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Edge Crack Studies in Rotating FGM Disks

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ABSTRACT

This article focused on the stress analysis of an edge crack in a thin hallow rotating functionally graded material (FGM) disk. The disk is assumed to be isotropic with exponentially varying elastic modulus in the radial direction. A comprehensive study is carried out for various combinations of the crack length and orientation with the different gradation of materials. The effect of non-uniform coefficient of thermal expansion on the distribution of stress intensity factor is also studied. The results which are normalized for the advantage of non-dimensional analysis show that the material gradation, the crack orientation and the crack length have significant influence on the amount of stress intensity factors.

Keywords : Functionally graded materials; Rotating discs; Edge crack, Stress intensity factor.

1 INTRODUCTION

FUNCTIONALLY graded materials have attracted much interest primarily as an alternative to thermal barrier coating (TBCs), which are used in aerospace and high temperature applications. The possibility of tailoring the desired thermo mechanical properties holds a wide range application potential for FGMs.

FGMs are multiphase materials in which the volume fractions of the constituents vary continuously as a function of position. Therefore, the mismatch of thermo mechanical properties near the bond line is minimized. Another application area of FGMs include their use as interfacial zone between two different layers, improves the bonding strength [1], and reduces the residual stresses, interfacial delamination [2] and stress concentration or stress intensity factors [3-4]. Because of their outstanding advantages over conventional composites and monolithic materials, these materials have received wide attention of engineers and researchers from different fields of interest. Kim and Paulino [5] have addressed a wide variety of FGMs applications. So far, the effects of material distribution on the characteristics of these materials under various loading conditions and for various geometries have been investigated from different points of view.

Because of the outstanding advantages of FGMs over conventional composites and monolithic materials, these materials have been extensively studied for potential applications as structural elements, such as FGM plates, cylinders and discs. Rotating discs are very common and useful parts of several engineering high speed rotating equipments such as compressors, cutters, grinding tools and brake disks that find extensive use in the process industrial today. Parallel to new industrial developments, it seems that the use of conventional materials in rotating discs is inadequate.

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Although FGM discs can be used for better performance in the unusual working conditions, generation of defects such as holes, cavity, and cracks in the material substructures during manufacturing or in-service conditions is inevitable. The fatigue failure of such components often develops from the propagation of surface defects. Therefore, consideration of fracture mechanic criterion in the design process of this equipment is essential for reliable application in the above mentioned equipments. Crack analysis of FGMs is an indispensable task in the optimization, reliable and durable design of functionally graded materials and structures in innovative engineering applications. For crack problems in FGMs with general geometrical and loading conditions, efficient and accurate numerical simulation tools are required due to the high mathematical complexity of the arising governing partial differential equations.

With the trend toward the analyses of cracked structural parts under centrifugal loading, much attention is being paid to investigate the strength and life of a rotating cracked disc. Unfortunately, the most studies are focused to homogeneous discs. For example, Tweed and Rooke [6] considered the homogeneous rotating disc with a radial and edge crack, and Isida [7] considered it for a crack in an arbitrary position. A rigorous elastodynamic hybrid displacement finite element procedure for a safety analysis of fast rotating discs with mixed mode cracks is considered by Chen and Lin [8]. Cho and Park [9] have investigated the thermo-elastic characteristics of functionally graded lathe cutting tools. Zenkour [10] considered a rotating FGM sandwich solid disk with material gradient in the thickness direction for the analysis of stress and displacement.

The problem of finite element analysis of thermo-elastic field in a thin circular FGM disk with exponentially variation of material properties in radial direction is considered by Afsar and Go [11]. Hosseini Tehrani and Talebi [12] studied the stress and temperature distribution in a functionally graded brake disk. Eskandari [13-14] considered the FGM rotating disk with cracks located at the radial and arbitrary positions.

