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Problem of Rayleigh Wave Propagation in Thermoelastic Diffusion

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ABSTRACT

In this work, the problem of Rayleigh wave propagation is considered in the context of the theory of thermoelastic diffusion. The formulation is applied to a homogeneous isotropic thermoelastic half space with mass diffusion at the stress free, isothermal, isoconcentrated boundary. Using the potential functions and harmonic wave solution, three coupled dilatational waves and a shear wave is obtained. After developing mathematical formulation, the dispersion equation is obtained, which results to be complex and irrational. This equation is converted into a polynomial form of higher degree. The roots of this polynomial equation are verified for not satisfying the original dispersion equation and therefore are filtered out and the remaining roots are checked with the property of decay with depth. Phase velocity and attenuation coefficient of the Rayleigh wave are computed numerically and depicted graphically. Behavior of particle motion of these waves inside and at the surface of the thermoelastic medium with mass diffusion is studied. Some particular cases are also deduced from the present investigation.

Keywords : Rayleigh waves; Thermoelastic; Phase velocity; Attenuation coefficient; Diffusion.

1 INTRODUCTION

DIFFUSION is defined as the spontaneous migration of particles from region of high concentration to the region of low concentration. It is not a tool, but rather it is a phenomenon which has to be dealt with. It has tremendous applications in geophysics [1]. Diffusion is used to form the base and emitter in bipolar transistors, integrated resistors and the source/drain regions in Metal Oxide Semiconductor (MOS) transistors, and dope poly-silicon gates in MOS transistors [2].

Gekas, Öste, and Lamberg [3] established in their experiments on diffusion of nutrients in potato tissue by showing that nutrient movement is hindered by the structure of the tissue, in the similar basic way that atomic structure hinders diffusion in metals. The processes governing powder metallurgy and one type of ceramic material processing are greatly dependent on diffusional processes that combine distinct powdered grains into one cohesive material. The dependence of life processes on diffusion mechanisms could not be more prevalent. Diffusion occurs throughout the human body, and without it, cells and body tissue could not get important nutrients for survival, the eyes would dry out, and many medicines could not be absorbed into the body.

Thermodiffusion in an elastic solid is due to the coupling of the fields of temperature, mass diffusion and that of strain. Podstrigach[4] was the first to consider the problem of thermodiffusion in classical elastic material and

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investigated the elemental corollaries and differential equations. Podstrigach and Pavlina[5] extended the work of Podstrigach[4] of thermodynamical processes for an n-component solid solution. Podstrigach [6] also presented the diffusion theory of strain of an isotropic solid medium.

Nowacki [7-9] developed the theory of thermoelastic diffusion using coupled thermoelastic model. Nowacki [10] derived the basic equations for generalized thermoelastic diffusion. Sherief and Saleh [11] developed the generalized theory of thermoelastic diffusion with one relaxation time which allows finite speeds of propagation of waves. Kumar and Kansal [12] derived the basic equations for generalized thermoelastic diffusion (G-L model) and discussed the Lamb waves. Sharma [13] and Sharma et al [14] investigated the plane harmonic generalized thermoelastic diffusive waves and elasto-thermodiffusive surface waves in heat conducting solids. Kumar and Gupta [15] studied the plane wave propagation and proved uniqueness and reciprocity theorem in thermoelastic diffusion medium with fractional order derivative. A two-dimensional problem for an infinitely long solid conducting circular cylinder in the context of generalized thermoelastic diffusion theory with one relaxation time was studied by Allam, Omar and Ramadan [16].

Kumar and Kansal [17-18] discussed the propagation of Rayleigh waves with and without rotation in a homogeneous transversely isotropic, generalized thermoelastic diffusive half-space. Abouelregal [19] illustrated the effect of coupling parameter and phase-lags on Rayleigh waves in a thermoelastic solid half space. Sharma [20] studied the propagation of Rayleigh waves in a generalized thermoelastic medium for isothermal or insulated surface. Inspite of the above study the wave travelling along the free surface of thermoelastic half-space with mass diffusion such that the disturbance is largely confined to the neighbourhood of the boundary has not been considered. Therefore, Rayleigh wave propagation in thermoelastic half-space with mass diffusion is studied in this paper. The phase velocity, attenuation coefficient and path of surface particles of Rayleigh wave propagation are obtained from the secular equations. The resulting quantities are computed numerically and presented graphically.

2 BASIC EQUATIONS

The basic equations for a homogeneous isotropic elastic half space with thermoelastic diffusion in the absence of body forces, heat sources and mass diffusion sources are:

The constitutive relations

$$\sigma_{ij} = 2\mu e_{ij} + \delta_{ij}\left[\lambda e_{kk} - \beta_1 T - \beta_2 C\right] \tag{1}$$

$$\rho T_0 S = \rho C_E T + \beta_1 T_0 e_{kk} + a T_0 C \tag{2}$$

$$P = -\beta_2 e_{kk} - aT + bC \tag{3}$$

Equations of motion

$$(\lambda+\mu)\nabla(\nabla.\mathrm{u}) + \mu\nabla^2\mathrm{u} - \beta_1\nabla T - \beta_2\nabla C = \rho\ddot{\mathrm{u}} \tag{4}$$

