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Optimization of Functionally Graded Beams Resting on Elastic Foundations

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ABSTRACT

In this study, two goals are followed. First, by means of the Generalized Differential Quadrature (GDQ) method, parametric analysis on the vibration characteristics of three-parameter Functionally Graded (FG) beams on variable elastic foundations is studied. These parameters include (a) three parameters of power-law distribution, (b) variable Winkler foundation modulus, (c) two-parameter elastic foundation modulus. Then, volume fraction optimization of FG beam with respect to the fundamental frequency is studied. Since the optimization process is so complicated and time consuming, Genetic Algorithm (GA), a computational algorithm based on Darwinian theories that allow to solve optimization problems without using gradient-based information on the objective functions and the constraints, is performed to obtain the best material profile through the thickness to maximize the first natural frequency. A proper Artificial Neural Network (ANN) is trained by training data sets obtained from GDQ method and then is applied as the objective function in genetic algorithm by reproducing the fundamental frequency for improving the speed of the optimization process. Finally, the optimized material profile for the maximum natural frequency of a FG beam resting on elastic foundations is presented.

Keywords: Functionally graded beam; Elastic foundations; Optimization; Genetic algorithm; Artificial neural network

1 INTRODUCTION

A NEW class of materials known as Functionally Graded Materials (FGMs) has attracted much attention as advanced structural materials in many structural members. FGMs are composite materials that are microscopically inhomogeneous, and the mechanical properties vary continuously in one (or more) direction(s). Recently, Tornabene [1] has used three-parameter power law distribution to study the dynamic behavior of functionally graded parabolic panels of revolution. One of the advantages of using three-parameter power law distribution is the ability of controlling the materials volume fraction of FG structures for considered applications. Beams and columns supported along their length are very common in structural configurations. Beams are often found to be resting on earth in various engineering applications. These include railway lines, geotechnical areas, highway pavement, building structures, etc. This motivated many researchers to analyze the behavior of beam structures on elastic foundations [2-6].

Optimization is the task of finding one or more solutions which correspond to minimizing (or maximizing) one or more specified objectives and which satisfy all constraints (if any). Optimization is implemented for various

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objective functions in mechanical problems, such as buckling loads, fundamental frequencies, deflection, weight (either as a constraint or as an objective to be minimized) [7-10] and etc. When the search space becomes large, enumeration is soon no longer feasible simply because it would take far too much time. In this it's needed to use a specific technique to find the optimal solution. Genetic Algorithm (GA) provides one of these methods. GA method was first introduced by John Holland in 1975 and has been applied to a wide range of engineering problems [7-10]. Genetic algorithm is one of the most approved heuristic methods to optimization problems where the extreme of the function cannot be computed analytically or it takes too much time. GA is a particular class of evolutionary algorithms that use techniques inspired by evolutionary biology such a inheritance, mutation, selection, and cross over (also called recombination). M. Abouhamze et al. [7] optimized stacking sequence of laminated cylindrical panels with respect to the first natural frequency and critical buckling load using genetic algorithm and neural networks. Artificial Neural Network (ANN) modeling is an equation-free, data-driven modeling technique that tries to emulate the learning process in the human brain by using many examples. ANN can be defined as a massive parallel-distributed information processing system that has a natural propensity for recognizing and modeling complicated input-output systems. The concept of neural networks has been introduced to different branches of engineering, analytical procedure of structural design, structural optimization problems and functionally graded materials [11-14].

However, this paper is motivated by the lack of studies in the technical literature concerning the analysis of three-parameter functionally graded beams resting on two-parameter elastic foundation. In this study, ceramic-metal graded beams resting on variable elastic foundations with three-parameter power-law variations of the volume fraction of the constituents in the thickness direction are considered. The effect of the power-law exponent, power-law distribution choice, variable Winkler foundation modulus and two-parameter elastic foundation modulus on the mechanical behavior of functionally graded beams is investigated. The frequency parameter of beam is obtained by using numerical technique termed the Generalized Differential Quadrature (GDQ) method based on the DQ technique [15]. The GDQ approach was developed by Shu and Coworkers [16, 17]. It approximates the spatial derivative of a function of given grid point as a weighted linear sum of all the functional value at all grid point in the whole domain. The computation of weighting coefficient by GDQ is based on an analysis of a high order polynomial approximation and the analysis of a linear vector space. The weighting coefficients of the first-order derivative are calculated by a simple algebraic formulation, and the weighting coefficient of the second-and higher-order derivatives are given by a recurrence relationship. The details of the GDQ method can be found in [16, 17]

As a second goal of this study, volume fraction optimization of three-parameter power law distribution is presented for maximizing the natural frequency parameter of FG beam. A nature inspired technique named genetic algorithm is applied to find the optimal solution. To speed the optimization process, a suitable ANN is implemented to increase the speed of the process of optimization by reproducing the fundamental frequency parameter.

2 PROBLEM DESCRIPTION

2.1 FG material properties

Consider a FG beam resting on two-parameter elastic foundation as shown in Fig.1 where $k(x)$, $k_1(x)$ are Winkler foundation modulus and second parameter foundation modulus respectively. In the present work, different models of Winkler elastic coefficient including constant, linear and parabolic types are considered as follows:

a: Winkler elastic foundation with constant modulus $k(x)=k_0$

b: Winkler elastic foundation with linear variation type $k(x)=k_0(1-\alpha x)$

c: Winkler elastic foundation with nonlinear variation type $k(x)=k_0(1-\beta x^2)$

Linear and parabolic types of Winkler elastic foundation are shown in Fig. 2.

