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Shear Waves Through Non Planar Interface Between Anisotropic Inhomogeneous and Visco-Elastic Half-Spaces

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ABSTRACT

A problem of reflection and transmission of a plane shear wave incident at a corrugated interface between transversely isotropic inhomogeneous and visco-elastic half-spaces is investigated. Applying appropriate boundary conditions and using Rayleigh's method of approximation expressions for reflection and transmission coefficients are obtained for the first and second order approximation of the corrugation. Further, closed form formulae of these coefficients are presented for a corrugated interface of periodic shape (cosine law interface). Numerical computations for this particular type of corrugated interface are performed and a number of graphs are plotted to illustrate the effect of different parameters of the both half-spaces on the reflection and transmission coefficients. It is found that these coefficients depend upon the amplitude of corrugation of the boundary, angle of incidence and frequency of the incident wave and are strongly influenced by the anisotropy, inhomogeneity and visco-elasticity of the half-spaces. Some special cases are also derived.

Keywords: SH-waves; Visco-elastic; Inhomogeneity; Anisotropy; Corrugated boundary.

1 INTRODUCTION

THE study of seismic waves generated from an earthquake origin provides the most trustworthy information about the complex internal structure of the Earth. The phenomena of reflection and refraction of seismic waves at the interface between two dissimilar media is a fundamental issue in many areas such as geophysics, seismology, earthquake engineering, non-destructive evaluation, etc. In the propagation of seismic waves through layered media, the boundaries play crucial role. Wave propagation in elastic medium with non-parallel boundaries is an important topic for geophysicists and seismologists to understand and predict the seismic activities at continental margins and mountain roots. SH-waves are seismic waves that cause horizontal shifting of the earth during the earthquake. The particle motion of SH-type waves forms a horizontal line perpendicular to the direction of propagation.

Seismic waves generated due to earthquakes require moving through many irregular geological structures like mountains roots, mountain basins, salt, ore bodies, etc. which do alter their nature of propagation. Also, it is well known fact that the Earth crust is not perfectly homogeneous and isotropic throughout; various forms of anisotropy and inhomogeneity always exist. The presence of inhomogeneity and anisotropy drastically affect the seismic wave propagation. Moreover, the discontinuities present in the Earth crust between the layers produce the reflection and

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transmission of SH-waves through the layers. Keeping this type of geophysical case in mind, in the present study a corrugated boundary between a visco-elastic solid half-space and a transversely isotropic inhomogeneous solid half-space is considered and the reflection and transmission of SH-waves are discussed.

An approximate method of solving problems of sound and electromagnetic waves scattering from a sinusoidal surface with small amplitude and slope was first given by Rayleigh [1]. In this method the equation of the interface is expressed in Fourier series and by using the appropriate boundary conditions of the problem, the unknown coefficients in the solutions are determined for any order of approximation. Afterward researchers of different areas used this method to interpret reflection and transmission phenomena of waves from non-flat boundaries. Using Rayleigh's method many problems concerning reflection and refraction phenomena of elastic waves at a corrugated interface between two uniform elastic half-spaces were solved by Asano [2-4]. Abubakar [5] and Dunkin and Eringen [6] studied the problem of reflection of body waves from a rough surface of a semi-infinite elastic solid. Abubakar [7, 8] studied the reflection and refraction of SH-waves at an irregular boundary between two uniform elastic solid half-spaces. Kaushik and Chopra [9] derived the reflection and transmission coefficients of plane SH-waves at a plane boundary between anisotropic and visco-elastic half-spaces. Gogna and Chander [10] investigated the reflection and refraction of shear waves at an interface between heterogeneous anisotropic elastic and visco-elastic half-spaces. Later researchers attempted different problems of reflection and transmission of seismic waves at a corrugated boundary between two different elastic solid half-spaces by using Rayleigh's method considering either vertical heterogeneity or both vertical and lateral heterogeneity. Gupta [11] analysed reflection and transmission of SH-waves in laterally and vertically inhomogeneous media at an irregular boundary. Kumar et al. [12] studied reflection and refraction of SH-waves at a corrugated interface between two different anisotropic and vertically heterogeneous elastic solid half-spaces. Tomar and Kaur [13] analysed reflection and transmission of SH-waves at a corrugated interface between two laterally and vertically heterogeneous anisotropic elastic half-spaces. Kaur et al. [14] attempted problem of reflection and refraction of SH-waves at a corrugated interface between two laterally and vertically heterogeneous visco-elastic solid half-spaces.

In last few years, noticeable amount of work considering the effect of irregular boundaries of different types in the propagation of seismic waves are done by Chattopadhyay and co-authors [15-18]. Chattopadhyay et al. [15] discussed the dispersion equation of SH-waves in a monoclinic layer over a semi-infinite elastic medium with an irregularity of rectangular type. Chattopadhyay et al. [16] studied reflection and refraction phenomena of plane quasi-P waves at a corrugated interface between distinct triclinic elastic half-spaces. Chattopadhyay et al. [17] studied the shear wave propagation in a visco-elastic layer over a semi-infinite visco-elastic half-space having irregularity of parabolic and triangular notch type in the visco-elastic layer. Chattopadhyay et al. [18] discussed dispersion of SH-waves in an irregular non-homogeneous self-reinforced crustal layer over a semi-infinite self-reinforced medium. Kumar et al. [19] investigated reflection and refraction of plane waves at an imperfect boundary of two different fibre-reinforced transversely isotropic thermo-elastic solid half-spaces under hydrostatic initial stress. Recently, Prasad et al. [20] studied reflection and refraction of SH-waves through non planar interface between visco-elastic and fibre-reinforced solid half-spaces. Some more recent works related to seismic waves of different types in heterogeneous, anisotropic and visco-elastic media with different geometries are considered by Kakar [21], Kumar et al. [22] and Vaishnav et al. [23].

In the present investigation, using Rayleigh's method of approximation, an attempt is made to consider the reflection and refraction of SH-waves at a corrugated interface between transversely isotropic inhomogeneous elastic solid and linear visco-elastic solid half-spaces. Formulae for reflection and transmission coefficients are obtained for the first and second order approximation of the corrugation. Further, the reflection and transmission coefficients for a periodic type interface are obtained in closed form for first order approximation and various graphs are drawn to show the effect of frequency and angle of incidence of the wave, corrugation of the interface and anisotropy, inhomogeneity and visco-elasticity factors of the half-spaces on the reflection and transmission coefficients.

2 FORMULATION OF THE PROBLEM, BASIC EQUATIONS AND THEIR SOLUTIONS

Geometry of the problem is shown in Fig. 1. Cartesian x and y axes are taken on the horizontal plane and the z axis is vertically downward. A visco-elastic solid half-space H_2 $(-\infty < z < 0)$ over a transversely isotropic inhomogeneous solid half-space H_1 $(0 < z < \infty)$ separated by a corrugated interface given by $z = \zeta$ is considered.

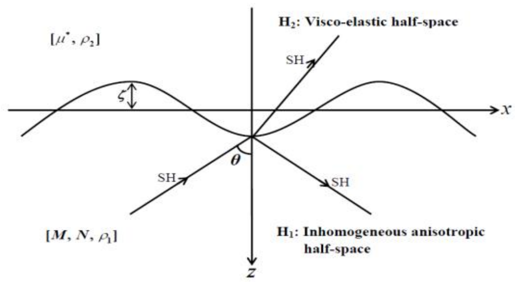


Fig.1
Geometry of the problem.

Here ζ is taken to be a periodic function of x and independent of y, whose average value is zero. In Fourier series ζ is represented as follows:

$$\zeta = \sum_{n=1}^{\infty}\left[\zeta_n \exp(inpx) + \zeta_{-n} \exp(-inpx)\right] \tag{1}$$

where ζ_n and ζ_{-n} are Fourier coefficients, n is the series expansion order and i is the imaginary unit.

Introducing the constants h, c_n and s_n as follows:

$$\zeta_1 = \zeta_{-1} = \frac{h}{2} \quad \text{and} \quad \zeta_{\pm n} = \frac{c_n \mp i s_n}{2}, \ \ n = 2,3,4.... \tag{2}$$

we get,

$$\zeta = h\cos px + \sum_{n=2}^{\infty}\left[c_n \cos npx + s_n \sin npx\right] \tag{3}$$

The special case, in which the geometry of the interface is expressed by one cosine term, that is, $\zeta = h\cos px$ (obtained by putting $\zeta_n = \zeta_{-n} = 0, \ n = 2,3,4,...$) the amplitude and the period of the corrugation are given by h and $2\pi/p$ respectively.

