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Neutrosophic-Cubic Analaytic Hierarchy Process with Applications

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Abstract

In this paper we extend fuzzy analytic hierarchy process into neutrosophic cubic environment. The neutrosophic cubic analytic hierarchy process can be used to manage more complex problems when the decision makers has a number of uncertainty, assigning preferences values to the considered object. We also define the concept of triangular neutrosophic cubic numbers and their operations laws. The advantages of the proposed methodology and the application of neutrosophic cubic analytic hierarchy process in decision making are shown by testing the numerical example in practical life.

1 Introduction

Analytic hierarchy process is initially developed by Saaty [13] and it held a very important place in the field of operation research while selecting the best alternatives. Analytic hierarchy process (AHP) is a multi criterion technique which is used to solve and analyze complex problems. AHP is also used to make mathematical and programming devices to select those alternatives [14]. AHP has various steps; In the first step, the problems are structured in order for clear understanding. This order is based on a particular pattern, it consists of the goal, decision making criteria, sub-criteria and in the last all accessible alternatives. When the hierarchy is structured, the decision makers construct pair wise comparison matrices. The scale that is used to measure criteria is called Satty's scale [15]. On the basis of measurement, alternative is first determined and then ranked. The AHP can predict both qualitative and quantitative elements. This quality is widely used in multi-criteria decision making technique. In practical life, decision criterion is a habitually hazy, difficult and conflicting. Along with that, there is uncertainty and using non-fuzzy value in a decision matrix sometimes gets inaccurate. The other thing is that the information available to the decision makers is indeterminant, see paper [16]. This is also a factor, in the way of accuracy. Many researchers started to use a new theory called the fuzzy set theory [24]. However, there were some drawbacks in that theory that it only considers truth membership degree. First time, Van Laarhoven and Pedrycz [20] introduced fuzzy AHP, in which the membership is taken in terms of triangular fuzzy number and used a logarithmic least squares method to obtain the fuzzy weight and fuzzy concert scores for ranking the alternatives. Through this, they were able to control the indeterminacy, inconsistency and inaccuracy.

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Notation	Description
X	Ground set
x	Element of ground set (X)
Υ	Fuzzy set
$\widetilde{\Upsilon}=\left[\Upsilon^{L},\Upsilon^{U}\right]$	Interval valued fuzzy set
$T_{N_C}, I_{N_C}, F_{N_C}$	Components of neutrosophic set
$\widetilde{T}_{N_C}, \widetilde{I}_{N_C}, \widetilde{F}_{N_C}$	Components of interval valued neutrosophic set
$\widetilde{T}_{N_C}, \widetilde{I}_{N_C}, \widetilde{F}_{N_C}, T_{N_C}, I_{N_C}, F_{N_C}$	Components of neutrosophic cubic set

Table 1: Some notations with their descriptions.

Then Buckley [2] extended the classical AHP with the trapezoidal fuzzy number and gets the fuzzy weight and fuzzy concert scores in the geometric mean method. After that, Chang [4] used row mean method to derived priority for similarity ratio in the perspective of triangular fuzzy numbers. The Chang method is comparatively easier than the other fuzzy AHP approach. Even though the computation of fuzzy AHP is tiresome it is appropriate to confine and represent the human assessment of haziness when the multifarious multi-criteria decision making is considerd. It has been practical as a depiction of the fuzzy AHP too many special areas, for instance public administration [8], airlines industry [12], manufacturing industry [6], textile industry [3], electronic industry [5], oil industry [7], entertainment industry [23], transportation industry [11] etc. Xu and Liao [21], provided the concept of intuitionistic fuzzy analytic hierarchy process, which is a new approach in AHP. Cubic set was developed by Jun in 2012 [9]. In 1998 Smarandache [17] define the neutrosophic set (NS). In neutrosophic set (NS) Samarndache add indeterminacy-membership function, i.e NS is composed of truth-membership $T(u)$, indeterminacy-membership $I(u)$ and falsity-membership $F(u)$, also see [18, 22]. Samarndache [19] also proved that neutrosophic set is a generalization of intuitionistic fuzzy set, inconsistent intuitionistic fuzzy set (picture fuzzy set, ternary fuzzy set), Pythagorean fuzzy set (Atanassov's intuitionistic fuzzy set of second type), q-Rung orthopair fuzzy set, spherical fuzzy set, and n-hyper spherical fuzzy set, while neutrosophication is a generalization of regret theory, grey system theory, and three-ways decision Jun et al [10], projected the new concept of neutrosophic cubic set, which have three grade of membership true, false and indeterrminacy. Basset [1], takes the fuzzy analytic hierarchy process into the neutrosophic analytic hierarchy process. In continuation of these efforts in this work we develop an algorithm for neutrosophic-cubic analytic hierarchy process for solving complicated multi-criteria decision making problems. The arrangement of remaining paper is as follows: Section 2: review of basic concepts and properties. Section 3: we present the triangular neutrosophic-cubic numbers and operation rules. Section 4: present the algorithm for neutrosophic-cubic analytic hierarchy process. Section 5: as an application we use this in a practical example of the daily life problem. Section 6: we compare our method to other existing methods and finally we write the conclusion of this paper in section 7.

2 Preliminaries

This section consists of two parts: Some notations with their descriptions and some previous definitions and results.

2.1 Notations

This section consists of some notations with their descriptions, as shown in Table 1.

2.2 Pre-defined Definitions

In this section, we add some important definitions of the cubic sets which are subsequently used to prove our results.

Definition 2.1. *[24]A mapping $\Upsilon : X \to [0,1]$ is called a fuzzy set, and $\Upsilon(x)$ is a membership function and denoted by Υ.*

Definition 2.2. *[9]A structure $C = \left\{ \left(x; \widetilde{\Upsilon}(x), \Upsilon(x) \mid x \in X \right) \right\}$ is a cubic set in X in which $\widetilde{\Upsilon}(x)$ is IVF in X,and $\Upsilon(x)$ is fuzzy set in X. this is simply denotd by $C = \left(\widetilde{\Upsilon}, \Upsilon \right)$. C^X denotes the collection of cubic sets in X.*

Definition 2.3. *[17]A neutrosophic set is a structure*

$$N = \{(x; T_N(x), I_N(x), F_N(x) \mid x \in X)\}$$

in X. Here $(T_N(x), I_N(x), F_N(x) \in [0,1])$ are called truth, indeterminacy and falsity functions, respectively. Simply denoted by $N = (T_N, I_N, F_N)$.

