Smart Vibration Control of Magnetostrictive Nano-Plate Using Nonlocal Continuum Theory

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ABSTRACT

In this research, a control feedback system is used to study the free vibration response of rectangular plate made of magnetostrictive material (MsM) for the first time. A new trigonometric higher order shear deformation plate theory are utilized and the results of them are compared with two theories in order to clarify their accuracy and errors. Pasternak foundation is selected to modelling of elastic medium due to considering both normal and shears modulus. Also in-plane forces are uniformly applied on magnetostrictive nano-plate (MsNP) in x and y directions. Nonlocal motion equations are derived using Hamilton's principle and solved by differential quadrature method (DQM) considering different boundary conditions. Results indicate the effect of various parameters such as aspect ratio, thickness ratio, elastic medium, compression and tension loads and small scale effect on vibration behaviour of MsNP especially the controller effect of velocity feedback gain to minimizing the frequency. These finding can be used to active noise and vibration cancellation systems in micro and nano smart structures.

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Keywords : Free vibration; Magnetostrictive rectangular nano-plate; A new trigonometric/tangential shear deformation theory; Control feedback system.

1   INTRODUCTION

MAGNETOSTRICTION refers to the phenomenon in which an applied magnetic field generates elastic strain in ferromagnetic materials. The most known magnetostrictive material is Terfenol-D which may generate strains of about 1.5×10^{-3} at a magnetic field intensity of 200KA/m and another magnetostrictive materials are iron, Ferrite, Nickel, Cobalt and their alloys which can be utilized for designing the magnetostrictive actuator, motor, transducer, sensor development of communications equipment and computers also one of the applications that today is investigating of a magnetostrictive actuator for contributing to the pollution reduction in novation of mechanical part designs in field of green energy [1-3].

Nanostructures have increased considerable attention among the experimental and theoretical research communities. One of the typical structures of nanosystems is rectangular nanoplates which are two dimensional nanostructures. Nanoplates have applications in various fields of nanotechnology, for example in nano-electro-mechanical devices, can be potentially exploited as bio and mechanical sensors, electro-catalysts, DNA detectors, drug deliverer, and energy storage systems [4-6].

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Therefore plates of MsM at the nanoscale can improve properties plates and it can be nano and magnetostrictive features together and have applications at various means in leading years. The literature contains some of papers that have been analyzed MsM and it also reviews previous studies on nanoplates with several shear deformation plate theories and lastly has been introduced some of papers about plate based on new trigonometric shear deformation plate theory.

Jia et al.[7] studied a novel magnetostrictive static force sensor with giant magnetostrictive material rod for realizing static force measuring and improving the sensitivity of magnetostrictive force sensor. They proposed a special structure surrounding Hall sensor to improve the sensitivity. They also developed the model based on the coupled linear magneto-mechanical constitutive equations and the experimental result shows that the model is good at reflecting the force.

A micro-mechanical analysis was offered for the prediction of the effective behavior and internal field distribution of multiphase magnetostrictive composites based on the homogenization technique for periodic composites by Jacob Aboudi et al. [8].

Li et al. [5] presented buckling and free vibration of magneto-electro-elastic nanoplate based on nonlocal Mindlin theory resting on Pasternak foundation. The governing equations of magneto-electro-elastic nanoplate were derived by using of Hamilton's principle. They concluded that the buckling load and free vibration frequency decreased with increasing nonlocal parameter and the buckling load decreases with increasing lateral load for a magneto-electro-elastic nanoplate.

The small scale effect on the vibration analysis of orthotropic single layered graphene sheets embedded in elastic medium was obtained using nonlocal elasticity and Classical shear deformation plate theory by Pradhan and Kumar [9]. Considering the principle of virtual work, the governing differential equations were derived and solved by differential quadrature method for various boundary conditions.

Pradhan and Phadikar [10] proposed vibration of single and double layered nanoplates based on Classical and first order shear deformation plate theory for simply supported boundary conditions. Navier's approach was used to solve the governing equations and investigate the effect of nonlocal parameter, length, height, elastic modulus and stiffness of Winkler foundation of the plate on non-dimensional vibration frequencies.

Malekzadeh and Shojaee [11] presented free vibration of nanoplates based on a nonlocal two-variable refined plate theory for some different types of boundary conditions. The effect of small scale parameter on the frequency were studied. The equations of motion was derived by the nonlocal differential constitutive relations of Eringen and solved by DQM.

Buckling of single layer graphene sheet based on nonlocal elasticity and higher order shear deformation theory for all edges simply supported boundary conditions was developed by Pradhan [12]. He utilized Navier's approach to solve the governing equations and analytical solutions for critical buckling loads. Effects of several parameters such as length, nonlocal parameter, thickness of the graphene sheets and higher order shear deformation theory on the critical buckling load were investigated in this paper.

Zenkour and Sobhy [13] analyzed thermal buckling of single-layered graphene sheets lying on elastic medium based on the nonlocal classical, first-order and sinusoidal shear deformation plate theories for various boundary conditions. Three types of thermal loading as uniform, linear and nonlinear temperature rise through the thickness of the plate formulated according to closed form solutions. They studied effect of different parameter such as plate aspect ratio, side-to-thickness ratio, nonlocal parameter and elastic foundation parameters on the thermal buckling temperature.

Mantari et al. [14] reported a layerwise finite element formulation of a new (tangential) trigonometric shear deformation theory for the flexure of thick multilayered plates. They resulted that the new theory performs very well as other existing higher order layerwise deformation theories for analyzing the global and inter-laminar mechanical behaviour of multilayered sandwich and composite plates. And then, Mantari et al. [15] used this theory to obtain shear deformation for advanced composite plates and optimize the shear strain function and bi-sinusoidal load.

One of the most important distinctive features of this paper deferred from the mentioned ones is that material of selected structure which is MsM. MsM due to the reciprocal nature has made the choice for smart control systems where magneto-mechanical coupling lead to changes in strains and magnetization of them, like electro-mechanical coupling in piezoelectric materials. This feature is used to create a control feedback system in order to study vibrational behaviour of MsNP. In fact, velocity feedback gain acts as a controller parameter for changing the natural frequency to desire values. In the other hand, nonlocal continuum theory is used to investigate the effect of small scale parameter on MsNP for the first time. The result of this study can be useful to design smart control system in order to reduce the damage caused by the destructive vibration.

2   MAIN THEORIES TO MODELING OF MsNP

An embedded MsNP system by two parameters foundation under the in-plane force N_x, N_y is considered in Fig.1 in which geometrical parameters of length *a*, width *b* and thickness *h* are indicated and the cartesian coordinate system (x,y,z), is introduced.
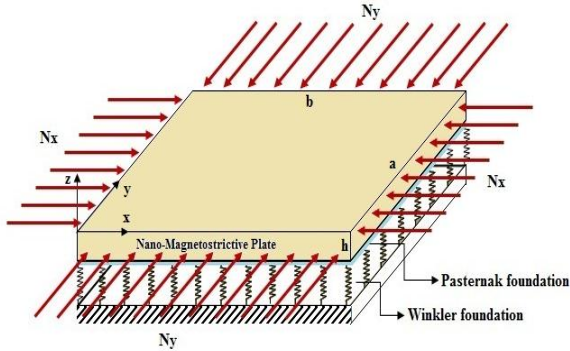

Fig.1
Geometry and coordinate of MsNP.

2.1 A new trigonometric/tangential shear deformation theory

A new trigonometric higher order shear deformation theory involved six unknown displacement functions and does not require shear correction factor. Mantari et al. [16] for the first time presented this theory, and then, this theory has expanded for an analytical solution to the static analysis of FG plates and to study the static response of advanced composite plates [15,17]. Results of this theory have a high accuracy and performs as good as the Reddy's and Touratier's shear deformation theories for analyzing the static behavior of isotropic plate especially for composite laminated and sandwich plates [16]. The displacement field is described in the following equations [17]:

$$\tilde{U}(x,y,z,t) = u_0(x,y,t) + z\left(p\theta_1(x,y,t) + p\frac{\partial}{\partial x}\theta_3(x,y,t) - \frac{\partial}{\partial x}w_0(x,y,t)\right) + \tan(mz)\theta_1(x,y,t)$$

$$\tilde{V}(x,y,z,t) = v_0(x,y,t) + z\left(p\theta_2(x,y,t) + p\frac{\partial}{\partial y}\theta_3(x,y,t) - \frac{\partial}{\partial y}w_0(x,y,t)\right) + \tan(mz)\theta_2(x,y,t) \qquad (1)$$

$$\tilde{W}(x,y,z,t) = w_0(x,y,t) + m\left(\sec^2(mz)\right)\theta_3(x,y,t)$$

where $u_0(\mathrm{x,y,t}), v_0(\mathrm{x,y,t}), w_0(\mathrm{x,y,t}), \theta_1(\mathrm{x,y,t}), \theta_2(\mathrm{x,y,t})$ and $\theta_3(\mathrm{x,y,t})$ are the six unknown displacement functions of middle surface of the plate along (x, y, z) direction and rotations, also $p = -m\sec^2(m\frac{h}{2}), m = \frac{1}{5h}$.