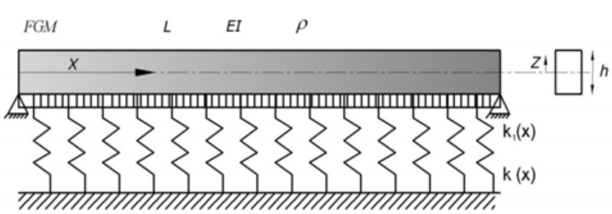

Fig. 1
FG beam supported on variable two-parameter elastic foundations.

Fig. 2
Various Winkler elastic foundations along the axial direction: (a) linear type (b) parabolic type.

The Young's modulus E_{fgm}, Poisson's ratio υ_{fgm} and mass density ρ_{fgm} of the functionally graded beam can be expressed as a linear combination [1]:

$$\begin{aligned} E_{fgm} &= (E_c - E_m)V_c + E_m \\ \rho_{fgm} &= (\rho_c - \rho_m)V_c + \rho_m \\ \nu_{fgm} &= (\nu_c - \nu_m)V_c + \nu_m \end{aligned} \tag{1}$$

where $\rho_m, E_m, \upsilon_m, V_m$ and $\rho_c, E_c, \upsilon_c, V_c$ represent mass density, Young's modulus, Poisson's ratio and volume fraction of the metal and ceramic constituent materials, respectively. In the present work, V_c is considered as follow [1];

$$V_c = \left[\frac{1}{2} - \frac{z}{h} + b\left(\frac{1}{2} + \frac{z}{h}\right)^c\right]^p, \qquad -\frac{1}{2} \le \eta = \frac{z}{h} \le \frac{1}{2} \tag{2}$$

where volume fraction index p $(0 \le p \le \infty)$ and the parameters b, c dictate the material variation profile through the FG beam thickness. It should be noticed that the values of parameters b and c must be chosen so that $0 \le V_c \le 1$. Some material profiles through the FG beam thickness are illustrated in Figs. 3-5. In Fig. 3, the classical volume fraction profiles are presented as special case of the general distribution laws (2) by setting b=0.

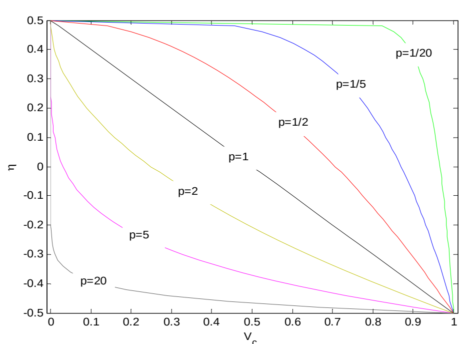

Fig. 3
Variations of the volume fractions of the matrix phase V_c through the thickness for different values of the power-law index p (b=0).

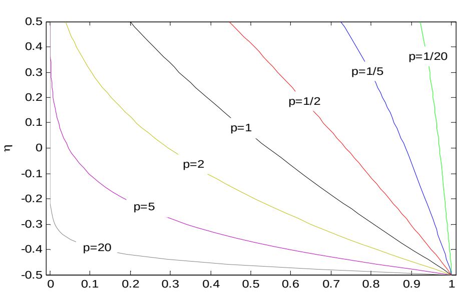

Fig. 4
Variations of the volume fractions of the matrix phase V_c through the thickness for different values of the power-law index p (b=0.2, c=2.2).

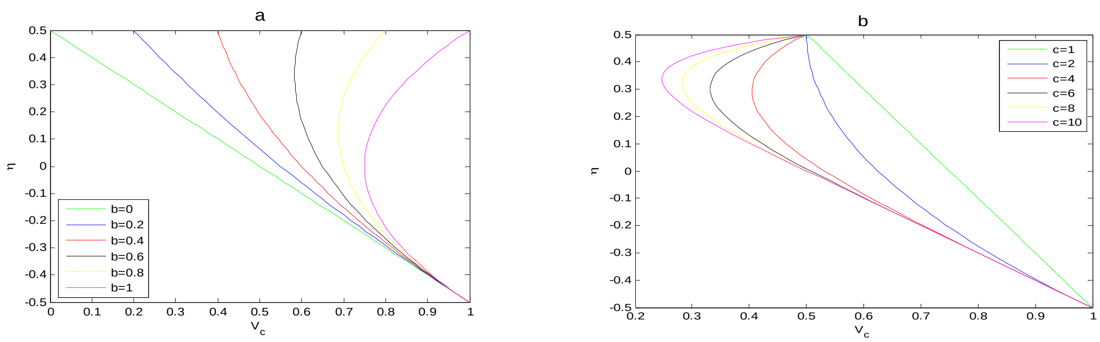

Fig. 5
Variations of the volume fractions of the ceramic constituent (V_c) through the thickness for $p=1$ (a : c=2 ; b: b=0.5).