Definition 2.4. *[22] An interval neutrosophic set is a structure*

$$N = \left\{ \left(x; \widetilde{T}_{N_C}(x), \widetilde{I}_{N_C}(x), \widetilde{F}_{N_C}(x) \mid x \in X \right) \right\}$$

where $\left(x; \widetilde{T}_{N_C}(x), \widetilde{I}_{N_C}(x), \widetilde{F}_{N_C}(x), x \in D[0,1] \right)$ is called truth, indeterminacy and falsity function in X, respectively.This can be simply denoted by $N = \left(\widetilde{T}_{N_C}, \widetilde{I}_{N_C}, \widetilde{F}_{N_C} \right)$.

Definition 2.5. *[10] A structure*

$$N_C = \left\{ \left(x; \widetilde{T}_{N_C}(x), \widetilde{I}_{N_C}(x), \widetilde{F}_{N_C}(x), T_{N_C}(x), I_{N_C}(x), F_{N_C}(x) \mid x \in X \right) \right\}$$

is neutrosophic cubic set in X. Here

$$\left(\widetilde{T}_{N_C} = [T^L_{N_C}, T^U_{N_C}], \widetilde{I}_{N_C} = [I^L_{N_C}, I^U_{N_C}], \widetilde{F}_{N_C}[F^L_{N_C}, F^U_{N_C}] \right)$$

is an interval neutrosophic set and $(T_{N_C}, I_{N_C}, F_{N_C})$ is a neutrosophic set in X. Simply denoted by

$$N_C = \left(\widetilde{T}_{N_C}, \widetilde{I}_{N_C}, \widetilde{F}_{N_C}, T_{N_C}, I_{N_C}, F_{N_C} \right),$$

$$[0,0] \le \widetilde{T}_{N_C} + \widetilde{I}_{N_C} + \widetilde{F}_{N_C} \le [3,3],$$
$$0 \le T_{N_C} + I_{N_C} + F_{N_C} \le 1.$$

N_C^X denotes the collection of neutrosophic-cubic sets in X.

3 Triangular Neutrosophic-Cubic Numbers

In this section we define the concept of neutrosophic-cubic numbers which is then used in next section.

Definition 3.1. *A neutrosophic-cubic number*

$$\begin{aligned} &N_C = \tilde{a} = \langle (a_1, a_2, a_3)\,; \\ &\widetilde{T}_{N_C} = [\alpha^L_{\tilde{a}}, \alpha^U_{\tilde{a}}], T_{N_C} = \alpha_{\tilde{a}}, \widetilde{I}_{N_C} = [\theta^L_{\tilde{a}}, \theta^U_{\tilde{a}}], I_{N_C} = \theta_{\tilde{a}}, \widetilde{F}_{N_C} = [\beta^L_{\tilde{a}}, \beta^U_{\tilde{a}}], F_{N_C} = \beta_{\tilde{a}} \rangle, \end{aligned}$$

such that $\langle \widetilde{T}_{N_C}, \widetilde{I}_{N_C}, \widetilde{F}_{N_C} \rangle = R \to D[0,1]$ *and* $\langle T_{N_C}, I_{N_C}, F_{N_C} \rangle = R \to [0,1]$ *is*

1. *a neutrosophic-cubic subset of the real lines,*
2. *Normal* $\exists\, x_0 \in R$ *such that* $\left\langle \widetilde{T}_{N_C} \right\rangle = [1,1], \left\langle \widetilde{I}_{N_C}, \widetilde{F}_{N_C} \right\rangle = [0,0]$ *and* $\langle T_{N_C} \rangle = 1, \langle I_{N_C}, F_{N_C} \rangle = [0,0]$.
3. *Convex for the truth membership: i.e*

$$\begin{aligned} \widetilde{T}_{N_C}(tx + (1-t)y) &\succcurlyeq \min\{\widetilde{T}_{N_C}(x), \widetilde{T}_{N_C}(y)\}, \\ T_{N_C}(tx + (1-t)y) &\geq \min\{T_{N_C}(x), T_{N_C}(y)\}, \end{aligned}$$

4. *concave for the falsity and indeterminacy membership : i.e*

$$\begin{aligned} \widetilde{I}_{N_C}(tx + (1-t)y) &\preccurlyeq \max\{\widetilde{I}_{N_C}(x), \widetilde{I}_{N_C}(y)\}, \\ I_{N_C}(tx + (1-t)y) &\leq \max\{I_{N_C}(x), I_{N_C}(y)\}, \\ \widetilde{F}_{N_C}(tx + (1-t)y) &\preccurlyeq \max\{\widetilde{F}_{N_C}(x), \widetilde{F}_{N_C}(y)\}, \\ F_{N_C}(tx + (1-t)y) &\leq \max\{F_{N_C}(x), F_{N_C}(y)\}. \end{aligned}$$

$\forall t \in [0,1]\; x, y \in R$, *whose truth memberships functions are:*

$$\widetilde{T}_{\tilde{a}}(x) = \left\{ \begin{array}{ll} \frac{x-d}{e-d}[T^L_{\tilde{a}}, T^U_{\tilde{a}}] & d \leqslant x < e \\ \frac{f-x}{f-e}[T^L_{\tilde{a}}, T^U_{\tilde{a}}] & e < x \leqslant f \\ 0 & \textit{otherwise} \end{array} \right\} \tag{3.1}$$

$$T_{\tilde{a}}(x) = \left\{ \begin{array}{ll} \frac{x-d}{e-d}T_{\tilde{a}} & d \leqslant x < e \\ \frac{f-x}{f-e}T_{\tilde{a}} & e < x \leqslant f \\ 0 & \textit{otherwise} \end{array} \right\} \tag{3.2}$$

indertermincy memberships functions:

$$\widetilde{I}_{\tilde{a}}(x) = \left\{ \begin{array}{ll} \frac{(e-x)+(x-d)}{e-d}[I^L_{\tilde{a}}, I^U_{\tilde{a}}] & d \leqslant x < e \\ \frac{(x-e)+(f-x)}{f-e}[I^L_{\tilde{a}}, I^U_{\tilde{a}}] & e < x \leqslant f \\ 1 & \textit{otherwise} \end{array} \right\} \tag{3.3}$$

$$I_{\tilde{a}}(x) = \left\{ \begin{array}{ll} \frac{(e-x)+(x-d)}{e-d}I_{\tilde{a}} & d \leqslant x < e \\ \frac{(x-e)+(f-x)}{f-e}I_{\tilde{a}} & e < x \leqslant f \\ 1 & \textit{otherwise} \end{array} \right\} \tag{3.4}$$

falsity memberships functions:

$$\widetilde{F}_{\tilde{a}}(x)=\left\{\begin{array}{cc} \frac{e-x}{e-d}[F_{\tilde{a}}^{L},F_{\tilde{a}}^{U}] & d\leqslant x<e \\ \frac{x-e}{f-e}[F_{\tilde{a}}^{L},F_{\tilde{a}}^{U}] & e<x\leqslant f \\ 1 & \textit{otherwise} \end{array}\right\} \tag{3.5}$$

$$F_{\tilde{a}}(x)=\left\{\begin{array}{cc} \frac{e-x}{e-d}F_{\tilde{a}} & d\leqslant x<e \\ \frac{x-e}{f-e}F_{\tilde{a}} & e<x\leqslant f \\ 1 & \textit{otherwise} \end{array}\right\} \tag{3.6}$$