In the present paper, an edge crack in a thin hollow circular FGM disk is considered (Fig. 1). The Young modulus and coefficient of the thermal expansion of homogeneous disk is assumed equal to 210 (GPa) and 7.4×10^{-6} (1/^{o}C) respectively. The mentioned properties are also adopted for inner radius of the FGM disk. A comprehensive study is carried out for various combinations of the crack length, orientation, and location with the different gradation of materials. The effect of non-uniform coefficient of thermal expansion on the distribution of stress intensity factor is also studied.

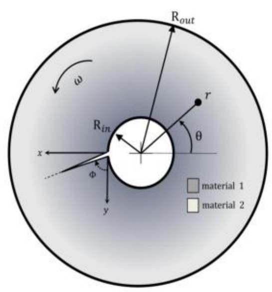


Fig.1
Rotating FGM disk containing an edge crack.

2 FINITE ELEMENT FORMULATION

Consider a rotating FGM disk with a concentric circular hole containing an edge crack as shown in Fig. 1. The FGM disk is considered to be made of two distinct material phases, which are respectively, represented by the dark and white colors as shown in the figure. The distribution of each material continuously varies along the radial direction. The radii of the hole and outer surface of the disk are designated by R_{in} and R_{out} , respectively. Further, the angular velocity of the disk is denoted by ω .

A finite element code can be used to account for spatial variation in material property of FGM disk. There are different ways of incorporating changes in material properties into a finite element program. Walter et al. [15] describe two commonly-used methods. An element base method where the desired spatial material property of each element based on its location is achieved through a finite element code. Another way is to compute the material property at each integration point for element stiffness matrix via the spatially varying in material property function.

In this study, a finite element code is used to account the material property changes for each element via its location. This section describes the details of the finite element formulation for stress and fracture analyses of FGM disk. Here, the singularity elements are applied around the crack tip and the isoparametric brick elements are used everywhere except near the crack tip. The singularity elements have square-root terms in their assumed displacement distribution and, therefore, produce a singular stress field at the crack front.

The material is assumed to be isotropic with exponentially varying elastic modulus and coefficient of thermal expansion (CTE), in radial direction as follow which are used in literature [16],

$$E(r) = E_{in} e^{e(r-R_{in})}$$
$$\alpha(r) = \alpha_{in} e^{\beta(r-R_{in})} \tag{1}$$

where R_{in} is the inner radius of the disk and e, β are the constants of material and thermal non-homogeneity which defined as [16]

$$e = \frac{1}{R_{in} - R_{out}} \ln(\frac{E_{in}}{E_{out}})$$
$$\beta = \frac{1}{R_{in} - R_{out}} \ln(\frac{\alpha_{in}}{\alpha_{out}}) \tag{2}$$

which R_{in} and R_{out} denote the inner and outer radius of the disk, E_{in}, α_{in} and E_{out}, α_{out} are the values of elastic modulus and CTE at the inner and outer radius of the disk, respectively. For a simple traceable solution, the dependency to the Poisson's ratio is neglected and it is assumed constant ($\nu = 0.3$) throughout the disk.

The stress intensity factor for the disk is considered in the non-dimensional form and is defined as [14]

$$(K_I)_{normalized} = \frac{K_I}{K_0}$$
$$(K_{II})_{normalized} = \frac{K_{II}}{K_0} \tag{3}$$

In which K_I is the calculated value of the first mode stress intensity factors in the crack tip. K_{II} is the calculated values of the second mode stress intensity factors in the crack tip.

The nominal stress intensity factor, K_0, for FGM disk is used as [14]

$$K_0 = \sigma_0 \sqrt{\pi\alpha} \tag{4}$$

where σ_0 is defined as [14]

$$\sigma_0 = \frac{3+\nu}{8} \rho V^2 \tag{5}$$

α is the crack length, ρ and V being the material density and the peripheral speed, respectively. For varying thermal expansion FGM disk, the stress intensity factors are normalized by

$$K_0 = \alpha E_1 T_0 \sqrt{\pi\alpha}$$

where T_0 is a reference value of temperature. The dimensionless crack length is known as:

$$\lambda = \frac{\alpha}{R_{in}} \tag{6}$$

The FGM disk considered in the present study is assumed to be fixed to a rotating shaft. The outer surface of the disk is free from any mechanical load. Thus, the boundary condition of the problem can be given by

$$\begin{aligned} r &= R_{in}, \qquad u_r = 0 \\ r &= R_{out}, \qquad \sigma_r = 0 \end{aligned} \tag{7}$$