Fig. 4 shows various power-law distributions obtained by modifying the parameters b, c, p with respect to the reference surface $(\eta = 0)$ of the beam. For another example, matrix phase profile for the different values of parameters b, c and p=1 is shown in Figs. 5a, b. In Figs. 5a, b, ceramic volume fraction on the lower surface is the same (V_c=1); however, volume fraction profile of ceramic constituent as well as volume fraction on the upper surface will change with changing the parameter b or c. Eq. (2) presents this fact that:

$$p = 0 \rightarrow V_c = 1\,, \quad V_m = 0 \rightarrow \rho(\eta) = \rho_c\,, \qquad E(\eta) = E_c\,, \quad \upsilon(\eta) = \upsilon_c$$
$$p = \infty \rightarrow V_c = 0\,, \quad V_m = 1 \rightarrow \rho(\eta) = \rho_m\,, \qquad E(\eta) = E_m\,, \quad \upsilon(\eta) = \upsilon_m$$

2.2 The basic formulation

For FG beam resting on two-parameter elastic foundation in the absence of body force, the governing equation can be expressed as [6]:

$$-D_{fgm}\frac{\partial^4 w}{\partial x^4} - \frac{\partial}{\partial x}\left(K_1(x)\frac{\partial w}{\partial x}\right) - K(x)w - \rho A\frac{\partial^2 w}{\partial t^2} = 0, \qquad 0 < x < L \tag{3}$$

in which

$$D_{fgm} = \int_{\frac{-h}{2}}^{\frac{h}{2}} Q_{11} z^2 \mathrm{d}z, \qquad Q_{11} = \frac{E_{fgm}}{1 - v_{fgm}{}^2}$$

To obtain the natural frequency, Eq. (3) is formulated as an eigenvalue problem by using the following periodic function $w(x,t) = W(x)e^{-i\omega t}$, where $W(x)$ is the mode shape of the transverse motion of the beam. Eq. (3) is a fourth-order ordinary differential equation. Thus, it requires four boundary conditions. The following two types of boundary conditions are considered.

Simply supported edge

$$W = 0, \quad \frac{\partial^2 W}{\partial x^2} = 0 \qquad \text{at} \qquad x = 0 \;\text{ or }\; x = L \tag{4a}$$

Clamped edge

$$W = 0, \quad \frac{\partial W}{\partial x} = 0 \qquad \text{at} \qquad x = 0 \;\text{ or }\; x = L \tag{4b}$$

3 GDQ SOLUTION OF GOVERNING EQUATION

The generalized differential quadrature (GDQ) approach is used to solve the governing equation of beam. In the GDQ method, the nth order of a continuous function $f(x,z)$ with respect to x at a given point x_i can be approximated as a linear sum of weighting values at all of the discrete point in the domain of x , i.e. [16, 17],

$$\frac{\partial f^{n(x_i,z)}}{\partial x^n} = \sum_{k=1}^{N} c_{ik}^n f(x_{ik}, z), \qquad i = 1, 2, ..., N, \quad n = 1, 2, N-1 \tag{5}$$

where N is the number of sampling points, and c_{ij}^n is the x^i dependent weight coefficients.

4 NEURAL NETWORK MODELING

The basic element of an NN is the artificial neuron as shown in Fig. 6 which consists of three main components namely as weights, bias, and an activation function. Each neuron receives inputs X^1, X^2, …, X^n, attached with a weight w^i which shows the connection strength for that input for each connection. Each input is then multiplied by the corresponding weight of the neuron connection. A bias b_i can be defined as a type of connection weight with a constant nonzero value added to the summation of inputs and corresponding weights u, given by

$$u_i = \sum_{j=1}^{H} w_{ij} x_j + b_i \tag{6}$$

The summation u_i is transformed using a scalar-to-scalar function called an ''activation or transfer function'', $F(u_i)$ yielding a value called the unit's ''activation'', given by:

$$Y_i = f(u_i) \tag{7}$$

Activation functions serve to introduce nonlinearity into NNs which makes NNs so powerful. NNs are commonly classified by their network topology (i.e. feedback, feed forward) and learning or training algorithms (i.e. supervised, unsupervised). There is no well-defined rule or procedure to have optimal network architecture. In this work, the feed forward Multi-Layer Perceptron (MLP) network has been applied. MLP networks are one of the most popular and successful neural network architectures which are suited to a wide range of applications such as prediction and process modeling. The neural network architecture adopted in the present work has two hidden layers which has high accuracy and has been used for various applications. Fig.7 illustrates the topology of a simple, fully connected four-layer MLP network.

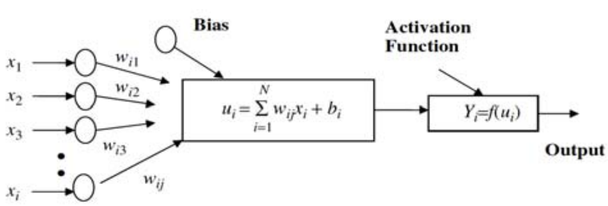

Fig. 6
Basic elements of an artificial neuron.

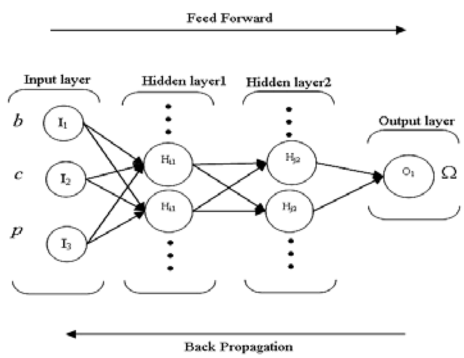

Fig. 7
Schematic diagram of ANN model.