The lower half-space H_1 is characterised by elastic constants M and N, density ρ_1 and the horizontal and vertical shear wave velocities:

$$\beta_h = \left(\frac{N}{\rho_1}\right)^{1/2}, \quad \beta_v = \left(\frac{M}{\rho_1}\right)^{1/2} \tag{4}$$

The upper half-space H_2 has density ρ_2, complex frequency dependent shear modulus μ^*, and complex frequency dependent shear wave velocity β^*.

Let the variations of elastic parameters in the medium H_1 be defined by

$$\{M, N, \rho_1\} = \{M_0, N_0, \rho_0\}\cosh^2\left(\frac{z}{a}\right) \tag{5}$$

where M_0, N_0, ρ_0 are constants and a is the inhomogeneity parameter having dimension of length.

The equation of SH-wave propagation in a transversely isotropic, inhomogeneous elastic medium with zero body forces can be written as:

$$\frac{\partial}{\partial x}\left(N\,\frac{\partial v_1}{\partial x}\right)+\frac{\partial}{\partial z}\left(M\,\frac{\partial v_1}{\partial z}\right)=\rho_1\frac{\partial^2 v_1}{\partial t^2} \tag{6}$$

where v_1 denotes the displacement in *y*-direction.

Consider the time harmonic wave and let

$$v_1 = X(x)Z(z)\exp(i\,\omega t) \tag{7}$$

where ω is the angular frequency. Using Eq. (7) in Eq. (6), we get,

$$\frac{d^2X}{dx^2}+b^2X=0,\quad M\frac{d^2Z}{dz^2}+\frac{dM}{dz}\frac{dZ}{dz}+\rho_1\omega^2\cos^2\theta\;Z=0 \tag{8}$$

where $b=\omega\sin\theta/\beta_h$ is the *x*-component of the wave number given by Gupta [24] and θ is the angle between the wave normal and the positive direction of the *z*-axis.

Putting $Z=Z^*/\sqrt{M}$ in the second equation of (8), we get

$$\frac{d^2Z^*}{dz^2}-q^2Z^*=0 \tag{9}$$

where

$$q^2=\frac{1}{a^2}-\frac{\omega^2}{\beta_h^2}\left(\frac{N}{M}\right)\cos^2\theta=\frac{1}{a^2}-\left(\frac{\omega\cos\theta}{\beta_v}\right)^2 \tag{10}$$

For a wave propagating in the positive direction of *x*-axis, the solution of Eq. (8) is given by

$$X=A\exp(-ibx),\qquad Z=\frac{1}{\sqrt{M}}\left[A_0\exp(qz)+B_0\exp(-qz)\right] \tag{11}$$

where A, A_0 and B_0 are constants.

Consider a plane SH-wave of unit amplitude and period $2\pi/\omega$, incident at the interface $z=\zeta$ from the lower half-space H_1 then with the help of Eqs. (11) and (7) we can write

$$v_1=\frac{1}{\sqrt{M_0}\cosh(z/a)}\left(\exp(qz)+B_0\exp(-qz)\right)\exp\left\{i\,\omega\left(t-\frac{x\sin\theta}{\beta_h}\right)\right\} \tag{12}$$

For the linear visco-elastic medium equation of motion is given by

$$\nabla^2 v_2=\frac{1}{\beta^{*2}}\frac{\partial^2 v_2}{\partial t^2} \tag{13}$$

where

$$\beta^{*2}=\frac{\mu^*}{\rho_2} \tag{14}$$

and v_2 denotes the *y*-component of the displacement.

The solution of Eq. (13) is given by (Schoenberg [25])

$$v_2 = D_0 \exp\left[i\,\omega \left(t - \frac{x \sin\theta^* - z \cos\theta^*}{\beta^*} \right) \right] \tag{15}$$

where θ^* is the complex angle of propagation.

Since the interface is corrugated, the reflection and refraction phenomena will be affected and following waves will be generated at the corrugated interface due to the incident SH-wave:

In the medium H_1 :

(i) A regularly reflected wave making an angle θ with *z*-axis

(ii) A spectrum of *nth* order of irregularly reflected waves at angle θ_n^+ in the left side of regularly reflected wave

(iii) A similar spectrum of irregularly reflected waves at angle θ_n^- in the right side of regularly reflected wave.

In the medium H_2 :

(i) A regularly refracted wave making an angle θ^* with *z*-axis

(ii) A spectrum of *nth* order of irregularly refracted waves at angle θ_n^{+*} in the left side of regularly refracted wave

(iii) A similar spectrum of irregularly refracted waves at angle θ_n^{-*} in the right side of regularly refracted wave.

Total displacement v_1 in the lower medium H_1 is the sum of the displacements due to incident, regularly reflected and all irregularly reflected waves:

$$v_1 = \frac{1}{\sqrt{M_0}\cosh(z/a)} \left[\left(\exp(qz) + B_0 \exp(-qz) \right) \exp\left\{ i\,\omega \left(t - \frac{x \sin\theta}{\beta_h} \right) \right\} + \sum_n B_n^+ \exp(-q_n^+ z) \exp\left\{ i\,\omega \left(t - \frac{x \sin\theta_n^+}{\beta_h} \right) \right\} + \sum_n B_n^- \exp(-q_n^- z) \exp\left\{ i\,\omega \left(t - \frac{x \sin\theta_n^-}{\beta_h} \right) \right\} \right] \tag{16}$$

where

$$\left(q_n^+\right)^2 = \frac{1}{a^2} - \left(\frac{\omega \cos\theta_n^+}{\beta_v} \right)^2, \qquad \left(q_n^-\right)^2 = \frac{1}{a^2} - \left(\frac{\omega \cos\theta_n^-}{\beta_v} \right)^2 \tag{17}$$

B_0 is the amplitude of the regularly reflected SH-wave, B_n^+ and B_n^- are the amplitude of the irregularly reflected SH-waves with angle of reflection θ_n^+ and θ_n^- respectively.

Similarly, the total displacement v_2 in the upper medium H_2 is the sum of the displacements due to regularly transmitted waves and all irregularly transmitted waves:

$$v_2 = D_0 \exp(irz) \exp\left\{ i\,\omega \left(t - \frac{x \sin\theta^*}{\beta^*} \right) \right\} + \sum_n D_n^+ \exp(ir_n^+ z) \exp\left\{ i\,\omega \left(t - \frac{x \sin\theta_n^{+*}}{\beta^*} \right) \right\} + \sum_n D_n^- \exp(ir_n^- z) \exp\left\{ i\,\omega \left(t - \frac{x \sin\theta_n^{-*}}{\beta^*} \right) \right\} \tag{18}$$

where D_0 is the amplitude of the regularly refracted wave, θ^* is the angle which the refracted wave makes with the normal, D_n^+ and D_n^- are the amplitudes of the irregularly refracted waves with refracted angles θ_n^{+*} and θ_n^{-*} respectively, and

$$r = \frac{\omega \cos\theta^*}{\beta^*}, \qquad r_n^+ = \frac{\omega \cos\theta_n^{+*}}{\beta^*}, \qquad r_n^- = \frac{\omega \cos\theta_n^{-*}}{\beta^*}. \tag{19}$$

The angles θ and θ^* are connected by Snell's law (Kaushik and Rana [26]):

$$\frac{\omega\sin\theta}{\beta_h}=\frac{\omega\sin\theta^*}{\beta^*} \tag{20}$$

Also by Spectrum theorem (Asano [2]) we have the following relations between the angles of the regular waves and the corresponding irregular waves:

$$\begin{aligned}
&\sin\theta_n^+ - \sin\theta = \frac{np\,\beta_h}{\omega}, \qquad \sin\theta_n^- - \sin\theta = -\frac{np\,\beta_h}{\omega},\\
&\sin\theta_n^{+*} - \sin\theta^* = \frac{np\,\beta^*}{\omega}, \qquad \sin\theta_n^{-*} - \sin\theta^* = -\frac{np\,\beta^*}{\omega}.
\end{aligned} \tag{21}$$

3 BOUNDARY CONDITIONS

The constants B_0, D_0, B_n^+, D_n^+, B_n^- and D_n^- can be determined with the help of boundary conditions satisfied at the interface $z=\zeta$, i.e. continuity of displacements and stresses (Gupta [11]):

(i) $v_1 = v_2$

(ii) $M\left(\frac{\partial v_1}{\partial z} - \zeta'\frac{N}{M}\frac{\partial v_1}{\partial x}\right) = \mu^*\left(\frac{\partial v_2}{\partial z} - \zeta'\frac{\partial v_2}{\partial x}\right)$

where ζ' denotes the derivative of ζ with respect to *x*.