3 CRACK TIP FIELDS IN FGMs

Material non-homogeneity has a significant influence on SIFs, which in turn will influence subsequent crack trajectory [17]. Williams [18] proposed the eigenfunction expansion technique to investigate the nature of the near-tip fields in a two-dimensional crack body. An extension of this conventional procedure has used by Eischen [19] to establish the general form of the stress and displacement fields near a crack tip in a nonhomogeneous material with a spatially varying material property. Eischen [19] solved the problem for materials with continuous, bounded, and differentiable property variations. He showed that the asymptotic fields for a crack in an FGM with continuous mechanical properties are same as those of a crack embedded in a homogeneous material. In addition, the asymptotic displacement expressions for the homogeneous materials can be used for FGMs on condition that the material properties are calculated at the crack-front location. Jin and Noda [20] further showed that this result is also valid for materials with piecewise differentiable property variation.

A crack in a continuously non-homogeneous, isotropic and linear elastic FGM body with applied boundary conditions on the body satisfies the equilibrium equation. As mentioned previously, for a simple traceable solution, the functional dependence to the Poisson's ratio is neglected and it is assumed constant throughout the analysis. Since the nature of the stress singularity for continuously non-homogenous, isotropic and linear elastic solid is precisely the same as the well-known form applicable to homogeneous materials, irrespective of the particular form of the Young's modulus variation [19], the stress intensity factors can be obtained from crack-opening-displacements (CODs) as [21]:

$$\begin{Bmatrix} K_I \\ K_{II} \\ K_{III} \end{Bmatrix} = \frac{\mu_{tip}\sqrt{2\pi}}{4(1-\nu)} \lim \frac{1}{\sqrt{\delta}} \begin{Bmatrix} \Delta u_{\varsigma}(\delta) \\ \Delta u_{\xi}(\delta) \\ (1-\nu)\Delta u_{\eta}(\delta) \end{Bmatrix} \tag{8}$$

where K_I, K_{II} and K_{III} are opening, sliding and tearing modes of SIFs, μ_{tip} is the shear modulus at the crack front, δ which approaches zero is a small distance between specified node at crack-surface and a node at crack-front, and $\Delta u_I(X) = [u_I(X\,\varepsilon\; upper\; crack\;\; surface) - u_I(X\,\varepsilon\; lower\; crack\;\; surface)]\; in\; which\;\; I = \varsigma, \xi\; and\;\; \eta$ are the CODs in the local coordinate systems.

4 THE VALIDATION OF THE METHOD

4.1 Stresses in a functionally graded strip

To justify the reliability of the FGM model, we consider the semi-infinite functionally graded strip shown in Fig. 2 (a) and apply the uniform tensile load. Young's modulus is an exponential function of *z,* i.e. $E(z) = E_1 e^{\ln(\frac{E_2}{E_1})\frac{z}{W}}$, while Poisson's ratio is assumed to be constant.

Fig. 2 (b) shows the normalized σ_{yy} stresses for different levels of material gradation. These results agree well with those of Erdogan and Wu [22]. Thus, such excellent results validate the present FEM implementation for elastic FGMs.