5 GENETIC ALGORITHM

Genetic algorithms operate on a set of possible solutions. Chromosomes represent solutions within the genetic algorithm. Chromosomes are grouped into population (set of solutions) on which the genetic algorithm operates. In each step (generation), the genetic algorithm selects chromosomes from a population and combines them to produce new chromosomes (offspring chromosomes). These offspring chromosomes form a new population in the hope that the new population will be better than the previous ones. Genetic algorithms produce new chromosomes (solutions) by combining existing chromosomes. This operation is called crossover. A crossover operation takes parts of solution encodings from two existing chromosomes (parents) and combines them into a single solution (new chromosome). A coupling operation defines how the selected chromosomes (parents) are paired for mating (mating is done by performing a crossover operation over the paired parents and applying a mutation operation to the newly produced chromosome). This operation gives better control over the production of new chromosomes, but it can be skipped and new chromosomes can be produced as the selection operation selects parents from the population. The next step performed by a genetic algorithm is the introduction of new chromosomes into a population. Offspring chromosomes can form a new population and replace the entire (previous) population (non-overlapping population), or they can replace only a few chromosomes in the current population (overlapping population). For overlapping populations, the replacement operation defines which chromosomes are removed (usually the worst chromosomes) from the current population and which offspring chromosomes are inserted. By replacing chromosomes, there is a chance that the genetic algorithm will lose the best chromosome(s). To prevent this, the concept of elitism is introduced into genetic algorithms. Elitism guarantees that the best chromosome(s) from the current generation is (are) going to survive to the next generation.

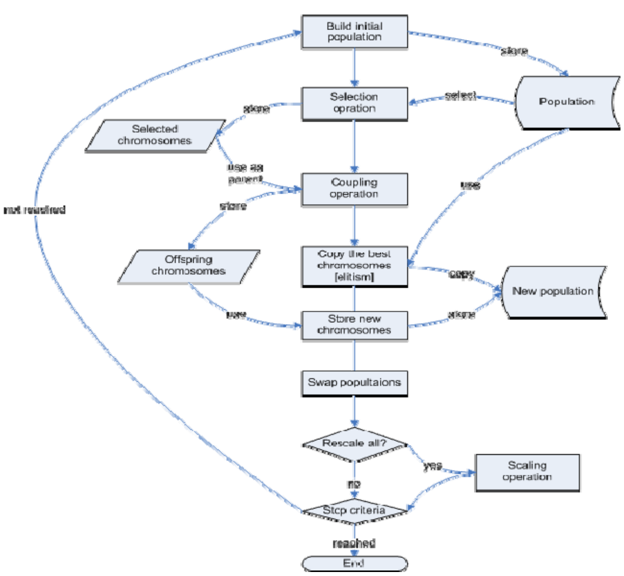

Fig. 8
Flowchart of GA.

An algorithm performs the previously described steps one by one in sequence, and when they have been performed, it is said that a generation has passed. At the end of each generation, the genetic algorithm checks the stop criteria. Because of the nature of genetic algorithms, most of the time, it is not clear when the algorithm should stop, so a criteria is usually based on statistical information such as the number of the generation, the fitness value of the best chromosome, or the average fitness value of the chromosomes in the population, the duration of the evolution process, etc. The flowchart of the proposed algorithm is shown in Fig. 8. More details about the algorithms are found in Haftka and Gurdal [18], Gurdal et al. [19], etc.

6 RESULTS AND DISCUSSION

6.1 Free vibration analysis

This section introduces some results and considerations about the free vibration problem of functionally graded beams on elastic foundations by means of GDQ method. First, validation study of the results is considered for an isotropic beam resting on Winkler elastic foundation in Table 1. As observed there is good agreement between the present results with similar one obtained by Zhou Ding [2]. In this study, ceramic and metal are particle mixed to form the functionally graded material. The relevant material properties for the constituent materials are shown in Table 2 [20]. The convergence and accuracy of the GDQ method is investigated in evaluating the normalized natural frequency, $\Omega = \omega L^2 \sqrt{\rho_m A / E_m I}$ (ρ_m , E_m are mechanical properties of aluminum). The non-dimensional forms of the elastic foundation coefficients are defined as $k = K\,L / D_{fgm}$ and $k_1 = K_1 / D_{fgm}\,L.$ From Fig. 9 fast rate of convergence of the method is evident at different boundary conditions and it is found that only ten DQ grid in the axial direction can yield accurate results. It is also observed for the considered system the formulation is stable while increasing the number of points and that the use of 50 points guarantees convergence of the procedure.

The influence of the index p on the natural frequency of simply supported FG beam on elastic foundations is shown in Table 3. As can be seen from this table, increasing the values of the parameter index p up to infinity reduces the contents of ceramic phase and at the same time increases the percentage of metal phase. In other words, by considering the relation (1), it is possible to obtain the homogeneous isotropic material when the power-law exponent is set equal to zero $(p = 0)$ or equal to infinity $(p = \infty)$. The influence of the index p on the natural frequency is also shown in Figs. 10 and 11 for three sets of boundary conditions, that is, simply supported-simply supported (S-S), clamped- simply supported (C-S) and clamped-clamped (C-C) conditions. In these figures, it is assumed that $k_1 = 10,\ k = 900\ (1 - 0.2\,x)$.

