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Plane Strain Deformation of a Poroelastic Half-Space Lying Over Another Poroelastic Half-Space

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ABSTRACT

The plane strain deformation of an isotropic, homogeneous, poroelastic medium caused by an inclined line-load is studied using the Biot linearized theory for fluid saturated porous materials. The analytical expressions for the displacements and stresses in the medium are obtained by applying suitable boundary conditions. The solutions are obtained analytically for the limiting case of undrained conditions in high frequency limit. The undrained displacements, stresses and pore pressure in poroelastic medium are plotted and discussed to draw the conclusions.

Keywords: Poroelastic; Line load; Undrained state; Displacements; Stresses

1 INTRODUCTION

POROELASTICITY is the mechanics of poroelastic solids with fluid filled pores. Fluid-saturated porous materials are often present on and below the surface of the Earth. The fluid in the pores of poroelastic solids plays an important role in the occurrence of earthquakes. Because of this, as well as the significance of liquid saturated porous medium in many engineering problems, the liquid saturated porous media are of great concern and have many applications in various fields such as earthquake engineering, seismology, geomechanics, soil mechanics, civil engineering, mechanical engineering etc. Biot [1, 2] developed linearized constitutive and field equations for poroelastic medium which has been used by many researchers (see e.g. Wang [13] and the references listed therein).

Problems, such as loading by a reservoir lake or seabed structure that is very extensive in one direction on the earth's surface, can be solved as two dimensional plane strain problem. Many researchers discussed the two dimensional problems by taking different models, e.g., Rudnicki [8], Rudnicki and Roeloffs [9], Singh and Rani [11] and Rani and Singh [6].Many researchers Kuo [5], Kumar and Rani [4], Kumar and Ailawalia [3], Sharma [12].

Discussed the deformation problems of different media by taking inclined loads. Deformation problems are studied by the researchers by taking different type of sources and the study of deformation by inclined load of a fluid saturated porous medium is very important because of its realistic nature. In the present paper, the plane strain deformation due to inclined load at the interface of poroelastic half-space lying over another poroelastic half-space has been discussed. Using Biot stress function given by Biot [2] and Roeloffs [7] and applying the Fourier transform, we find stresses, displacements and pore pressure for poroelastic unbounded medium in integral form. These integral form solutions are used to get the analytical solutions of the formulated problem. These integrals cannot be solved analytically for arbitrary values of the frequency. So, these integrals are evaluated for the limiting case of undrained conditions in high frequency limit. The undrained displacements, stresses and pore pressure for both poroelastic half-spaces are obtained and plotted for a particular model.

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2 FORMULATION OF THE PROBLEM

Consider a homogeneous, isotropic, poroelastic half-space lying over another homogeneous, isotropic, poroelastic half-space which is in welded contact at the interface with closed pore conditions. A rectangular Cartesian coordinate system *oxyz* is taken in such a way that plane $z = 0$ coincides with the intersecting surface of the two half-spaces. We take z-axis vertically downwards so that the lower half-space is treated as Medium-I $(z \geq 0)$ and upper half-space as Medium-II $(z \leq 0)$. Further an inclined line-load of magnitude F_0, per unit length, is acting along the y-direction with its inclination δ with z-direction. The geometry of the problem is as shown in Fig. 1 and it conforms to the two dimensional approximation. Considering the Cartesian coordinates (x, y, z) as (x_1, x_2, x_3), we have $\frac{\partial}{\partial x_2} \equiv 0$ and the displacement components (U_1, U_2, U_3) are independent of the Cartesian coordinate x_2 for the present two dimensional problem. Under this assumption, the plane strain problem $(U_2 = 0)$ and the antiplane strain problem $(U_1 = U_3 = 0)$ get decoupled, and can therefore be solved independently. Here, we consider only the plane strain problem.

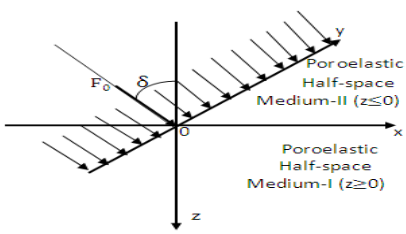

Fig. 1
An inclined line-load F_0 per unit length is acting at the interface along the y-axis with its inclination δ with z-direction in upper poroelastic half-space (Medium-II).

3 SOLUTION FOR POROELASTIC MEDIUM

Following Wang [13], a homogeneous, isotropic, poroelastic medium can be characterized by five poroelastic parameters: drained Poisson's ratio (υ), undrained Poisson's ratio (υ_μ), Shear modulus (G), hydraulic diffusivity (c) and Skempton's coefficient (B). Darcy conductivity (χ) and Biot-Willis coefficient α can be expressed in terms of these five parameters as:

$$\chi = \frac{9c(1-\upsilon_\mu)(\upsilon_\mu - \upsilon)}{2GB^2(1-\upsilon)(1+\upsilon_\mu)^2} \tag{1}$$

$$\alpha = \frac{3(\upsilon_\mu - \upsilon)}{B(1-2\upsilon)(1+\upsilon_\mu)} \tag{2}$$

The two dimensional plane strain problem for an isotropic poroelastic medium can be solved in terms of Biot's stress function F given by Biot [2] and Roeloffs [7] as:

$$\sigma_{11} = \frac{\partial^2 \mathrm{F}}{\partial z^2},\ \sigma_{33} = \frac{\partial^2 F}{\partial x^2},\ \sigma_{13} = -\frac{\partial^2 F}{\partial x \partial z} \tag{3}$$

Following Wang [13], using the above stress function in the governing equations, we get

$$\nabla^2\left(\nabla^2 F + 2\eta p\right) = 0 \tag{4}$$

and

$$\left(c\nabla^2 - \frac{\partial}{\partial t}\right)\left[\nabla^2 F + \frac{3}{\left(1+\upsilon_\mu\right)B} p\right] = 0 \tag{5}$$

where σ_{ij} denotes the total stress in the fluid saturated porous elastic material, p is the excess fluid pore pressure (compression negative) and

$$\eta = \frac{\left(1-2\upsilon\right)\alpha}{2\left(1-\upsilon\right)} \tag{6}$$

Is the poroelastic stress coefficient. Also, the constitutive relations are given as:

$$2G\varepsilon_{11} = \left(1-\upsilon\right)\sigma_{11} - \upsilon\sigma_{33} + \alpha_0 p \tag{7}$$

$$2G\varepsilon_{33} = \left(1-\upsilon\right)\sigma_{33} - \upsilon\sigma_{11} + \alpha_0 p \tag{8}$$

$$2G\varepsilon_{13} = \sigma_{13} \tag{9}$$

$$\varepsilon_{21} = \varepsilon_{22} = \varepsilon_{23} = 0 \tag{10}$$

where

$$\alpha_0 = \left(1-2\upsilon\right)\alpha \tag{11}$$

and ε_{ij} are the corresponding strains.Further,

$$\sigma_{21} = \sigma_{23} = 0 \tag{12}$$

and

$$\sigma_{22} = \upsilon\left(\sigma_{11} + \sigma_{33}\right) - \alpha_0 p \tag{13}$$