The linear strain relations are presented by the Eq. (2):

$$\varepsilon_{xx} = \frac{\partial}{\partial x}u_0(x,y,t) + z\left(p\frac{\partial}{\partial x}\theta_1(x,y,t) + p\frac{\partial^2}{\partial x^2}\theta_3(x,y,t) - \frac{\partial^2}{\partial x^2}w_0(x,y,t)\right) + \tan(mz)\frac{\partial}{\partial x}\theta_1(x,y,t) \qquad (2a)$$

$$\varepsilon_{yy} = \frac{\partial}{\partial y}v_0(x,y,t) + z\left(p\frac{\partial}{\partial y}\theta_2(x,y,t) + p\frac{\partial^2}{\partial y^2}\theta_3(x,y,t) - \frac{\partial^2}{\partial y^2}w_0(x,y,t)\right) + \tan(mz)\frac{\partial}{\partial y}\theta_2(x,y,t) \qquad (2b)$$

$$\varepsilon_{zz} = 2m^2\left(\sec(mz)\right)^2\theta_3(x,y,t)\tan(mz) \qquad (2c)$$

$$\varepsilon_{xy} = \frac{1}{2}\frac{\partial}{\partial x}v_0(x,y,t) + \frac{1}{2}\tan(mz)\frac{\partial}{\partial x}\theta_2(x,y,t) + \frac{1}{2}\frac{\partial}{\partial y}u_0(x,y,t) + \frac{1}{2}\tan(mz)\frac{\partial}{\partial y}\theta_1(x,y,t)$$

$$+\frac{1}{2}z\left(p\frac{\partial}{\partial x}\theta_2(x,y,t) + p\frac{\partial^2}{\partial y\,\partial x}\theta_3(x,y,t) - \frac{\partial^2}{\partial y\,\partial x}w_0(x,y,t)\right) \tag{2d}$$

$$+\frac{1}{2}z\left(p\frac{\partial}{\partial y}\theta_1(x,y,t) + p\frac{\partial^2}{\partial y\,\partial x}\theta_3(x,y,t) - \frac{\partial^2}{\partial y\,\partial x}w_0(x,y,t)\right)$$

$$\varepsilon_{xz} = \frac{1}{2}m\left(\sec(mz)\right)^2\frac{\partial}{\partial x}\theta_3(x,y,t) + \frac{1}{2}p\theta_1(x,y,t) + \frac{1}{2}p\frac{\partial}{\partial x}\theta_3(x,y,t) + \frac{1}{2}\left(1 + \left(\tan(mz)\right)^2\right)m\theta_1(x,y,t) \tag{2e}$$

$$\varepsilon_{yz} = \frac{1}{2}m\left(\sec(mz)\right)^2\frac{\partial}{\partial y}\theta_3(x,y,t) + \frac{1}{2}p\theta_2(x,y,t) + \frac{1}{2}p\frac{\partial}{\partial y}\theta_3(x,y,t) + \frac{1}{2}\left(1 + \left(\tan(mz)\right)^2\right)m\theta_2(x,y,t) \tag{2f}$$

Also Eq. (3) shows stress-strain relation for isotropic MsM [18,19]. The magneto-mechanical coupling in these materials can be observed in Eq. (3).

$$\begin{bmatrix}\sigma_{xx}\\\sigma_{yy}\\\sigma_{zz}\\\sigma_{xy}\\\sigma_{xz}\\\sigma_{yz}\end{bmatrix} = \begin{bmatrix}Q_{11}&Q_{12}&Q_{13}&0&0&0\\Q_{21}&Q_{22}&Q_{23}&0&0&0\\Q_{31}&Q_{32}&Q_{33}&0&0&0\\0&0&0&Q_{44}&0&0\\0&0&0&0&Q_{55}&0\\0&0&0&0&0&Q_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_{xx}\\\varepsilon_{yy}\\\varepsilon_{zz}\\\varepsilon_{xy}\\\varepsilon_{xz}\\\varepsilon_{yz}\end{bmatrix} - \begin{bmatrix}0&0&e_{31}\\0&0&e_{32}\\0&0&e_{34}\end{bmatrix}\begin{bmatrix}0\\0\\H_z\end{bmatrix} \tag{3}$$

where σ_{ij} and ε_{ij} are stress and strain respectively. Also Q_{ij} are the terms of engineering constants:

$$Q_{ij} = \begin{bmatrix}\dfrac{E(1-\upsilon)}{(1-2\upsilon)(1+\upsilon)}&\dfrac{E\upsilon}{(1-2\upsilon)(1+\upsilon)}&\dfrac{E\upsilon}{(1-2\upsilon)(1+\upsilon)}&0&0&0\\[2mm]\dfrac{E\upsilon}{(1-2\upsilon)(1+\upsilon)}&\dfrac{E(1-\upsilon)}{(1-2\upsilon)(1+\upsilon)}&\dfrac{E\upsilon}{(1-2\upsilon)(1+\upsilon)}&0&0&0\\[2mm]\dfrac{E\upsilon}{(1-2\upsilon)(1+\upsilon)}&\dfrac{E\upsilon}{(1-2\upsilon)(1+\upsilon)}&\dfrac{E(1-\upsilon)}{(1-2\upsilon)(1+\upsilon)}&0&0&0\\[2mm]0&0&0&\dfrac{E}{(1+\upsilon)}&0&0\\[2mm]0&0&0&0&\dfrac{E}{(1+\upsilon)}&0\\[2mm]0&0&0&0&0&\dfrac{E}{(1+\upsilon)}\end{bmatrix} \tag{4}$$

E and υ are Young modulus and Poisson's ratio, also e_{ij} are magnetostrictive coupling modules which determined as follow [18]:

$$e_{31} = \tilde{e}_{31}\cos^2\theta + \tilde{e}_{32}\sin^2\theta \qquad e_{32} = \tilde{e}_{31}\sin^2\theta + \tilde{e}_{32}\cos^2\theta \qquad e_{34} = (\tilde{e}_{31} - \tilde{e}_{32})\sin\theta\sin\theta \tag{5}$$

where θ represents the direction along which a given magnetic anisotropy may have been induced. H_z is the magnetic field intensity and can be expressed as follows [18, 20, 21]:

$$H_z = K_c I(x,y,t) = K_c C(t)\frac{\partial w(x,y,z,t)}{\partial t} \tag{6}$$

where $K_c, I(t)$ and $C(t)$ are the coil constant, coil current and the control gain in which $K_c C(t)$ is introduced as velocity feedback gain. Also $w(x, y, z, t)$ is displacement along z.

2.2 Nonlocal continuum theory

According to Eringen's nonlocal elasticity theory [22] the stress at a point in a body depends on the strain at that point and all other points of the body. Therefore the constitutive equation can be presented to [22, 23]:

$$(1 - \mu \nabla^2)\sigma_{ij}^{nl} = \sigma_{ij}^{l} \qquad for : i, j = x, y, z \qquad (7)$$

where σ_{ij}^{nl} and σ_{ij}^{l} are the nonlocal and local stress tensor components, $\mu = (e_0 l)^2$ is the nonlocal parameter for considering small scale effect that e_0 and l are Eringen's nonlocal elasticity constant suitable to each material and an internal characteristic length, also $\nabla^2 = (\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})$ is the Laplacian operator, respectively.