Placing the values of v_1 and v_2 given by Eqs. (16) and (18) into these boundary conditions and making use of relations (5), (20) and (21), we get

$$\begin{aligned}
&\left[\left(\exp(q\zeta)+B_0\exp(-q\zeta)\right)+\sum_n B_n^+\exp(-q_n^+\zeta)\exp(-inpx)+\sum_n B_n^-\exp(-q_n^-\zeta)\exp(inpx)\right]\\
&=\sqrt{M_0}\cosh\left(\frac{\zeta}{a}\right)\left[D_0\exp(ir\zeta)+\sum_n D_n^+\exp(ir_n^+\zeta)\exp(-inpx)+\sum_n D_n^-\exp(ir_n^-\zeta)\exp(inpx)\right]
\end{aligned} \tag{22}$$

and

$$\begin{aligned}
&\sqrt{M_0}\cosh\left(\frac{\zeta}{a}\right)\left[\left(\frac{i\zeta'\varepsilon\omega\sin\theta}{\beta_h}+q\right)\exp(q\zeta)+\left(\frac{i\zeta'\varepsilon\omega\sin\theta}{\beta_h}-q\right)B_0\exp(-q\zeta)\right.\\
&\left.+\sum_n B_n^+\left\{i\left(\frac{\omega\sin\theta}{\beta_h}+np\right)\zeta'\varepsilon-q_n^+\right\}\exp(-q_n^+\zeta)\exp(-inpx)+\sum_n B_n^-\left\{i\left(\frac{\omega\sin\theta}{\beta_h}-np\right)\zeta'\varepsilon-q_n^-\right\}\exp(-q_n^-\zeta)\exp(inpx)\right]\\
&-\frac{\sqrt{M_0}}{a}\sinh\left(\frac{\zeta}{a}\right)\left[\left(\exp(q\zeta)+B_0\exp(-q\zeta)\right)+\sum_n B_n^+\exp(-q_n^+\zeta)\exp(-inpx)+\sum_n B_n^-\exp(-q_n^-\zeta)\exp(inpx)\right]\\
&=i\,\mu^*\left[D_0\left(\frac{\zeta'\omega\sin\theta}{\beta_h}+r\right)\exp(ir\zeta)+\sum_n D_n^+\left\{\left(\frac{\omega\sin\theta}{\beta_h}+np\right)\zeta'+r_n^+\right\}\exp(ir_n^+\zeta)\exp(-inpx)\right.\\
&\left.\qquad+\sum_n D_n^-\left\{\left(\frac{\omega\sin\theta}{\beta_h}-np\right)\zeta'+r_n^-\right\}\exp(ir_n^-\zeta)\exp(inpx)\right]
\end{aligned} \tag{23}$$

where $\varepsilon=\frac{N_0}{M_0}$

4 SOLUTION FOR FIRST ORDER APPROXIMATION

For working out the approximate solutions, it is assumed that the corrugation of the surface $z=\zeta$ is so small that the terms of order higher than ζ may be neglected. Thus for first order corrugation,

$$\exp(\pm iq\zeta)=1\pm iq\zeta \tag{24}$$

Using Eqs. (1) and (24) into Eqs. (22) and (23), and comparing the terms independent of x and ζ in both sides of the resulting equations, we get

$$B_0=\frac{1-i\,\Delta}{1+i\,\Delta} \quad \text{and} \quad D_0=\frac{1}{\sqrt{M_0}}\left(\frac{2}{1+i\,\Delta}\right) \tag{25}$$

where

$$\Delta=\frac{\mu^* r}{M_0 q}=\frac{\rho_2}{\rho_0}\frac{\beta^*}{\beta_v}\frac{1}{P}\sqrt{1-\left(\frac{\beta^*}{\beta_h}\sin\theta\right)^2} \tag{26}$$

using Eqs. (4), (5), (10), (14) and (20)

$$P=\left(\frac{1}{a^2}\frac{\beta_v^2}{\omega^2}-\cos^2\theta\right)^{1/2} \tag{27}$$

Writing $\frac{\rho_2}{\rho_0}=\rho$, $\frac{\beta^*}{\beta_v}=Q_1-iQ_2$ and $\sqrt{1-\left(\frac{\beta^*}{\beta_h}\sin\theta\right)^2}=R_1-iR_2$ one can separate B_0 and D_0 into real and imaginary parts as $B_0=S-iS'$ and $D_0=\frac{1}{\sqrt{M_0}}\left[(1+S)-iS'\right]$, where

$$S=\frac{1-l^2-m^2}{(1+m)^2+l^2},\ S'=\frac{2l}{(1+m)^2+l^2},\ l=\frac{\rho(Q_1R_1+Q_2R_2)}{P},\ m=\frac{\rho(Q_1R_2-Q_2R_1)}{P} \tag{28}$$

Eq. (25) gives the reflection coefficient B_0 of SH-wave at a plane interface between transversely isotropic inhomogeneous and visco-elastic half-spaces, whereas the coefficient D_0 is related to the transmission coefficient T_0 by the relation $T_0=\sqrt{M_0}D_0$.

Equating the coefficients of $\exp(-inpx)$ for B_n^+ and D_n^+ on both sides of the Eqs. (22) and (23), we have

$$B_n^+-\sqrt{M_0}D_n^+=-(1-B_0)q\zeta_{-n}+i\sqrt{M_0}D_0 r\zeta_{-n} \tag{29}$$

$$q_n^+\sqrt{M_0}B_n^++ir_n^+\mu^*D_n^+=\sqrt{M_0}\zeta_{-n}(1+B_0)\left(\frac{\varepsilon np\omega\sin\theta}{\beta_h}+q^2-\frac{1}{a^2}\right)-\zeta_{-n}\mu^*D_0\left(\frac{np\omega\sin\theta}{\beta_h}-r^2\right) \tag{30}$$

Similarly, equating the coefficients of $\exp(inpx)$ for B_n^- and D_n^- on both sides of the Eqs. (22) and (23), we get

$$B_n^--\sqrt{M_0}D_n^-=-(1-B_0)q\zeta_n+i\sqrt{M_0}D_0 r\zeta_n \tag{31}$$

$$q_n^- \sqrt{M_0} B_n^- + i r_n^- \mu^* D_n^- = -\sqrt{M_0} \zeta_n \left(1 + B_0\right) \left(\frac{\varepsilon n p \omega \sin\theta}{\beta_h} - q^2 + \frac{1}{a^2} \right) + \zeta_n \mu^* D_0 \left(\frac{n p \omega \sin\theta}{\beta_h} + r^2 \right) \tag{32}$$

Solving Eqs. (29)–(30) and (31)–(32), one can obtain the coefficients B_n^+, B_n^-, D_n^+ and D_n^- for the first order approximation. B_n^+ and B_n^- are the reflection coefficients of the irregularly reflected waves and as in the case of regular waves the transmission coefficients T_n^+ and T_n^- of the irregularly refracted waves are given by the relation $T_n^+ = \sqrt{M_0} D_n^+$ and $T_n^- = \sqrt{M_0} D_n^-$.

5 SOLUTION FOR SECOND ORDER APPROXIMATION

For the solution of second order approximation the terms containing the third and higher powers of ζ are disregarded so that

$$\exp(\pm i q \zeta) = 1 \pm i q \zeta - \frac{(q\zeta)^2}{2} \tag{33}$$

Using Eqs. (1) and (33) into Eqs. (22) and (23), and comparing the terms independent of *x*, the coefficients of $\exp(-inpx)$, and those of $\exp(inpx)$ separately on both sides of the equations thus obtained, we get

$$\left(1 + \zeta_n \zeta_{-n} q^2\right)\left(1 + B_0\right) - \zeta_n q_n^+ B_n^+ - \zeta_{-n} q_n^- B_n^- = \sqrt{M_0} \left[\left(1 + \zeta_n \zeta_{-n} \left(\frac{1}{a^2} - r^2 \right)\right) D_0 + i \zeta_n r_n^+ D_n^+ + i \zeta_{-n} r_n^- D_n^- \right] \tag{34}$$

$$\begin{aligned} &\sqrt{M_0} \left[q \left(1 + \zeta_n \zeta_{-n} \left(q^2 - \frac{1}{a^2} \right)\right)(1 - B_0) \right] + \zeta_n \left(-np\varepsilon \left(\frac{\omega \sin\theta}{\beta_h} + np \right) + \left(q_n^+\right)^2 - \frac{1}{a^2} \right) B_n^+ + \zeta_{-n} \left(np\varepsilon \left(\frac{\omega \sin\theta}{\beta_h} - np \right) + \left(q_n^-\right)^2 - \frac{1}{a^2} \right) B_n^- \\ &= i \mu^* \left[r \left(1 - \zeta_n \zeta_{-n} r^2\right) D_0 + i \zeta_n \left(np \left(\frac{\omega \sin\theta}{\beta_h} + np \right) + \left(r_n^+\right)^2 \right) D_n^+ + i \zeta_{-n} \left(-np \left(\frac{\omega \sin\theta}{\beta_h} - np \right) + \left(r_n^-\right)^2 \right) D_n^- \right] \end{aligned} \tag{35}$$