From Eqs. (4) and (5), after eliminating F or p, we get the following decoupled equations

$$\left(c\nabla^2 - \frac{\partial}{\partial t}\right)\nabla^2 p = 0 \tag{14}$$

$$\left(c\nabla^2 - \frac{\partial}{\partial t}\right)\nabla^4 F = 0 \tag{15}$$

The general solution of Eq. (14) may be considered as the sum of two functions as:

$$p = p_1 + p_2 \tag{16}$$

where p_1 and p_2 satisfy the following equations:

$$c\nabla^2 p_1 = \frac{\partial p_1}{\partial t} \tag{17}$$

$$\nabla^2 p_2 = 0 \tag{18}$$

Similarly, the general solution of Eq. (15) may be taken as:

$$F = F_1 + F_2 \tag{19}$$

where F_1 and F_2 satisfy the following equations:

$$c\nabla^2 F_1 = \frac{\partial F_1}{\partial t} \tag{20}$$

$$\nabla^4 F_2 = 0 \tag{21}$$

Assuming the time dependence of the functions, p_1, p_2, F_1 and F_2 in the form $\exp(-i\omega t)$, Eqs . (17), (18), (20), (21) become

$$\nabla^2 p_1 + \frac{i\omega}{c} p_1 = 0 \tag{22}$$
$$\nabla^2 p_2 = 0 \tag{23}$$
$$\nabla^2 F_1 + \frac{i\omega}{c} F_1 = 0 \tag{24}$$
$$\nabla^4 F_2 = 0 \tag{25}$$

where p_1, p_2, F_1 and F_2 are now functions of x and z only.

Fourier transforms are now used to get suitable solutions of Eqs .(22)-(25) and hence the solution of Eqs .(14) and (15) can be written as:

$$p = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[A_1 e^{-mz} + A_2 e^{-|k|z} + A_3 e^{mz} + A_4 e^{|k|z}\right] e^{-ikx} dk \tag{26}$$
$$F = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_1 e^{-mz} + B_4 e^{mz} + \left(B_2 + B_3|k|z\right)e^{-|k|z} + \left(B_5 + B_6|k|z\right)e^{|k|z}\right] e^{-ikx} dk \tag{27}$$

Using the radiation conditions in Eqs. (26) and (27), the expressions are now obtained for medium-I $(z \geq 0)$ as:

$$p = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[A_1 e^{-mz} + A_2 e^{-|k|z}\right] e^{-ikx} dk \tag{28}$$
$$F = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_1 e^{-mz} + \left(B_2 + B_3|k|z\right)e^{-|k|z}\right] e^{-ikx} dk\ , \tag{29}$$

where A_1, A_2, B_1, B_2 and B_3 are the functions of k. Using Eqs.(28) and (29) in Eqs .(4)-(5), we obtain

$$A_1 = \frac{i\omega}{2\eta c} B_1, \quad A_2 = \frac{2}{3}\left(1+\upsilon_\mu\right) B k^2 B_3, \quad m = \left(\frac{ck^2 - i\omega}{c}\right)^{\frac{1}{2}}. \ \left(\operatorname{Re} m > 0\right) \tag{30}$$

Similarly, for medium-II $(z \le 0)$ in which the various parameters are described by '*' over the corresponding parameters of medium-I, pore pressure (p^*) and Biot stress function (F^*) obtained from Eqs. (26) and (27) can be written as:

$$p^* = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[A_3 e^{mz} + A_4 e^{|k|z}\right]e^{-ikx}dk \tag{31}$$

$$F^* = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_4 e^{mz} + \left(B_5 + B_6|k|z\right)e^{|k|z}\right]e^{-ikx}dk \tag{32}$$

From Eqs. (4), (5), (31) and (32), we get

$$A_3 = \frac{i\omega}{2\eta^* c^*}B_4, \quad A_4 = -\frac{2}{3}\left(1+\upsilon_\mu^*\right)B^* k^2 B_6 \tag{33}$$

where $\eta^*, c^*, \upsilon_\mu^*$ and B^* are poroelastic stress coefficient, hydraulic diffusivity, undrained Poisson's ratio and Skempton's coefficient, respectively for medium-II.

Using Eq. (29) in Eq. (3), the stresses in medium-I are obtained as:

$$\sigma_{11} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_1 m^2 e^{-mz} + \left(B_2 - 2B_3 + B_3|k|z\right)k^2 e^{-|k|z}\right]e^{-ikx}dk \tag{34}$$

$$\sigma_{33} = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_1 e^{-mz} + \left(B_2 + B_3|k|z\right)e^{-|k|z}\right]k^2 e^{-ikx}dk \tag{35}$$

$$\sigma_{13} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_1 m e^{-mz} + \left(B_2 - B_3 + B_3|k|z\right)|k|e^{-|k|z}\right](-ik)e^{-ikx}dk \tag{36}$$

Similarly, from Eqs. (3) and (32), the stresses in medium-II are given as:

$$\sigma_{11}^* = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_4 m^2 e^{mz} + \left(B_5 + 2B_6 + B_6|k|z\right)k^2 e^{|k|z}\right]e^{-ikx}dk \tag{37}$$

$$\sigma_{33}^* = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_4 e^{mz} + \left(B_5 + B_6|k|z\right)e^{|k|z}\right]k^2 e^{-ikx}dk \tag{38}$$

$$\sigma_{13}^* = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_4 m e^{mz} + \left(B_5 + B_6 + B_6|k|z\right)|k|e^{|k|z}\right](-ik)e^{-ikx}dk \tag{39}$$

The displacements in the medium-I are obtained by using the stresses given by Eqs. (34)-(36) in the constitutive relations Eqs. (7)-(9) as:

$$2Gu_1 = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_1 e^{-mz} + \left\{B_2 + B_3\left(2\upsilon_\mu - 2 + |k|z\right)\right\}e^{-|k|z}\right](-ik)e^{-ikx}dk \tag{40}$$

$$2Gu_3 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_1 m e^{-mz} + \left\{B_2 + B_3\left(1 - 2\upsilon_\mu + |k|z\right)\right\}|k|e^{-|k|z}\right]e^{-ikx}dk \tag{41}$$

Similarly, the displacements for medium II are obtained as:

$$2G_1u_1^* = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_4e^{mz} + \left\{B_5 + B_6\left(2 - 2\upsilon_{\mu_1} + |k|z\right)\right\}e^{|k|z}\right](-ik)e^{-ikx}dk \tag{42}$$

$$2G_1u_3^* = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_4me^{mz} + \left\{B_5 + B_6\left(2\upsilon_{\mu_1} - 1 + |k|z\right)\right\}|k|e^{|k|z}\right]e^{-ikx}dk \tag{43}$$

4 SOLUTION OF THE PROBLEM OF INCLINED LINE LOAD

We have taken the problem of an inclined line load acting at the interface between two poroelastic half spaces, medium-I $(z \geq 0)$ and medium II $(z \leq 0)$. The problem of inclined line load can be visualized as superposition of the problems of tangential and normal line loads. Therefore, firstly, we consider the two cases, i.e. Normal line load and Tangential line load.