4.2 Edge crack in a plate

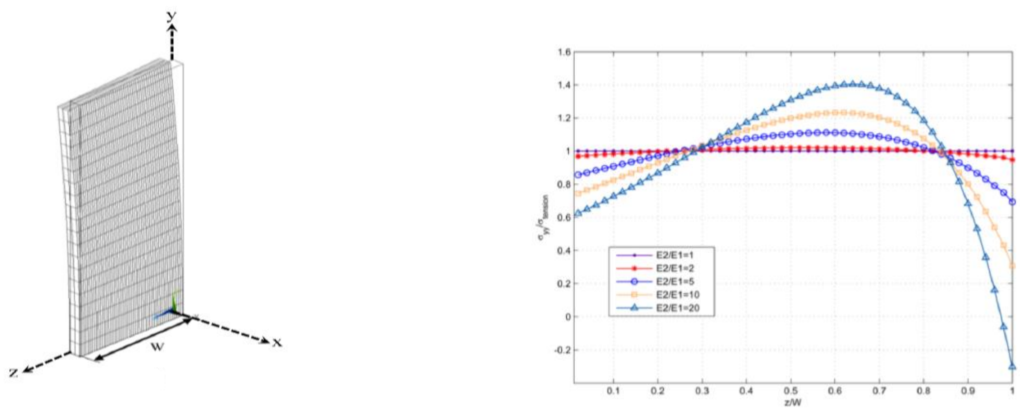
Fig.3 shows an edge crack of length a located in a finite two-dimensional functionally graded strip under tension loading. As in the case of stress analysis of uncracked strip, the same form of material property gradation is considered. Table 1. compares the normalized SIFs of current study with those reported by Chen et al. [23]. As can be seen from this table, good agreement is obtained between the different solutions for both homogeneous and FGM cases.

Table 1
Normalized stress intensity factors for an edge cracked FGM plate under tension.

Method	E_2/E_1	*a/W*				
		0.2	0.3	0.4	0.5	0.6
Chen *et al*.[19]	0.2	1.455	1.897	2.529	3.443	4.926
	1	1.408	1.698	2.178	2.933	4.237
	5	1.158	1.392	1.794	2.446	3.611
	10	1.032	1.249	1.614	2.223	3.337
Current study	0.2	1.455	1.915	2.539	3.442	4.880
	1	1.428	1.733	2.205	2.951	4.216
	5	1.182	1.430	1.826	2.472	3.603
	10	1.046	1.282	1.658	2.273	3.357

4.3 Arbitrary crack in a rotating solid disk

Consider the homogeneous elastic rotating disk with no hole. The stress intensity factors for right hand side of the crack at different positions of the solid disk are determined and compared with those reported by Isida [7]. We solved the problem by two different methods, i.e. the Displacement Correlation Technique (DCT) and the *J*-integral method. The results are shown in Fig. 4. As it can be seen from Fig. 4, the results are agree well with those reported in literature by Isida [7].



(a) (b)

Fig.2
(a) Finite-element model of uncracked functionally graded strip, (b) Normalized σ_{yy} stresses for different levels of material gradation – uniform tensile far-field stress.

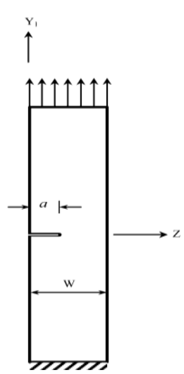


Fig.3
A functionally graded semi-infinite strip containing an edge crack.

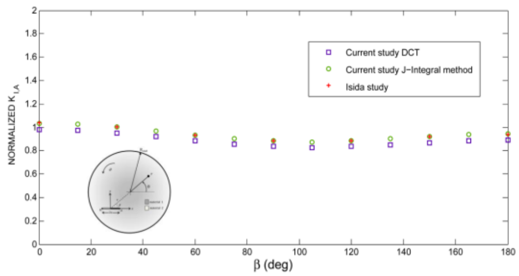


Fig.4
Variation of the normalized stress intensity factors with β for $\lambda = \varepsilon = 0.5$.

5 RESULTS AND DISCUSSION

5.1 Convergence study

In this case, convergence of the stress intensity factor is studied while the number of singular elements around the crack front in the finite element model ranged from 8 to 24.

The results of a convergence study on the stress intensity factors for an edge crack with $\varphi = 90, \dfrac{E_{in}}{E_{out}} = 20$ and in the absence of thermal parameters are shown at Fig. 5. The models were composed of either 8,12,16,20 or 24 equal wedges. The twenty-wedge model gave results less than 1% of those from the finest model. The results demonstrate rapid convergence of the solution and indicate that the twenty-wedge model may be used to obtain the stress intensity factor at the crack tip.