Equation of heat conduction

$$K\nabla^2 T = \rho C_E\dot{T} + \beta_1 T_0\dot{e}_{kk} + aT_0\dot{C} \tag{5}$$

Equation of mass diffusion

$$D\beta_2\nabla^2\left(\nabla.\mathrm{u}\right) + Da\nabla^2 T - Db\nabla^2 C + \dot{C} = 0 \tag{6}$$

where λ, μ are the Lame's constants, ρ is the density assumed to be independent of time, *D* is the diffusivity, *P* is the chemical potential per unit mass, *C* is the concentration, u_i are the components of displacement vector *u, K* is the coefficient of thermal conductivity, C_E is the specific heat at constant strain, $T = \Theta - T_0$ is small temperature increment, Θ is the absolute temperature of the medium; T_0 is the reference temperature of the body chose such that $\left|(T/T_0)\right| << 1$, a and b are respectively, the coefficients describing the measure of thermodiffusion and mass diffusion effects respectively, σ_{ij}, e_{ij} are the components of the stress and strain respectively, e_{kk} is the dilatation, S is the entropy per unit mass, $\beta_1 = (3\lambda + 2\mu)\alpha_t$ and $\beta_2 = (3\lambda + 2\mu)\alpha_c, \alpha_t$ is the coefficient of thermal linear expansion, α_c is the coefficient of linear diffusion expansion. In the above equations, a comma followed by a suffix denotes spatial derivative and a superposed dot denotes the derivative with respect to time.

3 FORMULATION AND SOLUTION OF THE PROBLEM

We consider a homogeneous isotropic elastic half space under thermoelastic diffusion, initially at uniform temperature T_0. The origin of the coordinate system (x_1, x_2, x_3) is taken at any point on the plane horizontal surface with x_3 – axis pointing vertically downward to the half space, which is thus represented by $x_3 \geq 0$. The surface $x_3 = 0$ is subjected to stress free isothermal/isoconcentrated boundary. We choose the x_1 axis in the direction of wave propagation in such a way that all the particles on a line parallel to the x_2 axis are equally displaced. Therefore, all field quantities are independent of the x_2 coordinate. For the two dimensional problem, we take

$$\mathrm{u} = (u_1, 0, u_3),\ \mathrm{T}(x_1, x_3, t),\ C(x_1, x_3, t) \tag{7}$$

The displacement components u_1 and u_3 are related to the potential functions as:

$$u_1 = \frac{\partial \phi}{\partial x_1} + \frac{\partial \phi_4}{\partial x_3}, u_3 = \frac{\partial \phi}{\partial x_3} - \frac{\partial \phi_4}{\partial x_1} \tag{8}$$

where ϕ_4 is the potential function of shear wave.

The general form of the potential function ϕ for combined dilatation in linear thermoelastic with mass diffusion is expressed as:

$$\phi = \phi_1 + \phi_2 + \phi_3 \tag{9}$$

Making use of Eqs. (8) in (4)-(6) and with the aid of Eq. (7), we have

$$(\lambda + 2\mu)\nabla^2\phi - \beta_1 T - \beta_2 C = \rho\ddot{\phi} \tag{10}$$

$$K\nabla^2 T - \beta_1 T_0 \nabla^2 \dot{\phi} - aT_0 \dot{C} = \rho C_E \dot{T} \tag{11}$$

$$D\beta_2 \nabla^4 \phi + Da\nabla^2 T - Db\nabla^2 C + \dot{C} = 0 \tag{12}$$

$$\nabla^2 \phi_4 = \frac{1}{\beta^2}\ddot{\phi}_4 \tag{13}$$

Eq. (13) represents the propagation of transverse wave with velocity $\beta = \sqrt{\frac{\mu}{\rho}}$. Real value of β shows that transverse wave travels without attenuation in thermoelastic diffusion medium. For the propagation of harmonic waves in $x_1 - x_3$ plane, we assume

$$\{\phi, T, C, \phi_4\}(x_1, x_3, t) = \{\bar{\phi}, \bar{T}, \bar{C}, \bar{\phi}_4\} e^{-i\omega t} \tag{14}$$

where ω is the angular frequency of vibrations of material particles.

Substituting the values of ϕ, T, C, ϕ_4 from Eq. (14) into the Eqs. (10)-(13) after simplification, we obtain

$$\left[A\nabla^6 + B\nabla^4 + C\nabla^2 + D\right]\bar{\phi} = 0 \tag{15}$$

$$\left[\nabla^2 + \frac{\omega^2}{V_4^{\,2}}\right]\phi_4 = 0 \tag{16}$$

where

$$\begin{aligned}
A &= (\lambda + 2\mu) DbK - DK\beta_2^2, \\
B &= i\omega(\lambda + 2\mu)\left(\rho C_E Db + K + a^2 DT_0\right) + \rho\omega^2 DbK + i\omega DT_0\beta_1\left(\beta_1 b + \beta_2 a\right) - i\omega D\beta_2\left(\rho C_E \beta_2 - aT_0\beta_1\right), \\
C &= -(\lambda + 2\mu)\rho C_E \omega^2 + i\rho\omega^3\left(\rho C_E Db + K + a^2 DT_0\right) - \omega^2 T_0 \beta_1^2, \\
D &= -\omega^4 \rho^2 C_E
\end{aligned}$$

and $V_4 = \beta$ is the velocity of transverse wave.