$$\zeta_{-n} q \left(1 - B_0\right) + \left(1 + \zeta_n \zeta_{-n} \left(q_n^+\right)^2\right) B_n^+ + \frac{\zeta_{-n}^2}{2} \left(q_n^-\right)^2 B_n^- = \sqrt{M_0} \left[i \zeta_{-n} r D_0 + \left(1 + \zeta_n \zeta_{-n} \left(\frac{1}{a^2} - \left(r_n^+\right)^2 \right)\right) D_n^+ + \frac{\zeta_{-n}^2}{2} \left(\frac{1}{a^2} - \left(r_n^-\right)^2 \right) D_n^- \right] \tag{36}$$

$$\begin{aligned} &\sqrt{M_0} \left[\zeta_{-n} \left(np\varepsilon \frac{\omega \sin\theta}{\beta_h} + q^2 - \frac{1}{a^2} \right)\left(1 + B_0\right) - q_n^+ \left(1 + \zeta_n \zeta_{-n} \left(\left(q_n^+\right)^2 - \frac{1}{a^2} \right)\right) B_n^+ - \zeta_{-n}^2 q_n^- \left(np\varepsilon \left(\frac{\omega \sin\theta}{\beta_h} - np \right) + \frac{1}{2} \left(\left(q_n^-\right)^2 - \frac{1}{a^2} \right)\right) B_n^- \right] \\ &= i \mu^* \left[i \zeta_{-n} \left(r^2 - np \frac{\omega \sin\theta}{\beta_h} \right) D_0 + r_n^+ \left(1 - \zeta_n \zeta_{-n} \left(r_n^+\right)^2\right) D_n^+ + \zeta_{-n}^2 r_n^- \left(np \left(\frac{\omega \sin\theta}{\beta_h} - np \right) - \frac{1}{2} \left(r_n^-\right)^2 \right) D_n^- \right] \end{aligned} \tag{37}$$

$$\zeta_n q \left(1 - B_0\right) + \frac{\zeta_n^2}{2} \left(q_n^+\right)^2 B_n^+ + \left(1 + \zeta_n \zeta_{-n} \left(q_n^-\right)^2\right) B_n^- = \sqrt{M_0} \left[i \zeta_n r D_0 + \frac{\zeta_n^2}{2} \left(\frac{1}{a^2} - \left(r_n^+\right)^2 \right) D_n^+ + \left(1 + \zeta_n \zeta_{-n} \left(\frac{1}{a^2} - \left(r_n^-\right)^2 \right)\right) D_n^- \right] \tag{38}$$

$$\begin{aligned} &\sqrt{M_0} \left[\zeta_n \left(-np\varepsilon \frac{\omega \sin\theta}{\beta_h} + q^2 - \frac{1}{a^2} \right)\left(1 + B_0\right) + \zeta_n^2 q_n^+ \left(np\varepsilon \left(\frac{\omega \sin\theta}{\beta_h} + np \right) - \frac{1}{2} \left(\left(q_n^+\right)^2 - \frac{1}{a^2} \right)\right) B_n^+ - q_n^- \left(1 + \zeta_n \zeta_{-n} \left(\left(q_n^-\right)^2 + \frac{1}{a^2} \right)\right) B_n^- \right] \\ &= i \mu^* \left[i \zeta_n \left(r^2 + np \frac{\omega \sin\theta}{\beta_h} \right) D_0 - \zeta_n^2 r_n^+ \left(np \left(\frac{\omega \sin\theta}{\beta_h} + np \right) + \frac{1}{2} \left(r_n^+\right)^2 \right) D_n^+ + r_n^- \left(1 - \zeta_n \zeta_{-n} \left(r_n^-\right)^2\right) D_n^- \right]. \end{aligned} \tag{39}$$

By solving the above system of equations one can obtain the reflection and transmission coefficients of the reflected and refracted waves for the second order approximation.

6 REFLECTION AND REFRACTION COEFFICIENTS FOR SIMPLE HARMONIC BOUNDARY

Now we shall evaluate the reflection and refraction coefficients for an interface of simple harmonic type. Putting $\zeta_n = \zeta_{-n} = 0,\ (n \neq 1);\ \zeta_1 = \zeta_{-1} = h/2$ in Eq. (1), the equation of the boundary surface is given by $\zeta = h\cos px$ and the amplitude and period of the corrugation in this case are given by h and $2\pi/p$ respectively. Putting $n=1$ in Eqs. (29), (30), (31) and (32) and solving the resulting equations the coefficients for the considered case for the first order approximation are obtained as follows:

$$B_1^+ = \frac{\delta_{B1}^+}{\delta_1^+},\quad D_1^+ = \frac{\delta_{D1}^+}{\delta_1^+},\quad B_1^- = \frac{\delta_{B1}^-}{\delta_1^-},\quad D_1^- = \frac{\delta_{D1}^-}{\delta_1^-} \tag{40}$$

where

$$\delta_{B1}^+ = \frac{h}{2}\left[-i\,(1-B_0)qr_1^+\frac{\mu^*}{M_0} - \sqrt{M_0}D_0 rr_1^+\frac{\mu^*}{M_0} + (1+B_0)\left(\frac{\varepsilon p\omega\sin\theta}{\beta_h} + q^2 - \frac{1}{a^2}\right) - D_0\sqrt{M_0}\left(\frac{p\omega\sin\theta}{\beta_h} - r^2\right)\frac{\mu^*}{M_0}\right],$$

$$\delta_{D1}^+ = \frac{h}{2}\left[\frac{1}{\sqrt{M_0}}(1-B_0)qq_1^+ - iD_0 q_1^+ r + \frac{1}{\sqrt{M_0}}(1+B_0)\left(\frac{\varepsilon p\omega\sin\theta}{\beta_h} + q^2 - \frac{1}{a^2}\right) - D_0\left(\frac{p\omega\sin\theta}{\beta_h} - r^2\right)\frac{\mu^*}{M_0}\right],$$

$$\delta_{B1}^- = \frac{h}{2}\left[-i\,(1-B_0)qr_1^-\frac{\mu^*}{M_0} - \sqrt{M_0}D_0 rr_1^-\frac{\mu^*}{M_0} - (1+B_0)\left(\frac{\varepsilon p\omega\sin\theta}{\beta_h} - q^2 + \frac{1}{a^2}\right) + D_0\sqrt{M_0}\left(\frac{p\omega\sin\theta}{\beta_h} + r^2\right)\frac{\mu^*}{M_0}\right],$$

$$\delta_{D1}^- = \frac{h}{2}\left[\frac{1}{\sqrt{M_0}}(1-B_0)qq_1^- - iD_0 q_1^- r - \frac{1}{\sqrt{M_0}}(1+B_0)\left(\frac{\varepsilon p\omega\sin\theta}{\beta_h} - q^2 + \frac{1}{a^2}\right) + D_0\left(\frac{p\omega\sin\theta}{\beta_h} + r^2\right)\frac{\mu^*}{M_0}\right],$$

$$\delta_1^+ = q_1^+ + ir_1^+\frac{\mu^*}{M_0} \quad \text{and} \quad \delta_1^- = q_1^- + ir_1^-\frac{\mu^*}{M_0}$$

B_1^+ and B_1^- are the reflection coefficients of the irregularly reflected waves for the first order approximation when the interface is simple harmonic type and the transmission coefficients T_1^+ and T_1^- of the irregularly refracted waves in this case are given by the relation $T_1^+ = \sqrt{M_0}D_1^+$ and $T_1^- = \sqrt{M_0}D_1^-$.

7 DISCUSSION OF PARTICULAR VISCO-ELASTIC MATERIALS

In this section Voigt visco-elastic material and Maxwell's visco-elastic material are taken into consideration.