Table 1
Comparison of the frequency parameters of an isotropic beam resting on parabolic type of Winkler elastic foundation
($K = 1000\ (1 - \beta\, x^2)$, $k_1 = 0$, $\lambda_i^{\,4} = \rho A\ L^4 \omega_i^{\,2} / EI_0$, $N = 21$)

β		λ_1	λ_2	λ_3
	Zhou[2]	5.597	7.022	9.675
0.4	Present	5.596	7.0231	9.674
	Zhou[2]	5.409	6.935	9.638
0.8	Present	5.410	6.935	9.638

Table 2
Material properties [20]

Material properties	
E_c (Gpa)	380
E_m (Gpa)	70
ρ_c (kg/m^3)	3800
ρ_m (kg/m^3)	2707

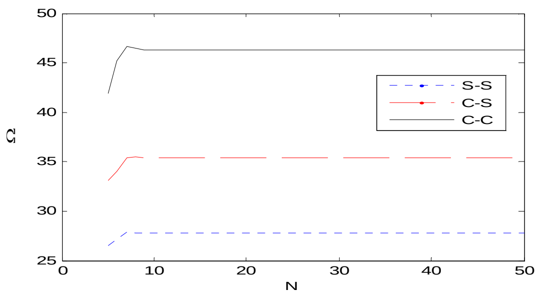

Fig. 9
Convergency of the normalized natural frequency
$(b=1,\ c=4,\ p=3,\ k_1=1,\ k=500)$

Table 3
The first non-dimensional natural frequency of FG beams resting on elastic foundation with simply supported ends
$(b=0.6,\ c=2\ ,k_1, k=k_0(1-0.2x))$

k_1, k_0	p = 0 (Ceramic beam)	p = 0.7	p = 1	p = 5	p=10	p = 20	p = ∞ (Metal beam)
1, 10	20.6735	19.6658	19.2785	16.1435	14.6866	13.4546	11.2211
1, 100	22.0248	21.1756	20.8521	18.2948	17.1280	16.1367	14.3842
10, 900	32.5760	32.6197	32.6389	32.7768	32.6910	32.4438	31.8620

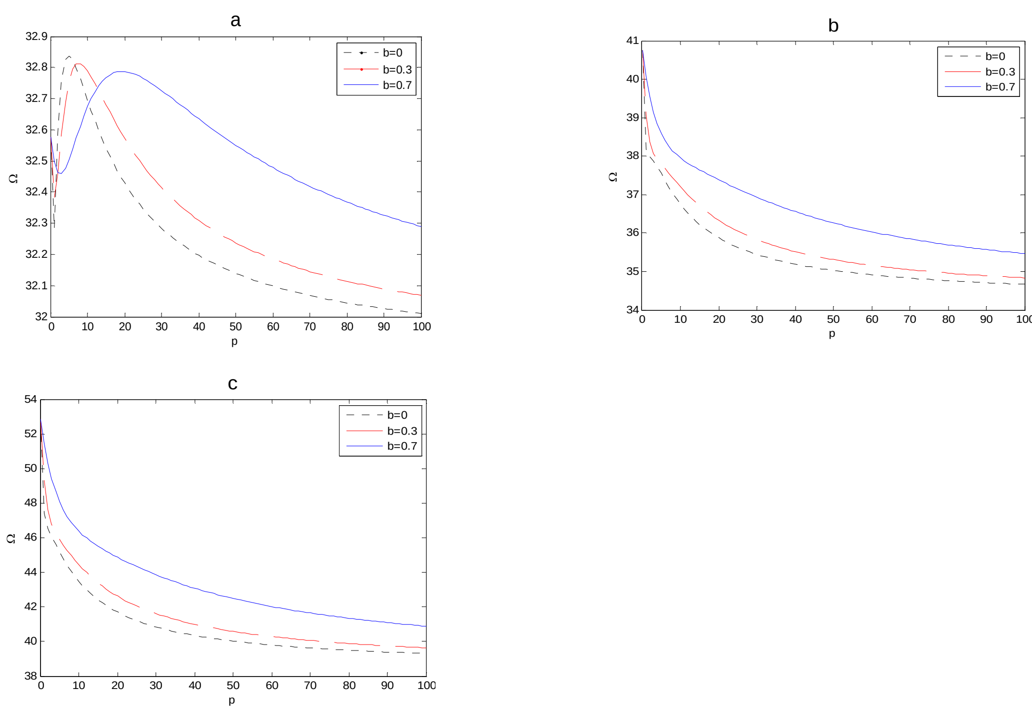

Fig. 10
Variation of the natural frequency parameter vs. the power-law exponent *p* for various values of the parameter *b* and different boundary conditions $(0\leq b\leq 0.7\ , c=1)$, (**a**: S-S, **b**: C-S, **c**: C-C).

The new and interesting result is that although it is expected to be the value of natural frequency parameter between the natural frequency parameter of the limit cases of homogeneous beams of alumina (*p*=0) and of aluminum (*p*=∞), as observed in Figs. 10 and 11, natural frequency parameter sometimes exceed the limit cases. Various parameters such as the boundary condition, the power-law distribution profile, mechanical properties of materials, elastic foundations modulus, etc can influence on this fact. Thus, it is possible to obtain dynamic characteristics similar or better than the isotropic ceramic limit case by choosing suitable values of the three parameters *b, c, p*.