The general solution of Eq. (15) can be written as:

$$\bar{\phi} = \bar{\phi}_1 + \bar{\phi}_2 + \bar{\phi}_3 \tag{17}$$

where the potentials $\bar{\phi}_i$, $\mathrm{i} = 1,2,3$ are solution of wave equations, given by

$$\left[\nabla^2 + \frac{\omega^2}{V_i^{\,2}}\right]\bar{\phi}_i = 0, \; i = 1,2,3 \tag{18}$$

Here V_1, V_2, V_3 are the velocities of three longitudinal waves namely qP wave, qT wave and a qMD wave in descending order of their real part and are derived from the roots of quadratic equations in V^2, given by

$$DV^6 - C\omega^2 V^4 + B\omega^4 V^2 - A\omega^6 = 0 \tag{19}$$

Here prefix "*q*" shows the coupling between elastic, thermal and diffusion fields. Also complex value of these velocities shows that these qP, qT and qMD waves are attenuated.

On using Eqs. (17)-(18) in Eqs. (10)-(12) and with the aid of Eq.(14), we have

$$\{\phi, T, C\} = \sum_{i=1}^{3} \{1, n_i, k_i\}\phi_i \tag{20}$$

where

$$\begin{aligned}
n_i &= \frac{\left(i\omega DT_0\left(a\beta_2 + b\beta_1\right) + V_i^{\,2}\beta_1 T_0\right)\omega^2}{\left(\rho C_E V_i^{\,4} + i\omega V_i^{\,2}\left(\rho C_E Db + K + a^2 DT_0\right) - DbK\omega^2\right)} \\
k_i &= \frac{\left(DK\beta_2\omega^2 - i\omega DV_i^{\,2}\left(\rho C_E\beta_2 - aT_0\beta_1\right)\right)\omega^2}{\left(\rho C_E V_i^{\,4} + i\omega V_i^{\,2}\left(\rho C_E Db + K + a^2 DT_0\right) - DbK\omega^2\right)V_i^{\,2}}, \; i = 1,2,3
\end{aligned}$$

For the propagation of plane harmonic waves with exponential decay in $x_1 - x_3$ plane, we take the displacement potentials as:

$$\phi_i = A_j e^{i\omega\left(\frac{x_1 + q_j x_3}{c} - t\right)}, \quad j = 1,2,3,4 \tag{21}$$

where, $q_j = \sqrt{\frac{c^2}{V_j^2} - 1}$, $(j = 1,2,3)$, $q_4 = \sqrt{\frac{c^2}{\beta^2} - 1}$ and C is the apparent phase velocity.

3.1 Boundary conditions

We consider the stress free isothermal and isoconcentrated surface, mathematically these can be written as:

(i) Mechanical conditions

$$\sigma_{33} = 0 \tag{22}$$

$$\sigma_{31} = 0 \tag{23}$$

(ii) Thermal condition for isothermal surface

$$T = 0 \tag{24}$$

(iii) Mass concentration condition for isoconcentrated surface

$$C = 0 \tag{25}$$

Making use the value of ϕ_i from Eq. (21) in the boundary conditions Eqs. (22)-(25) and with the aid of Eqs. (1), (7)-(9) and Eq. (20), we get a system of four homogeneous equations which can be written as:

$$\sum_{j=1}^{4} d_{ij} A_j = 0 \tag{26}$$

where,

$$d_{1j} = -\omega^2 \Pi_j + 2\frac{\omega^2}{h} - \beta_1 \frac{n_j}{\rho} - \beta_2 \frac{k_j}{\rho}, d_{2j} = 2q_j, d_{3j} = n_j, d_{4j} = k_j, \quad j = 1,2,3$$

$$d_{14} = 2\frac{\omega^2 q_4}{h}, d_{24} = q_4^2 - 1, d_{34} = d_{44} = 0$$

The system of Eq. (26) have a non-trivial solution if the determinant of the coefficients of this system vanishes, which yield the dispersion equation for propagation of Rayleigh waves as:

$$(2-h)\left[(2-\Pi_1 h) + p_1(2-\Pi_2 h) + p_2(2-\Pi_3 h)\right] = -4(q_1 + p_1 q_2 + p_2 q_3) q_4 \tag{27}$$

where $h = \frac{c^2}{\beta^2}, p_1 = \frac{n_3 k_1 - n_1 k_3}{n_2 k_3 - n_3 k_2}, p_2 = \frac{n_1 k_2 - n_2 k_1}{n_2 k_3 - n_3 k_2}$ and $\Pi_j = \frac{(\lambda + 2\mu)}{\rho V_j^2}, \quad j = 1,2,3$

