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On decomposability of projective curvature tensor in projective recurrent conformal Finsler space

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Abstract

M. S. Knebelman [1] has developed conformal geometry of generalised metric spaces. The decomposability of curvature tensor in a Finsler manifold was studied by P. N. Pandey [4]. The purpose of the present paper is to decompose the Projective curvature tensor and study the identities satisfied by projective curvature tensor in conformal Finsler space.

1 Introduction

Let us considered two distinct metric functions $F(x,\dot{x})$ and $\bar{F}(x,\dot{x})$ be defined over an n-dimentional space Fn, both of which satisfies the requisite conditions for a Finsler space. The corresponding two metric tensor $g_{ij}(x,\dot{x})$ and $\bar{g}_{ij}(x,\dot{x})$ resulting from these functions are called conformal. If there exist a factor of proportionality between two metric tensors, Knebelman has proved that the factor of proportionality between them is at most a point function. Thus we have

$$\bar{F}(x,\dot{x}) = e^{\sigma}F(x,\dot{x}), \tag{1.1}$$

$$\bar{g}_{ij}(x,\dot{x}) = e^{2\sigma}g_{ij}(x,\dot{x}), \tag{1.2}$$

$$\bar{g}^{ij}(x,\dot{x}) = e^{-2\sigma}g^{ij}(x,\dot{x}), \tag{1.3}$$

$$\sigma = \sigma(x). \tag{1.4}$$

The space equipped with quantities $\bar{F}(x,\dot{x})$ and $\bar{g}(x,\dot{x})$ etc is called a conformal Finsler space, it is denoted by $\bar{F}n$.

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The following geometric entities of the conformal Finsler space are given by [5] and [6].

$$\bar{G}^i(x,\dot{x}) = G^i(x,\dot{x}) - \sigma_m B^{im}(x,\dot{x}), \tag{1.5}$$

$$\bar{G}^i_{jk}(x,\dot{x}) = G^i_{jk}(x,\dot{x}) - \dot{\partial}_k\dot{\partial}_j B^{im}(x,\dot{x})\sigma_m, \tag{1.6}$$

$$\bar{G}^i_{jkh}(x,\dot{x}) = G^i_{jkh}(x,\dot{x}) - \dot{\partial}_h\dot{\partial}_k\dot{\partial}_j B^{im}(x,\dot{x})\sigma_m, \tag{1.7}$$

$$B^{ij}(x,\dot{x}) = \frac{1}{2}F^2 g^{ij} - \dot{x}^i\dot{x}^j. \tag{1.8}$$

Where $G^i_{jkh}(x,\dot{x})$ are the Berwald's's connection coefficients. They satisfy

$$\partial_j G^i_k(x,\dot{x}) = G^i_{jk}, \tag{1.9}$$

and the functions B^{ij} are homogeneous of the second degree in there directional arguments.

The tensor W^i_h and W^i_{kh} transform under the conformal change as follow [2].

$$\begin{aligned}
\bar{W}^i_h \;=\;& W^i_h - \sigma_m[2B^{im}_{(h)} - (\dot{\partial}_h B^{im})_{(r)}\dot{x}^r - \frac{1}{n-1}\delta^i_h\{2B^{pm}_{(p)} - (\dot{\partial}_p B^{pm})_{(r)}\dot{x}^r\} \\
& - \frac{\dot{x}^i}{n^2-1}\{2n-1)(\dot{\partial}_p B^{pm})_{(h)} - (n+1)(\dot{\partial}_h B^{pm})_{(p)} + 2(n-2)B^{rm}G^p_{rph} \\
& - (n-2)\dot{x}^r(\dot{\partial}_p\dot{\partial}_h B^{pm})_{(r)}\}] + \sigma_{m(r)}\dot{x}^r\{\dot{\partial}_h B^{im} - \frac{1}{n-1}\delta^i_h\dot{\partial}_p B^{pm} \\
& - \frac{n-2}{n^2-1}\dot{x}^i\dot{\partial}_h\dot{\partial}_p B^{pm}\} - \sigma_{m(h)}\{2B^{im} - \frac{2n-1}{n^2-1}\dot{x}^i\dot{\partial}_p B^{pm} \\
& + \sigma_{m(p)}\{\frac{2}{n-1}\delta^i_h B^{pm} - \frac{\dot{x}^i}{n-1}\dot{\partial}_h B^{pm}\} + \sigma_m\sigma_r[2B^{sm}\dot{\partial}_h\dot{\partial}_s B^{ir} \\
& - (\dot{\partial}_h B^{sm})\dot{\partial}_s B^{ir} - \frac{1}{n-1}\delta^i_h\{2B^{sm}\dot{\partial}_p\dot{\partial}_s B^{pr} - (\dot{\partial}_p B^{sm})\dot{\partial}_s B^{pr}\} \\
& + \frac{2\dot{x}^i}{n^2-1}\{(n+1)(\dot{\partial}_{[p}B^{sm})\dot{\partial}_{h]}\dot{\partial}_s B^{pr} + (n-2)B^{sm}\dot{\partial}_p\dot{\partial}_h\dot{\partial}_s B^{pr}\}],
\end{aligned} \tag{1.10}$$

$$
\begin{aligned}
\bar{W}^i_{kh} \;=\;& W^i_{kh} - 2\sigma_m[(\dot{\partial}_{[k}B^{im})_{(h)]} - \frac{\dot{x}^i}{n+1}\{(\dot{\partial}_p\dot{\partial}_{[k}B^{pm})_{(h)]} \\
& +(\dot{\partial}_{[k}B^{rm})G^p_{h]rp}\} + \frac{1}{n^2-1}\delta^i_{[k}\{(n+1)(\dot{\partial}_{h]}B^{pm})_{(p)} \\
& -n(\dot{\partial}_pB^{pm})_{(h)]} - (\dot{\partial}_{h]}\dot{\partial}_pB^{pm})_{(r)}\dot{x}^r + 2B^{rm}G^p_{h]rp}\}] \\
& +2\sigma_{m[(k)}\{\dot{\partial}_{h]}B^{im} - \frac{n}{n^2-1}\delta^i_{h]}\dot{\partial}_pB^{pm} - \frac{\dot{x}^i}{n+1}\dot{\partial}_{h]}\dot{\partial}_pB^{pm}\} \\
& +\frac{2}{n^2-1}\sigma_{m(p)}\delta^i_{[k}\{\dot{x}^p\dot{\partial}_{h]}\dot{\partial}_rB^{rm} - (n+1)\dot{\partial}_{h]}B^{pm}\} \\
& +2\sigma_m\sigma_r[(\dot{\partial}_{[k}B^{sm}\{\dot{\partial}_{h]}\dot{\partial}_sB^{ir} - \frac{\dot{x}^i}{n+1}\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_sB^{pr}\} \\
& \frac{1}{n-1}\delta^i_{[k}\{(\dot{\partial}_{h]}B^{sm})\dot{\partial}_p\dot{\partial}_sB^{pr} - (\dot{\partial}_{h]}\dot{\partial}_sB^{pr})\dot{\partial}_pB^{sm} \\
& +\frac{2}{n-1}B^{sm}\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_sB^{pr}\}].
\end{aligned}
\tag{1.11}
$$

R. B. Mishra [2] have introduced to obtained the conformal transformation of projective curvature tensor W^i_{jkh} by differentiating (1.11) with respect to $\dot{x}^j$.