According to Eq. (7) and the magneto-mechanical coupling for isotropic MsM, stress-strain relation can be observed in Eq. (8). [18,19,22,24]:

$$\begin{bmatrix} \sigma_{xx} - \mu \nabla^2 \sigma_{xx} \\ \sigma_{yy} - \mu \nabla^2 \sigma_{yy} \\ \sigma_{zz} - \mu \nabla^2 \sigma_{zz} \\ \tau_{xy} - \mu \nabla^2 \tau_{xy} \\ \tau_{xz} - \mu \nabla^2 \tau_{xz} \\ \tau_{yz} - \mu \nabla^2 \tau_{yz} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{21} & Q_{22} & Q_{23} & 0 & 0 & 0 \\ Q_{31} & Q_{32} & Q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{xy} \\ \varepsilon_{xz} \\ \varepsilon_{yz} \end{bmatrix} - \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{32} \\ 0 & 0 & e_{34} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ H_z \end{bmatrix} \qquad (8)$$

3 NONLOCAL GOVERNING EQUATIONS OF MOTION USING ENERGY METHOD
3.1 Strain energy

The strain energy of an elastic body for rectangular plate is expressed as [25]:

$$U = \frac{1}{2} \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_0^b \int_0^a \left(\sigma^{nl}_{xx} \varepsilon_{xx} + \sigma^{nl}_{yy} \varepsilon_{yy} + \sigma^{nl}_{zz} \varepsilon_{zz} + \tau^{nl}_{xy} \gamma_{xy} + \tau^{nl}_{xz} \gamma_{xz} + \tau^{nl}_{yz} \gamma_{yz} \right) dx \, dy \, dz \qquad (9)$$

Substituting Eq. (2) into Eq. (9), the strain energy of MsNP can be obtained.

3.2 Kinetic energy

Kinetic energy of the rectangular plate is calculated as [25]:

$$K = \frac{\rho_m h}{2} \int_0^b \int_0^a \left[\left(\frac{\partial \tilde{U}}{\partial t} \right)^2 + \left(\frac{\partial \tilde{V}}{\partial t} \right)^2 + \left(\frac{\partial \tilde{W}}{\partial t} \right)^2 \right] dx \, dy, \qquad (10)$$

where ρ_m and h is the thickness of MsNP.

3.3 In-plane forces

Rectangular plates are usually subjected to in-plane forces, therefore the in-plane stresses effects must be considered in their analysis and vibrations. Uniform in-plane forces N_x and N_y applied in x and y directions as shown in Fig. 1 and it is computed the following equation [26]:

$$F_I = N_x \left(\frac{\partial^2 W}{\partial x^2}\right) + N_y \left(\frac{\partial^2 W}{\partial y^2}\right) \tag{11}$$

3.4 Elastic medium

Pasternak foundation is capable to consider transverse shear loads and normal loads. The effect of surrounding elastic medium on the nanoplate which is simulated with Pasternak model is considered as follows [27]:

$$F_M = K_w W - K_G \left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2}\right) \tag{12}$$

where K_w and K_G are Winkler modulus for normal load and shear modulus for transverse shear loads, respectively.

Therefore, the external work due to in-plane forces and elastic medium is calculated as:

$$\Sigma = \frac{1}{2}\int_0^b\int_0^a F_I W \; dxdy \; + \frac{1}{2}\int_0^b\int_0^a F_M W \; dxdy \tag{13}$$

3.5 Hamilton's principle

In this step, Hamilton's principle is employed to obtain the motion equations and corresponding boundary conditions. This principle can be expressed as follows [25]:

$$\int_{t_1}^{t_2} \left[\delta U - (\delta K + \delta \Sigma)\right] dt = 0 \tag{14}$$

where $\delta U, \delta K$ and $\delta \Sigma$ are variation of strain energy, variation of Kinetic energy and variation of external work.

Once substituting Eqs. (9), (10) and (13) into Eq. (14), afterward using dimensional parameters which introduced in Eq. (15):

$$(\zeta, \eta) = \left(\frac{x}{a}, \frac{y}{b}\right), \quad (U, V, W) = \left(\frac{u_0}{a}, \frac{v_0}{b}, \frac{w_0}{h}\right), \quad (\alpha, \beta, \gamma) = \left(\frac{h}{a}, \frac{h}{b}, \frac{a}{b}\right), \quad \Theta_1 = \frac{\theta_1}{h}, \quad \Theta_2 = \frac{\theta_2}{h}, \quad \Theta_3 = \frac{\theta_3}{a.h},$$

$$G_{ij} = \frac{e_{ij} C(t) K_c}{\sqrt{E \rho_m}}, \quad \tau = \frac{t}{a}\sqrt{\frac{E}{\rho_m}}, \quad K_w^* = \frac{k_w h}{E}, \quad K_g^* = \frac{K_G}{a.E}, \quad \phi = \frac{\mu}{a^2}, \quad \psi = \frac{\mu}{b^2} = \phi.\gamma^2, \quad N_x^* = \frac{N_x}{a.E}, \quad N_y^* = \frac{N_y}{b.E}. \tag{15}$$

The non-dimensional form of motion equations are yield by setting the coefficient $\delta U, \delta V, \delta W, \delta\theta_1, \delta\theta_2, \delta\theta_3$ equal to zero as follow:

$$\delta U \; : \; -\frac{1}{2}\frac{\beta\gamma}{1+\upsilon}\frac{d^2}{d\zeta^2}U - \frac{(1-\upsilon)\alpha}{(1-2\upsilon)(1+\upsilon)}\frac{d^2}{d\zeta^2}U - \frac{1}{2}\frac{\alpha}{1+\upsilon}\frac{d^2}{d\eta d\zeta}V - \frac{\upsilon\alpha}{(1-2\upsilon)(1+\upsilon)}\frac{d^2}{d\eta d\zeta}V$$

$$+\alpha\frac{d^2}{d\tau^2}U - \phi\alpha\frac{d^4}{d\tau^2 d\zeta^2}U - \psi\alpha\frac{d^4}{d\tau^2 d\eta^2}U + \frac{1}{2}G_{31}\alpha^2\frac{d^2}{d\tau \mathrm{d}\zeta}W = 0 \tag{16}$$

$$\delta V: -\frac{1}{2}\frac{\beta}{1+\upsilon}\frac{d^2}{d\eta d\zeta}U - \frac{\upsilon\beta}{(1-2\upsilon)(1+\upsilon)}\frac{d^2}{d\eta d\zeta}U - \frac{1}{2}\frac{\alpha}{(1+\upsilon)\gamma}\frac{d^2}{d\zeta^2}V - \frac{(1-\upsilon)\beta}{(1-2\upsilon)(1+\upsilon)}\frac{d^2}{d\eta^2}V$$

$$+\frac{\alpha}{\gamma}\frac{d^2}{d\tau^2}V - \frac{\phi\alpha}{\gamma}\frac{d^4}{d\tau^2 d\zeta^2}V - \frac{\psi\alpha}{\gamma}\frac{d^4}{d\tau^2 d\eta^2}V + \frac{1}{2}G_{32}\alpha\beta\frac{d^2}{d\tau d\eta}W = 0 \qquad (17)$$

$$\frac{2s_3\upsilon\alpha^2\beta^2}{(1-2\upsilon)(1+\upsilon)}\frac{d^4}{d\zeta^2 d\eta^2}W + \frac{2s_3\alpha^2\beta^2}{1+\upsilon}\frac{d^4}{d\zeta^2 d\eta^2}W + \frac{s_3(1-\upsilon)\beta^4}{(1-2\upsilon)(1+\upsilon)}\frac{d^4}{d\eta^4}W$$

$$+\frac{s_3(1-\upsilon)\alpha^4}{(1-2\upsilon)(1+\upsilon)}\frac{d^4}{d\zeta^4}W - \frac{s_6(1-\upsilon)\alpha^3}{(1-2\upsilon)(1+\upsilon)}\frac{d^3}{d\zeta^3}\Theta_1 + \frac{1}{5}\frac{s_3(1-\upsilon)\alpha^3}{(1-2\upsilon)(1+\upsilon)}\frac{d^3}{d\zeta^3}\Theta_1$$

$$+\frac{1}{5}\frac{s_3\upsilon\alpha\beta^2}{(1-2\upsilon)(1+\upsilon)}\frac{d^3}{d\zeta d\eta^2}\Theta_1 - \frac{s_6\upsilon\beta^2\alpha}{(1-2\upsilon)(1+\upsilon)}\frac{d^3}{d\zeta d\eta^2}\Theta_1 - \frac{s_6\beta^2\alpha}{1+\upsilon}\frac{d^3}{d\zeta d\eta^2}\Theta_1$$