As Eq.(27) is an irrational, so cannot be solved through the algebraic methods. After three squaring and some algebraic manipulations, the equation is reduced to a polynomial form of degree 15, which can be written as follows:

$$\sum_{j=0}^{15} C_j h_j = 0 \tag{28}$$

where the coefficients C_j are defined as:

$$C_{15}=Y_1, C_{14}=Y_2, C_{13}=Y_3, C_{12}=Y_4, C_{11}=Y_5, C_{10}=Y_6, C_9=(Y_7-F_1), C_8=(Y_8+F_1-F_2),$$
$$C_7=(Y_9+F_2-F_3), C_6=(Y_{10}+F_3-F_4), C_5=(Y_{11}+F_4-F_5), C_4=(Y_{12}+F_5-F_6),$$
$$C_3=(Y_{13}+F_6-F_7), C_2=(Y_{14}+F_7-F_8), C_1=(Y_{15}+F_8-F_9), C_0=(Y_{16}+F_9-F_{10}),$$
$$F_0=4096p_1^2p_2^2, F_1=F_0D_1B_1, F_2=F_0(E_1B_1+D_1B_2), F_3=F_0(E_2B_1+E_1B_2+D_1B_3),$$
$$F_4=F_0(E_3B_1+E_2B_2+E_1B_3+D_1B_4), F_5=F_0(E_4B_1+E_3B_2+E_2B_3+E_1B_4),$$
$$F_6=F_0(E_4B_2+E_3B_3+E_2B_4+D_1B_5+E_1B_5-B_1), F_7=F_0(E_4B_3+E_3B_4+E_2B_5-B_2),$$
$$F_8=F_0(E_4B_4+E_3B_5-B_3), F_9=F_0(E_4B_5-B_4), F_{10}=-B_5,$$
$$E_1=D_2-2D_1, E_2=D_3+D_1-2D_2, E_3=D_2-1-2D_3, E_4=D_3+2,$$
$$D_1=\varepsilon_1\varepsilon_2\varepsilon_3, D_2=-\varepsilon_1\varepsilon_2-\varepsilon_2\varepsilon_3-\varepsilon_3\varepsilon_1, D_3=\varepsilon_1+\varepsilon_2+\varepsilon_3,$$
$$B_1=N_1^2, B_2=2N_1N_2, B_3=(N_2^2+2N_1N_3), B_4=2N_2N_3, B_5=N_3^2,$$
$$Y_1=S^8, Y_2=2R_1S^4, Y_3=R_1^2+2W_0S^4, Y_4=2W_0R_1+2W_1S^4, Y_5=W_0^2+2W_1R_1+2W_2S^4,$$
$$Y_6=2W_0W_1+2W_2R_1+2W_3S^4, Y_7=W_1^2+2W_0W_2+2W_3R_1+2W_4S^4, Y_8=2W_1W_2+2W_0W_3+2W_4R_1+2W_5S^4,$$
$$Y_9=W_2^2+2W_1W_3+2W_0W_4+2W_5R_1+2W_6S^4, Y_{10}=2W_1W_4+2W_0W_5+2W_6R_1+2W_2W_3,$$
$$Y_{11}=W_3^2+2W_0W_6+2W_1W_5+2W_2W_4, Y_{12}=2W_1W_6+2W_2W_5+2W_3W_4, Y_{13}=W_4^2+2W_2W_6+2W_3W_5,$$
$$Y_{14}=2W_3W_6+2W_4W_5, Y_{15}=W_5^2+2W_4W_6, Y_{16}=2W_5W_6, Y_{17}=W_6^2,$$
$$U_0=1024p_1^2p_2^2, W_0=R_2-H_1, W_1=R_3-H_2, W_2=R_4-U_0\varepsilon_2\varepsilon_3-H_3, W_3=R_5-U_0U_1-H_4,$$
$$W_4=R_6-U_0U_2-H_5, W_5=R_7-U_0U_3-H_6, W_6=Q_3^2-U_0-N_3^2,$$
$$U_1=-(\varepsilon_2+\varepsilon_3)^2, U_2=(1+2\varepsilon_2+2\varepsilon_3+\varepsilon_2\varepsilon_3), U_3=-(\varepsilon_2+\varepsilon_3+2),$$
$$V_1=T_1-N_1^2, V_2=T_2-T_1, V_3=T_3-T_2, V_4=N_3^2-T_3,$$
$$H_1=\varepsilon_1 N_1^2, H_2=\varepsilon_1 V_1-N_1^2, H_3=\varepsilon_1 V_2-V_1, H_4=\varepsilon_1 V_3-V_2, H_5=\varepsilon_1 V_4-V_3,$$
$$H_6=-(\varepsilon_1N_3^2+V_4), T_1=2N_1N_2, T_2=N_2^2+2N_1N_3, T_3=2N_2N_3,$$
$$R_1=2M_1S^2, R_2=M_1^2+2Q_1S^2, R_3=2M_1Q_1+2Q_2S^2, R_4=Q_1^2+2Q_3S^2+2M_1Q_2,$$
$$R_5=2Q_1Q_2+2M_1Q_3, R_6=2Q_1Q_3+Q_2^2, R_7=2Q_2Q_3,$$
$$N_1=-8S, N_2=8(2S+R), N_3=-16R, Q_1=M_2-16G_1+16\varepsilon_1, Q_2=M_3-16(\varepsilon_1+1)+16(G_1+G_2),$$
$$Q_3=4R^2+16-16G_2, M_1=-2S(2S+R), M_2=(2S+R)^2+4SR, M_3=-4R(2S+R),$$
$$G_1=\varepsilon_2p_1^2+\varepsilon_3p_2^2, G_2=p_1^2+p_2^2, R=2(1+p_1+p_2), S=\Pi_1+p_1\Pi_2+p_2\Pi_3, \varepsilon_j=\left(\frac{\beta}{\alpha_j^2}\right)^2, \ (j=1,2,3).$$