$$
\begin{aligned}
\bar{W}^i_{jkh} \;=\;& W^i_{jkh} + 2\sigma_m[(\dot{\partial}_{[k}B^{ir})G^m_{h]jr} - \dot{\partial}_j(\dot{\partial}_{[k}B^{im})_{(h)]} \\
& +\frac{\dot{x}^i}{n+1}\{\dot{\partial}_j\dot{\partial}_{[k}(\dot{\partial}_pB^{pm})_{(h)]}\} + \frac{\delta^i_j}{n+1}\{\dot{\partial}_p(\dot{\partial}_{[k}B^{pm})_{(h)]} \\
& -(\dot{\partial}_{[k}B^{pr})G^m_{h]pr}\} - \frac{\delta^i_{[k}}{n^2-1}\{n\dot{\partial}_j(\dot{\partial}_{h]}B^{pm})_{(p)} - n\dot{\partial}_j(\dot{\partial}_pB^{pm})_{(h)]} \\
& +\dot{\partial}_{h]}(\dot{\partial}_jB^{pm})_{(p)]} - \dot{\partial}_{h]}(\dot{\partial}_pB^{pm})_{(j)]}\} - \frac{\dot{x}^l\delta^i_{[k}}{n^2-1}\dot{\partial}_j\{\dot{\partial}_{h]}(\dot{\partial}_lB^{pm})_{(p)} \\
& -\dot{\partial}_{h]}(\dot{\partial}_pB^{pm})_{(l)}\}] + 2\sigma_{m[(k)}\{\dot{\partial}_{h]}(\dot{\partial}_jB^{im}) - \frac{\dot{x}^i}{n+1}\dot{\partial}_j(\dot{\partial}_{h]}\dot{\partial}_pB^{pm}) \\
& -\frac{\delta^i_j}{n+1}\dot{\partial}_{h]}(\dot{\partial}_pB^{pm})\} + \frac{2n\delta^i_{[k}}{n^2-1}\{\sigma_{m(h)]}(\dot{\partial}_j\dot{\partial}_pB^{pm}) \\
& -\sigma_{m(p)}(\dot{\partial}_j\dot{\partial}_{h]}B^{pm})\} + 2\sigma_m\sigma_r[\dot{\partial}_j(\dot{\partial}_{[k}B^{sm})\dot{\partial}_{h]}\dot{\partial}_sB^{ir} \\
& -\frac{\dot{x}^i}{n+1}\{\dot{\partial}_j\dot{\partial}_{[k}(\dot{\partial}_pB^{sm})\dot{\partial}_{h]}\dot{\partial}_sB^{pr} + \dot{\partial}_{[k}(\dot{\partial}_pB^{sm})\dot{\partial}_j(\dot{\partial}_{h]}\dot{\partial}_sB^{pr}) \\
& +\dot{\partial}_j(\dot{\partial}_{[k}B^{sm})\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_sB^{pr} + (\dot{\partial}_{[k}B^{sm})\dot{\partial}_j\dot{\partial}_{h]}\dot{\partial}_p(\dot{\partial}_sB^{pr})\} \\
& -\frac{\delta^i_j}{n+1}\{\dot{\partial}_p(\dot{\partial}_{[h}B^{sm})\dot{\partial}_{k]}\dot{\partial}_sB^{pr}\} + \frac{n\delta^i_{[k}}{n^2-1}\{(\dot{\partial}_j\dot{\partial}_{h]}B^{sm})\dot{\partial}_p\dot{\partial}_sB^{pr}
\end{aligned}
\tag{1.12}
$$

$$-(\dot{\partial}_j\dot{\partial}_p B^{sm})\dot{\partial}_{h]}\dot{\partial}_s B^{pr} + (\dot{\partial}_{h]} B^{sm})\dot{\partial}_j\dot{\partial}_p\dot{\partial}_s B^{pr} - (\dot{\partial}_p B^{sm})\dot{\partial}_j\dot{\partial}_{h]}\dot{\partial}_s B^{pr}\}$$
$$+\frac{\delta^i_{[k}}{n^2-1}\{\dot{\partial}_{h]}(\dot{\partial}_j B^{sm})\dot{\partial}_p\dot{\partial}_s B^{pr} - \dot{\partial}_{h]}(\dot{\partial}_p B^{sm})\dot{\partial}_j\dot{\partial}_s B^{pr} + (\dot{\partial}_j B^{sm})\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_s B^{pr}$$
$$-(\dot{\partial}_p B^{sm})\dot{\partial}_{[h}\dot{\partial}_j\dot{\partial}_s B^{pr}\} + \frac{\dot{x}^l\delta^i_{[k}}{n^2-1}[\dot{\partial}_j(\dot{\partial}_{h]}\dot{\partial}_l B^{sm})\dot{\partial}_p\dot{\partial}_s B^{pr}$$
$$-\dot{\partial}_j\dot{\partial}_{h]}(\dot{\partial}_p B^{sm})\dot{\partial}_l\dot{\partial}_s B^{pr} + \dot{\partial}_{h]}(\dot{\partial}_l B^{sm})\dot{\partial}_j\dot{\partial}_p\dot{\partial}_s B^{pr} - \dot{\partial}_{h]}(\dot{\partial}_p B^{sm})\dot{\partial}_j\dot{\partial}_l\dot{\partial}_s B^{pr}$$
$$+\dot{\partial}_j(\dot{\partial}_l B^{sm})\dot{\partial}_{h]}(\dot{\partial}_p\dot{\partial}_s B^{pr}) - \dot{\partial}_j(\dot{\partial}_p B^{sm})\dot{\partial}_{h]}(\dot{\partial}_l\dot{\partial}_s B^{pr}) + (\dot{\partial}_l B^{sm})\dot{\partial}_j\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_s B^{pr}$$
$$-(\dot{\partial}_p B^{sm})\dot{\partial}_j\dot{\partial}_{h]}\dot{\partial}_l\dot{\partial}_s B^{pr}\}] + \frac{2\delta^i_{[k}}{n^2-1}\{\sigma_{m(j)}\dot{\partial}_{h]}(\dot{\partial}_p B^{pm}) - \sigma_{m(p)}\dot{\partial}_{h]}(\dot{\partial}_j B^{pm})]$$
$$+\frac{2\dot{x}^l\delta^i_{[k}}{n^2-1}\{\sigma_{m(l)}\dot{\partial}_j(\dot{\partial}_{h]}\dot{\partial}_p B^{pm}) - \sigma_{m(p)}\dot{\partial}_j\dot{\partial}_{h]}(\dot{\partial}_l B^{pm})\}.$$

2 Decomposition of conformal projective curvature tensor

We considered the decomposition of projective curvature tensor in the form

$$W^i_{jkh} = X^i_j\phi_{kh}, \tag{2.1}$$

where X^i_j is non zero tensor and ϕ_{kh} is skew symmetric decomposition tensor.
Transvecting (2.1) by y_i, we get

$$y_i W^i_{jkh} = \lambda_j\phi_{kh}, \tag{2.2}$$

where

$$\begin{cases} a) \qquad y_i = \dot{x}^j g_{ij}\ , \\ b) \qquad \lambda_j = y_i X^i_j. \end{cases} \tag{2.3}$$

We may choose another vector V^j such that

$$V^j\lambda_j = 1. \tag{2.4}$$

Similar manner the decomposition of conformal projective curvature tensor $\bar{W}^i_{jkh}$ in the form

$$\bar{W}^i_{jkh} = \bar{X}^i_j\bar{\phi}_{kh}, \tag{2.5}$$

where $\bar{X}^i_j$ is non zero conformal tensor and $\bar{\phi}_{kh}$ is conformal decomposition tensor.

Transvecting (2.5) by $\bar{y}_i$, we get

$$\bar{y}_i\bar{W}^i_{jkh} = \bar{\lambda}_j\bar{\phi}_{kh}, \tag{2.6}$$

$$\begin{cases} a) \quad \bar{y}_i = \overline{\dot{x}^j}\bar{g}_{ij} , \\ b) \quad \bar{\lambda}_j = \bar{y}_i \bar{X}^i_j. \end{cases} \tag{2.7}$$

Where $\bar{\lambda}_j$ is non zero vector and choose another vector $\bar{V}^j$ such that

$$\bar{V}^j \bar{\lambda}_j = 1. \tag{2.8}$$

The projective recurrent conformal curvature tensor $\bar{W}^i_{jkh}$ is characterized by condition.

$$\bar{W}^i_{jkh(\bar{l})} = \bar{V}_l \bar{W}^i_{jkh}, \tag{2.9}$$

where

$$\bar{W}^i_{jkh} \neq 0. \tag{2.10}$$

The conformal covariant vector $\bar{V}_l$ is called the conformal recurrence vector. The space equipped by such conformal projective curvature tensor is called projective recurrent conformal Finsler space.

The conformal transformation of vector λ_j, V^j and tensor X^i_j and the directional argument $\dot{x}^j$ may be written as

$$\begin{cases} a) \quad \bar{X}^i_j = e^{-\sigma} X^i_j, \\ b) \quad \bar{\lambda}_j = e^{\sigma} \lambda_j, \\ c) \quad \bar{V}^j = e^{-\sigma} V^j, \\ d) \quad \overline{\dot{x}^j} = \dot{x}^j. \end{cases} \tag{2.11}$$

The projective curvature tensor satisfy the identity[3].