Definition 3.2. *Let*

$$\widetilde{a}=\langle(a_1,a_2,a_3),[T_{\tilde{a}}^{L},T_{\tilde{a}}^{U}],T_{\tilde{a}},[I_{\tilde{a}}^{L},I_{\tilde{a}}^{U}],I_{\tilde{a}}^{L},[F_{\tilde{a}}^{L},F_{\tilde{a}}^{U}],F_{\tilde{a}}^{U}\rangle,$$

and

$$\widetilde{b}=\langle(b_1,b_2,b_3),[T_{\widetilde{b}}^{L},T_{\widetilde{b}}^{U}],T_{\widetilde{b}},[I_{\widetilde{b}}^{L},I_{\widetilde{b}}^{U}],I_{\widetilde{b}}^{L},[F_{\widetilde{b}}^{L},F_{\widetilde{b}}^{U}],F_{\widetilde{b}}^{U}\rangle,$$

be two triangular neutrosophic cubic numbers and $\gamma\neq 0$ be any real number. Then:

1. *Addition of two TNCNs are*

$$\tilde{a}+\widetilde{b}=\left\{\begin{array}{c} \langle(a_1+b_1,a_2+b_2,a_3+b_3), \\ \left[\min(T_{\tilde{a}}^{L},T_{\widetilde{b}}^{L}),\min(T_{\tilde{a}}^{U},T_{\widetilde{b}}^{U})\right],\min(T_{\tilde{a}},T_{\widetilde{b}}), \\ \left[\max(I_{\tilde{a}}^{L},I_{\widetilde{b}}^{L}),\max(I_{\tilde{a}}^{U},I_{\widetilde{b}}^{U})\right],\max(I_{\tilde{a}},I_{\widetilde{b}}), \\ \left[\max(F_{\tilde{a}}^{L},F_{\widetilde{b}}^{L}),\max(F_{\tilde{a}}^{U},I_{\widetilde{b}}^{U})\right],\max(F_{\tilde{a}},F_{\widetilde{b}})\rangle \end{array}\right\} \tag{3.7}$$

2. *Subtraction of two TNCNs are*

$$\tilde{a}-\widetilde{b}=\left\{\begin{array}{c} \langle(a_1-b_3,a_2-b_2,a_3-b_1), \\ \left[\min(T_{\tilde{a}}^{L},T_{\widetilde{b}}^{L}),\min(T_{\tilde{a}}^{U},T_{\widetilde{b}}^{U})\right],\min(T_{\tilde{a}},T_{\widetilde{b}}), \\ \left[\max(I_{\tilde{a}}^{L},I_{\widetilde{b}}^{L}),\max(I_{\tilde{a}}^{U},I_{\widetilde{b}}^{U})\right],\max(I_{\tilde{a}},I_{\widetilde{b}}), \\ \left[\max(F_{\tilde{a}}^{L},F_{\widetilde{b}}^{L}),\max(F_{\tilde{a}}^{U},I_{\widetilde{b}}^{U})\right],\max(F_{\tilde{a}},F_{\widetilde{b}})\rangle \end{array}\right\} \tag{3.8}$$

3. *Multiplication of a TNCNs by a scalar value are*

$$\gamma\tilde{a}=\left\{\begin{array}{c} \langle(\gamma a_1,\gamma a_2,\gamma a_3),[T_{\tilde{a}}^{L},T_{\tilde{a}}^{U}],T_{\tilde{a}}, \\ [I_{\tilde{a}}^{L},I_{\tilde{a}}^{U}],I_{\tilde{a}}^{L},[F_{\tilde{a}}^{L},F_{\tilde{a}}^{U}],F_{\tilde{a}}^{U}\rangle \textit{ if } (\gamma>0) \\ \langle(\gamma a_3,\gamma a_2,\gamma a_1),[T_{\tilde{a}}^{L},T_{\tilde{a}}^{U}],T_{\tilde{a}},[I_{\tilde{a}}^{L},I_{\tilde{a}}^{U}], \\ I_{\tilde{a}}^{L},[F_{\tilde{a}}^{L},F_{\tilde{a}}^{U}],F_{\tilde{a}}^{U}\rangle \textit{ if } (\gamma<0) \end{array}\right\} \tag{3.9}$$

4. *Inverse of a TNCNs are*

$$\tilde{a}^{-1}=\left\{\begin{array}{c} \langle\left(\frac{1}{a_3},\frac{1}{a_2},\frac{1}{a_1}\right),[T_{\tilde{a}}^{L},T_{\tilde{a}}^{U}],T_{\tilde{a}}, \\ [I_{\tilde{a}}^{L},I_{\tilde{a}}^{U}],I_{\tilde{a}}^{L},[F_{\tilde{a}}^{L},F_{\tilde{a}}^{U}],F_{\tilde{a}}^{U}\rangle \textit{ where } (\tilde{a}\neq 0)\rangle \end{array}\right\} \tag{3.10}$$