$$+\frac{1}{5}\frac{s_3\alpha\beta^2}{1+\upsilon}\frac{d^3}{d\zeta d\eta^2}\Theta_1 - \frac{s_6\upsilon\alpha^2\beta}{(1-2\upsilon)(1+\upsilon)}\frac{d^3}{d\zeta^2 d\eta}\Theta_2 + \frac{1}{5}\frac{s_3\upsilon\alpha^2\beta}{(1-2\upsilon)(1+\upsilon)}\frac{d^3}{d\zeta^2 d\eta}\Theta_2$$

$$-\frac{s_6\alpha^2\beta}{1+\upsilon}\frac{d^3}{d\zeta^2 d\eta}\Theta_2 + \frac{1}{5}\frac{s_3\beta\alpha^2}{1+\upsilon}\frac{d^3}{d\zeta^2 d\eta}\Theta_2 + \frac{1}{5}\frac{s_3(1-\upsilon)\beta^3}{(1-2\upsilon)(1+\upsilon)}\frac{d^3}{d\eta^3}\Theta_2$$

$$-\frac{s_6(1-\upsilon)\beta^3}{(1-2\upsilon)(1+\upsilon)}\frac{d^3}{d\eta^3}\Theta_2 - \frac{2}{25}\frac{s_7\upsilon\gamma\beta}{(1-2\upsilon)(1+\upsilon)}\frac{d^2}{d\eta^2}\Theta_3 + \frac{2}{5}\frac{s_3\upsilon\alpha\beta^2}{(1-2\upsilon)(1+\upsilon)}\frac{d^4}{d\zeta^2 d\eta^2}\Theta_3$$

$$+\frac{1}{5}\frac{s_3(1-\upsilon)\beta^3\gamma}{(1-2\upsilon)(1+\upsilon)}\frac{d^4}{d\eta^4}\Theta_3 - \frac{2}{25}\frac{s_7\upsilon\alpha}{(1-2\upsilon)(1+\upsilon)}\frac{d^2}{d\zeta^2}\Theta_3 + \frac{1}{5}\frac{s_3(1-\upsilon)\alpha^3}{(1-2\upsilon)(1+\upsilon)}\frac{d^4}{d\zeta^4}\Theta_3$$

$$+\frac{2}{5}\frac{s_3\alpha\beta^2}{1+\upsilon}\frac{d^4}{d\zeta^2 d\eta^2}\Theta_3 + \frac{1}{2}G_{31}\alpha\frac{d^2}{d\zeta d\tau}U + \frac{1}{2}G_{32}\alpha\frac{d^2}{d\eta d\tau}V$$

$$+\alpha^2\frac{d^2}{d\tau^2}W - \psi\alpha^2\frac{d^4}{d\eta^2 d\tau^2}W - s_3\alpha^4\frac{d^4}{d\zeta^2 d\tau^2}W + \phi s_3\alpha^2\beta^2\frac{d^6}{d\zeta^2 d\eta^2 d\tau^2}W \qquad (18)$$

$$\delta W: +\psi s_3\alpha^2\beta^2\frac{d^6}{d\eta^4 d\tau^2}W - \phi\alpha^2\frac{d^4}{d\zeta^2 d\tau^2}W + \phi s_3\alpha^4\frac{d^6}{d\zeta^4 d\tau^2}W + \psi s_3\alpha^4\frac{d^6}{d\zeta^2 d\eta^2 d\tau^2}W$$

$$-s_3\alpha^2\beta^2\frac{d^4}{d\eta^2 d\tau^2}W + s_6\alpha^3\frac{d^3}{d\zeta d\tau^2}\Theta_1 - \frac{1}{5}s_3\alpha^3\frac{d^3}{d\zeta d\tau^2}\Theta_1 - \phi s_6\alpha^3\frac{d^5}{d\zeta^3 d\tau^2}\Theta_1$$

$$-\psi s_6\alpha^3\frac{d^5}{d\zeta d\eta^2 d\tau^2}\Theta_1 + \frac{1}{5}\phi s_3\alpha^3\frac{d^5}{d\zeta^3 d\tau^2}\Theta_1 + \frac{1}{5}\psi s_3\alpha^3\frac{d^5}{d\zeta d\eta^2 d\tau^2}\Theta_1$$

$$-\phi s_6\alpha^2\beta\frac{d^5}{d\zeta^2 d\eta d\tau^2}\Theta_2 - \psi s_6\alpha^2\beta\frac{d^5}{d\eta^3 d\tau^2}\Theta_2 + s_6\alpha^2\beta\frac{d^3}{d\eta d\tau^2}\Theta_2$$

$$-\frac{1}{5}s_3\alpha^2\beta\frac{d^3}{d\eta d\tau^2}\Theta_2 + \frac{1}{5}\psi s_3\alpha^2\beta\frac{d^5}{d\eta^3 d\tau^2}\Theta_2 + \frac{1}{5}\phi s_3\alpha^2\beta\frac{d^5}{d\zeta^2 d\eta d\tau^2}\Theta_2$$

$$+\frac{1}{5}\psi s_3\alpha\beta^2\frac{d^6}{d\eta^4 d\tau^2}\Theta_3 + \frac{1}{5}\phi s_3\alpha\beta^2\frac{d^6}{d\zeta^2 d\eta^2 d\tau^2}\Theta_3 - \frac{1}{5}s_3\alpha^3\frac{d^4}{d\zeta^2 d\tau^2}\Theta_3$$

$$+\frac{1}{5}s_2\alpha\frac{d^2}{d\tau^2}\Theta_3 - \frac{1}{5}\phi s_2\alpha\frac{d^4}{d\zeta^2 d\tau^2}\Theta_3 + \frac{1}{5}\psi s_3\alpha^3\frac{d^6}{d\zeta^2 d\eta^2 d\tau^2}\Theta_3$$

$$-\frac{1}{5}\psi s_2\alpha\frac{d^4}{d\eta^2 d\tau^2}\Theta_3 + \frac{1}{5}\phi s_3\alpha^3\frac{d^6}{d\zeta^4 d\tau^2}\Theta_3 - \frac{1}{5}s_3\alpha\beta^2\frac{d^4}{d\eta^2 d\tau^2}\Theta_3$$

$$+K_w^*W - \phi K_w^*\frac{d^2}{d\zeta^2}W - \psi K_w^*\frac{d^2}{d\eta^2}W - K_g^*\alpha\frac{d^2}{d\zeta^2}W + \phi K_g^*\alpha\frac{d^4}{d\zeta^4}W + \psi K_g^*\alpha\frac{d^4}{d\eta^2 d\zeta^2}W$$

$$-K_g^*\gamma\beta\frac{d^2}{d\eta^2}W + \phi K_g^*\gamma\beta\frac{d^4}{d\eta^2 d\zeta^2}W + \psi K_g^*\gamma\beta\frac{d^4}{d\eta^4}W + N_x^*\alpha\frac{d^2}{d\zeta^2}W - \phi N_x^*\alpha\frac{d^4}{d\zeta^4}W$$

$$-\psi N_x^*\alpha\frac{d^4}{d\eta^2 d\zeta^2}W + N_y^*\beta\frac{d^2}{d\eta^2}W - \phi N_y^*\beta\frac{d^4}{d\eta^2 d\zeta^2}W - \psi N_y^*\beta\frac{d^4}{d\eta^4}W = 0$$

$$-\frac{1}{5}\frac{(1-\upsilon)s_3\alpha^3}{(1-2\upsilon)(1+\upsilon)}\frac{d^3}{d\zeta^3}W +\frac{s_6(1-\upsilon)\alpha^3}{(1-2\upsilon)(1+\upsilon)}\frac{d^3}{d\zeta^3}W -\frac{1}{5}\frac{s_3\alpha\beta^2}{1+\upsilon}\frac{d^3}{d\zeta d\eta^2}W$$

$$+\frac{s_6\alpha\beta^2}{1+\upsilon}\frac{d^3}{d\zeta d\eta^2}W +\frac{s_6\upsilon\alpha\beta^2}{(1-2\upsilon)(1+\upsilon)}\frac{d^3}{d\zeta d\eta^2}W -\frac{1}{5}\frac{s_3\upsilon\alpha\beta^2}{(1-2\upsilon)(1+\upsilon)}\frac{d^3}{d\zeta d\eta^2}W$$