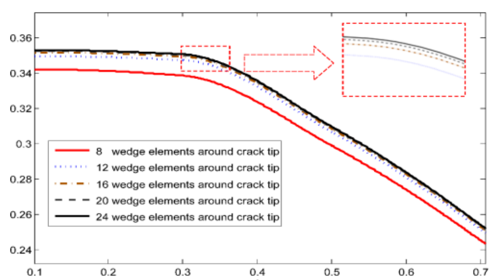


Fig.5
Convergence of stress intensity factor for an edge crack with $\varphi = 90$ in FGM disk with $\dfrac{E_{in}}{E_{out}} = 20$.

5.2 Stress intensity factors for an edge crack in a FGM disk

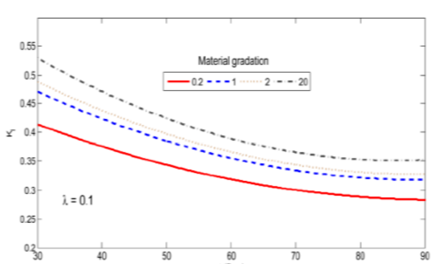
In this study, the problem of stress intensity factors analysis for an edge crack in a rotating FGM disk at different crack orientation angles is studied. The effect of crack length on values of the stress intensity factor is also studied. Variation of the normalized stress intensity factor with the crack orientation angle is plotted at Figs. 6 (a) through 6 (f). Each curve is plotted for a certain value of dimensionless crack length and different values of the material gradation, *i.e.,* $\dfrac{E_{in}}{E_{out}} = 0.2, 1, 2$ and 20. From Figs. 6, it is evident that for small cracks with $\lambda \leq 0.4$, higher the gradation of materials, i.e. $\dfrac{E_{in}}{E_{out}}$, higher the K_I . For large cracks with $\lambda \succ 0.4$, there exists a separation angle in which this trend is hold before it and apt to be reversed after that. For example, it can be seen from Figs. 6 (e) and 6 (f) that the separation angles for cracks with $\lambda = 0.5$ and 0.6 are happened at angles 52 and 42 degrees, respectively. As seen, for homogeneous materials and FGM's with $\dfrac{E_{in}}{E_{out}} \succ 1$, higher the orientation angle ϕ ., smaller the K_I . In other words, for FGM disks with $\dfrac{E_{in}}{E_{out}} \prec 1$ and $\lambda \succ 0.4$, higher the orientation angle ϕ , higher the K_I . Thus, the radial edge cracks are the safest ones in homogeneous and FGM's with $\frac{E_{in}}{E_{out}} \succ 1$.

Variation of normalized K_I with dimensionless crack length in the range of $0.1 \le \lambda \le 0.6$, for a radially edge crack in a FGM disk is plotted in Fig. 7. A piecewise cubic spline is exactly fitted on the data. As seen, there exist some points in which the curves have met. In the range of $0.1 \le \lambda \le 0.38$, the critical point of the stress intensity factor is occurred in the FGM with higher $\frac{E_{in}}{E_{out}}$. In this case, FGMs with smaller gradation are the safest ones. In other words, for cracks with λ between 0.4 to 0.57, higher the gradation of materials, safer the FGM disc.

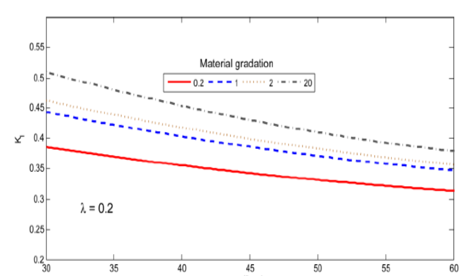
Fig. 8 shows the variation of K_I versus dimensionless crack length for different crack orientation in a FGM disk with gradation of 0.2. It can be seen that for edge cracks with $\lambda \le 0.42$, the orientation angle of 30^{o} is the critical ones. In other words, the radial edge cracks with $\lambda \succ 0.42$, have critical stress intensity factors.