$$W^i_{jkh} + W^i_{khj} + W^i_{hjk} = 0, \tag{2.12}$$

and also satisfy the identity[7]

$$W^i_{jkh(l)} + W^i_{jhl(k)} + W^i_{jlk(h)} = 0. \tag{2.13}$$

Transvecting (2.6) by $\bar{V}^j$, we get

$$\bar{\phi}_{kh} = \bar{V}^j \bar{y}_i \bar{W}^i_{jkh}, \tag{2.14}$$

using equation (1.12) in above equation and applying equations (2.7b), (2.11c),(2.11d) (2.5), (1.2) and using the symmetric property of the function G^i_{jkh} and B^{im}, we obtain

$$
\begin{aligned}
\bar{\phi}_{kh} \;=\;& e^{\sigma}V^j y_i W^i_{jkh} + 2e^{\sigma}V^j\sigma_m[\frac{F^2}{n+1}\dot{\partial}_j\dot{\partial}_{[k}(\dot{\partial}_p B^{pm})_{(h)]} - y_i\dot{\partial}_j(\dot{\partial}_{[k}B^{im})_{(h)]}\\
&+\frac{y_j}{n+1}\{\dot{\partial}_p(\dot{\partial}_{[k}B^{pm})_{(h)]}\} - \frac{y_{[k}}{n^2-1}\{n\dot{\partial}_j(\dot{\partial}_{h]}B^{pm})_{(p)} - n\dot{\partial}_j(\dot{\partial}_p B^{pm})_{(h)]}\\
&+\dot{\partial}_{h]}(\dot{\partial}_j B^{pm})_{(p)]} - \dot{\partial}_{h]}(\dot{\partial}_p B^{pm})_{(j)]}\} - \frac{\dot{x}^l y_{[k}}{n^2-1}\dot{\partial}_j\{\dot{\partial}_{h]}(\dot{\partial}_l B^{pm})_{(p)}\\
&-\dot{\partial}_{h]}(\dot{\partial}_p B^{pm})_{(l)}\}] + 2e^{\sigma}V^j[\sigma_{m[(k)}\{\dot{\partial}_{h]}(\dot{\partial}_j B^{im})y_i - \frac{y_j}{n+1}\dot{\partial}_{h]}(\dot{\partial}_p B^{pm})\}\\
&+n\frac{y_{[k}}{n^2-1}\{\sigma_{m(h)]}(\dot{\partial}_j\dot{\partial}_p B^{pm}) - \sigma_{m(p)}(\dot{\partial}_j\dot{\partial}_{h]}B^{pm})\}]\\
&+2\sigma_m\sigma_r e^{\sigma}V^j[\dot{\partial}_j(\dot{\partial}_{[k}B^{sm})\dot{\partial}_{h]}\dot{\partial}_s B^{ir} - \frac{F^2}{n+1}\{\dot{\partial}_j(\dot{\partial}_{[k}B^{sm})\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_s B^{pr}\\
&+(\dot{\partial}_{[k}B^{sm})\dot{\partial}_j\dot{\partial}_{h]}\dot{\partial}_p(\dot{\partial}_s B^{pr})\} - \frac{y_j}{n+1}\{\dot{\partial}_p(\dot{\partial}_{[h}B^{sm})\dot{\partial}_{k]}\dot{\partial}_s B^{pr}\}\\
&+\frac{ny_{[k}}{n^2-1}\{(\dot{\partial}_j\dot{\partial}_{h]}B^{sm})\dot{\partial}_p\dot{\partial}_s B^{pr} - (\dot{\partial}_j\dot{\partial}_p B^{sm})\dot{\partial}_{h]}\dot{\partial}_s B^{pr}\}\\
&+\frac{y_{[k}}{n^2-1}\{\dot{\partial}_{h]}(\dot{\partial}_j B^{sm})\dot{\partial}_p\dot{\partial}_s B^{pr} - \dot{\partial}_{h]}(\dot{\partial}_p B^{sm})\dot{\partial}_j\dot{\partial}_s B^{pr}\\
&+(\dot{\partial}_j B^{sm})\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_s B^{pr} + \dot{x}^l\dot{\partial}_j(\dot{\partial}_l B^{sm})\dot{\partial}_{h]}(\dot{\partial}_p\dot{\partial}_s B^{pr})\\
&-\dot{x}^l\dot{\partial}_j(\dot{\partial}_p B^{sm})\dot{\partial}_{h]}(\dot{\partial}_l\dot{\partial}_s B^{pr}) + \dot{x}^l(\dot{\partial}_l B^{sm})\dot{\partial}_j\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_s B^{pr}\\
&-\dot{x}^l(\dot{\partial}_p B^{sm})\dot{\partial}_j\dot{\partial}_{h]}\dot{\partial}_l\dot{\partial}_s B^{pr}\}]\frac{2e^{\sigma}V^j y_{[k}}{n^2-1}[\{\sigma_{m(j)}\dot{\partial}_{h]}(\dot{\partial}_p B^{pm})\\
&-\sigma_{m(p)}\dot{\partial}_{h]}(\dot{\partial}_j B^{pm})].
\end{aligned} \tag{2.15}
$$

Thus we state

Theorem 2.1. *Under the decomposition (2.5), the conformal decomposition tensor $\bar{\phi}_{kh}$ is expressed in the form (2.15).*

Theorem 2.2. *Under the decomposition (2.5), the conformal decomposition tensor $\bar{\phi}_{kh}$ Satisfy the condition*

$$\bar{\phi}_{kh} = -\bar{\phi}_{hk}. \tag{2.16}$$

Proof. Interchanging the indices k and h in (2.15) and adding the equation thus obtained to (2.15), we obtain

$$\bar{\phi}_{kh} + \bar{\phi}_{hk} = e^{\sigma}V^j y_i(W^i_{jkh} + W^i_{jhk}), \tag{2.17}$$

using the relation $W^i_{jkh} = -W^i_{jhk}$ [3].
we get the identity (2.16). □

Differentiating (2.5) covariantly with respect to x^l in the sence of Berwald's, we get

$$\bar{W}^i_{jkh(\bar{l})} = \bar{X}^i_{j(\bar{l})}\bar{\phi}_{kh} + \bar{\phi}_{kh(\bar{l})}\bar{X}^i_j, \tag{2.18}$$

applying (2.5) and (2.9) in the above equation, we find

$$\bar{V}_l \bar{X}^i_j \bar{\phi}_{kh} = \bar{X}^i_{j(\bar{l})} \bar{\phi}_{kh} + \bar{X}^i_j \bar{\phi}_{kh(\bar{l})}. \tag{2.19}$$

Let us assume that the conformal tensor $\bar{X}^i_j$ is covariant constant, then (2.19) reduces to

$$\bar{\phi}_{kh(\bar{l})} = \bar{V}_l \bar{\phi}_{kh}. \tag{2.20}$$

Conversely, if the above equation is true, then equation yields

$$\bar{X}^i_{j(\bar{l})} \bar{\phi}_{kh} = 0, \tag{2.21}$$

since $\bar{\phi}_{kh}$ is non zero conformal decomposition tensor, it implies

$$\bar{X}^i_{j(\bar{l})} = 0. \tag{2.22}$$

which shows that $\bar{X}^i_j$ is covariant constant in recurrent conformal Finsler space.
Thus we state:

Theorem 2.3. *In projective recurrent conformal Finsler space, the necessary and sufficient condition for the conformal decomposition tensor $\bar{\phi}_{kh}$ to be recurrent is that the conformal tensor $\bar{X}^i_j$ is covariant constant in the sense of Berwald's.*

If we suppose σ is constant, then the equation (2.15) reduces to

$$\bar{\phi}_{kh} = e^{\sigma} V^j y_i W^i_{jkh}. \tag{2.23}$$

The Berwald's covariant derivative of equation (35) with respect to x^l is given by

$$\bar{\phi}_{kh(l)} = e^{\sigma} V^j y_i W^i_{jkh(l)}. \tag{2.24}$$

Adding the equation obtained by the cyclic change of the indices k, h and l in equation (2.24) and using equation (2.13), we obtain

$$\bar{\phi}_{[kh(l)]} = 0. \tag{2.25}$$

Theorem 2.4. *Under the decompositon (2.5), If the mapping is homothetic then the conformal decompositon tensor $\bar{\phi}_{kh}$ satisfy the Bianchi identity (2.25).*

Theorem 2.5. *Under the Decomposition (2.5), the conformal projective curvature tensor satisfy the identity***.**

$$\bar{W}^i_{jkh} + \bar{W}^i_{khj} + \bar{W}^i_{hjk} = 0. \tag{2.26}$$

Proof. Applying equation (2.23), (2.11a), (2.3b) and (2.4) in equation (2.5), we obtain

$$\bar{W}^i_{jkh} = W^i_{jkh}. \tag{2.27}$$

Addin the equation obtained by the cyclic change of the indices j, k and h in equation (2.27), and using equation (2.12), we get the identity (2.7). □