$$+\frac{1}{5}\frac{s_6\beta^2}{1+\upsilon}\frac{d^2}{d\eta^2}\Theta_1 -\frac{1}{50}\frac{s_3\beta^2}{1+\upsilon}\frac{d^2}{d\eta^2}\Theta_1 -\frac{1}{2}\frac{s_1\beta^2}{1+\upsilon}\frac{d^2}{d\eta^2}\Theta_1 +\frac{1}{50}\frac{s_4}{1+\upsilon}\Theta_1$$

$$-\frac{(1-\upsilon)s_1\alpha^2}{(1-2\upsilon)(1+\upsilon)}\frac{d^2}{d\zeta^2}\Theta_1 -\frac{1}{25}\frac{(1-\upsilon)s_3\alpha^2}{(1-2\upsilon)(1+\upsilon)}\frac{d^2}{d\zeta^2}\Theta_1 +\frac{2}{5}\frac{s_6(1-\upsilon)\alpha^2}{(1-2\upsilon)(1+\upsilon)}\frac{d^2}{d\zeta^2}\Theta_1$$

$$-\frac{1}{2}\frac{s_1}{1+\upsilon}\frac{d^2}{d\zeta d\eta}\Theta_2 -\frac{s_1\upsilon\alpha\beta}{(1-2\upsilon)(1+\upsilon)}\frac{d^2}{d\zeta d\eta}\Theta_2 +\frac{2}{5}\frac{s_6\upsilon\beta\alpha}{(1-2\upsilon)(1+\upsilon)}\frac{d^2}{d\zeta d\eta}\Theta_2$$

$$-\frac{1}{25}\frac{s_3\upsilon\alpha\beta}{(1-2\upsilon)(1+\upsilon)}\frac{d^2}{d\zeta d\eta}\Theta_2 +\frac{1}{5}\frac{s_6\beta\alpha}{1+\upsilon}\frac{d^2}{d\zeta d\eta}\Theta_2 -\frac{1}{50}\frac{s_3\alpha\beta}{1+\upsilon}\frac{d^2}{d\zeta d\eta}\Theta_2$$

$$-\frac{1}{25}\frac{s_3\upsilon\beta^2}{(1-2\upsilon)(1+\upsilon)}\frac{d^3}{d\zeta d\eta^2}\Theta_3 -\frac{1}{50}\frac{s_1}{1+\upsilon}\frac{d}{d\zeta}\Theta_3 +\frac{1}{50}\frac{s_5}{1+\upsilon}\frac{d}{d\zeta}\Theta_3$$

$$+\frac{2}{125}\frac{s_7\upsilon}{(1-2\upsilon)(1+\upsilon)}\frac{d}{d\zeta}\Theta_3 -\frac{2}{25}\frac{s_5\upsilon}{(1-2\upsilon)(1+\upsilon)}\frac{d}{d\zeta}\Theta_3 -\frac{1}{25}\frac{(1-\upsilon)s_3\alpha^2}{(1-2\upsilon)(1+\upsilon)}\frac{d^3}{d\zeta^3}\Theta_3$$

$$\delta\theta_1:\ +\frac{1}{5}\frac{s_6\upsilon\beta^2}{(1-2\upsilon)(1+\upsilon)}\frac{d^3}{d\zeta d\eta^2}\Theta_3 +\frac{1}{5}\frac{s_6(1-\upsilon)\alpha^2}{(1-2\upsilon)(1+\upsilon)}\frac{d^3}{d\zeta^3}\Theta_3 -\frac{1}{25}\frac{s_3\alpha^2}{1+\upsilon}\frac{d^3}{d\zeta d\eta^2}\Theta_3 \qquad (19)$$

$$+\frac{1}{5}\frac{s_6\beta^2}{1+\upsilon}\frac{d^3}{d\zeta d\eta^2}\Theta_3 +\phi s_6\alpha^3\frac{d^5}{d\zeta^3 d\tau^2}W +\psi s_6\alpha^3\frac{d^5}{d\zeta d\eta^2 d\tau^2}W -\frac{1}{5}\phi s_3\alpha^3\frac{d^5}{d\zeta^3 d\tau^2}W$$

$$-\frac{1}{5}\psi s_3\alpha^3\frac{d^5}{d\zeta d\eta^2 d\tau^2}W -s_6\alpha^3\frac{d^3}{d\zeta d\tau^2}W +\frac{1}{5}s_3\alpha^3\frac{d^3}{d\zeta d\tau^2}W +\frac{2}{5}\psi s_6\alpha^2\frac{d^4}{d\eta^2 d\tau^2}\Theta_1$$

$$+\frac{2}{5}\phi s_6\alpha^2\frac{d^4}{d\zeta^2 d\tau^2}\Theta_1 -\frac{1}{25}\psi s_3\alpha^2\frac{d^4}{d\eta^2 d\tau^2}\Theta_1 -\frac{1}{25}\phi s_3\alpha^2\frac{d^4}{d\zeta^2 d\tau^2}\Theta_1 -\phi s_1\alpha^2\frac{d^4}{d\zeta^2 d\tau^2}\Theta_1$$

$$-\psi s_1\alpha^2\frac{d^4}{d\eta^2 d\tau^2}\Theta_1 +\frac{1}{25}s_3\alpha^2\frac{d^2}{d\tau^2}\Theta_1 +s_1\alpha^2\frac{d^2}{d\tau^2}\Theta_1 -\frac{2}{5}s_6\alpha^2\frac{d^2}{d\tau^2}\Theta_1$$

$$+\frac{1}{5}\psi s_6\alpha^2\frac{d^5}{d\zeta d\eta^2 d\tau^2}\Theta_3 +\frac{1}{5}\phi s_6\alpha^2\frac{d^5}{d\zeta^3 d\tau^2}\Theta_3 -\frac{1}{25}\phi s_3\alpha^2\frac{d^5}{d\zeta^3 d\tau^2}\Theta_3$$

$$-\frac{1}{25}\psi s_3\alpha^2\frac{d^5}{d\zeta d\eta^2 d\tau^2}\Theta_3 +\frac{1}{25}s_3\alpha^2\frac{d^3}{d\zeta d\tau^2}\Theta_3 -\frac{1}{5}s_6\alpha^2\frac{d^3}{d\zeta d\tau^2}\Theta_3 =0$$

$$-\frac{1}{5}\frac{(1-\upsilon)s_3\beta^3}{(1-2\upsilon)(1+\upsilon)}\frac{d^3}{d\eta^3}W +\frac{s_6(1-\upsilon)\beta^3}{(1-2\upsilon)(1+\upsilon)}\frac{d^3}{d\eta^3}W +\frac{s_6\beta\alpha^2}{1+\upsilon}\frac{d^3}{d\zeta^2 d\eta}W$$

$$-\frac{1}{5}\frac{s_3\beta\alpha^2}{1+\upsilon}\frac{d^3}{d\zeta^2 d\eta}W +\frac{s_6\upsilon\beta\alpha^2}{(1-2\upsilon)(1+\upsilon)}\frac{d^3}{d\zeta^2 d\eta}W -\frac{1}{5}\frac{s_3\upsilon\beta\alpha^2}{(1-2\upsilon)(1+\upsilon)}\frac{d^3}{d\zeta^2 d\eta}W$$

$$+\frac{2}{5}\frac{s_6\upsilon\beta\alpha}{(1-2\upsilon)(1+\upsilon)}\frac{d^2}{d\zeta d\eta}\Theta_1 -\frac{s_1\upsilon\alpha\beta}{(1-2\upsilon)(1+\upsilon)}\frac{d^2}{d\zeta d\eta}\Theta_1 -\frac{1}{50}\frac{s_3\alpha\beta}{1+\upsilon}\frac{d^2}{d\zeta d\eta}\Theta_1$$

$$\delta\theta_2:\ +\frac{1}{5}\frac{s_6\beta\alpha}{1+\upsilon}\frac{d^2}{d\zeta d\eta}\Theta_1 -\frac{1}{2}\frac{s_1\alpha\beta}{1+\upsilon}\frac{d^2}{d\zeta d\eta}\Theta_1 -\frac{1}{25}\frac{s_3\upsilon\alpha\beta}{(1-2\upsilon)(1+\upsilon)}\frac{d^2}{d\zeta d\eta}\Theta_1 +\frac{1}{50}\frac{s_4}{1+\upsilon}\Theta_2 \qquad (20)$$