In the fifteen roots of algebraic Eq. (28), some roots are those which are added while converting irrational Eq. (27) to the polynomial Eq. (28). These roots are identified for not satisfying the original dispersion Eq. (27). The remaining roots, which satisfy the Eq. (27) are again checked for the decay of the wavefield with increase in x_3 i.e. as we move away from the surface. The roots which satisfy both the checks represent the existence and propagation of Rayleigh waves in the elastic half space under thermoelastic diffusion. In this case only one root is obtained which satisfy both the checks. The value of phase velocity calculated from the root of Eq. (28) depends upon the frequency ω ensuring that Rayleigh wave is dispersive in elastic half space under thermoelastic diffusion. The complex value of *C* shows that Rayleigh waves are attenuated. Also it shows that these waves are inhomogeneous waves, which decay as we move away from the surface. For the complex *C*, the positive imaginary parts of the vertical slowness $\frac{q_j}{c},\left(j=1,2,3,4\right)$ in Eq. (20) ensures the decay of these waves in the region $x_3>0$.

The phase velocity and attenuation coefficient of Rayleigh wave is calculated by using the expression:
Phase velocity

$$V=\frac{\left|c^2\right|}{\operatorname{Re}\left(c\right)}=\frac{\beta\left|h\right|}{\operatorname{Re}\left(\sqrt{h}\right)}; \tag{29}$$

Attenuation coefficient

$$Q^{-1}=\frac{\operatorname{Im}\left(\frac{1}{c^2}\right)}{\operatorname{Re}\left(\frac{1}{c^2}\right)}=-\frac{\operatorname{Im}\left(h\right)}{\operatorname{Re}\left(h\right)}. \tag{30}$$

3.2 Path of surface particles

We shall now discuss the path of the particles at the surface $x_3=0$.

The displacement potentials Eq. (21) can be rewritten as:

$$\phi_i=A_1\gamma_j e^{i\left(kx_1-\omega t\right)+ikx_3q_j},\quad j=1,2,3,4, \tag{31}$$

where $k=\frac{\omega}{c}$ is the complex wave number and $\gamma_j=\frac{A_j}{A_1},\ j=1,2,3,4$ are the solution of system of Eq. (26).

where, $\gamma_1=1,\gamma_2=\frac{n_3\left(n_1k_2-n_2k_1\right)}{n_2\left(n_2k_3-n_3k_2\right)}-\frac{n_1}{n_2},\ \gamma_3=\frac{n_1k_2-n_2k_1}{n_2k_3-n_3k_2},\gamma_4=\frac{2q_1+2q_2\left(\frac{n_3}{n_2}p_2-\frac{n_1}{n_2}\right)+2q_3p_2}{1-q_4^2}$

Substituting the value of $\phi_i,\left(i=1,2,3,4\right)$ from Eq. (31) in Eq. (8) with the aid of Eq. (9), we have

$$\left(u_1,u_3\right)=\left(\left|U_0\right|e^{i\arg U_0},\left|W_0\right|e^{i\arg W_0}\right)e^{i\left(kx_1-\omega t\right)} \tag{32}$$

$$U_0=ikA_1\left(e^{ik_Rx_3\delta_1}+\gamma_2e^{ik_Rx_3\delta_2}+\gamma_3e^{ik_Rx_3\delta_3}+\gamma_4q_4e^{ik_Rx_3\delta_4}\right)e^{i\left(kx_1-\omega t\right)} \tag{33}$$

$$W_0=kA_1\left(q_1e^{ik_Rx_3\delta_1}+\gamma_2q_2e^{ik_Rx_3\delta_2}+\gamma_3q_3e^{ik_Rx_3\delta_3}-\gamma_4e^{ik_Rx_3\delta_4}\right)e^{i\left(kx_1-\omega t\right)} \tag{34}$$

where $\delta_j=\left(1-i\frac{c_I}{c_R}\right)q_j,\left(j=1,2,3,4\right)$. *R* and *I* denotes the real and imaginary part of the corresponding complex quantity. Similarly, from Eqs. (20) and (31), we have

$$\left(T,C\right)=\left(\left|\theta_0\right|e^{i\arg\theta_0},\left|C_0\right|e^{i\arg C_0}\right)e^{i\left(kx_1-\omega t\right)} \tag{35}$$

$$\theta_0 = A_1\left(n_1 e^{ik_R x_3 q_1} + n_2\gamma_2 e^{ik_R x_3 q_2} + n_3\gamma_3 e^{ik_R x_3 \delta_3} + n_4\gamma_4 e^{ik_R x_3 \delta_4}\right) \tag{36}$$

$$C_0 = A_1\left(k_1 e^{ik_R x_3 q_1} + k_2\gamma_2 e^{ik_R x_3 q_2} + k_3\gamma_3 e^{ik_R x_3 \delta_3} + k_4\gamma_4 e^{ik_R x_3 \delta_4}\right) \tag{37}$$

On the surface $x_3 = 0$, the Eq.(32) on retaining real parts leads to

$$\left.\begin{aligned} U &= \left|U_0\right| e^{-k_I x_1}\cos\left(\arg U_0 + \Phi\right) \\ W &= -\left|W_0\right| e^{-k_I x_1}\sin\left(\arg W_0 + \Phi\right) \end{aligned}\right\} \tag{38}$$

where $k_R\ (k_I)$ denotes the real (imaginary) parts of the complex wavenumber *K*. The parameter $\Phi\left(= k_R x - \omega t\right)$ is varied in $[0, 2\pi)$ to show the path traced at depth x_3. The parametric representation of curve shows that the surface particles trace elliptical path.