Here, we consider the effect of elastic foundations on the frequency parameters. First, the influence of the Winkler elastic foundation on the first three frequency parameters of FG beam with simply supported ends are investigated. In Fig. 12 the shearing layers elastic coefficient (k_1) is assumed to be 10 while Winkler elastic modulus (k) is considered to vary from 10 to 100,000. The effect of various Winkler elastic foundations on the natural frequency is studied. Fig.13 shows variations of the natural frequency parameter of FG beam with clamped ends resting on different type of Winkler elastic foundation. As observed, three different Winkler elastic foundations have the same effect on the natural frequency parameter of FG beam for Winkler elastic constant (k_0), ranges $10<k_0<1000$ and then for $k_0>1000$, the natural frequency parameter of a FG beam resting on a variable Winkler elastic foundation decreases from constant type to parabolic and then linear types. Fig. 14 shows the influence of k on frequency parameter for different types of boundary conditions. As observed for $k>10000$, the type of boundary conditions does not effect on natural frequency of FG beam. The effect of shearing layer coefficient on the natural frequency parameter of a FG beam with simply supported ends is illustrated in Fig. 15 for different Winkler elastic foundation coefficients. As it could be observed the natural frequency parameter converges with increasing the shearing layer elastic coefficient. For further study, the first natural frequency parameters of the FG beam on elastic foundations with various linear modulus (α) as well as parabolic modulus (β) is shown for different boundary conditions in Tables 4 and 5. As noticed, the natural frequency parameters decrease with the increase of linear and parabolic modulus. The results in these tables are for b=0.5, c=4, p=5.

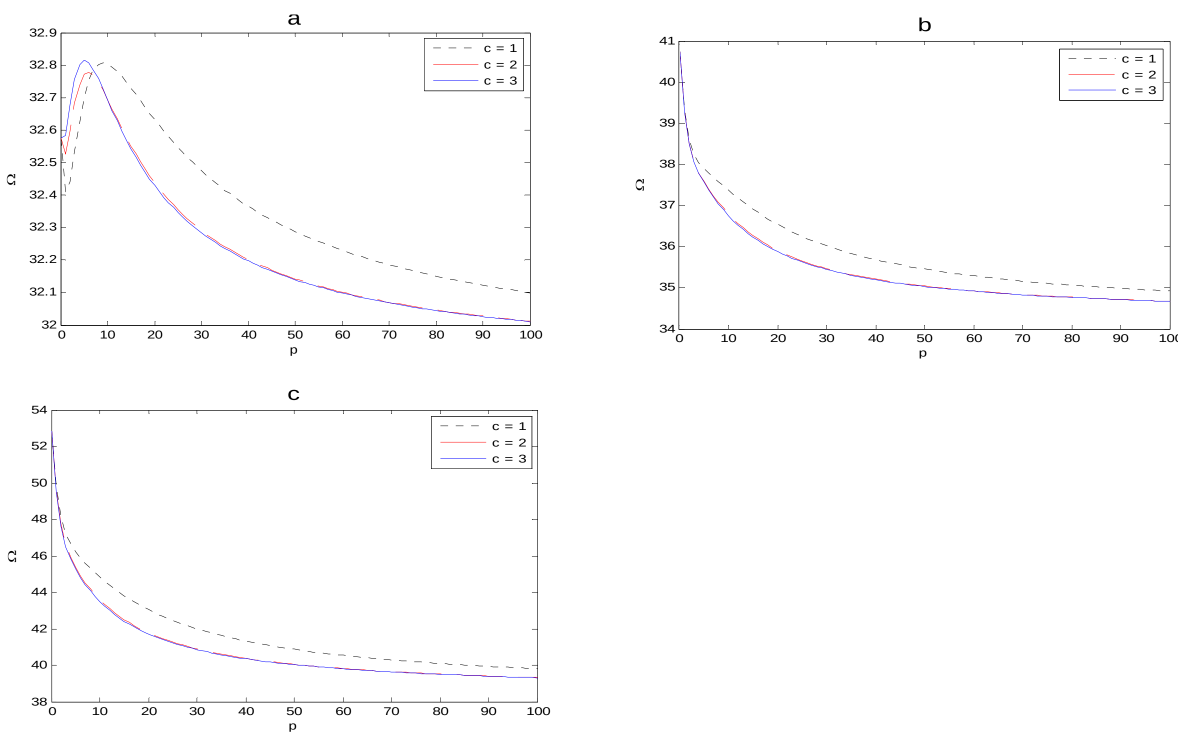

Fig. 11
Variation of the natural frequency parameter vs. the power-law exponent p for various values of the parameter b and different boundary conditions ($b = 0.4, 1 \le c \le 3$), **(a:** S-S, **b:** C-S**, c:** C-C)

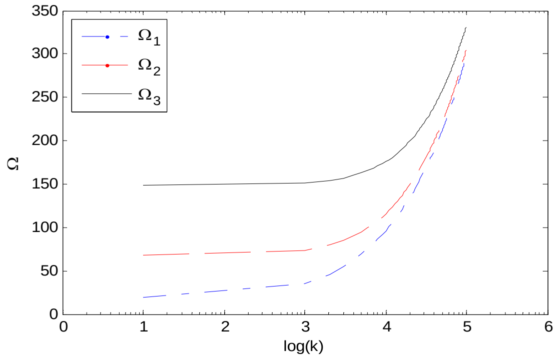

Fig. 12
Variation of the first three normalized natural frequency versus Winkler modulus (k).