5. Multiplication of two TNCNs are

$$\tilde{a}\widetilde{b} = \left\{ \begin{array}{c} \langle (a_1 b_1, a_2 b_2, a_3 b_3)\,; \\ \left[\min(T^L_{\tilde{a}}, T^L_{\widetilde{b}}), \min(T^U_{\tilde{a}}, T^U_{\widetilde{b}})\right] \min(T_{\tilde{a}}, T_{\widetilde{b}}), \\ \left[\max(I^L_{\tilde{a}}, I^L_{\widetilde{b}}), \max(I^U_{\tilde{a}}, I^U_{\widetilde{b}})\right] \max(I_{\tilde{a}}, I_{\widetilde{b}}), \\ \left[\max(F^L_{\tilde{a}}, F^L_{\widetilde{b}}), \max(F^U_{\tilde{a}}, I^U_{\widetilde{b}})\right] \max(F_{\tilde{a}}, F_{\widetilde{b}})\rangle \\ \textit{if}\,(a_3 > 0, b_3 > 0) \\ \langle (a_1 b_3, a_2 b_2, a_3 b_1)\,; \\ \left[\min(T^L_{\tilde{a}}, T^L_{\widetilde{b}}), \min(T^U_{\tilde{a}}, T^U_{\widetilde{b}})\right] \min(T_{\tilde{a}}, T_{\widetilde{b}}), \\ \left[\max(I^L_{\tilde{a}}, I^L_{\widetilde{b}}), \max(I^U_{\tilde{a}}, I^U_{\widetilde{b}})\right] \max(I_{\tilde{a}}, I_{\widetilde{b}}), \\ \left[\max(F^L_{\tilde{a}}, F^L_{\widetilde{b}}), \max(F^U_{\tilde{a}}, I^U_{\widetilde{b}})\right] \max(F_{\tilde{a}}, F_{\widetilde{b}})\rangle \\ \textit{if}\,(a_3 < 0, b_3 > 0) \\ \langle (a_1 b_3, a_2 b_2, a_3 b_1)\,; \\ \left[\min(T^L_{\tilde{a}}, T^L_{\widetilde{b}}), \min(T^U_{\tilde{a}}, T^U_{\widetilde{b}})\right] \min(T_{\tilde{a}}, T_{\widetilde{b}}), \\ \left[\max(I^L_{\tilde{a}}, I^L_{\widetilde{b}}), \max(I^U_{\tilde{a}}, I^U_{\widetilde{b}})\right] \max(I_{\tilde{a}}, I_{\widetilde{b}}), \\ \left[\max(F^L_{\tilde{a}}, F^L_{\widetilde{b}}), \max(F^U_{\tilde{a}}, I^U_{\widetilde{b}})\right] \max(F_{\tilde{a}}, F_{\widetilde{b}})\rangle \\ \textit{if}\,(a_3 < 0, b_3 < 0) \end{array} \right\} \tag{3.11}$$

6. Division of two TNCNs are

$$\frac{\tilde{a}}{\widetilde{b}} = \left\{ \begin{array}{c} \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right); \\ \left[\min(T^L_{\tilde{a}}, T^L_{\widetilde{b}}), \min(T^U_{\tilde{a}}, T^U_{\widetilde{b}})\right] \min(T_{\tilde{a}}, T_{\widetilde{b}}), \\ \left[\max(I^L_{\tilde{a}}, I^L_{\widetilde{b}}), \max(I^U_{\tilde{a}}, I^U_{\widetilde{b}})\right] \max(I_{\tilde{a}}, I_{\widetilde{b}}), \\ \left[\max(F^L_{\tilde{a}}, F^L_{\widetilde{b}}), \max(F^U_{\tilde{a}}, I^U_{\widetilde{b}})\right] \max(F_{\tilde{a}}, F_{\widetilde{b}}) \\ \textit{if}(a_3 > 0, b_3 > 0) \\ \left(\frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}\right); \\ \left[\min(T^L_{\tilde{a}}, T^L_{\widetilde{b}}), \min(T^U_{\tilde{a}}, T^U_{\widetilde{b}})\right] \min(T_{\tilde{a}}, T_{\widetilde{b}}), \\ \left[\max(I^L_{\tilde{a}}, I^L_{\widetilde{b}}), \max(I^U_{\tilde{a}}, I^U_{\widetilde{b}})\right] \max(I_{\tilde{a}}, I_{\widetilde{b}}), \\ \left[\max(F^L_{\tilde{a}}, F^L_{\widetilde{b}}), \max(F^U_{\tilde{a}}, I^U_{\widetilde{b}})\right] \max(F_{\tilde{a}}, F_{\widetilde{b}}) \\ \textit{if}\,(a_3 > 0, b_3 > 0) \\ \left(\frac{a_3}{b_1}, \frac{a_2}{b_2}, \frac{a_1}{b_3}\right); \\ \left[\min(T^L_{\tilde{a}}, T^L_{\widetilde{b}}), \min(T^U_{\tilde{a}}, T^U_{\widetilde{b}})\right] \min(T_{\tilde{a}}, T_{\widetilde{b}}), \\ \left[\max(I^L_{\tilde{a}}, I^L_{\widetilde{b}}), \max(I^U_{\tilde{a}}, I^U_{\widetilde{b}})\right] \max(I_{\tilde{a}}, I_{\widetilde{b}}), \\ \left[\max(F^L_{\tilde{a}}, F^L_{\widetilde{b}}), \max(F^U_{\tilde{a}}, I^U_{\widetilde{b}})\right] \max(F_{\tilde{a}}, F_{\widetilde{b}}) \\ \textit{if}\,(a_3 > 0, b_3 > 0) \end{array} \right. \tag{3.12}$$

7. Division of a TNCNs by a scalar value are

$$\frac{\tilde{a}}{\gamma} = \left\{ \begin{array}{l} \left(\frac{a_1}{\gamma}, \frac{a_2}{\gamma}, \frac{a_3}{\gamma}\right), [T^L_{\tilde{a}}, T^U_{\tilde{a}}], T_{\tilde{a}}, [I^L_{\tilde{a}}, I^U_{\tilde{a}}], I^L_{\tilde{a}}, [F^L_{\tilde{a}}, F^U_{\tilde{a}}], F^U_{\tilde{a}} \rangle \;\; \textit{if} \;\; (\gamma > 0) \\ \left(\frac{a_3}{\gamma}, \frac{a_2}{\gamma}, \frac{a_1}{\gamma}\right), [T^L_{\tilde{a}}, T^U_{\tilde{a}}], T_{\tilde{a}}, [I^L_{\tilde{a}}, I^U_{\tilde{a}}], I^L_{\tilde{a}}, [F^L_{\tilde{a}}, F^U_{\tilde{a}}], F^U_{\tilde{a}} \rangle \;\; \textit{if} \;\; (\gamma < 0) \end{array} \right\} \tag{3.13}$$

4 Neutrosophic Cubic Analytic Hierarchy Process

In this section, we introduced the AHP in the neutrosophic cubic environment. The Analytic Hierarchy Process includes three stages:

1. Decomposition,
2. Pair-wise comparison,
3. Synthesis of priorties.

Next a stepwise procedure is described for the model.