$$-\frac{1}{2}\frac{s_1\alpha^2}{1+\upsilon}\frac{d^2}{d\zeta^2}\Theta_2 -\frac{1}{50}\frac{s_3\alpha^2}{1+\upsilon}\frac{d^2}{d\zeta^2}\Theta_2 -\frac{(1-\upsilon)s1\beta^2}{(1-2\upsilon)(1+\upsilon)}\frac{d^2}{d\eta^2}\Theta_2 +\frac{1}{5}\frac{s_6\alpha^2}{1+\upsilon}\frac{d^2}{d\zeta^2}\Theta_2$$

$$-\frac{1}{25}\frac{(1-\upsilon)s_3\beta^2}{(1-2\upsilon)(1+\upsilon)}\frac{d^2}{d\eta^2}\Theta_2 +\frac{2}{5}\frac{s_6(1-\upsilon)\beta^2}{(1-2\upsilon)(1+\upsilon)}\frac{d^2}{d\eta^2}\Theta_2 -\frac{1}{25}\frac{s_3\upsilon\alpha\beta}{(1-2\upsilon)(1+\upsilon)}\frac{d^3}{d\zeta^2 d\eta}\Theta_3$$

$$+\frac{1}{5}\frac{s_6(1-\upsilon)\beta^2\gamma}{(1-2\upsilon)(1+\upsilon)}\frac{d^3}{d\eta^3}\Theta_3 -\frac{1}{25}\frac{s_3\alpha\beta}{1+\upsilon}\frac{d^3}{d\zeta^2 d\eta}\Theta_3 +\frac{1}{5}\frac{s_6\upsilon\beta\alpha}{(1-2\upsilon)(1+\upsilon)}\frac{d^3}{d\zeta^2 d\eta}\Theta_3$$

$$-\frac{1}{25}\frac{(1-\upsilon)s_3\beta^2\gamma}{(1-2\upsilon)(1+\upsilon)}\frac{d^3}{d\eta^3}\Theta_3+\frac{1}{5}\frac{s_6\beta\alpha}{1+\upsilon}\frac{d^3}{d\zeta^2 d\eta}\Theta_3-\frac{1}{50}\frac{s_1\gamma}{1+\upsilon}\frac{d}{d\eta}\Theta_3+\frac{1}{50}\frac{s_5\gamma}{1+\upsilon}\frac{d}{d\eta}\Theta_3$$

$$-\frac{2}{25}\frac{s_5\upsilon\gamma}{(1-2\upsilon)(1+\upsilon)}\frac{d}{d\eta}\Theta_3+\frac{2}{125}\frac{s_7\upsilon\gamma}{(1-2\upsilon)(1+\upsilon)}\frac{d}{d\eta}\Theta_3-s_6\alpha^2\beta\frac{d^3}{d\eta d\tau^2}W$$

$$-\frac{1}{5}\phi s_3\alpha^2\beta\frac{d^5}{d\zeta^2 d\eta d\tau^2}W-\frac{1}{5}\psi s_3\alpha^2\beta\frac{d^5}{d\eta^3 d\tau^2}W+\psi s_6\alpha^2\beta\frac{d^5}{d\eta^3 d\tau^2}W$$

$$+\phi s_6\alpha^2\beta\frac{d^5}{d\zeta^2 d\eta d\tau^2}W+\frac{1}{5}s_3\alpha^2\beta\frac{d^3}{d\eta d\tau^2}W+\frac{2}{5}\phi s_6\alpha^2\frac{d^4}{d\zeta^2 d\tau^2}\Theta_2$$

$$\delta\theta_2:\ +\frac{2}{5}\psi s_6\alpha^2\frac{d^4}{d\eta^2 d\tau^2}\Theta_2-\frac{1}{25}\phi s_3\alpha^2\frac{d^4}{d\zeta^2 d\tau^2}\Theta_2-\frac{1}{25}\psi s_3\alpha^2\frac{d^4}{d\eta^2 d\tau^2}\Theta_2 \tag{20}$$

$$-\psi s_1\alpha^2\frac{d^4}{d\eta^2 d\tau^2}\Theta_2-\phi s_1\alpha^2\frac{d^4}{d\zeta^2 d\tau^2}\Theta_2+s_1\alpha^2\frac{d^2}{d\tau^2}\Theta_2-\frac{2}{5}s_6\alpha^2\frac{d^2}{d\tau^2}\Theta_2$$

$$+\frac{1}{25}s_3\alpha^2\frac{d^2}{d\tau^2}\Theta_2-\frac{1}{5}s_6\alpha\beta\frac{d^3}{d\eta d\tau^2}\Theta_3+\frac{1}{5}\phi s_6\alpha\beta\frac{d^5}{d\zeta^2 d\eta d\tau^2}\Theta_3$$

$$+\frac{1}{25}s_3\beta\alpha\frac{d^3}{d\eta d\tau^2}\Theta_3-\frac{1}{25}\phi s_3\beta\alpha\frac{d^5}{d\zeta^2 d\eta d\tau^2}\Theta_3-\frac{1}{25}\psi s_3\beta\alpha\frac{d^5}{d\eta^3 d\tau^2}\Theta_3$$

$$+\frac{1}{5}\psi s_6\alpha\beta\frac{d^5}{d\eta^3 d\tau^2}\Theta_3=0$$

$$\frac{2}{5}\frac{s_3\upsilon\beta^2\alpha}{(1-2\upsilon)(1+\upsilon)}\frac{d^4}{d\zeta^2 d\eta^2}W-\frac{2}{25}\frac{\upsilon s_7\beta\gamma}{(1-2\upsilon)(1+\upsilon)}\frac{d^2}{d\eta^2}W+\frac{1}{5}\frac{(1-\upsilon)s_3\beta^3\gamma}{(1-2\upsilon)(1+\upsilon)}\frac{d^4}{d\eta^4}W$$

$$+\frac{2}{5}\frac{s_3\beta^2\alpha}{1+\upsilon}\frac{d^4}{d\zeta^2 d\eta^2}W+\frac{1}{5}\frac{(1-\upsilon)s_3\alpha^3}{(1-2\upsilon)(1+\upsilon)}\frac{d^4}{d\zeta^4}W-\frac{2}{25}\frac{\upsilon s_7\alpha}{(1-2\upsilon)(1+\upsilon)}\frac{d^2}{d\zeta^2}W$$

$$+\frac{1}{25}\frac{s_3\beta^2}{1+\upsilon}\frac{d^3}{d\zeta d\eta^2}\Theta_1-\frac{2}{125}\frac{\upsilon s_7}{(1-2\upsilon)(1+\upsilon)}\frac{d}{d\zeta}\Theta_1+\frac{1}{25}\frac{s_3\upsilon\beta^2}{(1-2\upsilon)(1+\upsilon)}\frac{d^3}{d\zeta d\eta^2}\Theta_1$$

$$+\frac{1}{25}\frac{(1-\upsilon)s_3\alpha^2}{(1-2\upsilon)(1+\upsilon)}\frac{d^3}{d\zeta^3}\Theta_1-\frac{1}{5}\frac{s_6\upsilon\beta^2}{(1-2\upsilon)(1+\upsilon)}\frac{d^3}{d\zeta d\eta^2}\Theta_1$$

$$-\frac{1}{5}\frac{s_6\beta^2}{1+\upsilon}\frac{d^3}{d\zeta d\eta^2}\Theta_1+\frac{2}{25}\frac{\upsilon s_5}{(1-2\upsilon)(1+\upsilon)}\frac{d}{d\zeta}\Theta_1+\frac{1}{25}\frac{s_3\upsilon\beta\alpha}{(1-2\upsilon)(1+\upsilon)}\frac{d^3}{d\zeta^2 d\eta}\Theta_2$$

$$+\frac{1}{25}\frac{s_3\alpha\beta}{1+\upsilon}\frac{d^3}{d\zeta^2 d\eta}\Theta_2-\frac{1}{50}\frac{s_5\gamma}{1+\upsilon}\frac{d}{d\eta}\Theta_2+\frac{1}{50}\frac{s_1\gamma}{1+\upsilon}\frac{d}{d\eta}\Theta_2$$

$$\delta\theta_3:\ +\frac{1}{25}\frac{(1-\upsilon)s_3\beta^2\gamma}{(1-2\upsilon)(1+\upsilon)}\frac{d^3}{d\eta^3}\Theta_2-\frac{1}{5}\frac{s_6\upsilon\alpha\beta}{(1-2\upsilon)(1+\upsilon)}\frac{d^3}{d\zeta^2 d\eta}\Theta_2-\frac{1}{5}\frac{s_6\alpha\beta}{1+\upsilon}\frac{d^3}{d\zeta^2 d\eta}\Theta_2 \tag{21}$$