4 NUMERICAL RESULTS AND DISCUSSION

We now represent some numerical results for copper material, the physical data for which is given below:

$\lambda = 7.76\times10^{10} Kgm^{-1}s^{-2}$, $\mu = 3.86\times10^{10} Kgm^{-1}s^{-2}, T_0 = 0.318\times10^3 K$,
$C_E = .3831\times10^3 JKg^{-1}K^{-1}$, $\alpha_t = 1.78\times10^{-5}K^{-1}$, $\alpha_c = 1.98\times10^{-4}Kg^{-1}m^3$,
$a = 1.2\times10^4 m^2s^{-2}K^{-1}, b = 9\times10^5 Kg^{-1}m^5s^{-2}, D = 0.85\times10^{-8} Kgsm^{-3}$,
$\rho = 8.954\times10^3 Kgm^{-3}$, $K = 0.383\times10^3 Wm^{-1}K^{-1}$

The diffusion parameters are taken as:

$\alpha_c = 1.98\times10^{-4} m^3Kg^{-1}, a = 1.2\times10^4 m^2s^{-2}K^{-1}$,
$b = 9\times10^5 Kg^{-1}m^5s^{-2}, D = 0.85\times10^{-8} Kg\ s\ m^{-3}$

The software Matlab 7.0.4 has been used to determine the values of phase velocity *V* and attenuation coefficient Q^{-1} defined in previous section for different values of frequency ω ranging from $2\times\pi$ Hz to $2\times\pi\times10$ Hz. In the Figs.1-8, the numerical data is varied around their reference values, to depict their effect on the phase velocity, attenuation coefficient and particle motion of Rayleigh wave propagation in elastic half space under thermoelastic diffusion.

Figs.1-4 show respectively the effect of α_t, α_c, a, b on the variation of phase velocity (*V*) with frequency (ω). In all these figs, phase velocity decreases with increase in frequency and shows negligible variation for $\omega \geq 8$. At the same frequency limit, attenuation shows an opposite behavior, its values initially increases and then becomes stationary. In Fig.1 increase in the value of coefficient of linear thermal expansion also increases the value of phase velocity whereas decreases the attenuation. Fig.2 depicts that increase in the value of coefficient of linear diffusion expansion, decreases the value of phase velocity whereas increases the attenuation. In Fig.3 increase in the value of coefficient *a*, describing the measure of thermodiffusion effects, also increases the value of phase velocity whereas decreases the attenuation. It is clear from Fig.4, increase in the value of coefficient *b*, which describes the measure of mass diffusion effects, also increases the value of phase velocity whereas decreases the attenuation.

In Fig. 5-8, the particle motion (*U,W*) is computed at different depths, i.e. $k_R z = 0, 50, 100$. The effect of thermal and diffusion coefficients is observed on the polarizations of the material particles there. Fig. 5 depicts the effect of coefficient of linear thermal expansion α_t on the particle motion of Rayleigh wave. It is observed that increase in the value of α_t increases the particle motion. Also particle motion tilts for value of $\alpha_t = 3.78\times10^{-5}K^{-1}$. Fig. 6

depicts the effect of coefficient of linear diffusion expansion α_c on the particle motion of Rayleigh wave. It is observed that, particle motion is strengthening with increase in α_c and tilts for $\alpha_c = 5.98\times10^{-4}\,\mathrm{Kg^{-1}m^3}$. Fig. 7 shows the effect of coefficient describing the measure of thermodiffusion *a* on the polarization of Rayleigh wave. It is noticed that, particle motion is expanding and also slants for the value of $a = 19.2\times10^4\,\mathrm{m^2s^{-2}K^{-1}}$. Fig. 8 depicts the effect of coefficient describing the measure of massdiffusion i.e. *b* on the polarization of Rayleigh wave. Here also, increase in *b* enhances the particle motion. It slants for $b = 49\times10^5\,\mathrm{Kg^{-1}m^5s^{-2}}$.

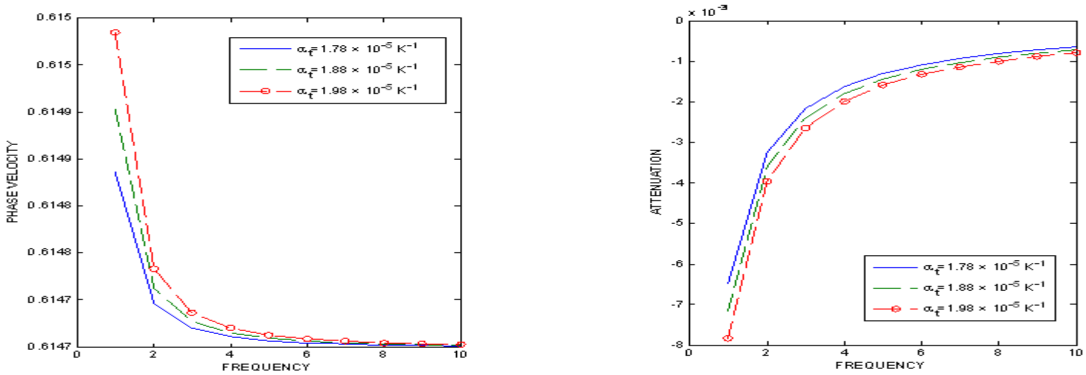

Fig.1
Variation of phase velocity (V) (m/s) and attenuation (Q^{-1}) with frequency (ω) (Hz) for different values of α_t.