4.1 Normal line-load

Consider a normal line load of magnitude F_1, per unit length, acting at the interface $z = 0$ along y-axis as shown in Fig. 2. The two poroelastic half spaces are assumed to be in welded contact under closed pore conditions along the plane $z = 0$, therefore, the continuity of stresses and displacements give the following boundary conditions:

$$u_1(x,z) = u_1^*(x,z) \tag{44}$$

$$u_3(x,z) = u_3^*(x,z) \tag{45}$$

$$\sigma_{33}(x,z) - \sigma_{33}^*(x,z) = -F_1\delta(x) \tag{46}$$

$$\sigma_{13}(x,z) = \sigma_{13}^*(x,z) \tag{47}$$

where $\delta(x)$ in Eq. (46) is the Dirac delta function and it satisfies the following properties

$$\int_{-\infty}^{\infty}\delta(x)dx = 1, \quad \delta(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-ikx}dx \tag{48}$$

Also, due to the closed pore conditions at the interface, i.e. impermeable situation, we have

$$\frac{\partial p}{\partial z} = 0 \tag{49}$$

$$\frac{\partial p^*}{\partial z} = 0 \tag{50}$$

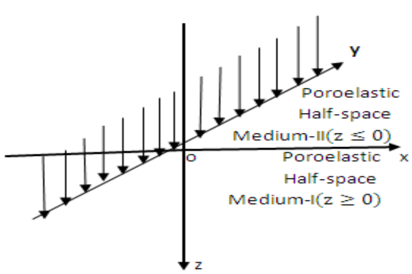

Fig. 2
A Normal line load of magnitude F_1 per unit length acting along y-axis in the direction of +ve z-axis at the interface $z = 0$.

Using Eqs. (28), (31), (35)-(36), (38)-(39) and (40)-(43) in boundary conditions (44)-(47) and (49)-(50), we get the following system of equations

$$B_1 + B_2 + B_3\left(2\upsilon_\mu - 2\right) - \theta B_4 - \theta B_5 - \theta B_6\left(2 - 2\upsilon_\mu^*\right) = 0 \tag{51}$$

$$B_1 m + B_2\left|k\right| + B_3\left|k\right|\left(1 - 2\upsilon_\mu\right) + \theta m B_4 + \theta\left|k\right| B_5 + \theta\left|k\right| B_6\left(2\upsilon_\mu^* - 1\right) = 0 \tag{52}$$

$$B_1 + B_2 - B_4 - B_5 = \frac{F_1}{k^2} \tag{53}$$

$$B_1 m + B_2\left|k\right| - B_3\left|k\right| + mB_4 + \left|k\right| B_5 + \left|k\right| B_6 = 0 \tag{54}$$

$$maB_1 + \left|k\right| bB_3 = 0 \tag{55}$$

$$meB_4 + \left|k\right| fB_6 = 0 \tag{56}$$

where

$$a = \frac{i\omega}{2\eta c},\ b = \frac{2}{3}\left(1 + \upsilon_\mu\right)Bk^2,\ e = \frac{i\omega}{2\eta_1 c_1},\ f = -\frac{2}{3}\left(1 + \upsilon_\mu^*\right)B^* k^2,\ \theta = \frac{G}{G^*} \tag{57}$$

Solving the system of Eqs. (51)-(56) for B_1, B_2, B_3, B_4, B_5 and B_6, we get

$$B_1 = \alpha W_1,\quad B_2 = -\gamma W_1 + \theta W_3 + \zeta W_2,\quad B_3 = W_1,\quad B_4 = \beta W_2,\quad B_5 = W_3,\quad B_6 = W_2 \tag{58}$$

where

$$W_1 = \frac{-2F_1\theta S - F_1\left(\theta - 1\right)Q}{k^2\left(PS - QR\right)},\qquad W_2 = \frac{-2F_1\theta - PW_1 k^2}{Qk^2},\qquad W_3 = \frac{\left(\gamma - p^1\right)W_1}{2\theta} - \frac{\left(\zeta + q^1\right)W_2}{2\theta} \tag{59}$$

$$P = \left(\gamma - p^1\right)\left(1 + \theta\right) + \left(p^1 - \alpha\right)\left(2\theta\right) \tag{60}$$

$$Q = \left(q^1 + \beta\right)\left(2\theta\right) - \left(\zeta + q^1\right)\left(1 + \theta\right) \tag{61}$$

$$R = 2\left(p^1 - \alpha\right) + \left(\alpha - r^1\right)\left(1 + \theta\right) \tag{62}$$

$$S = 2\left(q^1 + \beta\right) - \left(\beta + s^1\right)\left(1 + \theta\right) \tag{63}$$

$$p^1 = \frac{\alpha m}{\left|k\right|} + \left(1 - 2\upsilon_\mu\right),\qquad q^1 = \frac{\theta m\beta}{\left|k\right|} + \theta\left(2\upsilon_\mu^* - 1\right),\qquad r^1 = \frac{\alpha m}{\left|k\right|} - 1 \tag{64}$$

$$s^1 = \frac{m\beta}{\left|k\right|} + 1,\qquad \alpha = \frac{-\left|k\right| b}{ma},\qquad \beta = \frac{-\left|k\right| f}{me} \tag{65}$$

$$\gamma = \alpha + 2\upsilon_\mu - 2,\qquad \zeta = \theta\left(\beta + 2 - 2\upsilon_\mu^*\right) \tag{66}$$

Substituting the values of B_1, B_2 and B_3 from Eq. (58) into Eqs. (28), (34)-(36) and (40)-(41), we get the stresses, displacements and pore pressure for poroelastic half-space medium-I. Taking the undrained conditions in high frequency, i.e., $\omega \to \infty$, we get the stresses, displacements and pore pressure for normal line-load in medium-I as:

$$\sigma_{11}^{(N)}\left(x, z\right) = \frac{F_1\left(P_1 - 2P_2\right)}{\pi}\frac{z}{x^2 + z^2} + \frac{F_1 P_2}{\pi}\frac{z\left(z^2 - x^2\right)}{\left(x^2 + z^2\right)^2} \tag{67}$$

$$\sigma_{33}^{(N)}\left(x,z\right)=-\frac{F_1P_1}{\pi}\frac{z}{x^2+z^2}-\frac{F_1P_2}{\pi}\frac{z\left(z^2-x^2\right)}{\left(x^2+z^2\right)^2} \tag{68}$$

$$\sigma_{13}^{(N)}\left(x,z\right)=\frac{F_1\left(P_2-P_1\right)}{\pi}\frac{x}{x^2+z^2}-\frac{F_1P_2}{\pi}\frac{2xz^2}{\left(x^2+z^2\right)^2} \tag{69}$$

$$2Gu_1^{(N)}\left(x,z\right)=\frac{F_1\left\{P_1+P_2\left(2\upsilon_\mu-2\right)\right\}}{\pi}\tan^{-1}\frac{x}{z}+\frac{F_1P_2}{\pi}\frac{xz}{x^2+z^2} \tag{70}$$

$$2Gu_3^{(N)}\left(x,z\right)=-\frac{F_1P_1}{2\pi}\log\left(x^2+z^2\right)-\frac{F_1P_2\left(1-2\upsilon_\mu\right)\log\left(x^2+z^2\right)}{2\pi}+\frac{F_1P_2}{\pi}\frac{z^2}{x^2+z^2} \tag{71}$$

$$P^{(N)}\left(x,z\right)=\frac{2\left(1+\upsilon_\mu\right)BF_1P_2}{3\pi}\frac{z}{\left(x^2+z^2\right)^2} \tag{72}$$

where P_1 and P_2 are as follows

$$P_1=\frac{-\theta}{2\left(4\upsilon_\mu-3-\theta\right)}+\frac{\theta\left(4\upsilon_\mu^*-3\right)}{2\left(4\theta\upsilon_\mu^*-3\theta-1\right)},\qquad P_2=\frac{-\theta}{\left(4\upsilon_\mu-3-\theta\right)} \tag{73}$$

Here, we have used the standard integrals as given in Appendix and superscript (*N*) signifies the case of normal line load.

