Theory of Approximation and Applications

Vol. 12, No.2, (2018), 1-15

Numerical Solution of fuzzy differential equations of nth-order by Adams-Moulton method

Nouredin Parandin [a,*]

[a] *Depratment of Mathematics, Kermanshah Branch, Islmic Azad University, Kermanshah, Iran.*

Received 12 March 2018; accepted 19 October 2018

Abstract

So far, many methods have been presented to solve the first-order differential equations. But, not many studies have been conducted for numerical solution of high-order fuzzy differential equations. In this research, First, the equation by reducing time, we transform the first-order equation. Then we have applied Adams-Moulton multi-step methods for the initial approximation of one order differential equations. Finally, we examine the accuracy of method by presenting examples.

Key words: Fuzzy differential equations; Adams-Moulton

2010 AMS Mathematics Subject Classification : 03E72

* Corresponding author's E-mail:nparandin54@gmail.com(Nouredin Parandin)

1 Introduction

Fuzzy differential equations are very useful indifferent sciences such as physics, chemistry, biology and economy. It should be noted, that if the equations that appear to be uncertain, then take help of fuzzy logic at these equations. Considering that most of the time analytic solution of such equations and finding an exact solution has either high complexity or cannot be solved, we applied numerical methods for the solution. The topics of fuzzy differential equations have been rapidly growing in recent years.

The theory of fuzzy differential equations was treated byBuckley and Feuring [9], Kaleva [24, 25], Nieto [29], Ouyang andWu [32], Roman-Flores and Rojas- Medar [36], Seikkala [37], also recently there appeared the papers of Bede [7], Bede and Gal [8], Diamond [15, 16], Georgiou et al., [20] Nieto and Rodriguez-Lopez [29]. In the following, we have mentioned some numerical solution which have proposed by other scientists. Abbasbandy and Allahviranloo have solved fuzzy differential equations by Runge-Kutta and Taylor methods[1, 2]. Also, Allahviranloo et al. solved differential equations by predictor-corrector and transformation methods[4, 5, 6].

Ghazanfari and Shakerami developed Runge-Kutta like formulae of order 4 for solving fuzzy differential equations[19]. Nystrom method has been introduced for solving fuzzy differential equations[26]. Mosleh and Otadi (2012) simulated and evaluate fuzzy differential equations by fuzzy neural network[28]. Pederson and Sambandham (2008) applied Runge-Kutta method for solving hybrid fuzzy differential equations [34]. Runge-Kutta method has been used for solving fuzzy differential equations by Palligkinis et al. (2009)[33]. Also, Kim and Sakthivel could solve hybrid fuzzy differential equations using improved predictor-corrector method [27].

The paper is organized as follows. Section 2 includes preliminaries. In Section 3, we can see the main idea of this paper. In Section 4, the proposed method is illustrated by examples. The conclusion is in Section 5.

2 Required definitions and basic concepts

First, we review some initial basic notations and results about symmetric fuzzy numbers and fuzzy-number-valued functions.

Definition 2.1 *[17] A fuzzy number is a map* $u : \mathbb{R}^1 \to [0,1]$ *which satisfies the following conditions*

(i) u *is upper semicontinuous on* $\mathbb{R}$,
(ii) $u(x) = 0$ *outside some interval* $[c,d] \subset \mathbb{R}$,
(iii) There exist real numbers a, b *such that* $c \leq a \leq b \leq d$ *where*
  (1) $u(x)$ *is monotonic increasing on* $[c,a]$,
  (2) $u(x)$ *is monotonic decreasing on* $[b,d]$,
  (3) $u(x) = 1, a \leq x \leq b$.

The set of all such fuzzy numbers is represented by E^1.

Definition 2.2 *[17] An arbitrary fuzzy number in parametric form is an ordered pair functions* $(\underline{u}(r), \overline{u}(r))$, $r \in [0,1]$, *which satisfies the following requirements:*

(i) $\underline{u}(r)$ *is a bounded monotonic increasing left continuous function over* $[0,1]$,
(ii) $\overline{u}(r)$ *is a bounded monotonic decreasing left continuous function over* $[0,1]$,
(iii) $\underline{u}(r) \leq \overline{u}(r), \ r \in [0,1]$.

A crisp number α is simply represented by $\underline{u}(r) = \overline{u}(r) = \alpha$, $0 \leq r \leq 1$.