Step (1): Construct the structure of hierarchy of the problem, which has three levels (i) The first level is goal which organization wants to attained. (ii) .The second level that consists of the criteria and sub criteria. (iii) The third level is how the different alternatives are evaluated. The general hierarchy is presented in figure1.The next step is applied for the weighting criteria, sub-criteria and alternatives, according to expert's opinions

Step (1): First analyzing the intricate multi- criteria decision making problem into a hierarchical structure then use neutrosophic-cubic pair-wise comparison matrix. The vagueness of decision makers is represented by triangular neutrosophic -cubic number (TNCNs) and then construct neutrosophic- cubic pair-wise comparison matrix of criteria, sub-criteria and alternatives structured through the linguistic terms which are shown in Table 2. The neutrosophic cubic scale is obtained on the basis of expert's opinion. The neutrosophic cubic pair-wise comparison matrix are as follows of criteria, sub-criteria and alternatives:

$$\tilde{A} = \begin{bmatrix} \widetilde{1} & \widetilde{a}_{12} & . & . & . & \widetilde{a}_{1n} \\ \widetilde{a}_{21} & . & & & & . \\ . & . & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ \widetilde{a}_{n1} & \widetilde{a}_{n2} & . & . & . & \widetilde{1} \end{bmatrix} \tag{4.1}$$

where $\widetilde{a}_{ji} = \widetilde{a}_{ij}^{-1}$ is the triangular neutrosophic cubic number which is used to measures the imprecision in decision.

Step (3): For calculating overall preferences of each alternative and to conclude final ranking, we should first determine weights of each standard from the corresponding pair-wise comparison matrix, by transforming it to a deterministic matrix using the following equation. Let

$$\widetilde{a}_{ij} = \langle (a_1, a_2, a_3)\, ; [T^L_{\tilde{a}}, T^U_{\tilde{a}}], T_{\tilde{a}}, [I^L_{\tilde{a}}, I^U_{\tilde{a}}], I_{\tilde{a}}, [F^L_{\tilde{a}}, F^U_{\tilde{a}}], F_{\tilde{a}} \rangle$$

be a triangular neutrosophic-cubic number; then

Table 2.	Linguistic terms and the identical triangular neutrosophic-cubic numbers.	
Saaty Scale	Explanation	Neutrosophic-Cubic Triangular Scale
1	Equally influential	$\widetilde{1} = \langle(1,1,1); [0.50,0.50], 0.50, [0.50,0.50], 0.50, [0.50,0.50], 0.50\rangle$
3	Slighty influential	$\widetilde{3} = \langle(2,3,4); [0.25,0.35], 0.30, [0.55,0.65], 0.60, [0.65,0.75], 0.70\rangle$
5	Strongly influential	$\widetilde{5} = \langle(4,5,6); [0.65,0.75], 0.70, [0.15,0.25], 0.20, [0.25,0.35], 0.30\rangle$
7	Very Strongly influential	$\widetilde{7} = \langle(6,7,8); [0.85,0.95], 0.90, [0.20,0.20], 0.20, [0.20,0.20], 0.20\rangle$
9	Absolutely influential	$\widetilde{9} = \langle(9,9,9); [1.00,1.00], 1.00, [0.05,0.05], 0.05, [0.10,0.10], 0.10\rangle$
2		$\widetilde{2} = \langle(1,2,3); [0.50,0.50], 0.50, [0.50,0.50], 0.50, [0.50,0.50], 0.50\rangle$
4	intermediate values	$\widetilde{4} = \langle(3,4,5); [0.50,0.50], 0.50, [0.50,0.50], 0.50, [0.50,0.50], 0.50\rangle$
6		$\widetilde{6} = \langle(6,7,8); [0.50,0.50], 0.50, [0.50,0.50], 0.50, [0.50,0.50], 0.50\rangle$
8		$\widetilde{8} = \langle(7,8,9); [0.50,0.50], 0.50, [0.50,0.50], 0.50, [0.50,0.50], 0.50\rangle$

$$S(\widetilde{a}_{ij}) = \frac{1}{21}\left[a_1 + a_2 + a_3\right]\begin{pmatrix} \left[3 + T^L_{\tilde{a}} + I^L_{\tilde{a}} + F^L_{\tilde{a}}\right] \\ + \left[3 + T^U_{\tilde{a}} + I^U_{\tilde{a}} + F^U_{\tilde{a}}\right] + \left[3 + T_{\tilde{a}} + I_{\tilde{a}} + F_{\tilde{a}}\right] \end{pmatrix} \tag{4.2}$$

and

$$A(\widetilde{a}_{ij}) = \frac{1}{21}\left[a_1 + a_2 + a_3\right]\begin{pmatrix} \left[3 + T^L_{\tilde{a}} + I^L_{\tilde{a}} + F^L_{\tilde{a}}\right] \\ + \left[3 + T^U_{\tilde{a}} + I^U_{\tilde{a}} + F^U_{\tilde{a}}\right] - \left[3 + T_{\tilde{a}} + I_{\tilde{a}} + F_{\tilde{a}}\right] \end{pmatrix} \tag{4.3}$$

are the score function and accuracy function.

To acquire the score and accuracy function of $\widetilde{a}_{ji}$, we use the following equation

$$S(\widetilde{a}_{ji}) = \frac{1}{S(\widetilde{a}_{ij})} \tag{4.4}$$

$$A(\widetilde{a}_{ji}) = \frac{1}{A(\widetilde{a}_{ij})} \tag{4.5}$$

With compensation by score value of each triangular neutrosophic-cubic number in the neutrosophic-cubic pair-wise comparison matrix, we get the following deterministic matrix;

$$A = \begin{bmatrix} 1 & a_{12} & . & . & . & a_{1n} \\ a_{21} & . & & & & . \\ . & . & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ a_{n1} & a_{n2} & . & . & . & 1 \end{bmatrix} \tag{4.6}$$

Now we can easily find ranking of priorities namely Eigen vector X, from the above matrix.

1. Normalize the column entries by dividing each entry by the sum of the column.
2. Take the total of the row averages.

Step (4): To measure inconsistency; within the decision in each pair-wise comparison matrix the complete hierarch, AHP methodology gives a consistency index .Check if any inconsistency in a neutrosophic-cubic comparison matrix. AHP uses consistency index and consistency ratio, If consistency ratio is greater than 10, then judgment are unreliable because they are so closely uncertainty and the procedure is not correct and must be repeated.

To compute CI and CR do the following steps:

1. Multiply each value in the first column of the pair-wise comparison matrix by the priority of the first item; we will keep the process same for all the columns of the pair-wise comparison matrix. Sum the values across the rows to get a vector of values (weighted sum).

2. Get the elements of the weighted sum vector by the corresponding priority for each criterion.

3. Calculate the average of the values found in step 2; this average is denoted by λ max.

4. Calculate the consistency index (CI) as follows:

$$CI = \frac{\lambda \max - n}{n-1} \tag{4.7}$$

where n is the number of items which can be compared.

5. Calculate the consistency ratio, which is defined as:

$$CR = \frac{CI}{RI} \tag{4.8}$$

where RI is the consistency index of a randomly generated comparison matrix.