Similarly, by using the values of B_4, B_5 and B_6 from Eq. (58) in Eqs. (31), (37)-(39) and (42)-(43), we get the stresses, displacements and pore pressure for undrained conditions in medium-II as:

$$\sigma_{11}^{*(N)}\left(x,z\right)=-\frac{F_1\left(P_4+2P_5\right)}{\pi}\frac{z}{x^2+z^2}+\frac{F_1P_5}{\pi}\frac{z\left(z^2-x^2\right)}{\left(x^2+z^2\right)^2} \tag{74}$$

$$\sigma_{33}^{*(N)}\left(x,z\right)=\frac{F_1P_4}{\pi}\frac{z}{x^2+z^2}-\frac{F_1P_5}{\pi}\frac{z\left(z^2-x^2\right)}{\left(x^2+z^2\right)^2} \tag{75}$$

$$\sigma_{13}^{*(N)}\left(x,z\right)=\frac{F_1\left(P_4+P_5\right)}{\pi}\frac{x}{x^2+z^2}-\frac{2F_1P_5}{\pi}\frac{xz^2}{\left(x^2+z^2\right)^2} \tag{76}$$

$$2G_1u_1^{*(N)}\left(x,z\right)=\frac{F_1\left\{-P_4+P_5\left(2\upsilon_\mu-2\right)\right\}}{\pi}\tan^{-1}\frac{x}{z}+\frac{F_1P_5}{\pi}\frac{xz}{x^2+z^2} \tag{77}$$

$$2G_1u_3^{*(N)}\left(x,z\right)=\frac{F_1P_4}{2\pi}\log\left(x^2+z^2\right)-\frac{F_1P_5\left(1-2\upsilon_\mu\right)\log\left(x^2+z^2\right)}{2\pi}+\frac{F_1P_5}{\pi}\frac{z^2}{x^2+z^2} \tag{78}$$

$$P^{*(N)}\left(x,z\right)=\frac{2\left(1+\upsilon_\mu\right)B_0F_1P_5}{3\pi}\frac{z}{\left(x^2+z^2\right)^2} \tag{79}$$

where P_4 and P_5 are as follows

$$P_4=\frac{-\left(4\upsilon_\mu-3\right)}{2\left(4\upsilon_\mu-3-\theta\right)}+\frac{1}{2\left(4\theta\upsilon_\mu^*-3\theta-1\right)},\qquad P_5=\frac{-1}{\left(4\theta\upsilon_\mu^*-3\theta-1\right)} \tag{80}$$

4.2 Tangential line-load

A tangential line load of magnitude F_2 per unit length is acting at the interface z = 0 along y-axis in the +ve x-direction as shown in Fig. 3. The two poroelastic half spaces are assumed to be in welded contact with closed pore conditions along the plane z = 0. Therefore, due to the continuity of stresses and displacements, and closed pore conditions at the interface, boundary conditions at the interface z = 0 are

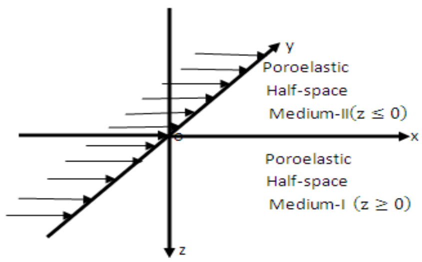

Fig. 3
A tangential line load of magnitude F_2 per unit length acting along y-axis in the direction of +ve x-axis at the interface.

$$u_1(x,z) = u_1^*(x,z) \tag{81}$$
$$u_3(x,z) = u_3^*(x,z) \tag{82}$$
$$\sigma_{33}(x,z) = \sigma_{33}^*(x,z) \tag{83}$$
$$\sigma_{13}(x,z) - \sigma_{13}^*(x,z) = -\mathrm{F}_2\,\delta(x) \tag{84}$$
$$\frac{\partial p}{\partial z} = 0 \tag{85}$$
$$\frac{\partial p^*}{\partial z} = 0 \tag{86}$$

Using Eqs. (28), (31), (35)-(36), (38)-(39), (40)-(43) in boundary conditions (81)-(86), we get the following system of equations:

$$B_1 + B_2 + B_3\left(2\upsilon_\mu - 2\right) - \theta B_4 - \theta B_5 - \theta B_6\left(2 - 2\upsilon_\mu^*\right) = 0 \tag{87}$$
$$B_1 m + B_2|k| + B_3|k|\left(1 - 2\upsilon_\mu\right) + \theta m B_4 + \theta|k|B_5 + \theta|k|B_6\left(2\upsilon_\mu^* - 1\right) = 0 \tag{88}$$
$$B_1 + B_2 - B_4 - B_5 = 0 \tag{89}$$
$$B_1 m + B_2|k| - B_3|k| + mB_4 + |k|B_5 + |k|B_6 = \frac{F_2}{Ik} \tag{90}$$
$$maB_1 + |k|bB_3 = 0 \tag{91}$$
$$meB_4 + |k|\,fB_6 = 0 \tag{92}$$

Solving the system of Eqs. (87)-(92) for B_1, B_2, B_3, B_4, B_5 and B_6, we get

$$\begin{aligned} &B_1 = \alpha W_1, && B_2 = -\gamma W_1 + \theta W_3 + \zeta W_2, && B_3 = W_1 \\ &B_4 = \beta W_2, && B_5 = W_3, && B_6 = W_2 \end{aligned} \tag{93}$$

where

$$W_1 = \frac{F_2(1+\theta)Q}{ik(PS-QR)}, \qquad W_2 = \frac{-PW_1}{Q}, \qquad W_3 = \frac{(\gamma - p^1)W_1}{2\theta} - \frac{(\zeta + q^1)W_2}{2\theta} \tag{94}$$

Substituting the values of B_1, B_2 and B_3 from Eq. (93) into Eqs .(28), (34)-(36) and (40)-(41), the stresses, displacements and pore pressure for poroelastic half-space medium-I on taking the undrained conditions in high frequency, i.e., $\omega \to \infty$, are obtained as:

$$\sigma_{11}^{(T)}(x,z) = \frac{-2F_2(Q_1 - 2Q_2)}{\pi}\frac{xz}{(x^2+z^2)^2} - \frac{2F_2Q_2}{\pi}\frac{xz(3z^2-x^2)}{(x^2+z^2)^3} \tag{95}$$

$$\sigma_{33}^{(T)}(x,z) = \frac{-2F_2Q_1}{\pi}\frac{xz}{(x^2+z^2)^2} + \frac{2F_2Q_2}{\pi}\frac{xz(3z^2-x^2)}{(x^2+z^2)^3} \tag{96}$$

$$\sigma_{13}^{(T)}(x,z) = -\frac{F_2(Q_1 - Q_2)}{\pi}\frac{(z^2-x^2)}{(x^2+z^2)^2} - \frac{2F_2Q_2}{\pi}\frac{z^2(z^2-3x^2)}{(x^2+z^2)^3} \tag{97}$$

$$2Gu_1^{(T)}(x,z) = \frac{F_2\{Q_1 + Q_2(2\upsilon_\mu - 2)\}}{\pi}\frac{z}{x^2+z^2} + \frac{F_2Q_2}{\pi}\frac{z(z^2-x^2)}{(x^2+z^2)^2} \tag{98}$$

$$2Gu_3^{(T)}(x,z) = -\frac{F_2\{Q_1 + Q_2(1-2\upsilon_\mu)\}}{\pi}\frac{x}{x^2+z^2} - \frac{2F_2Q_2xz^2}{\pi(x^2+z^2)^2} \tag{99}$$

$$p^{(T)}(x,z) = \frac{-4(1+\upsilon_\mu)BF_2Q_2}{3\pi}\frac{xz}{(x^2+z^2)^2} \tag{100}$$

where Q_1, Q_2 are as follows

$$Q_1 = \frac{\theta}{2(4\upsilon_\mu - 3 - \theta)} + \frac{\theta(4\upsilon_\mu^* - 3)}{2(4\theta\upsilon_\mu^* - 3\theta - 1)}, \qquad Q_2 = \frac{\theta}{(4\upsilon_\mu - 3 - \theta)} \tag{101}$$

Here, we have used the standard integrals as given in Appendix and superscript (T) signifies the case of tangential line load.