Definition 2.3 *[37] For arbitrary* $u = (\underline{u}(r), \overline{u}(r))$, $v = (\underline{v}(r), \overline{v}(r))$, $0 \leq r \leq 1$ *and* $\lambda \in \mathbb{R}$, *we define equality, addition and scalar product by* λ *as:*

(i) $u = v$ *if and only if* $\underline{u}(r) = \underline{v}(r)$ *and* $\overline{u}(r) = \overline{v}(r)$,
(ii) $u + v = (\underline{u}(r) + \underline{v}(r), \overline{u}(r) + \overline{v}(r))$,
(iii) $\lambda u = \begin{cases} (\lambda \underline{u}(r), \lambda \overline{u}(r)), & \lambda > 0, \\ (\lambda \overline{u}(r), \lambda \underline{u}(r)), & \lambda < 0. \end{cases}$

Definition 2.4 *[8] The Hausdorff metric of two fuzzy numbers given by $D : \mathbb{R} \times \mathbb{R} \to \mathbb{R}^+ \cup \{0\}$, is defined as follows*

$$\begin{aligned} D(u,v) &= \sup_{r\in[0,1]} \max\left\{ \left|\underline{u}(r) - \underline{v}(r)\right|, \left|\overline{u}(r) - \overline{v}(r)\right| \right\} \\ &= \sup_{r\in[0,1]} \left\{ d_H([u]_r, [v]_r) \right\}, \end{aligned}$$

where $[u]_r = [\underline{u}(r), \overline{u}(r)]$ and $[v]_r = [\underline{v}(r), \overline{v}(r)]$.

We denote $\|.\| = D(.,0)$.

Definition 2.5 *[37] Let I be a real interval. A mapping $x : I \to E$ is called a fuzzy process and it's r-level set is denoted by*

$$[x(t)]_r = [x_1(t;r), x_2(t;r)],\ t \in I,\ r \in (0,1],$$

and the derivative $x'(t)$ of a fuzzy process $x(t)$ is defined by

$$[x'(t)]_r = [x_1'(t;r), x_2'(t;r)],\ t \in I,\ r \in (0,1].$$

3 Adams-Moulton methods for solving fuzzy differential equations

Let us consider the nth-order fuzzy ordinary differential equations of the following form

$$\begin{aligned} &\tfrac{d^n x}{dt^n} = f\left(t, x, \tfrac{dx}{dt}, \tfrac{d^2x}{dt^2}, \tfrac{d^3x}{dt^3}, \dots, \tfrac{d^{n-1}x}{dt^{n-1}}\right),\ t \in [t_0, T] \\ &x(t_0) = x_0, x'(t_0) = x_0', x''(t_0) = x_0'', \dots, x^{(n-1)}(t_0) = x_0^{(n-1)} \quad . \end{aligned} \tag{3.1}$$

Eq. (3.1) of order n can be reduced to a system of n first-order simultaneous fuzzy differential equations

$$
\begin{aligned}
&\tfrac{dy_0}{dt} = y_1 = f_1(t, y_0, y_1, y_2, \ldots, y_{n-1}), \\
&\tfrac{dy_1}{dt} = y_2 = f_2(t, y_0, y_1, y_2, \ldots, y_{n-1}), \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad , t \in [t_0, T], \\
&\tfrac{dy_{n-1}}{dt} = y_n = f_n(t, y_0, y_1, y_2, \ldots, y_{n-1}), \\
&x(t_0) = y_{0,0}, x'(t_0) = y_{1,0}, \ldots, x^{(n-1)}(t_0) = y_{n,0},
\end{aligned}
\tag{3.2}
$$

where $y_{0,0}, y_{1,0}, \ldots$ and $y_{n,0}$ are fuzzy numbers. Assume that Eqs. (3.3) and (3.4) are the exact and approximate solutions of Eq. (3.2), respectively

$$
\begin{aligned}
[y_0(t)]_r &= [y_{0,1}(t;r), y_{0,2}(t;r)], \\
[y_1(t)]_r &= [y_{1,1}(t;r), y_{1,2}(t;r)], \\
&\vdots \\
[y_{n-1}(t)]_r &= [y_{n-1,1}(t;r), y_{n-1,2}(t;r)]
\end{aligned}
\tag{3.3}
$$

and

$$
\begin{aligned}
[w_0(t)]_r &= [w_{0,1}(t;r), w_{0,2}(t;r)], \\
[w_1(t)]_r &= [w_{1,1}(t;r), w_{1,2}(t;r)], \\
&\vdots \\
[w_{n-1}(t)]_r &= [w_{n-1,1}(t;r), w_{n-1,2}(t;r)].
\end{aligned}
\tag{3.4}
$$