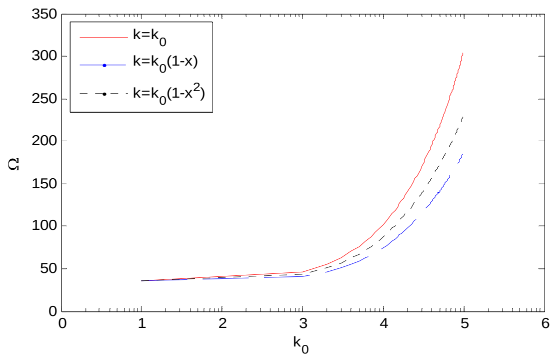

Fig. 13
Variations of the natural frequency parameter of a clamped FG beam resting on different kinds of Winkler elastic foundation $b=0.5$, $c=2$, $p=5$, $k_1=1$, $10 \le k_0 \le 10^5$.

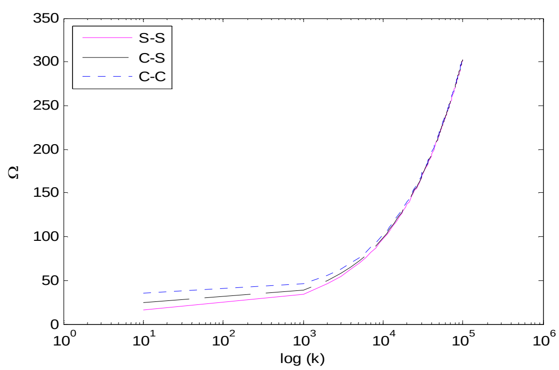

Fig. 14
Variations of the fundamental frequency parameter of a FG beam resting on different kinds of boundary conditions $(b=0.2,\ c=2,\ p=5,\ k_1=1,\ 10 \le k \le 10^5)$

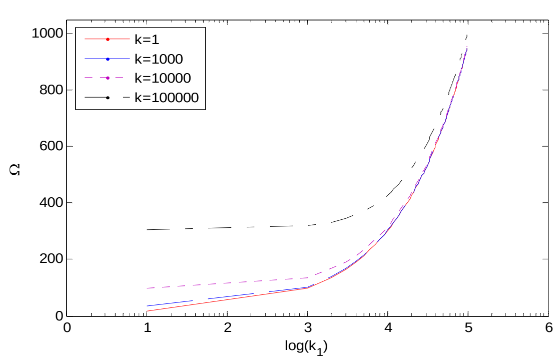

Fig. 15
The effect of shearing layer coefficient on the natural frequency parameter of a FG beam with simply supported ends for different Winkler elastic foundation coefficients $(b=0.2,\ c=2,\ p=5,\ 10 \le k_1 \le 10^5)$

Table 4
Comparison of the first three non-dimensional natural frequency parameters of FG beam on elastic foundations $(k = k_0(1-\alpha x),\ k_1 = 100\ ,\ N = 15)$.

k_0	α	S-S	C-S	C-C
1000	0.2	44.636	49.412	56.123
	0.4	43.571	48.353	55.283
	0.8	41.346	46.155	53.559
2000	0.2	53.219	57.201	63.167
	0.4	51.413	55.355	61.664
	0.8	47.551	51.440	58.526

Table 5
Comparison of the first three non-dimensional natural frequency parameters of FG beam on elastic foundations $(k = k_0(1-\beta x^2),\ k_1 = 100,\ N = 15)$

k_0	β	S-S	C-S	C-C
1000	0.2	45.088	49.826	56.499
	0.4	44.493	49.195	56.044
	0.8	43.264	47.899	55.118
2000	0.2	53.977	57.914	63.834
	0.4	52.969	56.817	63.023
	0.8	50.848	54.527	61.353

6.2 Optimization procedure

The objective of optimization in this paper is to find the best values of the parameters *b, c, p* in three-parameter power law distribution so that to maximize fundamental frequency parameter of FG beam. There is an important point that considered parameters must be obtained so that the ceramic volume fraction is between zero and one $(0 \le V_c \le 1)$. The boundary conditions of the FG beam are considered simply supported and the parameters are considered in the following ranges: $0 \le b \le 1,\ 0 \le c \le 30,\ \ 0 \le p \le 30,\ \ k = 900(1-0.2x),\ k_1 = 10.$

Therefore, the constrained optimization problem is defined as:

$$\begin{aligned} &\textit{Minimize} && f(b,c,p) = -\Omega \\ &\textit{Subject to} && \begin{cases} 0 \le V_c \le 1 \\ 0 \le b \le 1\ ,\ 0 \le c \le 30\ ,\ \ 0 \le p \le 30 \end{cases} \end{aligned} \tag{8}$$

It should be noticed that since GA minimizes the fitness function basically, fundamental frequency parameter has been multiplied by minus in the above equation. If GDQ method is applied for frequency parameters, the optimization process becomes so complicated and time consuming. For example, even if the increment of the parameters (*b, c, p*) is assumed 0.05, the formed discrete space contains more than 7,200,000 design choices to be searched for an optimum point. Also, if it is assumed that the process of one search takes 0.1 second in average, the optimization process takes about 100 hours. In the present work, therefore, ANN and GA are implemented for increasing the speed of optimization. The MLP network has been used having two hidden layers. A program was developed in MATLAB which handles the trial and error process automatically. The program tries varying number of hidden layers neurons is tested from two up to fifteen for a 1000 epochs for 5 times for different back propagation training algorithms. 8 neurons for first hidden layer and 10 neurons for second hidden layer, Levenberg-Marquardt (LM) algorithm are chosen for the network because it performs better than other cases. The ability of trained network to reproduce the fundamental frequency parameter is shown in Fig.16 for 50 test points which selected far from the training point randomly. In this figure, comparison is made between the ANN results for natural frequency with similar ones obtained from GDQ method. As noticed, the results are so close for all 50 test points. Since the neural network has been accurately designed, it can be implemented for fitness function in genetic algorithm by simulating fundamental frequency parameter. Table 6 shows the parameters of GA used to find the optimal solution.