Step (5): In the last step, calculate overall weight (priority) of each alternative. Final ranking of all the alternatives is calculated using equation [22].

$$T_{w_{Alt_i}} = \sum_{j=1}^{n} (w_j \otimes w_{ij}) \tag{4.9}$$

where $(j = 1, ..., n)$. From previous steps, we achieve the phases of neutrosophic-cubic analytic hierarchy process, as shown in Figure 1.

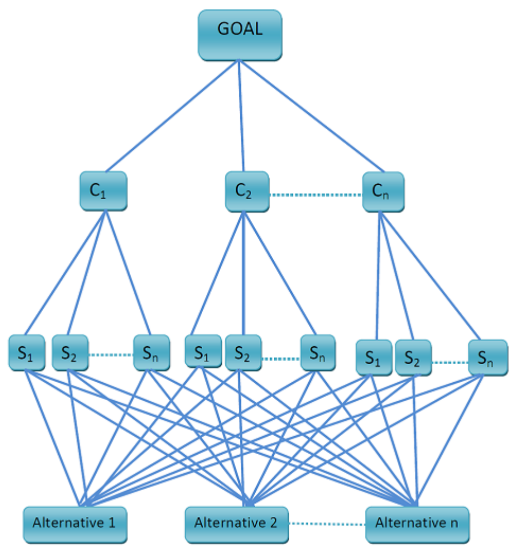

Figure 1. Structure of hierarchal problem.

5 Numerical Application

The model proposed in Section 5 is used to solve a real case study as follows.

The goal is to select the best among the alternatives. There are three alternatives which are Faisal Movers, Niazi Express and Bala Gujjar travels. The best alternative is selected on the basis of different criteria, which are defines as:

1. C_1:Flexibility

2. $\mathcal{C}_2$:Safety
3. $\mathcal{C}_3$: Affordable
4. $\mathcal{C}_4$:Punctuality
5. $\mathcal{C}_5$:Enjoyable

Now decomposed in hierarchy structure can be shown as in figure 2.

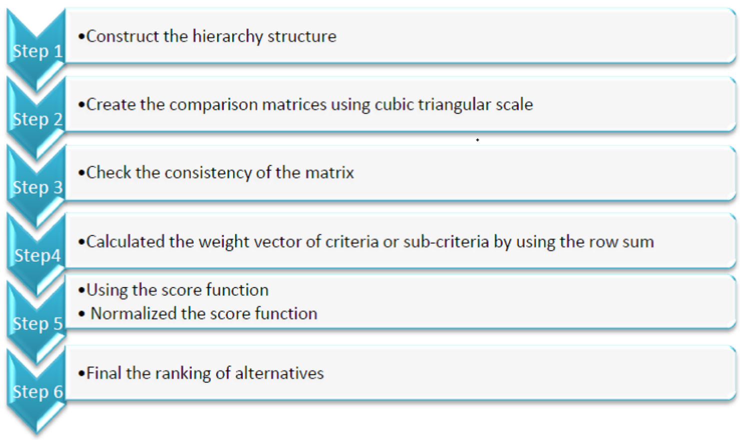

Figure 2 Cubic Anylatic Hierarchy Process (CAHP) diagram.

The hierarchy consists of four levels. The aim of objective is placed at Level 1, criteria are as Level 2, and the last, alternatives are as Level 3.

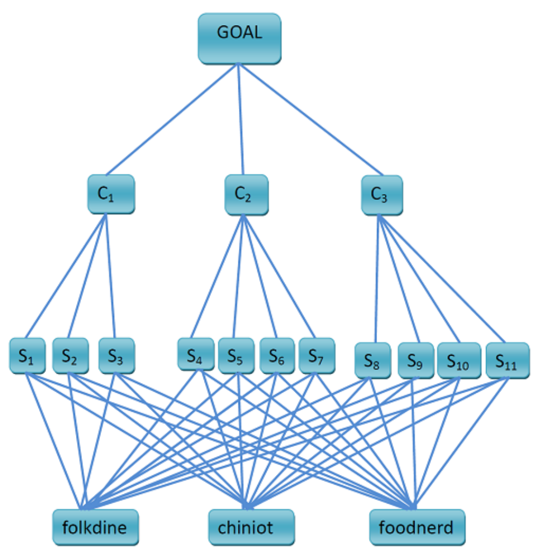

Figure 3 . Structure of hierarchy problem.

Criteria	Flexibility	Safety	Affordable	Punctuality	Enjoyable
Flexibility	$\widetilde{1}$	$\widetilde{2}$	$\widetilde{4}$	$\widetilde{9}$	$\widetilde{5}$
Safety	$\widetilde{2}^{-1}$	$\widetilde{1}$	$\widetilde{7}$	$\widetilde{6}$	$\widetilde{2}$
Affordable	$\widetilde{4}^{-1}$	$\widetilde{7}^{-1}$	$\widetilde{1}$	$\widetilde{5}$	$\widetilde{3}$
Punctuality	$\widetilde{9}^{-1}$	$\widetilde{6}^{-1}$	$\widetilde{5}^{-1}$	$\widetilde{1}$	$\widetilde{2}$
Enjoyable	$\widetilde{5}^{-1}$	$\widetilde{2}^{-1}$	$\widetilde{3}^{-1}$	$\widetilde{2}^{-1}$	$\widetilde{1}$

Table 2: The neutrosophic-cubic pair-wise comparison matrix of criteria

Criteria	Flexibility	Safety	Affordable	Punctuality	Enjoyable
Flexibility	1	1.3	2.45	5.3	3
Safety	.76	1	4.3	3.6	1.3
Affordable	.41	.23	1	3	2
Punctuality	.19	.27	.33	1	1.3
Enjoyable	.33	.76	.50	.76	1

Table 3: deterministic pair-wise comparison matrix of criteria

Step 3 Structure the cubic pair-wise comparison matrix of a criteria, sub-criteria and alternative, through the linguistic terms are shown in table 1. The values in table 2 are concern to a experts opinions. The pair-wise comparison matrix of criteria is presented in **Table** [3]**.**

Where;

$$\begin{aligned}
\widetilde{1} &= \langle (1,1,1);[0.50,0.50],0.50,[0.50,0.50],0.50,\ [0.50,0.50],0.50\rangle,\\
\widetilde{3} &= \langle (2,3,4);[0.25,0.35],0.30,[0.55,0.65],0.60,\ [0.65,0.75],0.70\rangle,\\
\widetilde{3} &= \langle (2,3,4);[0.25,0.35],0.30,[0.55,0.65],0.60,\ [0.65,0.75],0.70\rangle,\\
\widetilde{1} &= \langle (1,1,1);[0.50,0.50],0.50,[0.50,0.50],0.50,\ [0.50,0.50],0.50\rangle,
\end{aligned}$$