3 Identities satisfied by the conformally changed Projective curvature tensor.

Transvecting (1.12) by $\bar{g}_{iu}$, we get

$$\begin{aligned}\bar{W}^i_{jkh}\bar{g}_{iu} \;=\; & e^{2\sigma}g_{iu}W^i_{jkh}+2e^{2\sigma}\sigma_m g_{iu}[\dot{\partial}_{[k}B^{ir})G^m_{h]jr}-\dot{\partial}_j(\dot{\partial}_{[k}B^{im})_{(h)]}\\
&+\frac{\dot{x}^i}{n+1}\{\dot{\partial}_j\dot{\partial}_{[k}(\dot{\partial}_pB^{pm})_{(h)]}\}+\frac{\delta^i_j}{n+1}\{\dot{\partial}_p(\dot{\partial}_{[k}B^{pm})_{(h)]}\\
&-(\dot{\partial}_{[k}B^{pr})G^m_{h]pr}\}-\frac{\delta^i_{[k}}{n^2-1}\{n\dot{\partial}_j(\dot{\partial}_{h]}B^{pm})_{(p)}-n\dot{\partial}_j(\dot{\partial}_pB^{pm})_{(h)]}\\
&+\dot{\partial}_{h]}(\dot{\partial}_jB^{pm})_{(p)]}-\dot{\partial}_{h]}(\dot{\partial}_pB^{pm})_{(j)]}\}-\frac{\dot{x}^l\delta^i_{[k}}{n^2-1}\dot{\partial}_j\{\dot{\partial}_{h]}(\dot{\partial}_lB^{pm})_{(p)}\\
&-\dot{\partial}_{h]}(\dot{\partial}_pB^{pm})_{(l)}\}]+2e^{2\sigma}g_{iu}[\sigma_{m[k}\{\dot{\partial}_{h]}(\dot{\partial}_jB^{im})-\frac{\dot{x}^i}{n+1}\dot{\partial}_j(\dot{\partial}_{h]}\dot{\partial}_pB^{pm})\\
&-\frac{\delta^i_j}{n+1}\dot{\partial}_{h]}(\dot{\partial}_pB^{pm})\}+\frac{n\delta^i_{[k}}{n^2-1}\{\sigma_{m(h)]}(\dot{\partial}_j\dot{\partial}_pB^{pm})\\
&-\sigma_{m(p)}(\dot{\partial}_j\dot{\partial}_{h]}B^{pm})\}]+2e^{2\sigma}g_{iu}\sigma_m\sigma_r[\dot{\partial}_j(\dot{\partial}_{[k}B^{sm})\dot{\partial}_{h]}\dot{\partial}_sB^{ir}\\
&-\frac{\dot{x}^i}{n+1}\{\dot{\partial}_j\dot{\partial}_{[k}(\dot{\partial}_pB^{sm})\dot{\partial}_{h]}\dot{\partial}_sB^{pr}+\dot{\partial}_{[k}(\dot{\partial}_pB^{sm})\dot{\partial}_j(\dot{\partial}_{h]}\dot{\partial}_sB^{pr})\\
&+\dot{\partial}_j(\dot{\partial}_{[k}B^{sm})\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_sB^{pr}+(\dot{\partial}_{[k}B^{sm})\dot{\partial}_j\dot{\partial}_{h]}(\dot{\partial}_p\dot{\partial}_sB^{pr})\}\\
&-\frac{\delta^i_j}{n+1}\{\dot{\partial}_p(\dot{\partial}_{[h}B^{sm})\dot{\partial}_{k]}\dot{\partial}_sB^{pr}\}+\frac{n\delta^i_{[k}}{n^2-1}\{(\dot{\partial}_j\dot{\partial}_{h]}B^{sm})\dot{\partial}_p\dot{\partial}_sB^{pr}\\
&-(\dot{\partial}_j\dot{\partial}_pB^{sm})\dot{\partial}_{h]}\dot{\partial}_sB^{pr}+(\dot{\partial}_{h]}B^{sm})\dot{\partial}_j\dot{\partial}_p\dot{\partial}_sB^{pr}-(\dot{\partial}_pB^{sm})\dot{\partial}_j\dot{\partial}_{h]}\dot{\partial}_sB^{pr}\}\\
&+\frac{\delta^i_{[k}}{n^2-1}\{\dot{\partial}_{h]}(\dot{\partial}_jB^{sm})\dot{\partial}_p\dot{\partial}_sB^{pr}-\dot{\partial}_{h]}(\dot{\partial}_pB^{sm})\dot{\partial}_j\dot{\partial}_sB^{pr}\end{aligned} \quad (3.1)$$

$$\begin{aligned}&+(\dot{\partial}_jB^{sm})\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_sB^{pr}-(\dot{\partial}_pB^{sm})\dot{\partial}_{[h}\dot{\partial}_j\dot{\partial}_sB^{pr}\}\\
&+\frac{\dot{x}^l\delta^i_{[k}}{n^2-1}[\dot{\partial}_j(\dot{\partial}_{h]}\dot{\partial}_lB^{sm})\dot{\partial}_p\dot{\partial}_sB^{pr}-\dot{\partial}_j\dot{\partial}_{h]}(\dot{\partial}_pB^{sm})\dot{\partial}_l\dot{\partial}_sB^{pr}\\
&+\dot{\partial}_{h]}(\dot{\partial}_lB^{sm})\dot{\partial}_j\dot{\partial}_p\dot{\partial}_sB^{pr}-\dot{\partial}_{h]}(\dot{\partial}_pB^{sm})\dot{\partial}_j\dot{\partial}_l\dot{\partial}_sB^{pr}\\
&+\dot{\partial}_j(\dot{\partial}_lB^{sm})\dot{\partial}_{h]}(\dot{\partial}_p\dot{\partial}_sB^{pr})-\dot{\partial}_j(\dot{\partial}_pB^{sm})\dot{\partial}_{h]}(\dot{\partial}_l\dot{\partial}_sB^{pr})\\
&+(\dot{\partial}_lB^{sm})\dot{\partial}_j\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_sB^{pr}-(\dot{\partial}_pB^{sm})\dot{\partial}_j\dot{\partial}_{h]}\dot{\partial}_l\dot{\partial}_sB^{pr}\}]\\
&+\frac{2e^{2\sigma}g_{iu}\delta^i_{[k}}{n^2-1}[\{\sigma_{m(j)}\dot{\partial}_{h]}(\dot{\partial}_pB^{pm})-\sigma_{m(p)}\dot{\partial}_{h]}(\dot{\partial}_jB^{pm})\}\\
&+\{\dot{x}^l\sigma_{m(l)}\dot{\partial}_j(\dot{\partial}_{h]}\dot{\partial}_pB^{pm})-\dot{x}^l\sigma_{m(p)}\dot{\partial}_j\dot{\partial}_{h]}(\dot{\partial}_lB^{pm})\}]\end{aligned}$$

where

$$\bar{W}_{jukh}=\bar{g}_{iu}\bar{W}^i_{jkh} \quad (3.2)$$

We have the following identities.

Theorem 3.1. *When F_n and $\bar{F}_n$ are in conformal correspondence, we have*

$$
\begin{aligned}
2\bar{W}^i_{[j<k>h} = {} & 2W^i_{[j<k>h]} + 2\sigma_m[\dot{\partial}_{[j}(\dot{\partial}_{h]}B^{im})_{(h)]} - \dot{\partial}_{[j}(\dot{\partial}_k B^{im})_{(h)]} \\
& + \frac{\dot{x}^i}{n+1}\{\dot{\partial}_{[j}\dot{\partial}_k(\dot{\partial}_p B^{pm})_{(h)]} - \dot{\partial}_{[j}\dot{\partial}_{h]}(\dot{\partial}_{p]}B^{pm})_{(k)}\} \\
& + \frac{\delta^i_{[j}}{n^2-1}\{\dot{\partial}_p(\dot{\partial}_k B^{pm})_{(h)]} - \dot{\partial}_p(\dot{\partial}_{h]}B^{pm})_{(k)]} - (\dot{\partial}_k B^{pr})G^m_{h]pr} \\
& + (\dot{\partial}_{h]}B^{pr})G^m_{kpr}\} - \frac{\delta^i_k}{n^2-1}\{n\dot{\partial}_{[j}(\dot{\partial}_{h]}B^{pm})_{(p)} - n\dot{\partial}_{[j}(\dot{\partial}_p B^{pm})_{(h])} \\
& + \dot{\partial}_{[h}(\dot{\partial}_{j]}B^{pm})_{(p)} - \dot{\partial}_{[h}(\dot{\partial}_p B^{pm})_{(j)]}\} + \dot{x}^l\dot{\partial}_{[j}\dot{\partial}_{h]}(\dot{\partial}_l B^{pm})_{(p)} \\
& - \dot{x}^l\dot{\partial}_{[j}\dot{\partial}_{h]}(\dot{\partial}_p B^{pm})_{(l)}\} + \frac{\delta^i_{[h}}{n^2-1}\{n\dot{\partial}_{[h}\dot{\partial}_{j]}(\dot{\partial}_k B^{pm})_{(p)} \\
& - n\dot{\partial}_{j]}(\dot{\partial}_p B^{pm})_{(k)} + \dot{\partial}_k(\dot{\partial}_{j]}B^{pm})_{(p)} + \dot{\partial}_k(\dot{\partial}_p B^{pm} \\
& + \dot{x}^l\dot{\partial}_{j]}\dot{\partial}_k(\dot{\partial}_l B^{pm})_{(p)} - \dot{x}^l\dot{\partial}_{j]}\dot{\partial}_k(\dot{\partial}_p B^{pm})_{(l)}\} + 2\sigma_{m(k)}\{\dot{\partial}_{[h}(\dot{\partial}_{j]}B^{im}) \\
& - \frac{\dot{x}^i}{n+1}\dot{\partial}_{[j}(\dot{\partial}_{h]}(\dot{\partial}_p B^{pm}) - \frac{\delta^i_{[j}}{n^2-1}\dot{\partial}_{h]}(\dot{\partial}_p B^{pm}) \\
& - n\frac{\delta^i_{[h}}{n^2-1}\dot{\partial}_{j]}(\dot{\partial}_p B^{pm})\} - 2\sigma_{m[h}\{\dot{\partial}_k(\dot{\partial}_{j]}B^{im}) \\
& - \frac{\dot{x}^i}{n+1}\dot{\partial}_{j]}(\dot{\partial}_k(\dot{\partial}_p B^{pm}) - \frac{\delta^i_{j]}}{n^2-1}\dot{\partial}_k(\dot{\partial}_p B^{pm}) \\
& - n\frac{\delta^i_{[h}}{n^2-1}\dot{\partial}_{j]}(\dot{\partial}_p B^{pm})\} + 2n\sigma_{m(p)}\{\frac{\delta^i_{[h}}{n^2-1}\dot{\partial}_{j]}(\dot{\partial}_k B^{pm})
\end{aligned} \tag{3.3}
$$