So far, the stress intensity factors K_I for the opening mode deformation have been discussed. On the other hand, the shear mode stress intensity factor K_{II} are found to be rather small compared with K_I of the corresponding crack tips. This fact may be attributed to the biaxial tension state of the untracked disk. Fig. 9 shows the variation of normalized K_{II} values with crack orientation angle for the typical case when $\lambda = 0.1$, and they are shown to be fairly small compared with the corresponding K_I values given in Fig. 6 (a). Therefore, the shear mode deformation can be neglected in the fracture analysis of a rotating FGM disk.

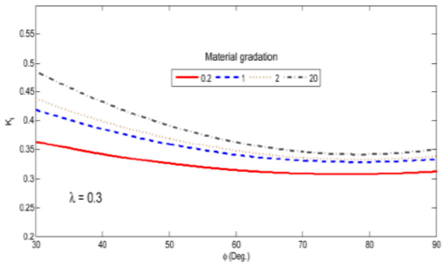
In order to investigate the effect of thermal expansion variations on the value of the stress intensity factor at the tip of the crack, we consider the above FGM disk with a radially edge crack, which is assumed to be isotropic with exponentially varying the thermal expansion (CTE) through the radial direction. Fig. 10 illustrates the variation of normalized K_I with dimensionless crack length (λ) in FGM disk with a radially edge crack for constant gradation of material, i.e. $\frac{E_{in}}{E_{out}} = 1$ and different values of the thermal expansion ($\frac{\alpha_{in}}{\alpha_{out}} = 0.2, 1, 2$ and 20). It can be seen that for materials with $\frac{\alpha_{in}}{\alpha_{out}} \ge 1$, higher the crack length, smaller the values of the K_I at the crack tips. In other words, for disks with $\frac{\alpha_{in}}{\alpha_{out}} \prec 1$, the values of K_I increases with increasing the crack length. It means that the maximum SIF for disks with $\frac{\alpha_{in}}{\alpha_{out}} \prec 1$ occurs the tips of the larger cracks and it occurs in tips of the lower cracks for disks with $\frac{\alpha_{in}}{\alpha_{out}} \ge 1$. It can be also concluded that, for cracks with $\lambda \ge 0.3$, smaller the gradation of the thermal expansion, higher the values of K_I at the crack tips. This tends to reverse for small cracks with $\lambda \le 0.18$.



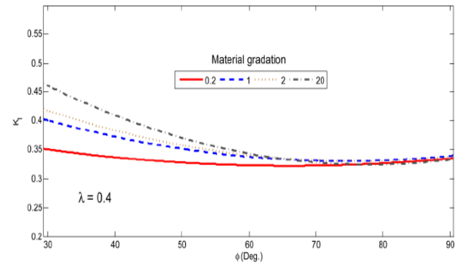
(a)



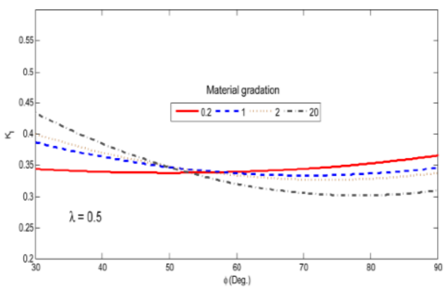
(b)



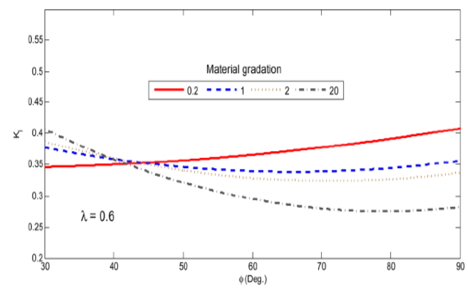
(c)



(d)



(e)



(f)

Fig.6
Variation of normalized K_I with ϕ for different gradation of materials and different dimensionless crack lengths.