7.1 Voigt visco-elastic material

If we take the half-space H_2 as Voigt's visco-elastic material then μ^* and β^* will be given by

$$\mu^* = \mu + i\,\omega\eta \quad \text{and} \quad \beta^* = \beta_0(1 + i\,\omega\eta/\mu)^{1/2} \tag{41}$$

where μ is the elastic modulus of the spring, η is the viscosity of the dashpot and $\beta_0 = (\mu/\rho_2)^{1/2}$ is the zero frequency shear wave speed. In this case the values of Q_1, Q_2, R_1 and R_2 of Eq. (28) are given by

$$Q_1 = \frac{\beta_0}{\beta_v}\sqrt{\frac{1}{2}(k_1+1)},\quad Q_2 = \frac{\beta_0}{\beta_v}\sqrt{\frac{1}{2}(k_1-1)},\quad R_1 = \sqrt{\frac{1}{2}(k_2+k_3)},\quad R_2 = \sqrt{\frac{1}{2}(k_3-k_2)} \tag{42}$$

where

$$k_1 = \sqrt{1 + \left(\frac{\omega\eta}{\mu}\right)^2}, \quad k_2 = 1 - \beta^2 \sin^2\theta, \quad k_3 = \sqrt{k_2^2 + \left(\frac{\omega\eta}{\mu}\right)^2 \beta^4 \sin^4\theta} \quad \text{and} \quad \beta = \frac{\beta_0}{\beta_h} \tag{43}$$

7.2 Maxwell's visco-elastic material

Taking the half-space H_2 as Maxwell's visco-elastic material μ^* and β^* can be expressed as follows:

$$\mu^* = \mu / (1 - i\,\mu/\omega\eta) \quad \text{and} \quad \beta^* = \beta_\infty (1 - i\,\mu/\omega\eta)^{-1/2} \tag{44}$$

where μ is the elastic modulus of the spring , η is the viscosity of the dashpot and $\beta_\infty = (\mu/\rho_2)^{1/2}$ is the infinite frequency shear wave speed. The values of Q_1, Q_2, R_1 and R_2 are given by

$$Q_1 = \frac{\beta_\infty}{\beta_v l_1}\sqrt{\frac{1}{2}(l_1+1)}, \quad Q_2 = \frac{\beta_\infty}{\beta_v l_1}\sqrt{\frac{1}{2}(l_1-1)}, \quad R_1 = \frac{1}{l_1}\sqrt{\frac{1}{2}(l_2+l_3)}, \quad R_2 = \frac{1}{l_1}\sqrt{\frac{1}{2}(l_3-l_2)} \tag{45}$$

where

$$l_1 = \sqrt{1 + \left(\frac{\mu}{\omega\eta}\right)^2}, \quad l_2 = 1 - \beta^2 \sin^2\theta, \quad l_3 = \sqrt{l_2^2 + \left(\frac{\mu}{\omega\eta}\right)^2 \beta^4 \sin^4\theta} \quad \text{and} \quad \beta = \frac{\beta_\infty}{\beta_h} \tag{46}$$

8 PARTICULAR CASES

a) When the inhomogeneity of the half-space H_1 is removed:

In this case $a^{-1} = 0$ and the problem reduces to the problem of reflection and transmission of SH-waves at a corrugated interface between transversely isotropic and visco-elastic solid half-spaces.

b) When the anisotropy and inhomogeneity of the half-space H_1 are removed:

In this case, we have $M = N = \mu_1$, $\beta_h = \beta_v = \beta_1$, $a^{-1} = 0$ and the problem reduces to the problem of reflection and transmission of SH-waves at a corrugated interface between isotropic and visco-elastic solid half-spaces.

c) When the inhomogeneity of the half-space H_1 and viscosity of the half-space H_2 are removed:

In this case, we have $a^{-1} = 0$, $Q_2 = R_2 = 0$, $\eta = 0$ and the problem reduces to the problem of reflection and transmission of SH-waves at a corrugated interface between transversely isotropic and uniform elastic solid half-spaces.

d) When the anisotropy and inhomogeneity of the half-space H_1 and viscosity of the half-space H_2 are removed:

In this case, we have $M = N = \mu_1$, $\beta_h = \beta_v = \beta_1$, $a^{-1} = 0$, $\eta = 0$ and the problem reduces to the problem of reflection and transmission of SH-waves at a corrugated interface between two uniform elastic half-spaces of different material properties (Asano [3]).

e) When the corrugation of the interface, anisotropy and inhomogeneity of the half-space H_1 and viscosity of the half-space H_2 are removed:

In this case, we have $\zeta = 0$, $M = N = \mu_1$, $\beta_h = \beta_v = \beta_1$, $a^{-1} = 0$, $\eta = 0$ and the problem reduces to the problem of reflection and transmission of SH-waves at a plane boundary between two uniform elastic half-spaces of different material properties (Savarensky [27])

9 NUMERICAL RESULTS AND DISCUSSION

In order to examine the effect of frequency and angle of incidence of the wave, irregularity of the boundary and different parameters of the both the half-spaces on the behavior of reflection and transmission of plane SH-wave when it incident obliquely at a corrugated interface between the two medium, we have computed the modulus of reflection and transmission coefficients numerically for an interface of simple harmonic type given by $\zeta = h\cos px$. We have taken Voigt's material as visco-elastic material and calculation are done for reflection and transmission coefficients for different values of inhomogeneity, anisotropy, visco-elasticity factors and frequency. For numerical illustrations following values are used (Gubbins [28]).

For the lower half-space:

$$M_0 = 3.99\times10^{10}\,N/\text{m}^2,\quad N_0 = 5.82\times10^{10}\,N/\text{m}^2,\quad \rho_0 = 4500\,kg/\text{m}^3$$

For the upper half-space:

$$\mu = 7.1\times10^{10}\,N/\text{m}^2,\quad \rho_2 = 3321\,kg/\text{m}^3$$

Moreover, unless otherwise stated the others relevant parameters and values are taken as:

$$\eta/\mu = 0.02\,\text{s}^{-1},\ \ h/a = 0.01,\ \omega h/\beta_0 = 0.02,\ p = 0.0125\ ,\ h = 1$$

The variations of modulus of reflection and transmission coefficients of regular and irregular waves for different values of anisotropy factor, inhomogeneity parameter, frequency etc., are shown through Figs. 2 to 7. In the Figs. 2-7 we have used the notations B, T, B1, T1, B2, and T2 respectively for modulus of the coefficients B_0, T_0, B_1^+, T_1^+, B_1^- and T_1^-, and hereafter we shall use the same.

Figs. 2(a-f) show the variation of B, T, B1, T1, B2 and T2 respectively with angle of incidence for different values of the inhomogeneity parameter h/a. Curve labelled as 2, 3 and 4 corresponds for $h/a = 0.005$, 0.010 and 0.015 respectively, whereas curve 1 corresponds to the case when the lower half-space is homogeneous. We observe that the effect of inhomogeneity on all the coefficients except T1 is more at higher values of angle of incidence, whereas the effect of inhomogeneity on T1 is high at lower value of angle of incidence. In case when the lower half-space is homogeneous B attains its maximum value at $\theta = 90^\circ$ and all other coefficients attains their minimum value at $\theta = 90^\circ$, whereas in the inhomogeneous case points for maximum value of B and minimum value of all other coefficients lies prior to $\theta = 90^\circ$. Higher is the value of inhomogeneity factor lower is the value of angle of incidence for maximum or minimum values. Also at these points moduli of all the coefficients change their nature from increasing to decreasing and from decreasing to increasing. Besides these points of maxima and minima there also exist angles at which curves sharply change their nature.