$$
\begin{aligned}
&-\frac{\delta^i_{[k}}{n^2-1}\dot\partial_{[j}(\dot\partial_{h]}B^{pm})\}2\sigma_m\sigma_r[\dot\partial_{[j}(\dot\partial_k B^{sm})\dot\partial_{h]}\dot\partial_s B^{ir}\\
&-\frac{\dot x^i}{n+1}\{\dot\partial_{[j}\dot\partial_k(\dot\partial_p B^{sm}\dot\partial_{h]}\dot\partial_s B^{pr}-\dot\partial_{[h}(\dot\partial_p B^{sm})\dot\partial_{j]}\dot\partial_k\dot\partial_s B^{pr}\\
&+\dot\partial_{[j}(\dot\partial_k B^{sm})\dot\partial_{h]}\dot\partial_p\dot\partial_s B^{pr}+(\dot\partial_{[j}B^{sm})\dot\partial_{h]}\dot\partial_k(\dot\partial_p B^{pr})\}\\
&+\frac{\delta^i_{[j}}{n^2-1}\{\dot\partial_p(\dot\partial_{h]}B^{sm})\dot\partial_k\dot\partial_s B^{pr}-\dot\partial_p(\dot\partial_k B^{sm})\dot\partial_{h]}\dot\partial_s B^{pr}\}\\
&-\frac{\delta^i_{[h}}{n^2-1}\{n\dot\partial_{j]}\dot\partial_k B^{sm})\dot\partial_p\dot\partial_s B^{pr}-n\dot\partial_{j]}(\dot\partial_p B^{sm})\dot\partial_k\dot\partial_s B^{pr}\\
&+n(\dot\partial_k B^{sm})\dot\partial_{j]}\dot\partial_p\dot\partial_s B^{pr}-n(\dot\partial_{[j}\dot\partial_k B^{sm})\dot\partial_p\dot\partial_s B^{pr}\\
&+\dot\partial_k(\dot\partial_{j]}B^{sm})\dot\partial_p\dot\partial_s B^{pr}-\dot\partial_k(\dot\partial_p B^{sm})\dot\partial_{j]}\dot\partial_s B^{pr}\\
&+\dot\partial_{j]}B^{sm}\dot\partial_k\dot\partial_p\dot\partial_s B^{pr}-(\dot\partial_p B^{sm})\dot\partial_k\dot\partial_{[j}\dot\partial_s B^{pr}-\dot x^l\dot\partial_{j]}\dot\partial_k(\dot\partial_l B^{sm})(\dot\partial_p\dot\partial_s B^{pr})\\
&+\dot x^l\dot\partial_{j]}\dot\partial_k(\dot\partial_p B^{sm})(\dot\partial_l\dot\partial_s B^{pr})+\dot x^l\dot\partial_k(\dot\partial_l B^{sm})\dot\partial_{j]}(\dot\partial_p\dot\partial_s B^{pr})\\
&-\dot x^l\dot\partial_k(\dot\partial_p B^{sm})\dot\partial_{j]}(\dot\partial_l\dot\partial_s B^{pr})+\dot x^l\dot\partial_{j]}(\dot\partial_l B^{sm})\dot\partial_k\dot\partial_p\dot\partial_s B^{pr})\\
&-\dot x^l\dot\partial_{j]}(\dot\partial_p B^{sm})\dot\partial_k\dot\partial_l\dot\partial_s B^{pr})-\dot x^l\dot\partial_k(\dot\partial_p B^{sm})\dot\partial_{j]}(\dot\partial_l\dot\partial_s B^{pr})\\
&+\dot x^l\dot\partial_{j]}(\dot\partial_l B^{sm})\dot\partial_k\dot\partial_p\dot\partial_s B^{pr})-\dot x^l\dot\partial_{j]}(\dot\partial_p B^{sm})\dot\partial_k\dot\partial_l\dot\partial_s B^{pr})\\
&+\dot x^l(\dot\partial_l B^{sm})\dot\partial_{j]}\dot\partial_k\dot\partial_p\dot\partial_s B^{pr}-\dot x^l(\dot\partial_p B^{sm})\dot\partial_{j]}\dot\partial_k\dot\partial_l\dot\partial_s B^{pr}\}\\
&-\frac{\delta^i_k}{n^2-1}\{n\dot\partial_{[j}(\dot\partial_p B^{sm})(\dot\partial_{h]}\dot\partial_s B^{pr})-n(\dot\partial_{[h}B^{sm})\dot\partial_{j]}\dot\partial_p\dot\partial_s B^{pr}\\
&-\dot\partial_{[h}(\dot\partial_p B^{sm})\dot\partial_{j]}\dot\partial_s B^{pr}-(\dot\partial_{[j}B^{sm})\dot\partial_{h]}\dot\partial_p\dot\partial_s B^{pr}-\dot x^l\dot\partial_{[h}(\dot\partial_l B^{sm})\dot\partial_{j]}(\dot\partial_p\dot\partial_s B^{pr})\\
&+\dot x^l\dot\partial_{[h}(\dot\partial_p B^{sm})\dot\partial_{j]}\dot\partial_l\dot\partial_s B^{pr}-\dot x^l\dot\partial_{[j}(\dot\partial_l B^{sm})\dot\partial_{h]}\dot\partial_p\dot\partial_s B^{pr}\\
&+\dot x^l\dot\partial_{[j}(\dot\partial_p B^{sm})\dot\partial_{h]}(\dot\partial_l\dot\partial_s B^{pr})\}]+2[\sigma_{m[(j)}\{\frac{\delta^i_k}{n^2-1}\dot\partial_{h]}(\dot\partial_p B^{pm})\\
&-\frac{\delta^i_{[h}}{n^2-1}\dot\partial_k(\dot\partial_p B^{pm})\}+\frac{\delta^i_{[h}}{n^2-1}\{\sigma_{m(p)}\dot\partial_k(\dot\partial_{j]}B^{pm})-\dot x^l\sigma_{m(l)}\dot\partial_{j]}(\dot\partial_k\dot\partial_p B^{pm})\}\\
&-\dot x^l\frac{\delta^i_{[j}}{n^2-1}\sigma_{m(p)}\dot\partial_{h]}(\dot\partial_k\dot\partial_l B^{pm})\}
\end{aligned}
$$

Proof. Interchanging the indices k and h in (1.12) and subtracting the equation thus obtained to (1.12) and using the symmetric property of the function G^i_{jkh}, we get the result (3.3). □

Theorem 3.2. *When F_n and $\bar F_n$ are in conformal correspondence,we have*

$$
\begin{aligned}
\bar W_{jukh}-\bar W_{jhku} \;=\;& e^{2\sigma}(W_{jukh}-W_{jhku})+2e^{2\sigma}\sigma_m g_{i[u}[\{(\dot\partial_k B^{ir})G^m_{h]jr}\\
&-\dot\partial_{h]}B^{ir})G^m_{kjr}-\dot\partial_j(\dot\partial_k B^{im})_{(h)]}+\dot\partial_j(\dot\partial_{h]}B^{im})_{(k)}
\end{aligned}
\tag{3.4}
$$