By using the Adams-Moulton method, the approximate solution is calculated as follows

$$
\begin{aligned}
w_{j,1}(t_{i+1};r) \;=\; & a_{j,m-1}w_{j,1}(t_i;r) + a_{j,m-2}w_{j,1}(t_{i-1};r) + \cdots + a_{j,0}w_{j,1}(t_{i+1-m};r) \\
& + h\Big[b_{j,m-1}\tilde{f}_j\big(t_{i+1}, w_0(t_{i+1};r), w_1(t_{i+1};r), \ldots, w_{n-1}(t_{i+1};r)\big) \\
& + b_{j,m-2}\tilde{f}_j\big(t_i, w_0(t_i;r), w_1(t_i;r), \ldots, w_{n-1}(t_i;r)\big) + \ldots \\
& + b_{j,0}\tilde{f}_j\big(t_{i+1-m}, w_0(t_{i+1-m};r), w_1(t_{i+1-m};r), \ldots, w_{n-1}(t_{i+1-m};r)\big)\Big],
\end{aligned}
\tag{3.5}
$$

and

$$
\begin{aligned}
w_{j,2}(t_{i+1};r) = & a_{j,m-1}w_{j,2}(t_i;r) + a_{j,m-2}w_{j,2}(t_{i-1};r) + \dots \\
& +a_{j,0}w_{j,2}(t_{i+1-m};r) + h\Big[b_{j,m-1}\tilde{f}_j\Big(t_{i+1}, w_0(t_{i+1};r), \\
& w_1(t_{i+1};r), \dots, w_{n-1}(t_{i+1};r)\Big) + b_{j,m-2}\tilde{f}_j\Big(t_i, w_0(t_i;r), \\
& w_1(t_i;r), \dots, w_{n-1}(t_i;r)\Big) + \dots + b_{j,0}\tilde{f}_j\Big(t_{i+1-m}, \\
& w_0(t_{i+1-m};r), w_1(t_{i+1-m};r), \dots, w_{n-1}(t_{i+1-m};r)\Big)\Big],
\end{aligned}
\tag{3.6}
$$

where

$$
\begin{aligned}
&\tilde{f}_j\Big(t_i + ., w_0(t_i + .;r), w_1(t_i + .;r), \dots, w_{n-1}(t_i + .;r)\Big) = \\
&\begin{cases} f_{j,1}\Big(t_{i+.}, w_0(t_{i+.};r), w_1(t_{i+.};r), \dots, w_{n-1}(t_{i+.};r)\Big), & b_{j,.} > 0 \\ f_{j,2}\Big(t_{i+.}, w_0(t_{i+.};r), w_1(t_{i+.};r), \dots, w_{n-1}(t_{i+.};r)\Big), & b_{j,.} < 0 \end{cases}
\end{aligned}
\tag{3.7}
$$

that

$$
\begin{aligned}
&f_{j,1}\Big(t_{i+.}, w_0(t_{i+.};r), w_1(t_{i+.};r), \dots, w_{n-1}(t_{i+.};r)\Big) = \\
&\min\{f_j(t, u_0, u_1, \dots, u_{n-1}|u_0 \in [w_{0,1}(t_{i+.};r), w_{0,2}(t_{i+.};r)], \\
&u_1 \in [w_{1,1}(t_{i+.};r), w_{1,2}(t_{i+.};r)], \dots, u_{n-1} \in [w_{n-1,1}(t_{i+.};r), w_{n-1,2}(t_{i+.};r)]\}
\end{aligned}
\tag{3.8}
$$

and

$$
\begin{aligned}
&f_{j,2}\Big(t_{i+.}, w_0(t_{i+.};r), w_1(t_{i+.};r), \dots, w_{n-1}(t_{i+.};r)\Big) \\
&= \max f_j(t, u_0, u_1, \dots, u_{n-1}|u_0 \in [w_{0,1}(t_{i+.};r), w_{0,2}(t_{i+.};r)] \\
&, u_1 \in [w_{1,1}(t_{i+.};r), w_{1,2}(t_{i+.};r)], \dots, u_{n-1} \in [w_{n-1,1}(t_{i+.};r), w_{n-1,2}(t_{i+.};r)]
\end{aligned}
\tag{3.9}
$$

where $j = 0, 1, \dots, n-1$, $i = 0, 1, ..., N$.
Using of some lemmas (see lemmas 5.7 and 5.8 in [12]), we can show that the approximate solution converges to the exact solution as the following

