2CV_2V_3}$ namely $T_{L_1}(h) = h\alpha + \frac{a_1}{V_1}(\beta - \sqrt{\beta^2 - \frac{\gamma}{h_0^{(1)}}})$ then T_{L_1} is a function with respect h and for to find its critical points we use derivation. $\frac{dT_{L_1}}{dh} = \frac{2\alpha V_1\sqrt{\beta^2h^4-\gamma h^3}-\gamma a_1}{2V_1\sqrt{\beta^2h^4-\gamma h^3}}$ With (18), $2V_1\sqrt{\beta^2h^4-\gamma h^3} > 0$ therefore critical points of T_{L_1} are roots of $2\alpha V_1\sqrt{\beta^2h^4-\gamma h^3} - \gamma a_1 = 0$ or $\beta^2h^4 - \gamma h^3 - (\frac{\gamma a_1}{2\alpha V_1})^2 = 0$ and if let $\eta = \frac{\gamma a_1}{2\alpha V_1}$, then we have:

$$\beta^2h^4 - \gamma h^3 - \eta^2 = 0 \tag{2.20}$$

We will prove (20) has only one real positive root, only one real negative root and two complex roots. And we will see that the strict minimum value of T_{L_1} occurs in real positive root of (19).
Let $f(t) = \beta^2h^4 - \gamma h^3 - \eta^2$, $f(t)$ is continues on $(-\infty, +\infty)$ and $\lim_{h\to\infty} f(h) = +\infty, \lim_{h\to-\infty} f(h) = +\infty, f(0) = -\eta^2$, therefore with the Intermediate value Theorem there is at least one number $h_0 \in (0, +\infty)$ such that $f(h_0) = 0$ and there is at least one number $h_{00} \in (-\infty, 0)$ such that $f(h_{00}) = 0$. Now if $\hat{h}_0 \in (0, +\infty)$ such that $\hat{h}_0 \neq h_0$ and $f(\hat{h}_0) = 0$, then with Roll's Theorem there is at least one number c between h_0 and h_{00} where $\acute{f}(c) = 0$, but $\acute{f}(h) = 4\beta^2h^3 - 3\gamma h^2 = h^2(4\beta^2h - 3\gamma)$ that with (18) must $\acute{f}(h) > 0$ therefore $\hat{h}_0 = h_0$. So there is one and only one real positive root for the equation $f(h) = 0$. With the same way, there is one and only one real negative root for the equation $f(h) = 0$. But f has four root, then other roots of f are complex. We need only $h_0 \in (0, +\infty)$ such that $f(h_0) = 0$. In other hand with use Second derivative test for maximum and minimum in $(0, +\infty)$, since $\frac{d^2T_{L_1}}{dh^2} = \frac{\gamma a_1 h^2(4\beta^2h-3\gamma)}{4V_1\sqrt{(\beta^2h^4-\gamma h^3)^3}}$ then $\frac{d^2T_{L_1}}{dh^2} > 0$ (with (18)), so the strict minimum value of T_{L_1} occurs in $h_0 \in (0, +\infty)$.
Results:

- If $h > h_0$ then $f(h) > 0$ and graph of T_{L_1} is strictly increasing.
- If $0 < h < h_0$ then $f(h) < 0$ and graph of T_{L_1} is strictly decreasing.
- If $h = h_0$ then $f(h) < 0$ and $\frac{d^2T_{L_1}}{dh^2} > 0$, then the point $(h_0, T_{L_1}(h_0))$ is strict minimum T_{L_1} in $(0, +\infty)$.

Remark:If in (19) we continue with plus sign, then T_{L_1} doesn't have any real positive critical point.
Final results: The total time of controlling the motion of the gripper X along the trajectory L_1 is computed as following:

$$T_{L_1} = \frac{h_0^{(1)}}{2}(\frac{V_2+CV_3}{CV_2V_3}) + \frac{a_1}{V_1}(\frac{V_2+2CV_3}{2CV_3} - \sqrt{(\frac{V_2+2CV_3}{2CV_3})^2 - \frac{1}{h_0^{(1)}}(\frac{a_2(1+CV_3)}{CV_3}})) \tag{2.21}$$

$$n_1=\frac{V_2+2CV_3}{2CV_3}-\sqrt{(\frac{V_2+2CV_3}{2CV_3})^2-\frac{1}{h_0^{(1)}}(\frac{a_2(1+CV_3)}{CV_3})} \tag{2.22}$$

Same above calculations, the total time of controlling the motion of the gripper X along the trajectory L_2 is computed as following:

$$T_{L_2}=\frac{h_0^{(2)}}{2}(\frac{V_1+CV_3}{CV_1V_3})+\frac{a_2}{V_2}(\frac{V_1+2CV_3}{2CV_3}-\sqrt{(\frac{V_1+2CV_3}{2CV_3})^2-\frac{1}{h_0^{(2)}}(\frac{a_1(1+CV_3)}{CV_3}))} \tag{2.23}$$

$$n_2=\frac{V_1+2CV_3}{2CV_3}-\sqrt{(\frac{V_1+2CV_3}{2CV_3})^2-\frac{1}{h_0^{(2)}}(\frac{a_1(1+CV_3)}{CV_3})} \tag{2.24}$$

For given parameters $a_1, a_2, V_1, V_2, V_3, C$ and use numerical methods and computer programming (we have used MAPLE 10) the values $(h_0^{(1)})^*$ and $(h_0^{(2)})^*$ are found from (21) and (23) respectively. Then with $(h_0^{(1)})^*, (h_0^{(2)})^*$, (22) and (24) are calculated n_1 and n_2, but $(n_1)^*$ and $(n_2)^*$ are selected from following:(see (11), (12))

$$T_{L_1}(a_1,a_2,h_0^{(1)},[n_1])<T_{L_1}(a_1,a_2,h_0^{(1)},[n_1+1])\Rightarrow n_1^*=[n_1]$$

$$T_{L_1}(a_1,a_2,h_0^{(1)},[n_1])>T_{L_1}(a_1,a_2,h_0^{(1)},[n_1+1])\Rightarrow n_1^*=[n_1]+1$$

$$T_{L_2}(a_1,a_2,h_0^{(2)},[n_2])<T_{L_2}(a_1,a_2,h_0^{(2)},[n_2+1])\Rightarrow n_2^*=[n_2]$$

$$T_{L_2}(a_1,a_2,h_0^{(2)},[n_2])>T_{L_2}(a_1,a_2,h_0^{(2)},[n_2+1])\Rightarrow n_2^*=[n_2]+1$$

Now q_1^* and q_2^* are calculated with these relations:(see (8))

$$q_1^*=\frac{2(h_0^{(1)})^*-2a_2(n_1^*)^{-1}}{n_1^*-1};q_2^*=\frac{2(h_0^{(2)})^*-2a_1(n_2^*)^{-1}}{n_2^*-1}$$

Then for the found parameters $(h_0^{(i)})^*, n_i^*, q_i^*$ controls $u_{L_1}^*$(9) and $u_{L_2}^*$(10), as well as the corresponding functions $T_{L_1}^*$ and $T_{L_2}^*$ are computed. After that, from (13), we fined the time

$$T^*(a_1,a_2)=\min_{(h_0^i)^*,n_i^*}\{T_{L_1}(a_1,a_2,(h_0^{(1)})^*,n_1^*),T_{L_2}(a_1,a_2,(h_0^{(2)})^*,n_2^*)\}$$

and one of controls $u_{L_1}^*$ or $u_{L_2}^*$, which corresponds to this time.

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