Similarly, by using the values of B_4, B_5 and B_6 from Eq. (93) in Eqs. (31), (37)-(39) and (42)-(43), we get the stresses, displacements and pore pressure in medium-II, with undrained conditions in high frequency, as:

$$\sigma_{11}^{*(T)}(x,z) = \frac{2F_2(Q_4 + 2Q_5)}{\pi}\frac{xz}{(x^2+z^2)^2} - \frac{2F_2Q_5}{\pi}\frac{xz(3z^2-x^2)}{(x^2+z^2)^3} \tag{102}$$

$$\sigma_{33}^{*(T)}(x,z) = \frac{-2F_2Q_4}{\pi}\frac{xz}{(x^2+z^2)^2} + \frac{2F_2Q_5}{\pi}\frac{xz(3z^2-x^2)}{(x^2+z^2)^3} \tag{103}$$

$$\sigma_{13}^{*(T)}(x,z) = \frac{F_2(Q_4 + Q_5)}{\pi}\frac{(z^2-x^2)}{(x^2+z^2)^2} - \frac{2F_2Q_5z^2(z^2-3x^2)}{\pi(x^2+z^2)^3} \tag{104}$$

$$2G_1u_1^{*(T)}(x,z) = \frac{-F_2\{Q_4 + Q_5(2-2\upsilon_\mu^*)\}}{\pi}\frac{z}{x^2+z^2} + \frac{F_2Q_5}{\pi}\frac{z(z^2-x^2)}{(x^2+z^2)^2} \tag{105}$$

$$2G_1 u_3^{*(T)}(x,z) = \frac{F_2\left\{Q_4 + Q_5\left(2\upsilon_\mu^* - 1\right)\right\}}{\pi}\frac{x}{x^2+z^2} - \frac{F_2 Q_5}{\pi}\frac{2xz^2}{\left(x^2+z^2\right)^2} \tag{106}$$

$$p^{*(T)}(x,z) = \frac{-4\left(1+\upsilon_\mu^*\right)B_0 Q_5 F_2}{3\pi}\frac{xz}{\left(x^2+z^2\right)^2} \tag{107}$$

where Q_4 and Q_5 are as follows

$$Q_4 = \frac{\left(4\upsilon_\mu - 3\right)}{2\left(4\upsilon_\mu - 3 - \theta\right)} + \frac{1}{2\left(4\theta\upsilon_\mu^* - 3\theta - 1\right)}, \qquad Q_5 = \frac{-1}{\left(4\theta\upsilon_\mu^* - 3\theta - 1\right)} \tag{108}$$

Now for an inclined line-load of magnitude F_0 per unit length acting along the *y*-axis with its inclination to *z*-direction, δ , following Saada [10],we get the stresses, displacements and pore pressure as:

$$u_1^{(IN)}(x,z) = u_1^{(N)}(x,z) + u_1^{(T)}(x,z) \tag{109}$$

$$u_3^{(IN)}(x,z) = u_3^{(N)}(x,z) + u_3^{(T)}(x,z) \tag{110}$$

$$\sigma_{13}^{(IN)}(x,z) = \sigma_{13}^{(N)}(x,z) + \sigma_{13}^{(T)}(x,z) \tag{111}$$

$$\sigma_{33}^{(IN)}(x,z) = \sigma_{33}^{(N)}(x,z) + \sigma_{33}^{(T)}(x,z) \tag{112}$$

$$\sigma_{11}^{(IN)}(x,z) = \sigma_{11}^{(N)}(x,z) + \sigma_{11}^{(T)}(x,z) \tag{113}$$

$$p^{(IN)}(x,z) = p^{(N)}(x,z) + p^{(T)}(x,z) \tag{114}$$

where

$$F_1 = F_0\cos\delta,\ F_2 = F_0\sin\delta \tag{115}$$

Using the expressions of the stresses, displacements and pore pressure for the two cases as obtained in Eqs. (67)-(72) and (95)-(100), in the Eqs. (109)-(114), we obtain the stresses, displacements and pore pressure in medium-I for the case of Inclined line load as:

$$\sigma_{11}^{(IN)}(x,z) = \left\{\frac{F_0\left(P_1 - 2P_2\right)}{\pi}\frac{z}{x^2+z^2} + \frac{F_0 P_2 z\left(z^2-x^2\right)}{\pi\left(x^2+z^2\right)^2}\right\}\cos\delta + \left\{\frac{F_0\left(4Q_2 - 2Q_1\right)xz}{\pi\left(x^2+z^2\right)^2} - \frac{2F_0 Q_2}{\pi}\frac{xz\left(3z^2-x^2\right)}{\left(x^2+z^2\right)^3}\right\}\sin\delta \tag{116}$$

$$\sigma_{33}^{(IN)}(x,z) = \left\{-\frac{F_0 P_1}{\pi}\frac{z}{x^2+z^2} - \frac{F_0 P_2 z\left(z^2-x^2\right)}{\pi\left(x^2+z^2\right)^2}\right\}\cos\delta + \left\{\frac{2F_0 Q_1 xz}{\pi\left(x^2+z^2\right)^2} + \frac{2F_0 Q_2}{\pi}\frac{xz\left(3z^2-x^2\right)}{\left(x^2+z^2\right)^3}\right\}\sin\delta \tag{117}$$

$$\sigma_{13}^{(IN)}(x,z) = \left\{\frac{F_0\left(P_2 - P_1\right)x}{\pi\left(x^2+z^2\right)} - \frac{2F_0 P_2 xz^2}{\pi\left(x^2+z^2\right)^2}\right\}\cos\delta + \left\{-\frac{2F_0 Q_2}{\pi}\frac{\left(z^2-3x^2\right)}{\left(x^2+z^2\right)^3} - \frac{F_0\left(Q_1-Q_2\right)\left(z^2-x^2\right)}{\pi\left(x^2+z^2\right)^2}\right\}\sin\delta \tag{118}$$

$$2Gu_1^{(IN)}(x,z) = \left\{F_0\left(\frac{P_1 + P_2\left(2\upsilon_\mu - 2\right)}{\pi}\right)\tan^{-1}\frac{x}{z} + \frac{F_0 P_2 xz}{\pi\left(x^2+z^2\right)}\right\}\cos\delta + \left\{\frac{F_0\left(Q_1 z + Q_2\left(2\upsilon_\mu - 2\right)z\right)}{\pi\left(x^2+z^2\right)} + \frac{F_0 Q_2}{\pi}\frac{z\left(z^2-x^2\right)}{\left(x^2+z^2\right)^2}\right\}\sin\delta \tag{119}$$