form

$$
\begin{aligned}
\lim_{h\to 0} w_{0,1}(t;r) &= y_{0,1}(t;r), & \lim_{h\to 0} w_{0,2}(t;r) &= y_{0,2}(t;r),\\
\lim_{h\to 0} w_{1,1}(t;r) &= y_{1,1}(t;r), & \lim_{h\to 0} w_{1,2}(t;r) &= y_{1,2}(t;r),\\
&\vdots & &\vdots\\
\lim_{h\to 0} w_{n-1,1}(t;r) &= y_{n-1,1}(t;r), & \lim_{h\to 0} w_{n-1,2}(t;r) &= y_{n-1,2}(t;r).
\end{aligned} \tag{3.10}
$$

Definition 3.1 *The fuzzy function* $f(t, y_0, y_1, \ldots, y_{n-1})$ *defined on the set*

$$D = \{(t, u_0, u_1, \ldots, u_{n-1}) | a \le t \le b, \quad -\infty < u_i < \infty, \quad \forall i = 1, 2, \ldots, n-1\},$$

is said to satisfy a Lipschitz condition on D *in the variables* $u_0, u_1, \ldots, u_{n-1}$ *is a constant* $L > 0$ *exists with*

$$D\left(f(t, u_0, u_1, \ldots, u_{n-1}), f(t, v_0, v_1, \ldots, v_{n-1})\right) \le L \sum_{i=0}^{n-1} D(u_i, v_i),$$

for all $(t, u_0, u_1, \ldots, u_{n-1})$ *and* $(t, v_0, v_1, \ldots, v_{n-1})$ *in* D.

Theorem 1 *Suppose that*

$$D = \{(t, u_0, u_1, \ldots, u_{n-1}) | a \le t \le b, \quad -\infty < u_i < \infty, \quad \forall i = 1, 2, \ldots, n-1\},$$

and let $f_i(t, u_0, u_1, \ldots, u_{n-1})$*, for each* $i = 1, 2, \ldots, n-1$*, be continuous and satisfies a Lipschitz condition on* D*. The system of first order differential equations (3.2), subject to the initial conditions, has a unique solution* $u_0(t), u_1(t), \ldots, u_{n-1}(t)$*, for each* $a \le t \le b$.

Proof. see [12]

4 Numerical experiments

In order to illustrate the performance and accuracy of the Adams-Moulton method in solving the high order linear fuzzy differential equations, we present two numerical examples. In Figures 1 to 2, the approximate solutions and the exact solutions are compared and are found to be in high agreement. All numerical computations were done on the computer using Mathematica software package.

Example 4.1 *Let us consider the following fuzzy differential equation*

$$x'' - 4x' + 4x = 0, \qquad t \in [0, 1]$$

with the initial conditions $x(0) = (2 + r, 4 - r)$, $x'(0) = (5 + r, 7 - r)$ *and the exact solution is*

$$\underline{x}(t; r) = -e^{2t}(rt - r - t - 2),$$
$$\overline{x}(t; r) = e^{2t}(rt - r - t + 4).$$

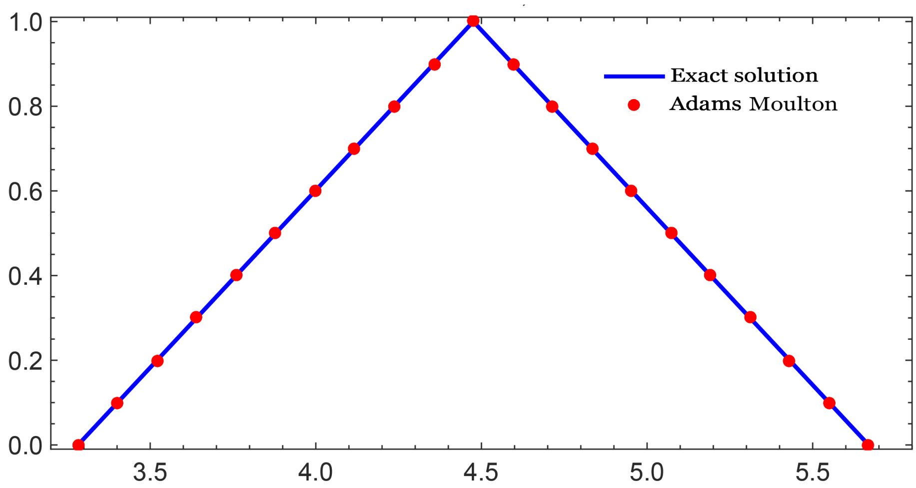

Fig. 1. Numerical results for AM_2 method and Exact solution at $t = 0.2$ of Example 4.1