$$
\begin{aligned}
&+\frac{\dot{x}^i}{n+1}\{\dot{\partial}_j\dot{\partial}_k(\dot{\partial}_p B^{pm})_{h]}\}-\dot{\partial}_j\dot{\partial}_{h]}(\dot{\partial}_p B^{pm})_k\}\\
&+\frac{\delta^i_j}{n^2-1}\{\dot{\partial}_p(\dot{\partial}_k B^{pm})_{(h)]}-\dot{\partial}_p(\dot{\partial}_{h]}B^{pm})_{(k)}-(\dot{\partial}_k B^{pr})G^m_{h]pr}\\
&+(\dot{\partial}_{h]}B^{pr})G^m_{kpr}\}+\frac{\delta^i_k}{n^2-1}\{n\dot{\partial}_j(\dot{\partial}_p B^{pm})_{(h)]}-n(\dot{\partial}_j\dot{\partial}_{h]}B^{pm})_{(p)}\\
&-\dot{\partial}_{h]}(\dot{\partial}_j B^{pm})_{(p)]}+\dot{\partial}_{h]}(\dot{\partial}_p B^{pm})_{(j)}-\dot{x}^l\dot{\partial}_j(\dot{\partial}_{h]}(\dot{\partial}_l B^{pm})_{(p)}\\
&+\dot{x}^l\dot{\partial}_j\dot{\partial}_{h]}(\dot{\partial}_p B^{pm})_{(l)}\}+\frac{\delta^i_{h]}}{n^2-1}\{n\dot{\partial}_j(\dot{\partial}_k B^{pm})_{(p)}\\
&-n\dot{\partial}_j(\dot{\partial}_p B^{pm})_{(k)}+\dot{\partial}_k(\dot{\partial}_j B^{pm})_{(p)]}-\dot{\partial}_k(\dot{\partial}_p B^{pm})_{(j)}\\
&+\dot{x}^l\dot{\partial}_j(\dot{\partial}_k(\dot{\partial}_l B^{pm})_{(p)}+\dot{x}^l\dot{\partial}_j\dot{\partial}_k(\dot{\partial}_p B^{pm})_{(l)}\}\\
&+2e^{2\sigma}g_{i[u}[\sigma_{m(k)}\{\dot{\partial}_{h]}(\dot{\partial}_j B^{im})-\frac{\dot{x}^i}{n+1}\dot{\partial}_j\dot{\partial}_{h]}(\dot{\partial}_p B^{pm})\\
&-\frac{\delta^i_j}{n^2-1}\dot{\partial}_{h]}(\dot{\partial}_p B^{pm})-n\frac{\delta^i_{h]}}{n^2-1}\dot{\partial}_j(\dot{\partial}_p B^{pm})\}-\sigma_{m(h)]}\{\dot{\partial}_k(\dot{\partial}_j B^{im})\\
&-\frac{\dot{x}^i}{n+1}\dot{\partial}_j\dot{\partial}_k(\dot{\partial}_p B^{pm})-\frac{\delta^i_j}{n^2-1}\dot{\partial}_k(\dot{\partial}_p B^{pm})-n\frac{\delta^i_k}{n^2-1}\dot{\partial}_j(\dot{\partial}_p B^{pm})\}\\
&-\sigma_{m(p)}\{\frac{n\delta^i_k}{n^2-1}\dot{\partial}_j(\dot{\partial}_{h]}B^{pm})+\frac{n\delta^i_{h]}}{n^2-1}\dot{\partial}_j(\dot{\partial}_k B^{pm})\}]\\
&+2e^{2\sigma}g_{i[u}\sigma_m\sigma_r[\dot{\partial}_j(\dot{\partial}_k B^{sm})\dot{\partial}_{h]}\dot{\partial}_s B^{ir}-\dot{\partial}_j(\dot{\partial}_{h]}B^{sm})\dot{\partial}_k\dot{\partial}_s B^{ir}\\
&-\frac{\dot{x}^i}{n+1}\{\dot{\partial}_j\dot{\partial}_k(\dot{\partial}_p B^{sm})\dot{\partial}_{h]}\dot{\partial}_s B^{pr}-\dot{\partial}_j\dot{\partial}_{h]}(\dot{\partial}_p B^{sm})\dot{\partial}_k\dot{\partial}_s B^{pr}\\
&-\dot{\partial}_j\dot{\partial}_k(\dot{\partial}_p B^{sm})\dot{\partial}_{h]}(\dot{\partial}_s B^{pr})+\dot{\partial}_j\dot{\partial}_{h]}(\dot{\partial}_p B^{sm})(\dot{\partial}_k\dot{\partial}_s B^{pr})\\
&+\dot{\partial}_j(\dot{\partial}_k B^{sm})\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_s B^{pr}-\dot{\partial}_j(\dot{\partial}_{h]}B^{sm})\dot{\partial}_k\dot{\partial}_p\dot{\partial}_s B^{pr}\\
&+(\dot{\partial}_k B^{sm})\dot{\partial}_j\dot{\partial}_{h]}\dot{\partial}_p(\dot{\partial}_s B^{pr})-(\dot{\partial}_{h]}B^{sm})\dot{\partial}_j\dot{\partial}_k\dot{\partial}_p(\dot{\partial}_s B^{pr})\}\\
&+\frac{\delta^i_j}{n+1}\{\dot{\partial}_p(\dot{\partial}_{h]}B^{sm})\dot{\partial}_k\dot{\partial}_s B^{pr}-\dot{\partial}_p(\dot{\partial}_k B^{sm})\dot{\partial}_{h]}\dot{\partial}_s B^{pr}\}\\
&+\frac{\delta^i_k}{n^2-1}\{n\dot{\partial}_j(\dot{\partial}_{h]}B^{sm})\dot{\partial}_p\dot{\partial}_s B^{pr}-n\dot{\partial}_j(\dot{\partial}_p B^{sm})\dot{\partial}_{h]}\dot{\partial}_s B^{pr}\\
&+n\dot{\partial}_{h]}(\dot{\partial}_p B^{sm})\dot{\partial}_j\dot{\partial}_s B^{pr}-n(\dot{\partial}_p B^{sm})\dot{\partial}_j\dot{\partial}_{h]}\dot{\partial}_s B^{pr}\\
&+\dot{\partial}_{h]}(\dot{\partial}_j B^{sm})\dot{\partial}_p\dot{\partial}_s B^{pr}-\dot{\partial}_{h]}(\dot{\partial}_p B^{sm})\dot{\partial}_j\dot{\partial}_s B^{pr}\\
&+(\dot{\partial}_j B^{sm})\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_s B^{pr}-(\dot{\partial}_p B^{sm})\dot{\partial}_{h]}\dot{\partial}_j\dot{\partial}_s B^{pr}\\
&+\dot{x}^l\dot{\partial}_j\dot{\partial}_{h]}(\dot{\partial}_l B^{sm})\dot{\partial}_p\dot{\partial}_s B^{pr}-\dot{x}^l\dot{\partial}_j\dot{\partial}_{h]}(\dot{\partial}_p B^{sm})\dot{\partial}_l\dot{\partial}_s B^{pr}\\
&+\dot{x}^l\dot{\partial}_{h]}((\dot{\partial}_l B^{sm})\dot{\partial}_j\dot{\partial}_p\dot{\partial}_s B^{pr}-\dot{x}^l\dot{\partial}_{h]}(\dot{\partial}_p B^{sm})\dot{\partial}_j\dot{\partial}_l\dot{\partial}_s B^{pr}\\
&-\dot{x}^l\dot{\partial}_j\dot{\partial}_{h]}(\dot{\partial}_p B^{sm})\dot{\partial}_l\dot{\partial}_s B^{pr}+\dot{x}^l\dot{\partial}_{h]}((\dot{\partial}_l B^{sm})\dot{\partial}_j\dot{\partial}_p\dot{\partial}_s B^{pr}
\end{aligned}
$$

$$-\dot{x}^l\dot{\partial}_{h]}(\dot{\partial}_p B^{sm})\dot{\partial}_j\dot{\partial}_l\dot{\partial}_s B^{pr} + \dot{x}^l\dot{\partial}_j(\dot{\partial}_l B^{sm})\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_s B^{pr} - \dot{x}^l\dot{\partial}_j(\dot{\partial}_p B^{sm})\dot{\partial}_{h]}\dot{\partial}_l\dot{\partial}_s B^{pr}$$
$$+\dot{x}^l(\dot{\partial}_l B^{sm})\dot{\partial}_j\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_s B^{pr} - \dot{x}^l(\dot{\partial}_p B^{sm})\dot{\partial}_j\dot{\partial}_{h]}\dot{\partial}_l\dot{\partial}_s B^{pr}\} - \frac{\delta^i_{h]}}{n^2-1}\{n\dot{\partial}_j(\dot{\partial}_k B^{sm})\dot{\partial}_p\dot{\partial}_s B^{pr}$$
$$-n\dot{\partial}_j(\dot{\partial}_p B^{sm})\dot{\partial}_k\dot{\partial}_s B^{pr}$$

Proof. Interchanging the indices u and h in (3.1) and subtracting the equation thus obtained to (3.1), we obtain the identity (3.4). □