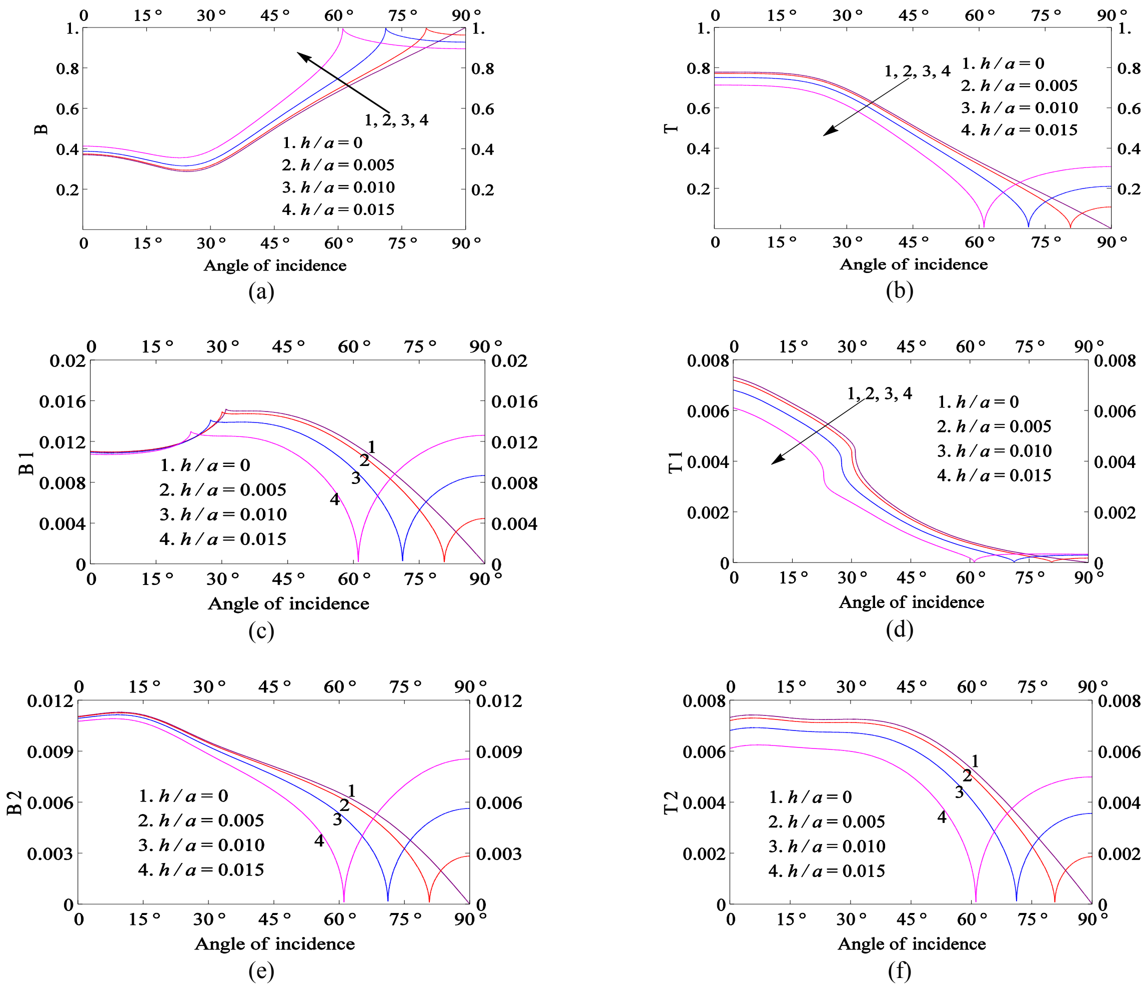
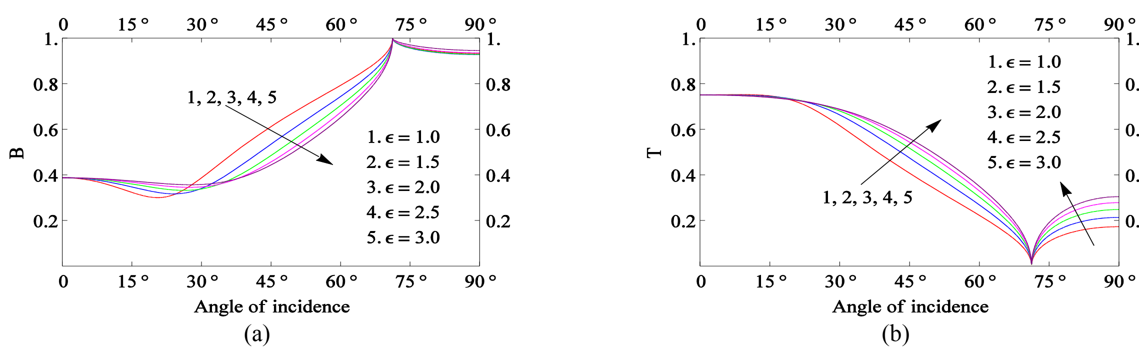


Fig.2
Modulus of reflection and refraction coefficients versus angle of incidence demonstrating the effect of inhomogeneity parameter.

Figs. 3(a-f) show the effect of anisotropy factor $\varepsilon\left(=N_0/M_0\right)$ of the lower half-space on the modulus of different coefficients. Curves labelled as 2, 3, 4 and 5 corresponds to $\varepsilon = 1.5, 2, 2.5$ and 3, whereas curve 1 corresponds to isotropic case. We see that anisotropy factor has significant effect on the moduli of all the coefficients. It is noticeable that the effect of anisotropy factor on B at angle where B attain its maximum value and the effect of anisotropy factor on the modulus of all other coefficients at angle where they attains their minimum value is nil. It is also observed that anisotropy factor has no effect on the modulus of reflection and transmission coefficients of regularly reflected and transmitted waves at $\theta = 0^{\circ}$.



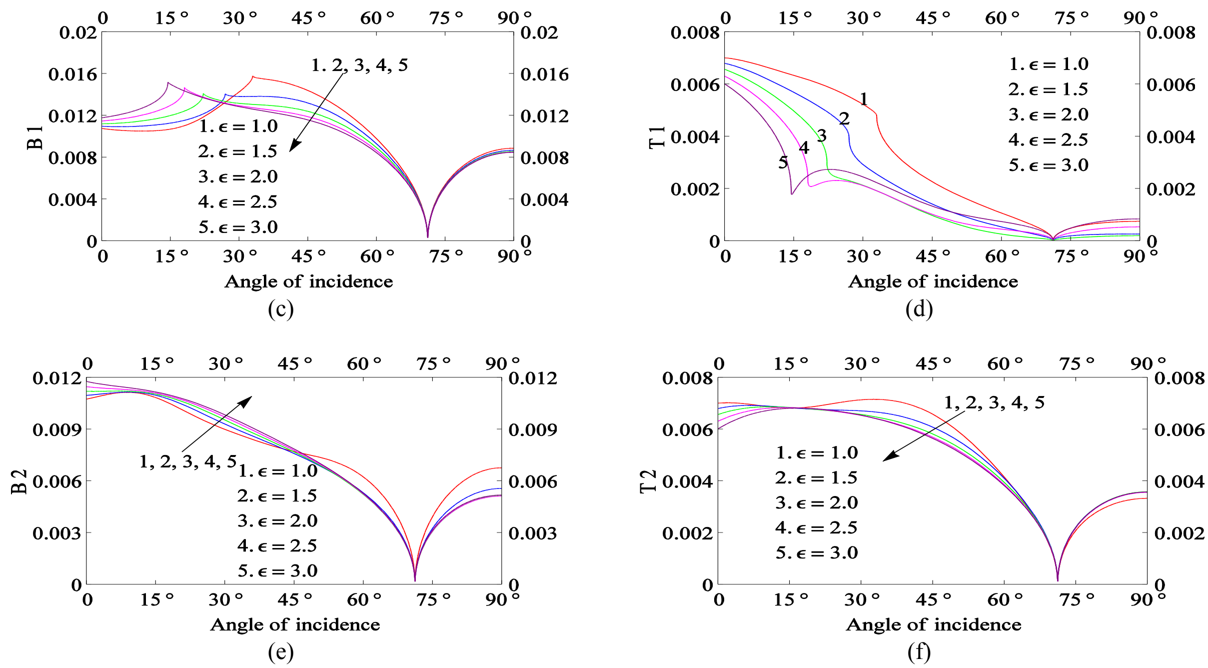
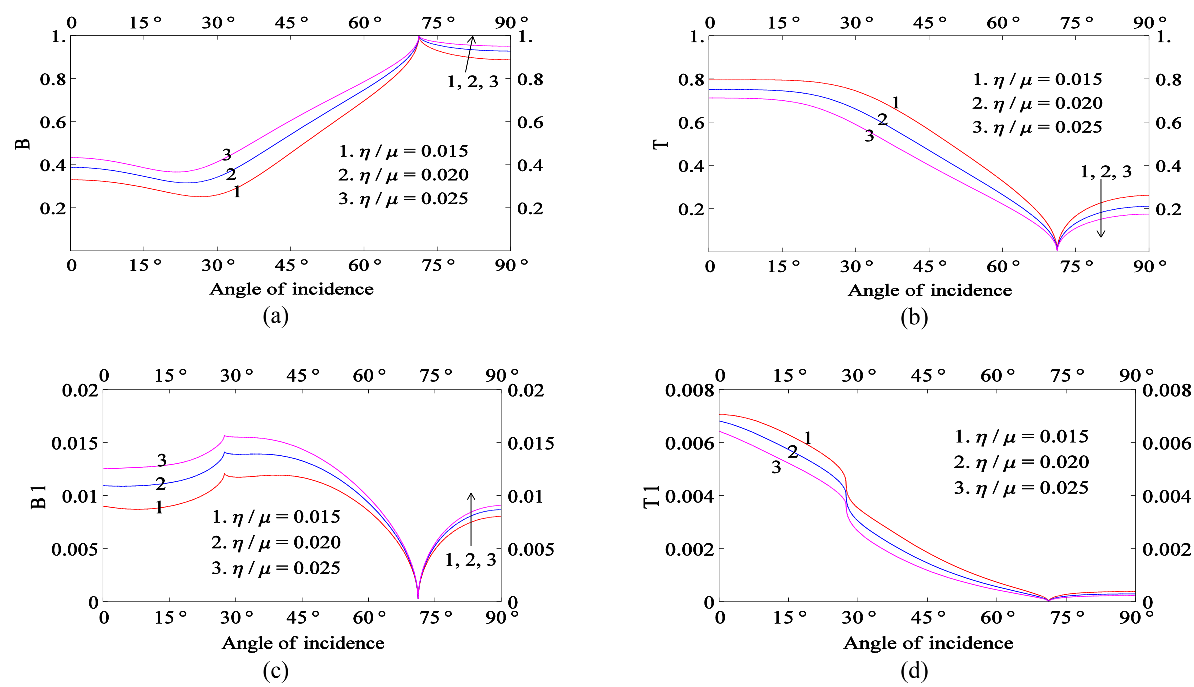


Fig.3
Modulus of reflection and refraction coefficients versus angle of incidence demonstrating the effect of anisotropy parameter ($M_0 = 3.99\times10^{10}\, N/\mathrm{m}^2$).