In this example, the exact and the approximate solutions of the equation were obtained by using Adams-Moulton 2-step and the system of the first differential equations was constructed for t=0.2 and h=0.1. The above ODE of order 2 *is equivalent to the following system*

$$y_1' = y_2$$
$$y_2' = 2y_1.$$

Table 1
Comparison between the exact and approximate solution of Example 4.1

	Exact	Exact	AM_2	AM_2	Absolute error	Absolute error
r	$\underline{x}(0.2, r)$	$\overline{x}(0.2, r)$	$w_{0,1}(0.2, r)$	$w_{0,2}(0.2, r)$	$\lvert\underline{x} - w_{0,1}\rvert$	$\lvert\overline{x} - w_{0,2}\rvert$
0.0	3.28201	5.66893	3.28234	5.66909	3.278×10^{-4}	1.53×10^{-4}
0.1	3.40136	5.54959	3.40168	5.54975	3.191×10^{-4}	1.617×10^{-4}
0.2	3.52071	5.43024	3.52102	5.43041	3.103×10^{-4}	1.705×10^{-4}
0.3	3.64005	5.31090	3.64035	5.31108	3.016×10^{-4}	1.792×10^{-4}
0.4	3.75940	5.19155	3.75969	5.19174	2.928×10^{-4}	1.88×10^{-4}
0.5	3.87874	5.07220	3.87903	5.07240	2.841×10^{-4}	1.967×10^{-4}
0.6	3.99809	4.95286	3.99837	4.95306	2.754×10^{-4}	2.054×10^{-4}
0.7	4.11744	4.83351	4.11770	4.83373	2.666×10^{-4}	2.142×10^{-4}
0.8	4.23678	4.71417	4.23704	4.71439	2.579×10^{-4}	2.229×10^{-4}
0.9	4.35613	4.59482	4.35638	4.59505	2.491×10^{-4}	2.317×10^{-4}
1.0	4.47547	4.47547	4.47571	4.47571	2.404×10^{-4}	2.404×10^{-4}

Table 2
Comparison between derivative of the exact and approximate solution of Example 4.1

	Exact	Exact	AM_2	AM_2	Absolute error	Absolute error
r	$\underline{x}'(0.2, r)$	$\overline{x}'(0.2, r)$	$w_{1,1}(0.2, r)$	$w_{1,2}(0.2, r)$	$\lvert\underline{x}' - w_{1,1}\rvert$	$\lvert\overline{x}' - w_{1,2}\rvert$
0.0	8.05585	9.84604	8.05659	9.84627	7.357×10^{-4}	2.259×10^{-4}
0.1	8.14536	9.75653	8.14607	9.75678	7.102×10^{-4}	2.514×10^{-4}
0.2	8.23487	9.66702	8.23556	9.66730	6.847×10^{-4}	2.769×10^{-4}
0.3	8.32438	9.57751	8.32504	9.57782	6.592×10^{-4}	3.023×10^{-4}
0.4	8.41389	9.48801	8.41453	9.48833	6.337×10^{-4}	3.278×10^{-4}
0.5	8.50340	9.39850	8.50401	9.39885	6.083×10^{-4}	3.533×10^{-4}
0.6	8.59291	9.30899	8.59349	9.30936	5.828×10^{-4}	3.788×10^{-4}
0.7	8.68242	9.21948	8.68298	9.21988	5.573×10^{-4}	4.043×10^{-4}
0.8	8.77193	9.12997	8.77246	9.13040	5.318×10^{-4}	4.298×10^{-4}
0.9	8.86144	9.04046	8.86194	9.04091	5.063×10^{-4}	4.553×10^{-4}
1.0	8.95095	8.95095	8.95143	8.95143	4.808×10^{-4}	4.808×10^{-4}

The initial conditions of original equation are equivalent to the following initial conditions to the first order system

$$y_1(0) = (2 + r, 4 - r), \; y_2(0) = (5 + r, 7 - r).$$

The exact solution, approximate solution and their derivative have been shown in Tables 1 and 2 respectively.

We only show a few of the approximate results for $t = 0.2$, obtained by the Adams Moulton two-step method in Figure 1.