Theorem 3.3. *When F_n and $\bar{F}_n$ are in conformal correspondence, we have*

$$\begin{aligned}
\bar{W}_{jukh} + \bar{W}_{ujhk} \;=\; & e^{2\sigma}(W_{jukh} + W_{ujhk}) \qquad (3.5)\\
& +4e^{2\sigma}\sigma_m g_{i(u}[\{(\dot{\partial}_{[k}B^{ir})G^m_{h]j)r} - \dot{\partial}_{j)}(\dot{\partial}_{[k}B^{im})_{(h)]}\\
& +\frac{\dot{x}^i}{n+1}\dot{\partial}_{j)}\dot{\partial}_{[k}(\dot{\partial}_p B^{pm})_{(h)]} + \frac{\delta^i_{j)}}{n+1}\{\dot{\partial}_p(\dot{\partial}_{[k}B^{pm})_{(h)]}\\
& -(\dot{\partial}_{[k}B^{pr})G^m_{h]pr}\} - \frac{\delta^i_{[k}}{n^2-1}\{n\dot{\partial}_{j)}(\dot{\partial}_{h]}B^{pm})_{(p)} - n\dot{\partial}_{j)}(\dot{\partial}_p B^{pm})_{(h)]}\\
& +\dot{\partial}_{h]}(\dot{\partial}_{j)}B^{pm})_{(p)} - \dot{\partial}_{h]}(\dot{\partial}_p B^{pm})_{(j))} + \dot{x}^l\dot{\partial}_{j)}\dot{\partial}_{h]}(\dot{\partial}_l B^{pm})_{(p)}\\
& -\dot{\partial}_{j)}\dot{\partial}_{h]}(\dot{\partial}_p B^{pm})_{(l)}\}] + 4e^{2\sigma}\sigma_{m[(k)}g_{i(u}[\{\dot{\partial}_{h]}(\dot{\partial}_{j)}B^{im})\\
& -\frac{\dot{x}^i}{n+1}\dot{\partial}_{j)}(\dot{\partial}_{h]}\dot{\partial}_p B^{pm}) - \frac{\delta^i_{j)}}{n^2-1}\dot{\partial}_{h]}(\dot{\partial}_p B^{pm})\}\\
& +\frac{n\delta^i_{[k}}{n^2-1}\{\sigma_{m(h)]}\dot{\partial}_{j)}(\dot{\partial}_p B^{pm}) - \sigma_{m(p)}(\dot{\partial}_{j)}\dot{\partial}_{h]}B^{pm})\}]\\
& +4e^{2\sigma}\sigma_m\sigma_r g_{i(u}[\{\dot{\partial}_{j)}(\dot{\partial}_{[k}B^{sm})\dot{\partial}_{h]}\dot{\partial}_s B^{ir}\\
& -\frac{\dot{x}^i}{n+1}\dot{\partial}_{j)}\dot{\partial}_{[k}(\dot{\partial}_p B^{sm})\dot{\partial}_{h]}\dot{\partial}_s B^{pr} + \dot{\partial}_{[k}(\dot{\partial}_p B^{sm})\dot{\partial}_{j)}(\dot{\partial}_{h]}\dot{\partial}_s B^{pr})\\
& +\dot{\partial}_{j)}(\dot{\partial}_{[k}B^{sm})\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_s B^{pr} + (\dot{\partial}_{[k}B^{sm})\dot{\partial}_{j)}\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_s B^{pr}\\
& +\frac{\delta^i_{j)}}{n^2-1}\dot{\partial}_p(\dot{\partial}_{[h}B^{sm})\dot{\partial}_{k]}\dot{\partial}_s B^{pr} + \frac{\delta^i_{[k}}{n^2-1}\{n\dot{\partial}_{j)}(\dot{\partial}_{h]}B^{sm})\dot{\partial}_p\dot{\partial}_s B^{pr}\\
& -n\dot{\partial}_{j)}(\dot{\partial}_p B^{sm})\dot{\partial}_{h]}\dot{\partial}_s B^{pr} + n(\dot{\partial}_{h]}B^{sm})\dot{\partial}_{j)}\dot{\partial}_p\dot{\partial}_s B^{pr}\\
& -n(\dot{\partial}_p B^{sm})\dot{\partial}_{j)}\dot{\partial}_{h]}\dot{\partial}_s B^{pr}) + \dot{\partial}_{h]}(\dot{\partial}_{j)}B^{sm})\dot{\partial}_p\dot{\partial}_s B^{pr}\\
& -\dot{\partial}_{h]}(\dot{\partial}_p B^{sm})\dot{\partial}_{j)}\dot{\partial}_s B^{pr} + (\dot{\partial}_{j)}B^{sm})\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_s B^{pr}\\
& -(\dot{\partial}_p B^{sm})\dot{\partial}_{h]}\dot{\partial}_{j)}\dot{\partial}_s B^{pr} + \dot{x}^l\dot{\partial}_{j)}\dot{\partial}_{h]}(\dot{\partial}_l B^{sm})\dot{\partial}_p\dot{\partial}_s B^{pr}\\
& -\dot{\partial}_{j)}\dot{\partial}_{h]}(\dot{\partial}_p B^{sm})\dot{\partial}_l\dot{\partial}_s B^{pr} + \dot{\partial}_{h]}(\dot{\partial}_l B^{sm})\dot{\partial}_{j)}\dot{\partial}_p\dot{\partial}_s B^{pr}\\
& -\dot{\partial}_{h]}(\dot{\partial}_p B^{sm})\dot{\partial}_{j)}\dot{\partial}_l\dot{\partial}_s B^{pr} + \dot{\partial}_{j)}(\dot{\partial}_l B^{sm})\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_s B^{pr}\\
& -\dot{\partial}_{j)}(\dot{\partial}_p B^{sm})\dot{\partial}_{h]}\dot{\partial}_l\dot{\partial}_s B^{pr} + (\dot{\partial}_l B^{sm})\dot{\partial}_{j)}\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_s B^{pr}
\end{aligned}$$

$$-(\dot\partial_p B^{sm})\dot\partial_{j)}\dot\partial_{h]}\dot\partial_l\dot\partial_s B^{pr})\}] + 4e^{2\sigma}\frac{g_{i(u}\delta^i_{[k}}{n-1}[\{\sigma_{m(j))}\dot\partial_{h]}(\dot\partial_p B^{pm})$$
$$-\sigma_{m(p)}\dot\partial_{h]}(\dot\partial_{j)}B^{pm})\} + \dot x^l\{\sigma_{m(l)}\dot\partial_{j)}(\dot\partial_{h]}\dot\partial_p B^{sm})$$
$$-\sigma_{m(p)}\dot\partial_{j)}(\dot\partial_{h]}\dot\partial_l B^{pm})\}]$$

Proof. By interchanging the indices in each pair (j,u) and (k,h) in equation (3.1) and adding the equation thus obtained from (3.1), we obtain the identity (3.5). □