$$-\frac{1}{5}\frac{(1-\upsilon)s_6\beta^2\gamma}{(1-2\upsilon)(1+\upsilon)}\frac{d^3}{d\eta^3}\Theta_2+\frac{1}{25}\frac{(1-\upsilon)s_3\beta^2\gamma^2}{(1-2\upsilon)(1+\upsilon)}\frac{d^4}{d\eta^4}\Theta_3-\frac{1}{50}\frac{1}{1+\upsilon}\frac{d^2}{d\zeta^2}\Theta_3$$

$$-\frac{4}{125}\frac{\upsilon s_7\gamma^2}{(1-2\upsilon)(1+\upsilon)}\frac{d^2}{d\eta^2}\Theta_3+\frac{1}{25}\frac{s_2}{1+\upsilon}\frac{d^2}{d\zeta^2}\Theta_3-\frac{1}{50}\frac{s_9}{1+\upsilon}\frac{d^2}{d\zeta^2}\Theta_3$$

$$-\frac{1}{50}\frac{\gamma^2}{1+\upsilon}\frac{d^2}{d\eta^2}\Theta_3+\frac{2}{25}\frac{s_3\beta^2}{1+\upsilon}\frac{d^4}{d\zeta^2 d\eta^2}\Theta_3-\frac{1}{50}\frac{s_9\gamma^2}{1+\upsilon}\frac{d^2}{d\eta^2}\Theta_3$$

$$+\frac{1}{5}\psi s_3\alpha^3\frac{d^6}{d\zeta^2 d\eta^2 d\tau^2}W-\frac{1}{5}s_3\alpha\beta^2\frac{d^4}{d\eta^2 d\tau^2}W+\frac{1}{5}\psi s_3\alpha\beta^2\frac{d^6}{d\eta^4 d\tau^2}W$$

$$+\frac{1}{5}\phi s_3\alpha\beta^2\frac{d^6}{d\zeta^2 d\eta^2 d\tau^2}W-\frac{1}{5}s_3\alpha^3\frac{d^4}{d\zeta^2 d\tau^2}W+\frac{1}{5}s_2\alpha\frac{d^2}{d\tau^2}W+\frac{1}{5}\phi s_3\alpha^3\frac{d^6}{d\zeta^4 d\tau^2}W$$

$$-\frac{1}{5}\phi s_2\alpha\frac{d^4}{d\zeta^2 d\tau^2}W-\frac{1}{5}\psi s_2\alpha\frac{d^4}{d\eta^2 d\tau^2}W+\frac{1}{25}\phi s_3\alpha^2\frac{d^5}{d\zeta^3 d\tau^2}\Theta_1$$

$$+\frac{1}{25}\psi s_3\alpha^2\frac{d^5}{d\zeta d\eta^2 d\tau^2}\Theta_1-\frac{1}{5}\phi s_6\alpha^2\frac{d^5}{d\zeta^3 d\tau^2}\Theta_1-\frac{1}{5}\psi s_6\alpha^2\frac{d^5}{d\zeta d\eta^2 d\tau^2}\Theta_1$$

$$+\frac{1}{5}s_6\alpha^2\frac{d^3}{d\zeta d\tau^2}\Theta_1-\frac{1}{25}s_3\alpha^2\frac{d^3}{d\zeta d\tau^2}\Theta_1+\frac{1}{25}\phi s_3\beta\alpha\frac{d^5}{d\zeta^2 d\eta d\tau^2}\Theta_2$$

$$+\frac{1}{5}s_6\alpha\beta\frac{d^3}{d\eta d\tau^2}\Theta_2-\frac{1}{25}s_3\beta\alpha\frac{d^3}{d\eta d\tau^2}\Theta_2-\frac{1}{5}\psi s_6\alpha\beta\frac{d^5}{d\eta^3 d\tau^2}\Theta_2$$

$$\delta\theta_3:\quad -\frac{1}{5}\phi s_6\alpha\beta\frac{d^5}{d\zeta^2 d\eta d\tau^2}\Theta_2+\frac{1}{25}\psi s_3\beta\alpha\frac{d^5}{d\eta^3 d\tau^2}\Theta_2-\frac{1}{25}\phi s_9\frac{d^4}{d\zeta^2 d\tau^2}\Theta_3$$

$$-\frac{1}{25}\psi s_9\frac{d^4}{d\eta^2 d\tau^2}\Theta_3-\frac{1}{25}s_3\beta^2\frac{d^4}{d\eta^2 d\tau^2}\Theta_3+\frac{1}{25}s_9\frac{d^2}{d\tau^2}\Theta_3$$

$$-\frac{1}{25}s_3\alpha^2\frac{d^4}{d\zeta^2 d\tau^2}\Theta_3+\frac{1}{25}\phi s_3\alpha^2\frac{d^6}{d\zeta^4 d\tau^2}\Theta_3+\frac{1}{25}\psi s_3\beta^2\frac{d^6}{d\eta^4 d\tau^2}\Theta_3$$

$$+\frac{1}{25}\psi s_3\alpha^2\frac{d^6}{d\zeta^2 d\eta^2 d\tau^2}\Theta_3+\frac{1}{25}\frac{s_2\gamma^2}{1+\upsilon}\frac{d^2}{d\eta^2}\Theta_3+\frac{1}{25}\phi s_3\beta^2\frac{d^6}{d\zeta^2 d\eta^2 d\tau^2}\Theta_3=0$$

(21)

In which:

$$s_i=\int_{-\frac{h}{2}}^{\frac{h}{2}}\left(\frac{\tan^2(mz)}{h},\frac{\sec^2(mz)}{h},\frac{z^2}{h^3},\frac{\tan^4(mz)}{h},\frac{\tan^2(mz).\sec^2(mz)}{h},\right.$$

$$\left.\frac{z.\tan(mz)}{h^2},\frac{z.\tan(mz).\sec^2(mz)}{h^2},\frac{\tan^2(mz).\sec^4(mz)}{h},\frac{\sec^4(mz)}{h}\right)dz\qquad(i=1,...,9)$$

(22)

4 SOLUTION PROCEDURE

There is no doubt that sophisticated coupled motion equations alongside with general boundary conditions are difficultly analysed by exact method, while theses equations can be numerically solved. One of the numerical methods is DQM with high rapid solution for linear and nonlinear partial differential equations in which its results is very close to exact solution. Before utilizing DQ method, the Navier solution is used considering following form:

$$U(\zeta,\eta,\tau)=U(\zeta)\sin(n\pi\eta)\,e^{\omega\tau}$$
$$V(\zeta,\eta,\tau)=V(\zeta)\cos(n\pi\eta)e^{\omega\tau}$$
$$W(\zeta,\eta,\tau)=W(\zeta)\sin(n\pi\eta)e^{\omega\tau}$$
$$\Theta_1(\zeta,\eta,\tau)=\Theta_1(\zeta)\sin(n\pi\eta)e^{\omega\tau}$$
$$\Theta_2(\zeta,\eta,\tau)=\Theta_2(\zeta)\cos(n\pi\eta)e^{\omega\tau}$$
$$\Theta_3(\zeta,\eta,\tau)=\Theta_3(\zeta)\sin(n\pi\eta)e^{\omega\tau}$$

(23)

In which ω and n are the dimensionless frequency and integer number which introduced as wave numbers. Eq. (23) is used when two edges of plate are simply supported. In DQ method F is a function representing, u,v,w,θ_1,θ_2 and θ_3 with respect to variables ξ in the domain of $(0<\xi<L)$ [28]:

$$\frac{\partial^k F}{\partial\zeta^k}=\sum_{k=1}^{N}A_{pq}^{(K)}\,F(\zeta_i),$$

(24)

where $A_{pq}^{(k)}$ is the weighting coefficients associated with k^{th}-order partial derivative of F, and N is the number of grid points in longitudinal direction where Chebyshev polynomials [28] are selected for positions of the grid points. Substituting Eq.(24) into motion equations, the standard form of vibrational motion equation set

$(M\ddot{X} + C\dot{X} + KX = 0)$ are obtained and by considering boundary condition an eigenvalue problem is derived in which the eigen-values of state-space matrix $\left([\text{state} - \text{space}] = \begin{bmatrix} [0] & [I] \\ -[MK] & -[MC] \end{bmatrix} \right)$ are the dimensionless frequency. It is worth to mention that M is the mass matrix, C is the damping matrix and K is the stiffness, $[I]$ and $[0]$ are the unitary and zero matrixes.