Figs. 4(a-f) show the effect of the parameter η/μ of the visco-elastic half-space on the modulus of the different coefficients. We observe that modulus of the reflection coefficients of regularly and irregularly reflected waves increase with increase in the value of η/μ, whereas modulus of the transmission coefficients of regularly and irregularly refracted waves decreases with increase in the value of η/μ. As in Fig. 3 it is noticed that the effect of the parameter η/μ on B at angle where B attain its maximum value and the effect of parameter η/μ on the modulus of all other coefficients at angle where they attains their minimum value is nil.



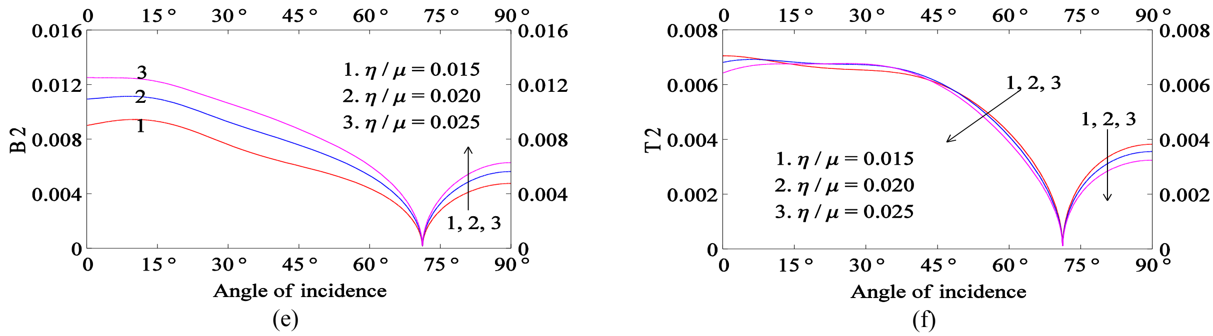
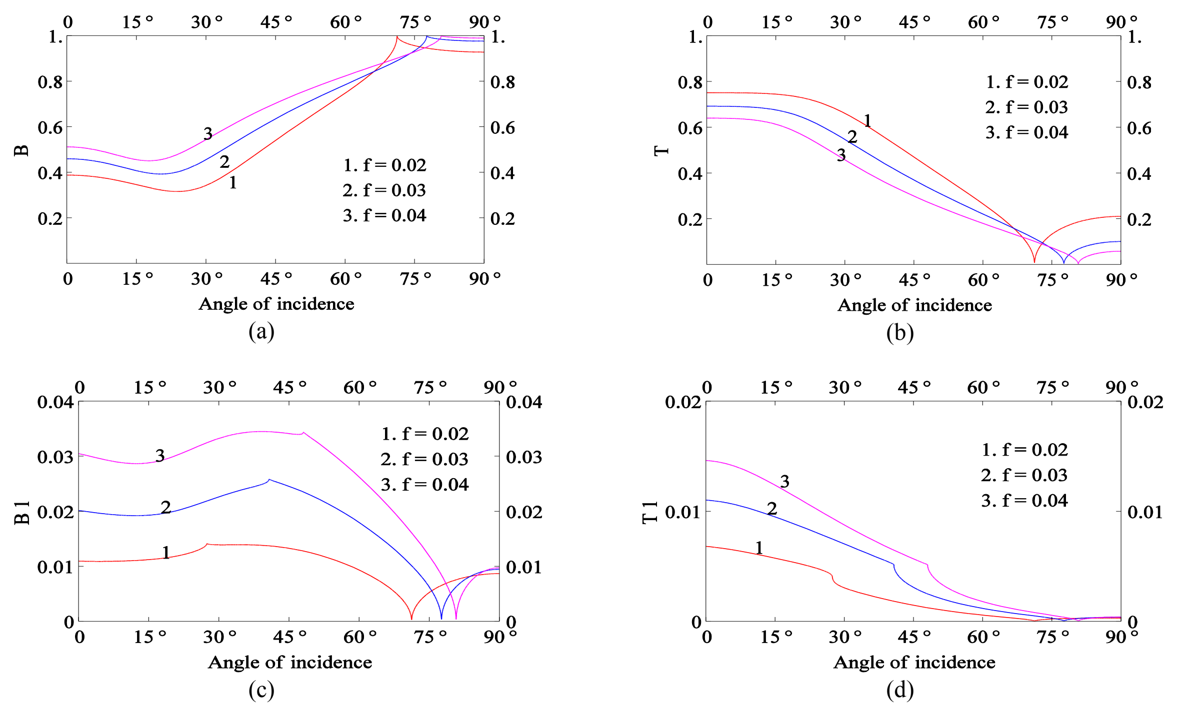


Fig.4
Modulus of reflection and refraction coefficients versus angle of incidence demonstrating the effect of parameter η/μ .

Figs. 5 and 6 depict the effect of dimensionless frequency parameter $\omega h/\beta_0$ $(=f$ say$)$ on the modulus of reflection and transmission coefficients of regular and irregular waves. Figs. 5 (a-f) show the variation of these coefficients with angle of incidence for different values of *f*. Curves labelled as 1, 2 and 3 corresponds to $f\ = 0.02,\, 0.03$ and 0.04 respectively. We observe that, except for higher angle of incidence modulus of all the coefficients except transmission coefficient of the regularly refracted waves increase with increase in frequency. Modulus of all the coefficients change their nature from increasing to decreasing and from decreasing to increasing at some higher values of θ . Higher the frequency, higher is the value of θ at which curves change their nature.

Figs. 6(a-f) show the variation of modulus of all coefficients with frequency for different values of angle of incidence. Dotted curves correspond to the case when the lower half-space is homogeneous, whereas solid curves correspond to inhomogeneous case. We see that as the frequency increases the effect of inhomogeneity decreases.



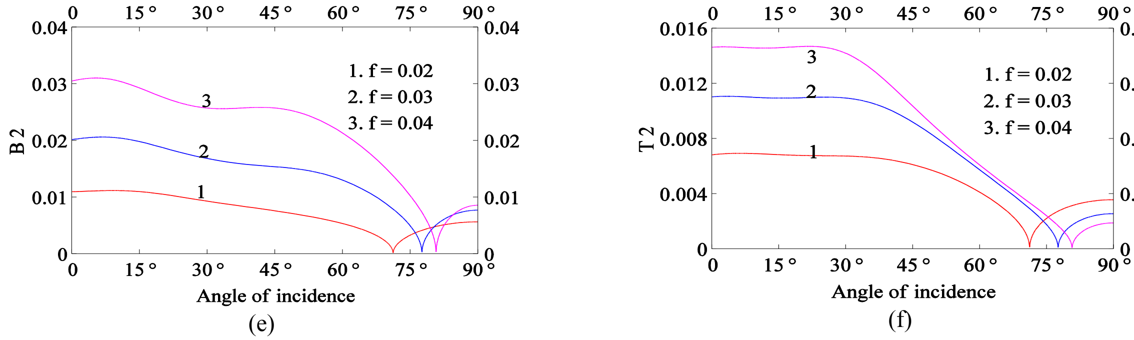


Fig.5
Modulus of reflection and refraction coefficients versus angle of incidence demonstrating the effect of frequency parameter $f \ (= \omega h/\beta_0)$.