Example 4.2 *Consider the following fuzzy differential equation*

$$x'' - 2x' = 0, \qquad t \in [0, 1]$$

with the initial conditions $x(0) = (r, 2 - r)$, $x'(0) = (2 + r, 4 - r)$ and the exact solution is

$$\underline{x}(t; r) = \frac{1}{2}\left(re^{2t} + r + 2e^{2t} - 2\right),$$
$$\overline{x}(t; r) = \frac{1}{2}\left(r\left(-e^{2t}\right) - r + 4e^{2t}\right).$$

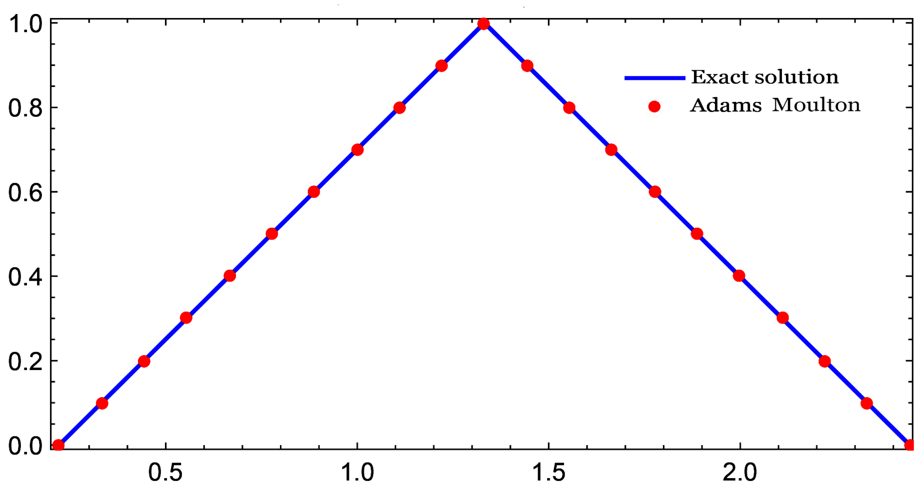

Fig. 2. Numerical results for AM_4 method and Exact solution at $t = 0.1$ of Example 4.2

In this example, the exact and the approximate solutions of the equation were obtained by using Adams-Moulton 4-step and the system of the first differential equations was constructed for t=0.1 and h=0.1. The above ODE of order 2 is equivalent to the following system

$$y_1' = y_2$$
$$y_2' = 2y_1.$$

Table 3
Comparison between the exact and approximate solution of Example 4.2

	Exact	Exact	AM_4	AM_4	Absolute error	Absolute error
r	$\underline{x}(0.1, r)$	$\overline{x}(0.1, r)$	$w_{0,1}(0.1, r)$	$w_{0,2}(0.1, r)$	$\lvert\underline{x} - w_{0,1}\rvert$	$\lvert\overline{x} - w_{0,2}\rvert$
0.0	0.221403	2.44281	0.22140	2.44280	2.758×10^{-6}	5.516×10^{-6}
0.1	0.332473	2.33174	0.33247	2.33173	2.896×10^{-6}	5.378×10^{-6}
0.2	0.443543	2.22067	0.44354	2.22066	3.034×10^{-6}	5.241×10^{-6}
0.3	0.554613	2.10960	0.55461	2.10959	3.172×10^{-6}	5.103×10^{-6}
0.4	0.665683	1.99852	0.66568	1.99852	3.310×10^{-6}	4.965×10^{-6}
0.5	0.776753	1.88745	0.77675	1.88745	3.448×10^{-6}	4.827×10^{-6}
0.6	0.887824	1.77638	0.88782	1.77638	3.586×10^{-6}	4.689×10^{-6}
0.7	0.998894	1.66531	0.99889	1.66531	3.724×10^{-6}	4.551×10^{-6}
0.8	1.109960	1.55424	1.10996	1.55424	3.861×10^{-6}	4.413×10^{-6}
0.9	1.221030	1.44317	1.22103	1.44317	3.999×10^{-6}	4.275×10^{-6}
1.0	1.33210	1.33210	1.33210	1.33210	4.137×10^{-6}	4.137×10^{-6}

Table 4
Comparison between derivative of the exact and approximate solution of Example 4.2