$$2Gu_3^{(IN)}(x,z)=\left\{-F_0\left(\frac{P_1+P_2\left(1-2\upsilon_\mu\right)}{2\pi}\right)\log\left(x^2+z^2\right)+\frac{F_0P_2}{\pi}\frac{z^2}{\left(x^2+z^2\right)}\right\}\cos\delta+\left\{-F_0\left(\frac{Q_1+Q_2\left(1-2\upsilon_\mu\right)}{\mu}\right)\left(\frac{x}{x^2+z^2}\right)-\frac{2F_0Q_2xz^2}{\pi\left(x^2+z^2\right)^2}\right\}\sin\delta \tag{120}$$

$$p^{(IN)}(x,z)=\left\{\frac{2\left(1+\upsilon_\mu\right)BF_1P_2}{3\pi}\frac{z}{\left(x^2+z^2\right)^2}\right\}\cos\delta+\left\{\frac{-4\left(1+\upsilon_\mu\right)BF_2Q_2}{3\pi}\frac{xz}{\left(x^2+z^2\right)^2}\right\}\sin\delta \tag{121}$$

Similarly, the stresses, displacements and pore pressure in medium-II are

$$\sigma_{11}^{*(IN)}(x,z)=\left\{-\frac{F_1\left(P_4+2P_5\right)}{\pi}\frac{z}{x^2+z^2}+\frac{F_1P_5}{\pi}\frac{z\left(z^2-x^2\right)}{\left(x^2+z^2\right)^2}\right\}\cos\delta+\left\{\frac{2F_2\left(Q_4+2Q_5\right)}{\pi}\frac{xz}{\left(x^2+z^2\right)^2}-\frac{2F_2Q_5}{\pi}\frac{xz\left(3z^2-x^2\right)}{\left(x^2+z^2\right)^3}\right\}\sin\delta \tag{122}$$

$$\sigma_{33}^{*(IN)}(x,z)=\left\{\frac{F_1P_4}{\pi}\frac{z}{x^2+z^2}-\frac{F_1P_5}{\pi}\frac{z\left(z^2-x^2\right)}{\left(x^2+z^2\right)^2}\right\}\cos\delta+\left\{\frac{-2F_2Q_4}{\pi}\frac{xz}{\left(x^2+z^2\right)^2}+\frac{2F_2Q_5}{\pi}\frac{xz\left(3z^2-x^2\right)}{\left(x^2+z^2\right)^3}\right\}\sin\delta \tag{123}$$

$$\sigma_{13}^{*(IN)}(x,z)=\left\{\frac{F_1\left(P_4+P_5\right)}{\pi}\frac{x}{x^2+z^2}-\frac{2F_1P_5}{\pi}\frac{xz^2}{\left(x^2+z^2\right)^2}\right\}\cos\delta+\left\{\frac{F_2\left(Q_4+Q_5\right)}{\pi}\frac{\left(z^2-x^2\right)}{\left(x^2+z^2\right)^2}-\frac{2F_2Q_5z^2\left(z^2-3x^2\right)}{\pi\left(x^2+z^2\right)^3}\right\}\sin\delta \tag{124}$$

$$2Gu_1^{*(IN)}(x,z)=\left\{\frac{F_1\left\{-P_4+P_5\left(2\upsilon_\mu-2\right)\right\}}{\pi}\tan^{-1}\frac{x}{z}+\frac{F_1P_5}{\pi}\frac{xz}{x^2+z^2}\right\}\cos\delta+\left\{\frac{-F_2\left\{Q_4+Q_5\left(2-2\upsilon_{\mu_1}\right)\right\}}{\pi}\frac{z}{x^2+z^2}+\frac{F_2Q_5z\left(z^2-x^2\right)}{\pi\left(x^2+z^2\right)^2}\right\}\sin\delta \tag{125}$$

$$\begin{aligned}2Gu_3^{*(IN)}(x,z)=&\left\{\frac{F_1P_4}{2\pi}\log\left(x^2+z^2\right)-\frac{F_1P_5\left(1-2\upsilon_{\mu_1}\right)\log\left(x^2+z^2\right)}{2\pi}+\frac{F_1P_5}{\pi}\frac{z^2}{x^2+z^2}\right\}\cos\delta\\&+\left\{\frac{F_2\left\{Q_4+Q_5\left(2\upsilon_{\mu_1}-1\right)\right\}}{\pi}\frac{x}{x^2+z^2}-\frac{F_2Q_5}{\pi}\frac{2xz^2}{\left(x^2+z^2\right)^2}\right\}\sin\delta\end{aligned} \tag{126}$$

$$p^{*(IN)}(x,z)=\left\{\frac{2\left(1+\upsilon_{\mu_1}\right)B_0F_1P_5}{3\pi}\frac{z}{\left(x^2+z^2\right)}\right\}\cos\delta+\left\{\frac{-4\left(1+\upsilon_{\mu_1}\right)B_0Q_5F_2}{3\pi}\frac{xz}{\left(x^2+z^2\right)^2}\right\}\sin\delta \tag{127}$$

5 NUMERICAL RESULTS AND DISCUSSION

To discuss the theoretical results obtained above, the numerical results are calculated by considering a particular model of Berea Sandstone half-space lying over Ruhr Sandstone half-space, the elastic parameters for these media are given by Wang [13] as:

$$G=13,\qquad \upsilon_\mu=0.31,\qquad B=0.88G^*=6,\qquad \upsilon_\mu^*=0.33,\qquad B^*=0.62 \tag{128}$$

To convert the results in dimensionless form, the following non-dimensional quantities are taken

$$X=\frac{x}{h},Z=\frac{z}{h},U_i=\frac{u_i}{h},U_i^*=\frac{u_i^*}{h},\Sigma_{ij}=\frac{\sigma_{ij}}{G},\Sigma_{ij}^*=\frac{\sigma_{ij}^*}{G}.\quad i,j=1,3 \tag{129}$$

For numerical computations and plotting the results, a computer program in MATLAB software is used. Numerical values of the dimensionless stresses, displacements and pore pressure are calculated in both the media, by taking non-dimensional depth Z = 1/4 in medium-I and Z = –1/4 in medium-II with the following values of $\delta : \delta = 0°, 45°, 60°, 90°$ for different range of values of dimensionless distance X. The results are also obtained with variation in depth, i.e., Z-axis for X = 1/4. The results are shown graphically in Figs. 4-19 as discussed below:

It is observed from the figures that all the displacements and stresses in both the half spaces tend to or approach to zero value with the increase in either the horizontal distance or the vertical distance for all the values of δ, except in the case of the variation of vertical displacement $\left(U_3\right)$ in both the media. Also, it is observed that the variations are almost symmetric with respect to either YZ plane or w.r.t. origin. Further, the magnitudes of variations are large in medium-I as compared to medium-II in almost all the cases. All these observations show that the results obtained conform to the properties of considered media and the nature of source applied and will be of significance in further studies.

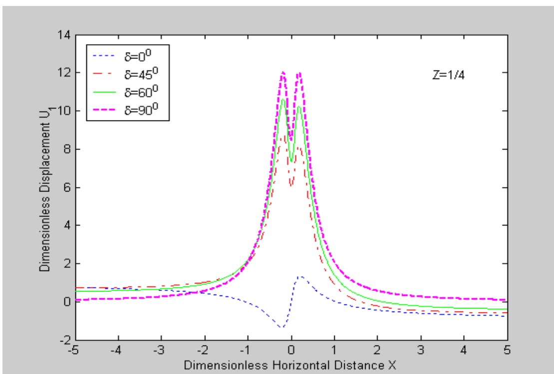

Fig. 4
Variation of tangential displacement w.r.t. horizontal distance in medium-I.