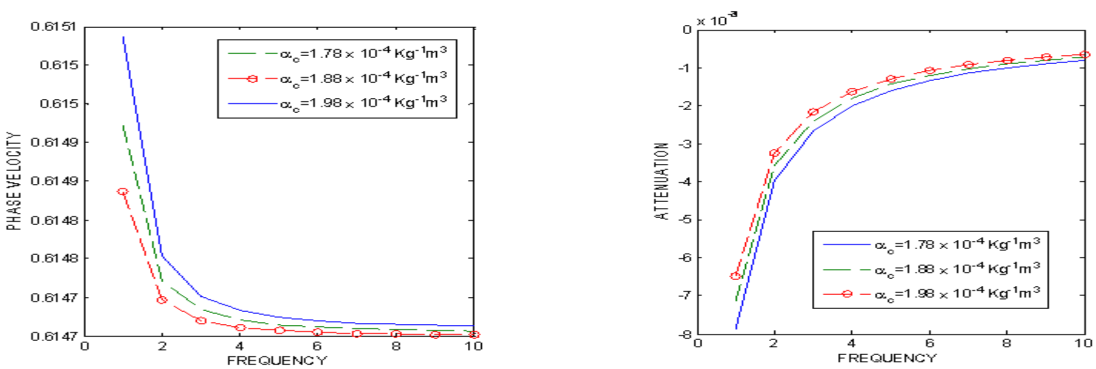

Fig.2
Variation of phase velocity (V) (m/s) and attenuation (Q^{-1}) with frequency (ω) (Hz) for different values of α_c.

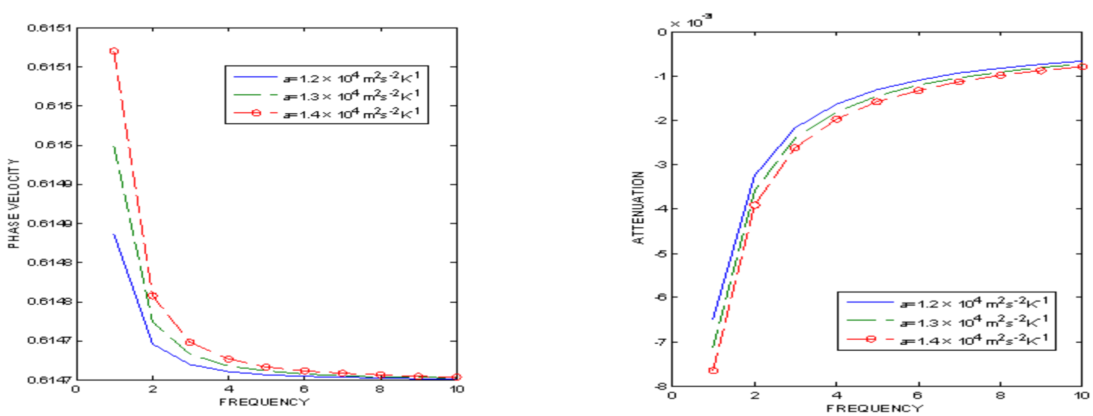

Fig.3
Variation of phase velocity (V) (m/s) and attenuation (Q^{-1}) with frequency (ω) (Hz) for different values of *a*.

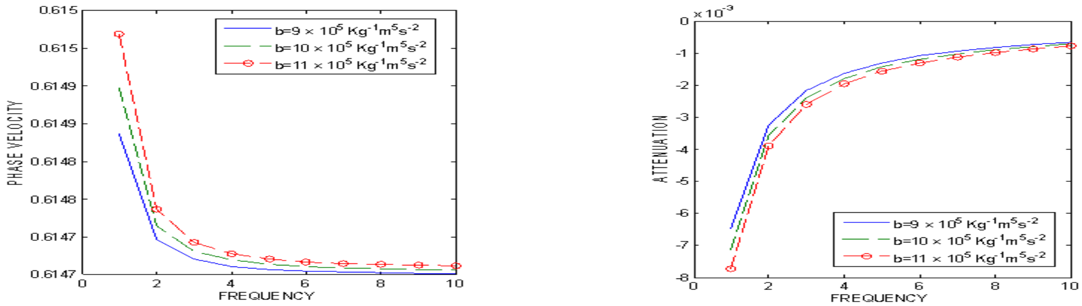

Fig.4
Variation of phase velocity (V) (m/s) and attenuation (Q^{-1}) with frequency (ω) (Hz) for different values of b.