	Exact	Exact	AM_4	AM_4	Absolute error	Absolute error
r	$\underline{x}'(0.1, r)$	$\overline{x}'(0.1, r)$	$w_{1,1}(0.1, r)$	$w_{1,2}(0.1, r)$	$\lvert\underline{x}' - w_{1,1}\rvert$	$\lvert\overline{x}' - w_{1,2}\rvert$
0.0	2.44281	4.88561	2.44280	4.88560	5.516×10^{-6}	1.103×10^{-5}
0.1	2.56495	4.76347	2.56494	4.76346	5.792×10^{-6}	1.076×10^{-5}
0.2	2.68709	4.64133	2.68708	4.64132	6.068×10^{-6}	1.048×10^{-5}
0.3	2.80923	4.51919	2.80922	4.51918	6.344×10^{-6}	1.021×10^{-5}
0.4	2.93137	4.39705	2.93136	4.39704	6.620×10^{-6}	9.929×10^{-6}
0.5	3.05351	4.27491	3.05350	4.27490	6.895×10^{-6}	9.654×10^{-6}
0.6	3.17565	4.15277	3.17564	4.15276	7.171×10^{-6}	9.378×10^{-6}
0.7	3.29779	4.03063	3.29778	4.03062	7.447×10^{-6}	9.102×10^{-6}
0.8	3.41993	3.90849	3.41992	3.90848	7.723×10^{-6}	8.826×10^{-6}
0.9	3.54207	3.78635	3.54206	3.78634	7.999×10^{-6}	8.550×10^{-6}
1.0	3.66421	3.66421	3.6642	3.6642	8.274×10^{-6}	8.274×10^{-6}

The initial conditions of original equation are equivalent to the following initial conditions to the first order system

$$y_1(0) = (r, 2 - r),\ \ y_2(0) = (2 + r, 4 - r).$$

The exact solution, approximate solution and their derivative have been shown in Tables 3 and 4 respectively.

We can see the computational results for $t = 0.1$, obtained by the Adams Moulton 4-step technique in Figure 2.

5 Conclusion

In this paper, the Adams-Moulton method numerically extended to solve the nth-order fuzzy ordinary differential equations. In this method, the nth-order of fuzzy differential equations converted to a system of first-order fuzzy differential equations. Then, these equations were solved by Adams-Moulton method. This method, which takes advantage of single-step methods to approximate the point is that new uses previous points. So we expect the error to a single-stage method is minimally.

Acknowledgment

The author would like to thank the Kermanshah Branch, Islamic Azad University for the financial support of this research project.

References

[1] S. Abbasbandy and T. Allahviranloo, Numerical solution of fuzzy differential equation by Runge-Kutta method, J. Sci. Teacher Training University,1(3), 2002.

[2] S. Abbasbandy and T. Allahviranloo, Numerical solutions of fuzzy differential equations by Taylor method, Journal of Computational Methods in Applied Mathematics, 2 (2002), 113-124.

[3] S. Abbasbandy, T. Allahviranloo, O. Lopez-Pouso, and J. J. Nieto, Numerical methods for fuzzy differential inclusions, Journal of Computer and Mathematics with Applications, 48 (2004), 1633-1641.

[4] T. Allahviranloo, N. Ahmady, and E. Ahmady, Numerical solution of fuzzy differential equations by predictor-corrector method, Information Sciences, 177 (2007), 1633-1647.

[5] T. Allahviranloo, E. Ahmady, and N. Ahmady, Nth-order fuzzy linear differential equations, Information Sciences, 178 (2008), 1309-1324.

[6] T. Allahviranloo, N. A. Kiani, and N. Motamedi, Solving fuzzy differential equations by differential transformation method, Information Sciences, 179 (2009), 956-966.

[7] B. Bede, A note on two-point boundary value problems associated with non-linear fuzzy differential equations, Fuzzy Sets and Systems, 157 (2006), 986-989.

[8] B. Bede and S. G. Gal, Generalizations of the differntiability of fuzzy number-valued functions with applications to fuzzy differential equations, Fuzzy Sets and Systems, 151 (2005), 581-599.

[9] J. J. Buckley and T. Feuring, Fuzzy differential equations, Fuzzy Sets and Systems, 110 (2000), 43-54.

[10] C.J. Budd, A. Iserles, Geometric integration: numerical solution of differential equations on manifolds, philosophical transactions: mathematical, Physical and Engineering Sciences: Mathematical 357 (1999) 945–956.

[11] J. Behlke, O. Ristau, A new approximate whole boundary solution of the Lamm differential equation for the analysis of sedimentation velocity experiments, Biophysical Chemistry 95 (2002) 59–68.

[12] R.L. Burden, J.D. Faires, Numerical Analysis, Brooks/Cole, USA (2001).

[13] G.A. Bocharov, F.A. Rihan, Numerical modelling in biosciences using delay differential equations, Journal of Computational and Applied Mathematics 125 (2000) 183–199.