Using the equation(15) , the above neutrosophic-cubic pair-wise comparison matrix transformed into deterministic pair-wise comparison matrix as shown in $Table$ [4]

Next determined the ranking of the Criteria,namely the Eigen Vector X, from the previous matrix, as illustrated previously in the detailed steps of the proposed model. The normalized comparison matrix of Criteria is presented in $Table$ [5]. Now taking the total of the row averages:

Criteria	Flexibility	Safety	Affordable	Punctuality	Enjoyable
Flexibility	.37	.36	.30	.40	.35
Safety	.28	.28	.50	.26	.15
Affordable	.15	.07	.17	.22	.23
Punctuality	.07	.08	.04	.07	.15
Enjoyable	.12	.21	.06	.06	.12

Table 4: The normalized comparison matrix of Criteria

0	2	3	4	5	6	7	8	9	10
0	0	.58	.90	1.12	1.24	1.32	1.4	1.45	1.49

Table 5: Saaty table for calculating consistency ratio

Criteria	Faisal Movers	Niazi Express	Bala Gujjar Travels
Faisal Movers	$\widetilde{1}$	$\widetilde{5}^{-1}$	$\widetilde{2}^{-1}$
Niazi Express	$\widetilde{5}$	$\widetilde{1}$	$\widetilde{5}$
Bala Gujjar Travels	$\widetilde{2}$	$\widetilde{5}^{-1}$	$\widetilde{1}$

Table 6

$$X = \begin{bmatrix} .356 \\ .294 \\ .164 \\ .081 \\ .114 \end{bmatrix}$$

We should make sure that all the inputs given by the expert's are very consistent, now we take the consistency test as follows: Firstly calculate consistency index and consistency ratio as follows:

$$A = \begin{pmatrix} 1 & 1.3 & 2.45 & 5.3 & 3 \\ .76 & 1 & 4.3 & 3.6 & 1.3 \\ .41 & .23 & 1 & 3 & 2 \\ .19 & .27 & .33 & 1 & 1.3 \\ .33 & .76 & .50 & .76 & 1 \end{pmatrix} \begin{pmatrix} .356 \\ .294 \\ .164 \\ .081 \\ .114 \end{pmatrix} = \begin{pmatrix} 1.922 \\ .3285 \\ .1447 \\ .0755 \\ .1317 \end{pmatrix}$$

from the equation (20) and (21) ,

$$\lambda \max = \text{average} \left\{ \begin{array}{c} \frac{1.922}{0.356}, \frac{0.3285}{0.294}, \frac{0.1447}{0.164}, \\ \frac{0.0755}{0.081}, \frac{0.1317}{0.114} \end{array} \right\} = 5.399$$

$$\text{and } CI = \frac{\lambda \max - n}{n-1} = \frac{5.399 - 5}{5-1} = 0.089975.$$

For the calculation of value of consistency ratio, we used $Table$ [6] which was taken from Saaty book. The upper row is in random matrix order, and the lower row is the corresponding index of consistency for random decision.

$$CR = \frac{CI}{RI} = \frac{0.09975}{1.12} = 0.089 = 8.9\%$$

so evaluation are consistent i.e

$$8.9\% < 10\%.$$

The neutrosophic cubic pair-wise comparison matrix of decision criteria with respect to $\mathcal{C}_1$ is presented in $Table$ [7]

Criteria	Faisal Movers	Niazi Express	Bala Gujjar Travels
Faisal Movers	1	.33	.76
Niazi Express	3	1	3
Bala Gujjar Travels	1.3	.33	1

Table 7

Criteria	Faisal Movers	Niazi Express	Bala Gujjar Travels
Faisal Movers	.18	.18	.16
Niazi Express	.57	.63	.63
Bala Gujjar Travels	.25	.18	.21

Table 8

The deterministic pair-wise comparison matrix of decision criteria w.r.t C_1 shown in $Table$ [8]and the normalized comparison matrix in shown in $Table$ [9] Now taking the total of the row averages;

$$X = \begin{bmatrix} .17 \\ .61 \\ .22 \end{bmatrix}$$

The neutrosophic–cubic pair-wise comparison matrix of decision criteria with respect to $\mathcal{C}_2$ is presented in $Table$ [10]. The deterministic pair-wise comparison matrix of decision criteria w.r.t C_2 shown in table $Table$ [11] and the normalized comparison matrix in shown in $Table$ [12] Now taking the total of the row averages;

$$X = \begin{bmatrix} .41 \\ .47 \\ .12 \end{bmatrix}$$

The neutrosophic–cubic pair-wise comparison matrix of decision criteria with respect to $\mathcal{C}_3$ is presented in $Table$ [13] The deterministic pair-wise comparison matrix of decision criteria w.r.t C_3 shown in $Table$ [14]and the normalized comparison matrix in shown in $Table$ [15] Now taking the total of the row averages;

Criteria2	Faisal Movers	Niazi Express	Bala Gujjar Travels
Faisal Movers	$\widetilde{1}$	$\widetilde{1}$	$\widetilde{5}^{-1}$
Niazi Express	$\widetilde{1}$	$\widetilde{1}$	$\widetilde{6}$
Bala Gujjar Travels	$\widetilde{5}$	$\widetilde{6}^{-1}$	$\widetilde{1}$

Table 9

Criteria	Faisal Movers	Niazi Express	Bala Gujjar Travels
Faisal Movers	1	1	3
Niazi Express	1	1	4.3
Bala Gujjar Travels	.33	.23	1

Table 10

Criteria2	Faisal Movers	Niazi Express	Bala Gujjar Travels
Faisal Movers	.43	.45	.36
Niazi Express	.43	.45	.52
Bala Gujjar Travels	.13	.10	.12

Table 11

Criteria3	Faisal Movers	Niazi Express	Bala Gujjar Travels
Faisal Movers	$\widetilde{1}$	$\widetilde{3}^{-1}$	$\widetilde{4}$
Niazi Express	$\widetilde{3}$	$\widetilde{1}$	$\widetilde{5}$
Bala Gujjar Travels	$\widetilde{4}^{-1}$	$\widetilde{5}^{-1}$	$\widetilde{1}$

Table 12

Criteria	Faisal Movers	Niazi Express	Bala Gujjar Travels
Faisal Movers	1	.50	2.45
Niazi Express	2	1	3
Bala Gujjar Travels	.41	.33	1

Table 13

Criteria3	Faisal Movers	Niazi Express	Bala Gujjar Travels
Faisal Movers	.29	.27	.38
Niazi Express	.58	.55	.46
Bala Gujjar Travels	.12	.17	.16