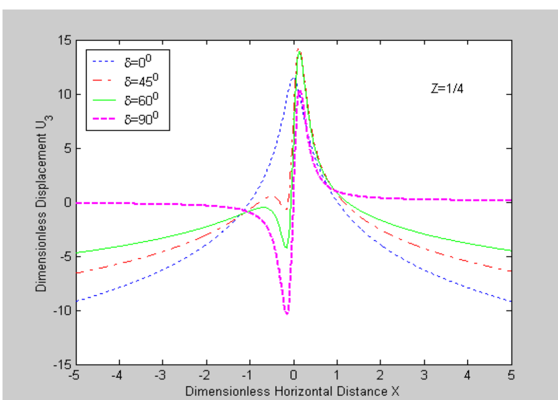

Fig. 5
Variation of normal displacement w.r.t. horizontal distance in medium-I.

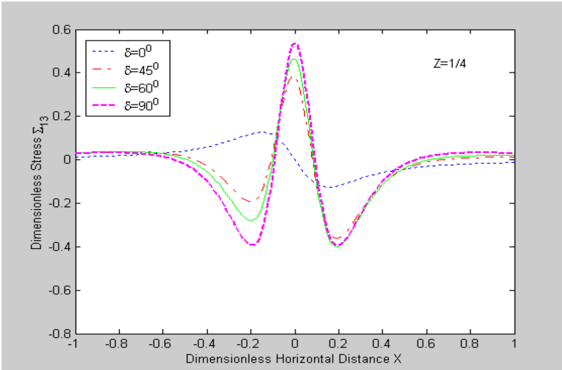

Fig. 6
Variation of tangential stress w.r.t. horizontal distance in medium-I.

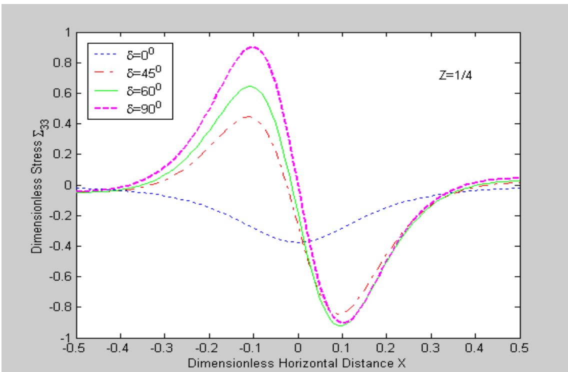

Fig. 7
Variation of normal stress w.r.t. horizontal distance in medium-I.

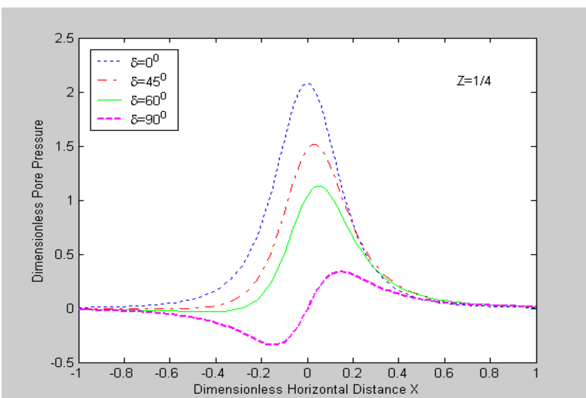

Fig. 8
Variation of pore pressure w.r.t. horizontal distance in medium-I.

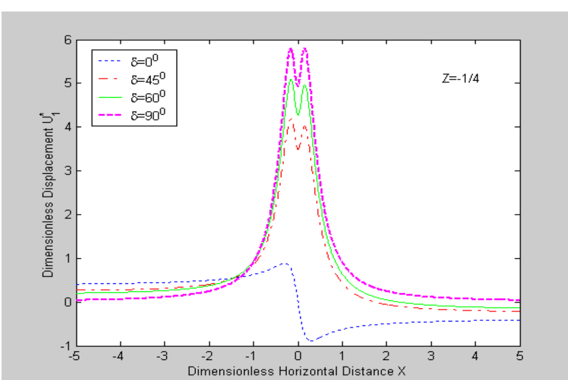

Fig. 9
Variation of tangential displacement w.r.t. horizontal distance in medium-II.

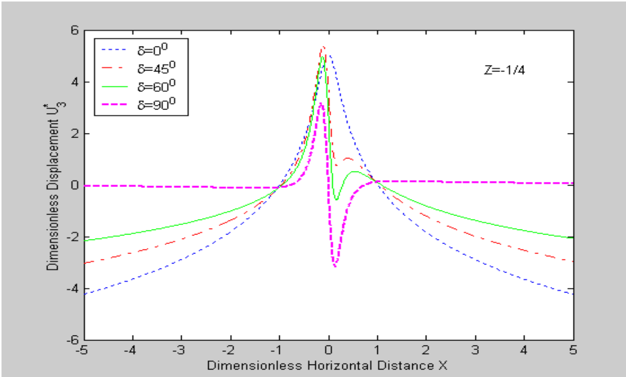

Fig. 10
Variation of normal displacement w.r.t. horizontal distance in medium-II.

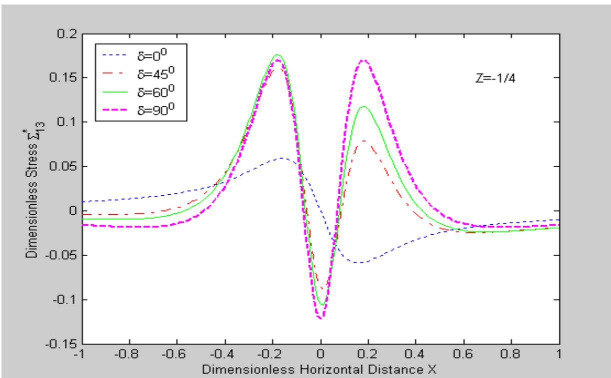

Fig. 11
Variation of tangential stress w.r.t. horizontal distance in medium-II.

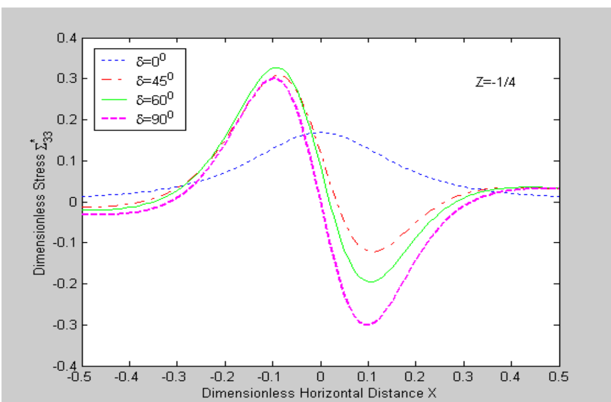

Fig. 12
Variation of normal stress w.r.t. horizontal distance in medium-II.