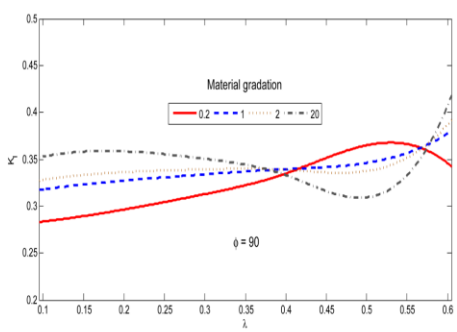


Fig.7
Variation of normalized K_I with dimensionless crack length in a FGM disk with a radially edge crack for different gradation of materials, i.e. $\dfrac{E_{in}}{E_{out}} = 0.2, 1, 2$ and 20.

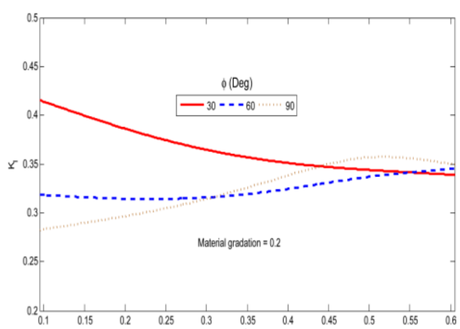


Fig.8
Variation of normalized K_I with dimensionless crack length in a FGM disk with an edge crack at different orientation.

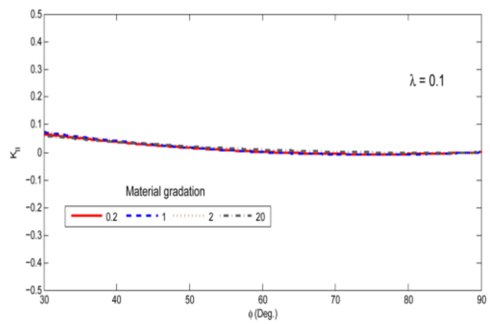


Fig.9
Variation of normalized K_{II} with the crack orientation angle for different values of material gradation.

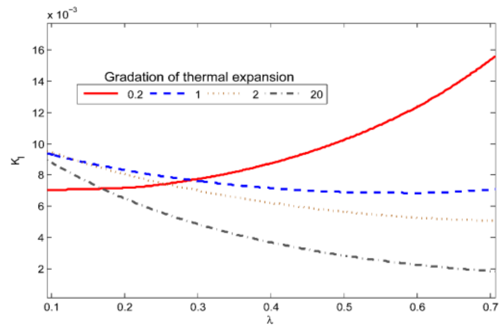


Fig.10
Variation of normalized K_I with dimensionless crack length (λ) in an exponentially varying thermal expansion FGM disk with a radially edge crack for constant gradation of material, i.e. $\frac{E_{in}}{E_{out}} = 1$.

6 CONCLUSIONS

In this study the stress analysis of edged cracks in a thin hallow rotating FGM disk is carried out. The disk is assumed to be isotropic with exponentially varying elastic modulus in the radial direction. A comprehensive study is carried out for various combinations of the crack lengths and orientations with the different gradation of materials. Then the variation in the coefficient of thermal expansion (CTE's) for radially edge cracks is also considered. The results which are normalized for the advantage of non-dimensional analysis show that the material and thermal expansion gradation, the crack orientation and the crack length have significant influence on the amount of stress intensity factors. The critical values of stress intensity factors and their orientation in homogeneous and FGM disks are obtained. In the absence of the thermal expansion coefficient, larger the cracks, larger the stress intensity factors in homogeneous disks. In general, this is not valid in FGM disks. Numerical results are given to assess the safety of the FGM and homogeneous cracked disks.