[14] R.V. Culshaw, S. Ruan, A delay-differential equation model of HIV infection of CD4+ T-cells, Mathematical Biosciences 165 (2000) 27–39.

[15] P. Diamond, Stability and periodicity in fuzzy differential equations, IEEE Trans, Fuzzy Systems, 8 (2000), 583-590.

[16] P. Diamond, Brief note on the variation of constants formula for fuzzy differential equations, Fuzzy Sets and Systems, 129 (2002), 65-71.

[17] Ming Ma, M. Friedman, and A. Kandel, Numerical solutions of fuzzy differential equations, Fuzzy Sets and Systems, 105 (1999), 133-138.

[18] M. Friedman, M.Ming, and A. Kandel, Numerical solutions of fuzzy differential and integral equations, Fuzzy Sets and Systems, 106 (1999), 35-48.

[19] B. Ghazanfari and A. Shakerami, Numerical solution of fuzzy differential equations by extended Runge-Kutta-like formulae of order 4, Fuzzy Sets and Systems, 189 (2011), 74-91.

[20] D. N. Georgiou, J. J. Nieto, and R. Rodriguez-Lopez, Initial value problems for higher-order fuzzy differential equations, Nonlinear Anal., 63 (2005), 587-600.

[21] H. Gang, H. Kaifen, Controlling chaos in systems described by partial differential equations, Phys. Rev. Lett. 71 (1993) 3794–3797.

[22] A.R. Its, A.G. Izergin, V.E. Korepin, N.A. Slavnov, Differential equations for quantum correlation functions, International Journal of Modern Physics B 4 (1990) 1003–1037.

[23] A.V. Kotikov, Differential equations method: the calculation of vertex-type Feynman diagrams, Physics Letters B 259 (1991) 314–322.

[24] O. Kaleva, Fuzzy differential equations, Fuzzy Sets and Systems, 24 (1987), 301-317.

[25] O. Kaleva, The Cauchy problem for fuzzy differential equations, Fuzzy Sets and Systems, 35 (1990), 389-396.

[26] A. Khastan and K. Ivaz, Numerical solution of fuzzy differential equations by Nystrom method. Chaos, Solitons and Fractals, 41 (2009), 859-868.

[27] H. Kim and R. Sakthivel, Numerical solution of hybrid fuzzy differential equations using improved predictor-corrector method, Commun Nonlinear Sci Numer Simulat, 17 (2012), 3788-3794.

[28] M. Mosleh and M. Otadi, Simulation and evalution of fuzzy differential equations by fuzzy neural network, Applied Soft Computing, 12 (2012), 2817-2827.

[29] J. J. Nieto and R. Rodriguez-Lopez, Bounded solutions for fuzzy differential and integral equations, Chaos, Solitons and Fractals, 27 (2006), 1376-1386.

[30] [30] J. J. Nieto, The Cauchy problem for continuous fuzzy differential equations, Fuzzy Sets and Systems, 102 (1999), 259-262.

[31] R. Norberg, Differential equations for moments of present values in life insurance, Insurance: Mathematics and Economics 17 (1995) 171–180.

[32] H. Ouyang and Y. Wu, On fuzzy differential equations, Fuzzy Sets and Systems, 32 (1989), 321-325.

[33] S. Ch. Palligkinis, G. Papageorgiou, and I. Th. Famelis, Runge-Kutta methods for fuzzy differential equations, Applied Mathematics and Computation, 209 (2009), 97-105.

[34] S. Pederson and M. Sambandham, The Runge-Kutta method for hybrid fuzzy differential equations, Nonlinear Analysis: Hybrid Systems, 2 (2008), 626-634.

[35] Y.Z. Peng, Exact solutions for some nonlinear partial differential equations, Physics Letters A 314 (2003) 401–408.

[36] H. Roman-Flores and M. Rojas-Medar, Embedding of level-continuous fuzzy sets on Banach spaces, Information Sciences, 144 (2002), 227-247.

[37] S. Seikkala, On the fuzzy initial value problem, Fuzzy Sets and Systems, 24 (1987), 319-330.

[38] U. Salzner, P. Otto, J. Ladik, Numerical solution of a partial differential equation system describing chemical kinetics and diffusion in a cell with the aid of compartmentalization, Journal of Computational Chemistry 11 (1990) 194–204.

[39] J.G. Verwer, J.G. Blom, M. van Loon, E.J. Spee, A comparison of stiff ODE solvers for atmospheric chemistry problems, Atmospheric Environment 30 (1996) 49–58.