Table 14

Criteria3	Faisal Movers	Niazi Express	Bala Gujjar Travels
Faisal Movers	$\widetilde{1}$	$\widetilde{3}^{-1}$	$\widetilde{5}^{-1}$
Niazi Express	$\widetilde{3}$	$\widetilde{1}$	$\widetilde{1}$
Bala Gujjar Travels	$\widetilde{5}$	$\widetilde{1}$	$\widetilde{1}$

Table 15

Criteria4	Faisal Movers	Niazi Express	Bala Gujjar Travels
Faisal Movers	1	.50	.33
Niazi Express	2.45	1	1
Bala Gujjar Travels	3	1	1

Table 16

$$X = \begin{bmatrix} .31 \\ .53 \\ .16 \end{bmatrix}$$

The neutrosophic–cubic pair-wise comparison matrix of decision criteria with respect to $\mathcal{C}_4$ is presented in $Table$ [16] The deterministic pair-wise comparison matrix of decision criteria w.r.t C_4 shown in $Table$ [17]and the normalized comparison matrix in shown in $Table$ [18] Now taking the total of the row averages;

$$X = \begin{bmatrix} .15 \\ .41 \\ .44 \end{bmatrix}$$

The neutrosophic–cubic pair-wise comparison matrix of decision criteria with respect to $\mathcal{C}_5$ is presented in $Table$ [19] The deterministic pair-wise comparison matrix of decision criteria w.r.t C_5 shown in $Table$ [20]and the normalized comparison matrix in shown in $Table$ [21] Now taking the total of the row averages;

Criteria3	Faisal Movers	Niazi Express	Bala Gujjar Travels
Faisal Movers	.15	.17	.13
Niazi Express	.38	.41	.43
Bala Gujjar Travels	.47	.41	.43

Table 17

Criteria3	Faisal Movers	Niazi Express	Bala Gujjar Travels
Faisal Movers	$\widetilde{1}$	$\widetilde{2}^{-1}$	$\widetilde{3}$
Niazi Express	$\widetilde{2}$	$\widetilde{1}$	$\widetilde{4}$
Bala Gujjar Travels	$\widetilde{3}^{-1}$	$\widetilde{4}^{-1}$	$\widetilde{1}$

Table 18

Criteria4	Faisal Movers	Niazi Express	Bala Gujjar Travels
Faisal Movers	1	.76	2
Niazi Express	1.3	1	2.45
Bala Gujjar Travels	.5	.41	1

Table 19

Criteria3	Faisal Movers	Niazi Express	Bala Gujjar Travels
Faisal Movers	.36	.35	.37
Niazi Express	.46	.46	.45
Bala Gujjar Travels	.18	.19	.18

Table 20

	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$	$\mathcal{C}_5$	Total Weight	Total Ranking of alternatives
Weights	0.356	0.294	0.164	0.081	0.114		
Faisal Movers	.17	0.41	0.31	0.15	.36	.285	2nd
Niazi Express	.61	0.47	0.53	0.41	.46	.52	1st
Bala Gujjar Travels	.22	0.12	0.16	0.44	.18	.196	3rd

Table 21

Table 22

$$X = \begin{bmatrix} .36 \\ .46 \\ .18 \end{bmatrix}$$

In $Table$ [22], Now the Total priority (weight) vector of the three alternatives with respect to Goal: Hence the final ranking of the best alternative are

Niazi Express > Faisal Movers > Bala Gujjar Travels,

presented in figure 3. Hence the final ranking of the best alternative are

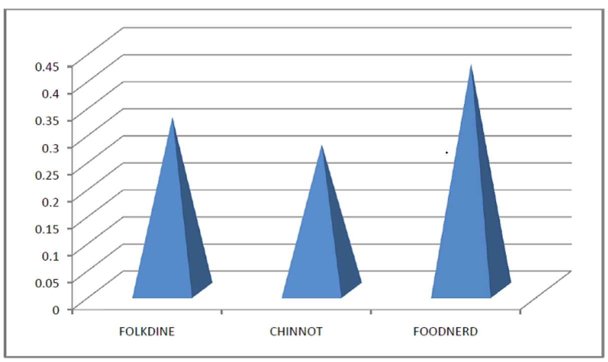

Figure 4. The final ranking of alternatives

Niazi Express > Faisal Movers > Bala Gujjar Travels,

presented in figure 3.

6 Comparsion Analysis

The comparison with other existing method is used to estimate the quality of the proposed model. In AHP we use non-fuzzy value to build a comparison matrix, which are not accurate because of ambiguity, inconsistent and doubtful information of decision maker. An idea of crisp information can be extended by many researchers in fuzzy environment for fulfilling the drawbacks of classical AHP to get the desired result, in which he takes

Methods	Final ranking
AHP [14]	Faisal movers $\geq$ Bala gujjar travels $\geq$ Niazi express
F-AHP [2]	Bala gujjar travels $\geq$ Niazi express $\geq$ Faisal movers
IF-AHP[21]	Bala gujjar travels $\geq$ Niazi express $\geq$ Faisal movers
N-AHP[1]	Niazi express $\geq$ Bala gujjar travels $\geq$ Faisal movers
NC-AHP Our porposed method	Niazi express $\geq$ Faisal movers $\geq$ Bala gujjar travels

Table 23: Comparsion of other existing methods

triangular fuzzy numbers but he fails to describe the relative weights among it. In cubic environment they are only describe the membership in terms of interval valued and fuzzy point and not discuss the non-membership and indeterminacy. To overcome the drawback of fuzzy AHP the cubic AHP are introduced, which is more reliable then fuzzy AHP. Then the neutrosophic-cubic set are the generalization of neutrosophic and interval neutrosophic set, and considered the truth, false and indeterminacy membership in the forms of intervals and also points, which is a best to representation for a vague and unsure information that exist in real life. We are the first to introduce the AHP in the neutrosophic-cubic environment. We can apply this method to various fields for the solution of different problems for achieving desired goals. in Table [23]. We compare the result of the NC-AHP with the other existing methods, which are

7 Conclusion and future work

Decision making is vital in daily life, dealing with real life problems; one who makes decision faces the data which is incomplete and not clear. The trails of neutrosophic-cubic enable that very situation. In the result of this, for every situation we define triangle neutrosophic-cubic number (TNCNs) and their operation laws. Mostly it is difficult to compare the neutrosophic-cubic value. The score and accuracy functions are defined for the comparison of neutrosophic-cubic values. By using these operations, we define analytic hierarchy process (AHP) in neutrosophic-cubic environment, which is the more generalized version of neutrosophic sets and interval neutrosophic sets. Since neutrosophic-cubic setsare is placed a better way to express inconsistent, indeterminate and obscure data. This method can be applied to the problems of daily life effectively and can be solved many intricate problems.

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