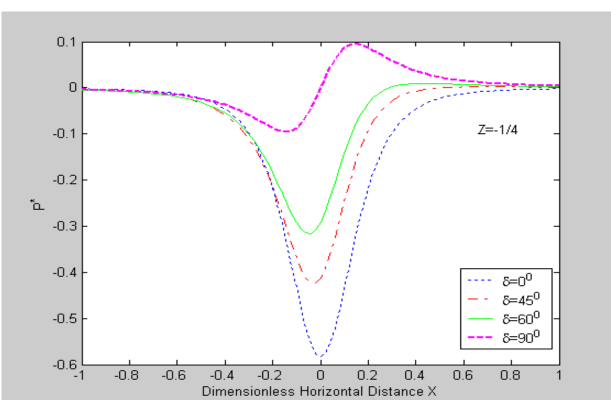

Fig. 13
Variation of pore pressure w.r.t. horizontal distance in medium-II.

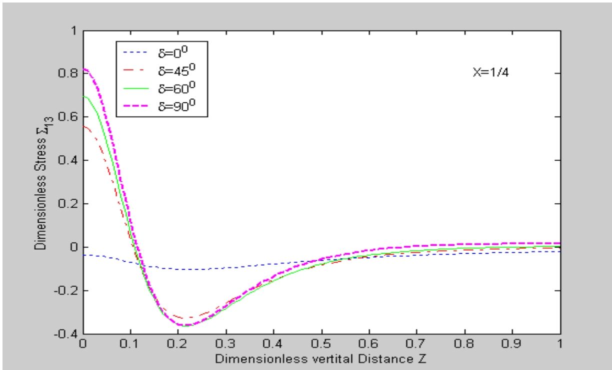

Fig. 14
Variation of tangential stress w.r.t. vertical distance in medium-I.

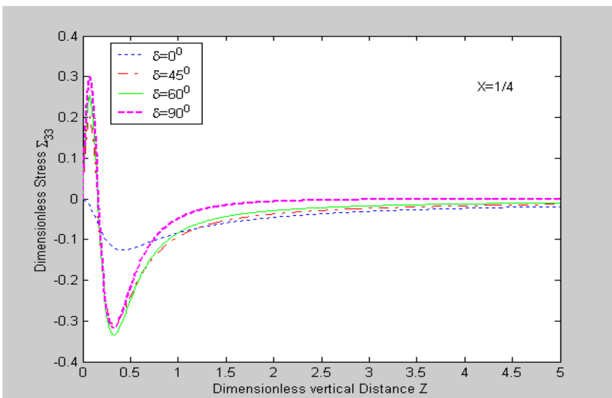

Fig. 15
Variation of normal stress w.r.t. vertical distance in medium-I.

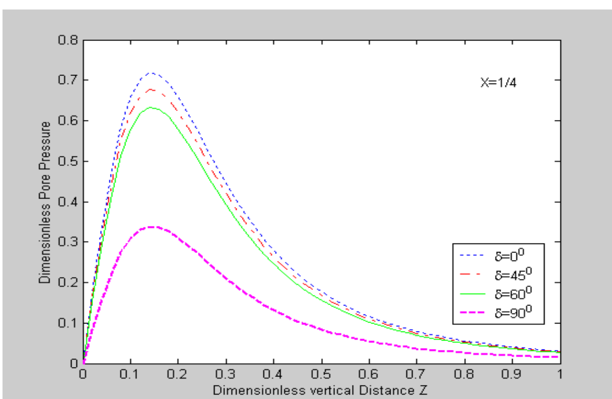

Fig. 16
Variation of pore pressure w.r.t. vertical distance in medium-I.

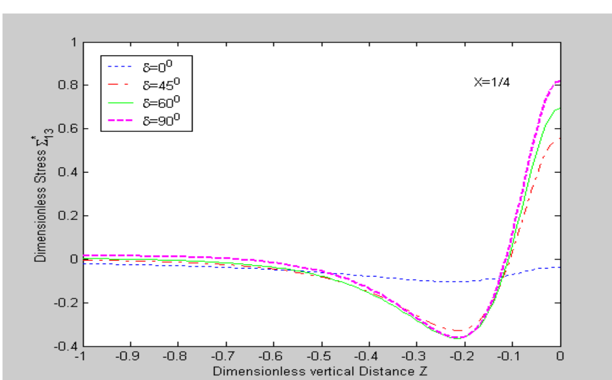

Fig. 17
Variation of tangential stress w.r.t. vertical distance in medium-II.

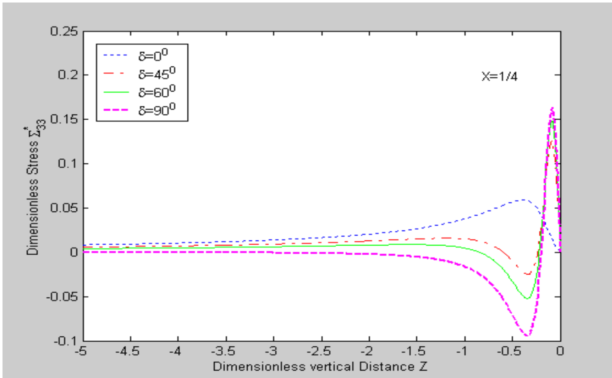

Fig. 18
Variation of normal stress w.r.t. vertical distance in medium-II.

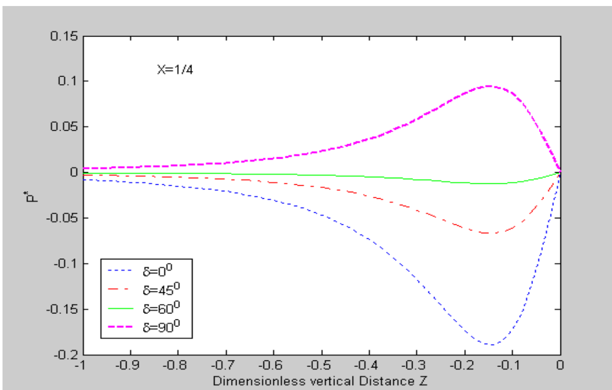

Fig. 19
Variation of pore pressure w.r.t. vertical distance in medium-I.

APPENDIX ($x > 0$)

$$\int_{-\infty}^{\infty} e^{-|k|x} e^{-iky} dk = \frac{2x}{y^2 + x^2}, \qquad \int_{-\infty}^{\infty} |k| e^{-|k|x} e^{-iky} dk = \frac{2\left(x^2 - y^2\right)}{\left(y^2 + x^2\right)^2},$$

$$\int_{-\infty}^{\infty} \frac{k}{|k|} e^{-|k|x} e^{-iky} dk = \frac{-2iy}{y^2 + x^2}, \qquad \int_{-\infty}^{\infty} k e^{-|k|x} e^{-iky} dk = \frac{-4ixy}{\left(y^2 + x^2\right)^2},$$

$$\int_{-\infty}^{\infty} k|k| e^{-|k|x} e^{-iky} dk = \frac{-4iy\left(3x^2 - y^2\right)}{\left(y^2 + x^2\right)^3}, \qquad \int_{-\infty}^{\infty} k^2 e^{-|k|x} e^{-iky} dk = \frac{4x\left(x^2 - 3y^2\right)}{\left(y^2 + x^2\right)^3},$$

$$\int_{-\infty}^{\infty} \frac{1}{k} e^{-|k|x} e^{-iky} dk = -2i \tan^{-1}\left(\frac{y}{x}\right), \qquad \int_{-\infty}^{\infty} \frac{1}{|k|} e^{-|k|x} e^{-iky} dk = -\log\left(y^2 + x^2\right). \tag{A.1}$$

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