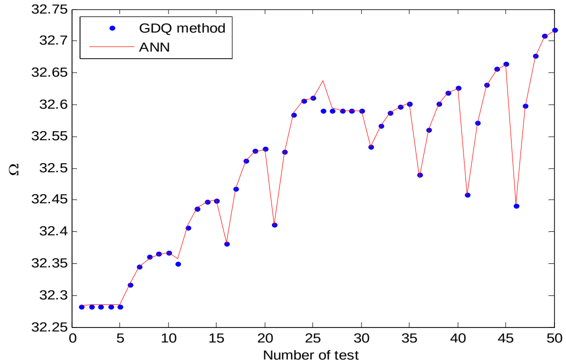

Fig. 16
Comparison of various predicted values of fundamental frequency parameter versus GDQ data.

Table 6
Parameters of GA approach

Parameters	Value/type
Population size	30
Generations	200
Selections	stochastic uniform
Crossover options	Scattered
Mutations options	Constraints dependent

Table 7
Optimization result for the fundamental frequency parameter of the simply supported FG beam on elastic foundation $(k = 900(1-0.2x),\ k_1 = 10)$

Optimum parameters			Exact value of Ω	Predicted by ANN	Relative error	Density kg/m^3	Volume fraction of ceramic
b	*c*	*p*					
1	2.856	4.801	33.46	33.45	0.03 %	3061.6	32.44%

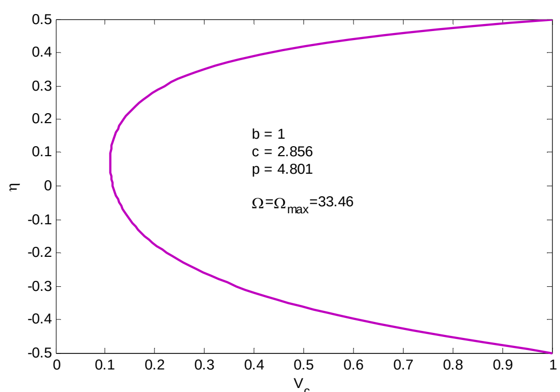

Fig. 17
Optimized material profile for maximum natural frequency.

Here, optimization is investigated for the FG beam and result is shown in Table 7. The algorithm reached to the optimal values after 51 generations. The (b), (c) and (p) parameters for the optimized profile are 1, 2.856 and 4.801 respectively. Fig 17 shows the optimized material profile for the maximum frequency parameter. As observed, the ceramic volume fraction decreases from 1 at η=- 0.5 as far as middle surface of the beam (η=0) and then increases to 1 for η=0.5. In other words, the profile is so close to parabolic one. For further verification on the accuracy of the trained network, the results obtained by ANN and GDQ have been compared in Table 7. As observed, the results are so close to each other and relative error is about 0.03 %. It should be mentioned that the process of optimization by GA took less than 2 minutes (CPU time was reduced by a considerable amount).

7 CONCLUSIONS

In this research, free vibrations of three-parameter FG beam on variable elastic foundation including Winkler elastic foundation with constant modulus, linear and parabolic types is studied through using GDQ method. The effect of the power-law exponent, power-law distribution choice, variable Winkler foundation modulus and two-parameter elastic foundation modulus on the natural frequencies of FG beams is investigated. Interesting result shows that although frequency parameter of the ceramic beam is more than the metal one, the frequency parameter of the FG beam does not necessarily increase with the increase of ceramic volume fraction. In other words, by choosing suitable values of b, c, *p*, frequency parameter can be obtained more than the frequency parameter of the similar beam made of 100% ceramic and at the same time lighter. This result is against the expected one for the frequency parameter of FG beam to fall between those for p=0 (100% Ceramic) and $p=\infty$ (100% metal). In the next step, volume fraction optimization of FG beam resting on elastic foundations with respect to first natural frequency was studied. Genetic algorithm was performed to obtain the best material profile through the thickness to maximize the fundamental natural frequency. A numerical method able to solve the free vibration was developed and was used in training artificial neural network (with much fewer runs) and then the trained network was implemented as the fitness function in the GA. Also, it was concluded that using the combination of NN and GA reduces the CPU time by a considerable amount with losing negligible accuracy. Comparing Table 3 (if $p=0, \Omega=\Omega_c=32.5760$) and then $\rho=\rho_c=3800\,\text{kg/m}^3$ with Table 7, one can come to this conclusion that by choosing suitable values of (b), (c), (p), frequency parameter can be obtained more than the frequency parameter of the similar beam made of 100% ceramic and at the same time lighter (about 740 kg/m^3). As observed for this frequency parameter, the volume fraction of the ceramic constituent is 32.44%.

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