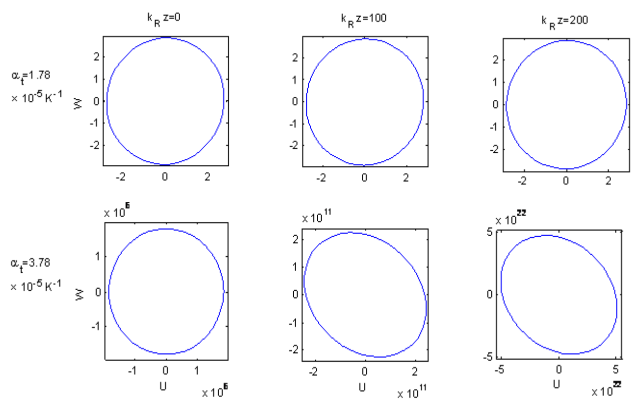

Fig.5
Variation of Particle motion (U,W) with depth $k_R z$ for different values of α_t.

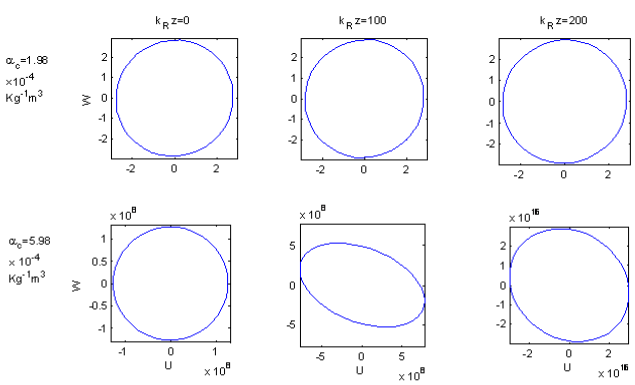

Fig.6
Variation of Particle motion (U,W) with depth $k_R z$ for different values of α_c.

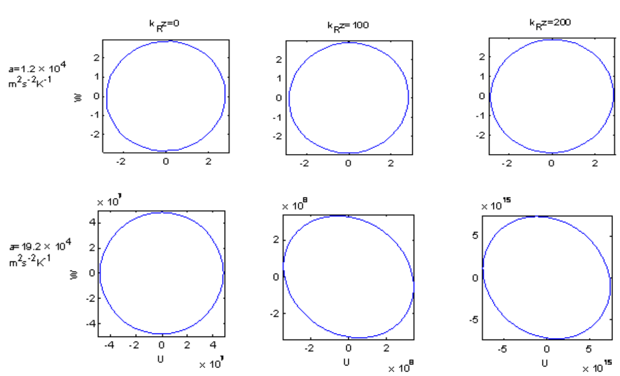

Fig.7
Variation of Particle motion (U,W) with depth $k_R z$ for different values of a.

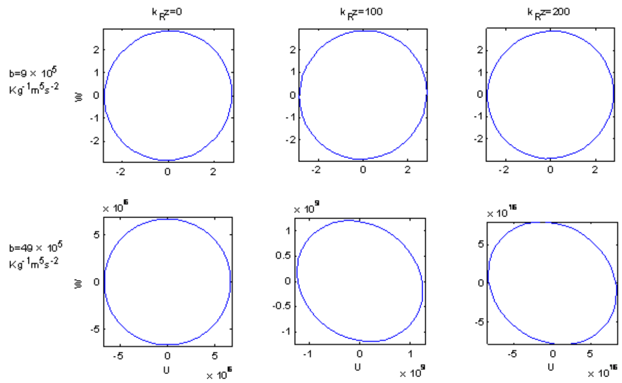

Fig.8
Variation of Particle motion (U,W) with depth $k_R z$ for different values of b.

4.1 Particular cases

If we take $k_3 = 1,\ n_3 = 0,\ p_2 = 0$ in the dispersion Eq. (27), we obtain the corresponding equation for the propagation of Rayleigh wave at the isothermal boundary of thermoelastic solid as:

$$(2-h)\left[(2-\Pi_1 h)-p(2-\Pi_2 h)\right]=-4(q_1-pq_2)q_3$$

which is similar to that obtained in Sharma [20] in absence of relaxation times.

If we take $\Pi_1 = 1,\ p_1 = 0,\ p_2 = 0$ in the dispersion Eq. (27), we obtain the corresponding equation for the propagation of Rayleigh wave in perfectly elastic solid as:

$$(2-h)^2 = -4q_1 q_4$$

which is similar to that given in Ewing et al.[21].

5 CONCLUSIONS

In the present work, propagation of Rayleigh waves is studied in a homogeneous isotropic thermoelastic half space with mass diffusion. Dispersion equations in form of complex irrational expression for Rayleigh wave propagation are obtained for isothermal and isoconcentrated surface. This equation is rationalized to a polynomial equation. Some of the roots of this polynomial equation are checked for not satisfying the original dispersion equation and thus are filtered out. The obtained roots are checked for their property of decay with depth. The dispersive character of Rayleigh waves and their inhomogeneous nature is ensured.

The analysis of the graphs gives some concluding remarks:

The phase velocity is found to be inversely proportional to the frequency while attenuation is directly proportional to the frequency.

Increase in the value of thermal and diffusion coefficients strengthens the particle motion of Rayleigh wave.

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