Theorem 3.4. *When F_n and $\bar F_n$ are in conformal correspondence, we have*

$$\bar W_{jukh} + \bar W_{jhku} + \bar W_{khju} + \bar W_{kujh} = e^{2\sigma}(W_{jhku} + W_{jukh} + W_{kujh} + W_{khju}) \tag{3.6}$$
$$+4e^{2\sigma}\sigma_m g_{i(u}[\dot\partial_{[k}B^{ir})G^m_{h)j]r} - \dot\partial_{[j}(\dot\partial_{k]}B^{im})_{(h))} + \dot\partial_{[j}(\dot\partial_{h)}B^{im})_{(k)]}$$
$$+\frac{\dot x^i}{n+1}\{\dot\partial_{[j}\dot\partial_{k]}(\dot\partial_p B^{pm})_{(h))} - \{\dot\partial_{[j}\dot\partial_{h)}(\dot\partial_p B^{pm})_{(k)]}\} + \frac{\delta^i_{[j}}{n+1}\{\dot\partial_p(\dot\partial_{k]}B^{pm})_{(h))}$$
$$-\dot\partial_p(\dot\partial_{h)}B^{pm})_{(k)]} - (\dot\partial_{k]}B^{pr})G^m_{h)pr} + (\dot\partial_{h)}B^{pr})G^m_{k]pr}\} - \frac{\delta^i_{[k}}{n^2-1}\{n\dot\partial_{j]}(\dot\partial_{h)}B^{pm})_{(p)}$$
$$-n\dot\partial_{j]}(\dot\partial_p B^{pm})_{(h))} + \dot\partial_{h)}(\dot\partial_{j]}B^{pm})_{(p)} - \dot\partial_{h)}(\dot\partial_p B^{pm})_{(j)]}\} + \frac{\delta^i_{h)}}{n^2-1}\{n\dot\partial_{[j}(\dot\partial_{k]}B^{pm})_{(p)}$$
$$-n\dot\partial_{[j}(\dot\partial_p B^{pm})_{(k)]} + \dot\partial_{[k}(\dot\partial_{j]}B^{pm})_{(p)} - \dot\partial_{[k}(\dot\partial_p B^{pm})_{(j)]}\} - \frac{\dot x^l\delta^i_{[k}}{n^2-1}\{\dot\partial_{j]}\dot\partial_{h)}(\dot\partial_l B^{pm})_{(p)}$$
$$-\dot\partial_{j]}\dot\partial_{h)}(\dot\partial_p B^{pm})_{(l)]}\} + \frac{\dot x^l\delta^i_{h)}}{n^2-1}\{\dot\partial_{[j}\dot\partial_{k]}(\dot\partial_l B^{pm})_{(p)} - \dot\partial_{[j}\dot\partial_{k]}(\dot\partial_p B^{pm})_{(l)]}\}]$$
$$+4e^{2\sigma}g_{i(u}[\sigma_{m[k}\{\dot\partial_{h)}(\dot\partial_{j]}B^{im}) - \frac{\dot x^i}{n+1}\dot\partial_{j]}(\dot\partial_{h)}\dot\partial_p B^{pm}) - \frac{\delta^i_{j]}}{n+1}\dot\partial_{h)}(\dot\partial_p B^{pm})\}$$
$$-\sigma_{m(h))}\{\dot\partial_{[k}(\dot\partial_{j]}B^{im}) - \frac{\dot x^i}{n+1}\dot\partial_{[j}(\dot\partial_{k]}\dot\partial_p B^{pm}) - \frac{\delta^i_{[j}}{n+1}\dot\partial_{k]}(\dot\partial_p B^{pm})\}$$
$$+\frac{n\delta^i_{[k}}{n^2-1}\{\sigma_{m(h))}(\dot\partial_{j]}\dot\partial_p B^{pm}) - \sigma_{m(p)}(\dot\partial_{j]}\dot\partial_{h)}B^{pm})\} - \frac{n\delta^i_{h)}}{n^2-1}\{\sigma_{m[(k)}(\dot\partial_{j]}\dot\partial_p B^{pm})$$
$$-\sigma_{m(p)}(\dot\partial_{[j}\dot\partial_{k]}B^{pm})\} + 4e^{2\sigma}\sigma_m\sigma_r g_{i(u}[\dot\partial_{[j}(\dot\partial_{k]}B^{sm})\dot\partial_{h)}\dot\partial_s B^{ir} - \dot\partial_{[j}(\dot\partial_{h)}B^{sm})\dot\partial_{k]}\dot\partial_s B^{ir}$$
$$-\frac{\dot x^i}{n+1}\{\dot\partial_{[j}\dot\partial_{k]}(\dot\partial_p B^{sm})\dot\partial_{h)}\dot\partial_s B^{pr} - \dot\partial_{[j}\dot\partial_{h)}(\dot\partial_p B^{sm})\dot\partial_{k]}\dot\partial_s B^{pr}$$
$$-\dot\partial_{[k}(\dot\partial_p B^{sm})\dot\partial_{j]}(\dot\partial_{h)}\dot\partial_s B^{pr}) - \dot\partial_{h)}(\dot\partial_p B^{sm})\dot\partial_{[j}(\dot\partial_{k]}\dot\partial_s B^{pr}) + \dot\partial_{[j}(\dot\partial_{k]}B^{sm})\dot\partial_{h)}\dot\partial_p\dot\partial_s B^{pr}$$
$$-\dot\partial_{[j}(\dot\partial_{h)}B^{sm})\dot\partial_{k]}\dot\partial_p\dot\partial_s B^{pr} - (\dot\partial_{[k}B^{sm})\dot\partial_{j]}\dot\partial_{h)}(\dot\partial_p\dot\partial_s B^{pr}) - (\dot\partial_{h)}B^{sm})\dot\partial_{[j}\dot\partial_{k]}(\dot\partial_p\dot\partial_s B^{pr})\}$$
$$-\frac{\delta^i_{[j}}{n+1}\{\dot\partial_p(\dot\partial_{h)}B^{sm})\dot\partial_{k]}\dot\partial_s B^{pr} - \dot\partial_p(\dot\partial_{k]}B^{sm})\dot\partial_{h)}\dot\partial_s B^{pr}\}$$
$$+\frac{\delta^i_{[k}}{n^2-1}\{n(\dot\partial_{j]}\dot\partial_{h)}B^{sm})\dot\partial_p\dot\partial_s B^{pr} - n(\dot\partial_{j]}\dot\partial_p B^{sm})\dot\partial_{h)}\dot\partial_s B^{pr} + n(\dot\partial_{h)}B^{sm})\dot\partial_{j]}\dot\partial_p\dot\partial_s B^{pr}$$

$$-n(\dot\partial_p B^{sm})\dot\partial_{j]}\dot\partial_{h)}\dot\partial_s B^{pr}+\dot\partial_{h)}(\dot\partial_{j]}B^{sm})\dot\partial_p\dot\partial_s B^{pr}-\dot\partial_{h)}(\dot\partial_p B^{sm})\dot\partial_{j]}\dot\partial_s B^{pr}$$
$$+(\dot\partial_{j]}B^{sm})\dot\partial_{h)}\dot\partial_p\dot\partial_s B^{pr}-(\dot\partial_p B^{sm})\dot\partial_{h)}\dot\partial_{j]}\dot\partial_s B^{pr}\}-\frac{\delta^i_{h)}}{n^2-1}\{n(\dot\partial_{[j}\dot\partial_{k]}B^{sm})\dot\partial_p\dot\partial_s B^{pr}$$
$$-n(\dot\partial_{[j}\dot\partial_p B^{sm})\dot\partial_{k]}\dot\partial_s B^{pr}+n(\dot\partial_{[k}B^{sm})\dot\partial_{j]}\dot\partial_p\dot\partial_s B^{pr}-n(\dot\partial_p B^{sm})\dot\partial_{[j}\dot\partial_{k]}\dot\partial_s B^{pr}$$
$$+\dot\partial_{[k}(\dot\partial_{j]}B^{sm})\dot\partial_p\dot\partial_s B^{pr}-\dot\partial_{[k}(\dot\partial_p B^{sm})\dot\partial_{j]}\dot\partial_s B^{pr}+(\dot\partial_{[j}B^{sm})\dot\partial_{k]}\dot\partial_p\dot\partial_s B^{pr}$$
$$-(\dot\partial_p B^{sm})\dot\partial_{[k}\dot\partial_{j]}\dot\partial_s B^{pr}\}+\frac{\dot x^l\delta^i_{[k}}{n^2-1}\{\dot\partial_{j]}(\dot\partial_{h)}\dot\partial_l B^{sm})\dot\partial_p\dot\partial_s B^{pr}-\dot\partial_{[j}\dot\partial_{h)}(\dot\partial_p B^{sm})\dot\partial_l\dot\partial_s B^{pr}$$
$$+\dot\partial_{h)}(\dot\partial_l B^{sm})\dot\partial_{j]}\dot\partial_p\dot\partial_s B^{pr}-\dot\partial_{h)}(\dot\partial_p B^{sm})\dot\partial_{j]}\dot\partial_l\dot\partial_s B^{pr}+\dot\partial_{j]}(\dot\partial_l B^{sm})\dot\partial_{h)}(\dot\partial_p\dot\partial_s B^{pr})$$
$$-\dot\partial_{j]}(\dot\partial_p B^{sm})\dot\partial_{h)}(\dot\partial_l\dot\partial_s B^{pr})+(\dot\partial_l B^{sm})\dot\partial_{j]}\dot\partial_{h)}\dot\partial_p\dot\partial_s B^{pr}-(\dot\partial_p B^{sm})\dot\partial_{j]}\dot\partial_{h)}\dot\partial_l\dot\partial_s B^{pr}\}$$
$$-\frac{\dot x^l\delta^i_{h)}}{n^2-1}\{\dot\partial_{[j}(\dot\partial_{k]}\dot\partial_l B^{sm})\dot\partial_p\dot\partial_s B^{pr}-\dot\partial_{[j}\dot\partial_{k]}(\dot\partial_p B^{sm})\dot\partial_l\dot\partial_s B^{pr}+\dot\partial_{[k}(\dot\partial_l B^{sm})\dot\partial_{j]}\dot\partial_p\dot\partial_s B^{pr}$$
$$-\dot\partial_{[k}(\dot\partial_p B^{sm})\dot\partial_{j]}\dot\partial_l\dot\partial_s B^{pr}+\dot\partial_{[j}(\dot\partial_l B^{sm})\dot\partial_{k]}(\dot\partial_p\dot\partial_s B^{pr})-\dot\partial_{[j}(\dot\partial_p B^{sm})\dot\partial_{k]}(\dot\partial_l\dot\partial_s B^{pr})$$
$$+(\dot\partial_l B^{sm})\dot\partial_{[j}\dot\partial_{k]}\dot\partial_p\dot\partial_s B^{pr}-(\dot\partial_p B^{sm})\dot\partial_{[j}\dot\partial_{k]}\dot\partial_l\dot\partial_s B^{pr}\}]$$
$$+4e^{2\sigma}g_{i(u}[\frac{\delta^i_{[k}}{n^2-1}\{\sigma_{m(j)]}\dot\partial_{h)}(\dot\partial_p B^{pm})-\sigma_{m(p)}\dot\partial_{h)}(\dot\partial_{j]}B^{pm})+\dot x^l\sigma_{m(l)}\dot\partial_{j]}(\dot\partial_{h)}\dot\partial_p B^{pm})$$
$$-\dot x^l\sigma_{m(p)}\dot\partial_{j]}\dot\partial_{h)}(\dot\partial_l B^{pm})\}-\frac{\delta^i_{h)}}{n^2-1}\{\sigma_{m[(j)}\dot\partial_{k]}(\dot\partial_p B^{pm})-\sigma_{m(p)}\dot\partial_{[k}(\dot\partial_{j]}B^{pm})$$
$$+\dot x^l\sigma_{m(l)}\dot\partial_{[j}(\dot\partial_{k]}\dot\partial_p B^{pm})-\dot x^l\sigma_{m(p)}\dot\partial_{[j}\dot\partial_{k]}(\dot\partial_l B^{pm})\}]$$

Proof. The proof follows in consequence of (3.1) and (3.4). □

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