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REFERENCES

[1] Kurihara K., Sasaki K., Kawarada M., 1990, Adhesion improvement of diamond films, *Proceedings of the First International Symposium on Functionally Gradient Materials*, Tokyo.
[2] Lee Y. D., Erdogan F., 1995, Residual/thermal stresses in FGM and laminated thermal barrier coatings, *International Journal of Fracture* **69**(2): 145-165.
[3] Erdogan F., 1995, Fracture mechanics of functionally graded materials, *Composites Part B: Engineering* **5**(7): 753-770.
[4] Kim J. H., Paulino G.H., 2002, Isoparametric graded finite elements for nonhomogeneous isotropic and orthotropic materials, *Journal of Applied Mechanics* **69**(4): 502-514.
[5] Kim J. H., Paulino G. H., 2002, Finite element evaluation of mixed mode stress intensity factors in functionally graded materials, *International Journal for Numerical Methods in Engineering* **53**: 1903-1935.

[6] Rooke D. P., Tweed J., 1973, The stress intensity factor of an edge crack in a finite rotating elastic disc, *International Journal of Engineering Science* **11**: 279-283.
[7] Isida M., 1981, Rotating disk containing an internal crack located at an arbitrary position, *Engineering Fracture Mechanics* **14**: 549-555.
[8] Chen W. H., Lin T. C., 1983, A mixed-mode crack analysis of rotating disk using finite element method, *Engineering Structures* **18**(1): 133-143.
[9] Cho J. R., Park H. J., 2002, High strength FGM cutting tools: finite element analysis on thermoelastic characteristics, *Journal of Materials Processing Technology* **130**: 351-356.
[10] Zenkour A. M., 2009, Stress distribution in rotating composite structures of functionally graded solid disks, *Journal of Materials Processing Technology* **209**: 3511-3517.
[11] Afsar A.M., Go J., 2010, Finite element analysis of thermoelastic field in a rotating FGM circular disk, *Applied Mathematical Modelling* **34**: 3309-3320.
[12] Hosseini Tehrani P., Talebi M., 2012, Stress and temperature distribution study in a functionally graded brake disk, *International Journal of Automotive Engineering* **2**(3): 172-179.
[13] Eskandari H., 2014, Stress intensity factors for crack located at an arbitrary position in rotating FGM disks, *Jordan Journal of Mechanical and Industrial Engineering* **8**(1): 27-34.
[14] Eskandari H., 2014, Stress intensity factor for radial cracks in rotating hollow FGM disks, *Composites: Mechanics, Computations, Applications: An International Journal* **5**(3): 207-217.
[15] Walters M. C., Paulino G. H., Dodds R. H., 2004, Stress-intensity factors for surface cracks in functionally graded materials under mode-I thermomechanical loading, *International Journal of Solids and Structures* **41**: 1081-1118.
[16] Nami M.R., Eskandari H., 2012, Three-dimensional investigations of stress intensity factors in a thermo-mechanically loaded cracked FGM hollow cylinder , *International Journal of Pressure Vessels and Piping* **89**: 222-229.
[17] Kim J. H., 2003, Mixed-mode crack propagation in functionally graded materials, Ph.D. Thesis, University of Illinoisat Urbana-Champaign, Illinois.
[18] Williams M. L., 1957, On the stress distribution at the base of a stationary crack, *Journal of Applied Mechanics* **24**: 109 -114.
[19] Eischen J. W., 1987, Fracture of non-homogeneous materials, *International Journal of Fracture* **34**: 3-22.
[20] Jin Z. H., Noda N., 1994, Crack tip singular fields in non-homogeneous materials, *Journal of Applied Mechanics* **61**(3): 738-740.
[21] Zhang C., Cui M., Wang J., Gao X. W. , Sladek J., Sladek V., 2010, 3D crack analysis in functionally graded materials, *Engineering Fracture Mechanics* **78**: 585-604.
[22] Erdogan F., Wu B. H., 1997, The surface crack problem for a plate with functionally graded properties, *Journal of Applied Mechanics* **64**: 449-456.
[23] Chen J., Wu L., Du S., 2000, A modified J integral for functionally graded materials, *Mechanics Research Communications* **27**(3): 301-306.