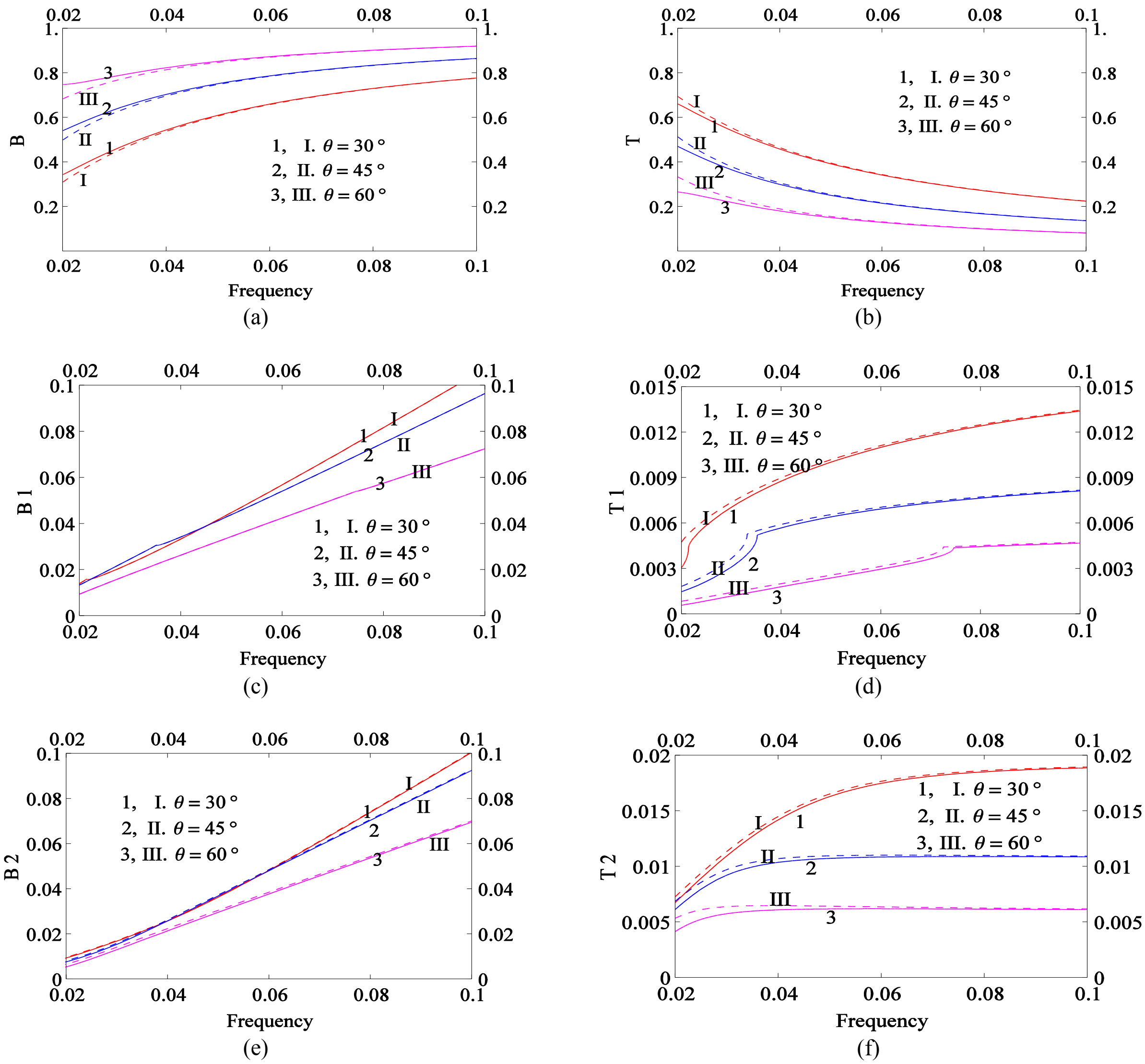


Fig.6
Variation of modulus of reflection and transmission coefficients with frequency $f \ (= \omega h/\beta_0)$ for different values of angle of incidence (solid curve: inhomogeneous case, dotted curve: homogeneous case).

Figs. 7(a-d) show the variation of modulus of reflection and transmission coefficient of irregularly reflected and transmitted waves for different value of *p*. We observe that except for B2 the effect of *p* is high at lower value of angle of incidence. We also note that the parameter *p* has no effect on these coefficients at that value of θ where the curve changes their nature.

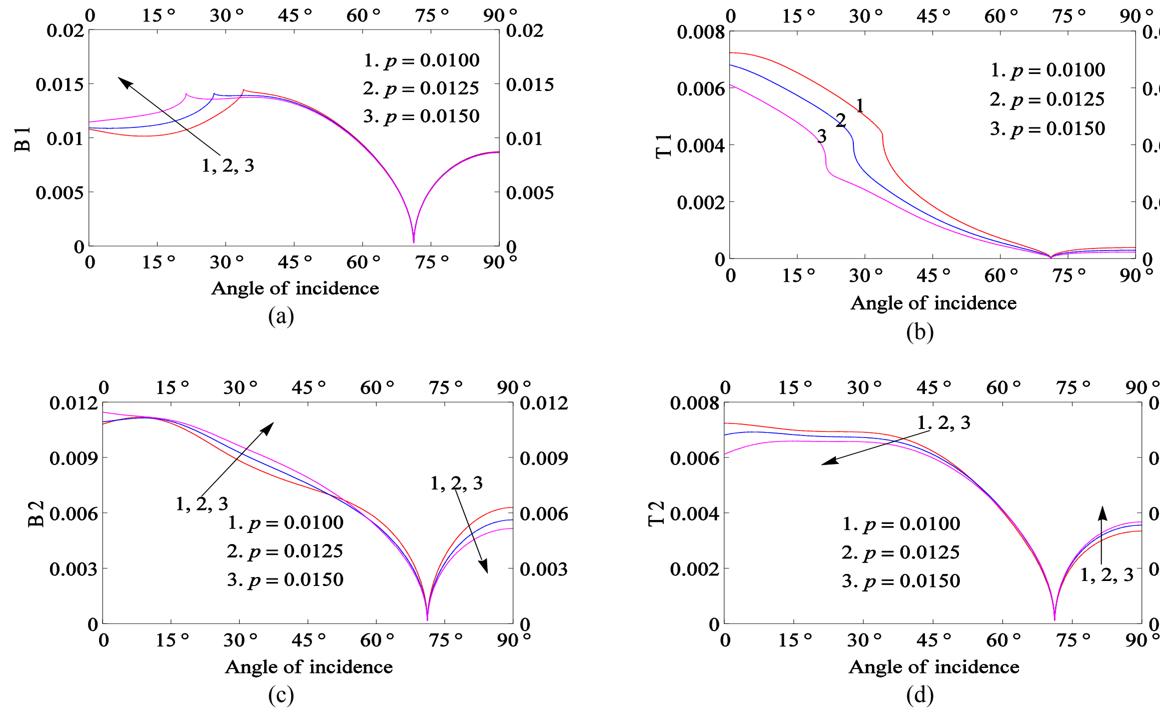


Fig.7
Modulus of reflection and refraction coefficients versus angle of incidence demonstrating the effect of parameter *p*.

10 CONCLUSIONS

Using Rayleigh's method of approximation reflection and transmission coefficients of a plane SH-wave incident at a corrugated boundary between transversely isotropic inhomogeneous and visco-elastic solid half-spaces are obtained for first and second order of approximation of corrugation. Further, these coefficients are obtained for an interface of periodic type for first order of approximation of corrugation. It is concluded that:

(i) The reflection and transmission coefficients of the irregularly reflected and refracted waves are proportional to the amplitude of the corrugated interface and are strongly influenced by the period of the corrugation.

(ii) The reflection and transmission coefficients of the irregularly reflected and refracted waves are very small in comparison to that of the regular waves.

(iii) Reflection and transmission coefficients are strongly influenced by inhomogeneity and anisotropy factors of the lower half-space and the parameter η/μ of the upper visco-elastic half-space. Modulus of all the coefficients except transmission coefficient of the regularly refracted waves increase with increase in the value of η/μ, whereas transmission coefficient of the regularly refracted waves decreases with increase in the value of η/μ. In case the lower half-space is homogeneous, modulus of the reflection coefficient of the regularly reflected waves attain its maximum value at $\theta = 90^\circ$, whereas modulus of all other coefficients attain their minimum value at $\theta = 90^\circ$. But in the non-homogeneous case, the angle of incidence for maximum value of modulus of the reflection coefficient of the regularly reflected and minimum value of modulus of all the other coefficients are less than 90°. This angle depends on the inhomogeneity of the lower half-space. Higher the value of inhomogeneity parameter, lower is the value of angle of incidence for maxima of modulus of the reflection coefficient of the regularly reflected waves and minima of modulus of all the other coefficients. It is also observed that at this particular angle anisotropy factor of the lower half-space, the parameter η/μ of the upper visco-elastic half-space and the period of the corrugation have no effect on the modulus of the different coefficients. Furthermore, the effect of the anisotropy factor of the lower half-space on the modulus of the reflection and transmission coefficients of the regularly reflected and refracted waves is nil at $\theta = 0^\circ$.

(iv) It is found that the reflection and transmission coefficients are influenced by the frequency parameter. Except for higher angle of incidence modulus of all the reflection and transmission coefficients except transmission coefficient of regularly refracted waves increases with increase in frequency, whereas transmission coefficient of regularly refracted waves decreases with increase in frequency.

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