5   NUMERICAL RESULTS AND DISCUSSION

In this study, a new trigonometric higher order shear deformation theory are used to derive the motion equation of embedded MsNP using nanlocal continuum theory. MsNP subjected to uniform magnetic field and undergoes in-plane forces. The results of this study that included the effect of stimulus factors such as small scale parameter, velocity feedback gain, elastic medium and … report in this section along with corresponding figures. The plate has been made of Terfnol-D that listed its properties at following Table:

Table 1
Elastic properties of Terfenol-D [18].

Properties	E	υ	ρ_m	$e_{31} = e_{32}$
Terfenol-D	$30e9 \, Pa$	0.25	$9.25 \times 10^3 \, kg \, / \, m^3$	$442.55 N \, / \, (m.A)$

Fig. 2 shows the variation of dimensionless frequency versus nonlocal parameter in different thickness ratios. α introduces the ratio of thickness to length of MsNP where changes from 0.1 to 0.3 for thick plates. The figure has been plotted for constant values of elastic medium and velocity feedback gain. As can be seen from the figure increasing the nonlocal parameter lead to significant decreases in natural frequency of MsNP. It is asuumed that the MsNP becomes softer with considering nonlocal parameter.
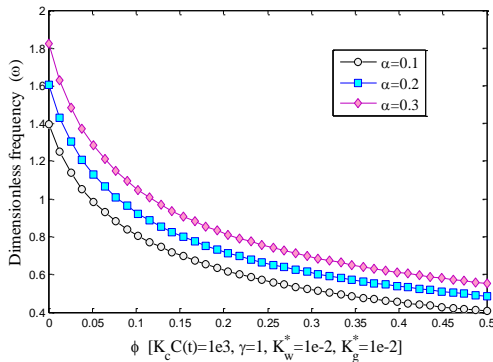

Fig.2
Variation of dimensionless frequency versus nanlocal parameters in different thickness ratios.

Fig. 3 shows the variation of dimensionless frequency versus nonlocal parameter in different aspect ratios. Aspect ratio (γ) introduces the length to width ratio of MsNP where changes from 0.5 to 2 for thick plate with $\alpha = 0.2$. Like Fig. 3 natural frequency of MsNP decreases when the nonlocal parameter increases. Then, it can be said that in the same values of dimensionless parameters, the natural frequency of local system is greater than its non-local where the values on the horizontal axes in $\phi = 0$ show the local frequencies.
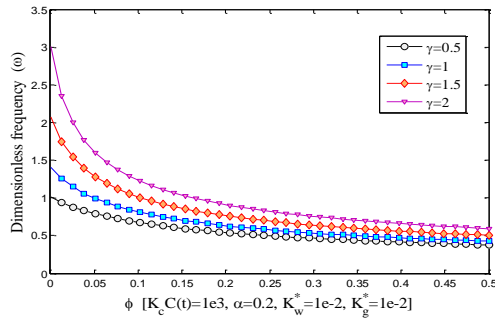
Fig.3
Variation of dimensionless frequency versus nanlocal parameters in different aspect ratios.

Fig. 4 shows the variation of dimensionless frequency versus nonlocal parameter in different boundary conditions. The results have been reported for special conditions of MsNP in which $K_w^* = 0.01$, $K_g^* = 0.01$, $\alpha = 0.2$, $\gamma = 1$. Three boundary conditions of CSCS, SCSS and SSSS have been compared in Fig.4 where the value of natural frequency for CSCS is larger than the others. One of the important results that can be concluded from Figs.2 to 4 is the role of small scale parameter in Eringen's theory on instability of system. In fact, nan-local parameter with decreasing natural frequency helps the system to instability.
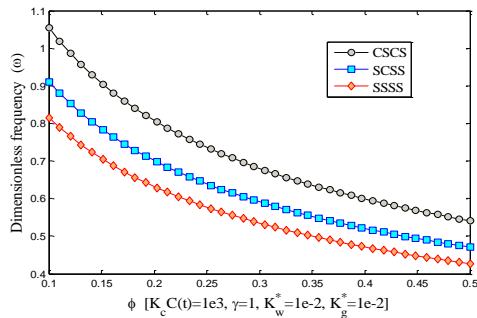

Fig.4
Variation of dimensionless frequency versus nanlocal parameters in different boundary conditions for two theories.

Fig. 5 shows the variation of dimensionless frequency versus nonlocal parameter in different elastic medium. The lowest curves ($K_g^* = 0, K_w^* = 0$) introduces the case of without elastic medium where the natural frequency has the minimum value to other cases. The second curve is belong to the Winkler foundation with $K_w^* = 0.01$, this value introduces the spring constant of elastic medium. Pasternak foundation includes two normal and shear modules and is more effective than Winkler type. The role of elastic medium in stability of system is observed in Fig.5 where increasing of Pasternak modules creates the larger values for natural frequency of MsNP. It is worth to mention that increasing elastic medium parameters follow this trend upward.
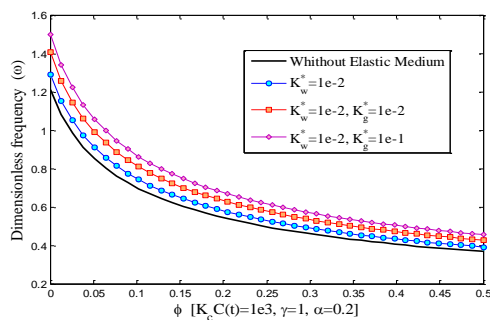

Fig.5
Variation of dimensionless frequency versus nanlocal parameters in different elastic medium.

Effect of velocity feedback gain is especially studied in Figs.6 in the presence of in-plane forces. The result shows that in-plane forces change effectively the vibration response of embedded MsNP in $\gamma = 1$ *and* $\alpha = 0.2$. Since the in-plane forces are vector quantity, the positive value indicates compression force and negative value shows the

extensional or tension force where in-plane compression forces decrease the dimensionless frequency and cause the instability of system, but tension force increases the frequency and system stability. Fig. 6 (a) displays the effect of unidirectional in-plane force on vibration response of MsNP. Increasing of tension force leads to increase of natural frequency while compression force has contrary effect. Fig. 6b depicts that both cases $(N_x^* = 0, N_y^* = 0)$ and $(N_x^* = -1e-2, N_y^* = 1e-2)$ report the same results. Also, both cases $N_x^* \succ 0, N_y^* \succ 0$ and $N_x^* \prec 0, N_y^* \prec 0$ introduce the pure compression and pure tension, respectively.
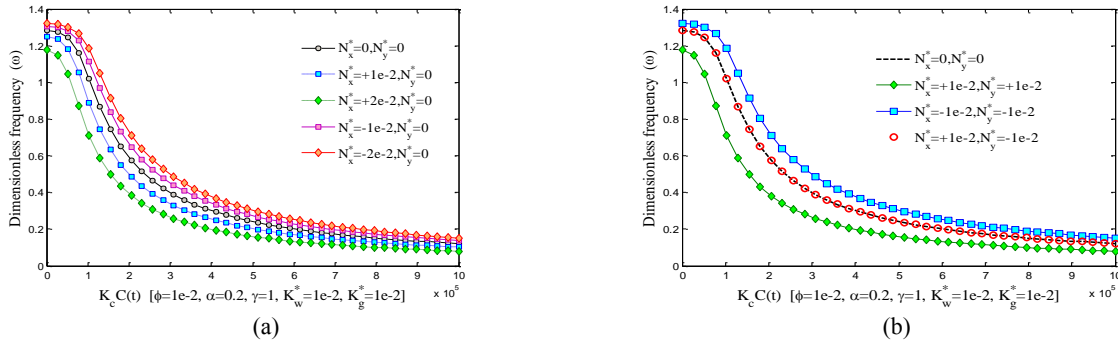

Fig.6
Variation of dimensionless frequency versus velocity feedback gain in different in-plane forces for two cases (a) and (b).

A 3D plot of variation natural frequency versus velocity feedback gain and Pasternak module have been presented simultaneously in Fig.7 (a). This figure shows the effect of two parameters with contrary turnovers. The velocity feedback gain decreases the natural frequency while elastic medium increases it. In the other hand, Fig. 7 (a) shows that it can be utilized several parameter with different effects to achieve desire value or appropriative state in the systems. It is worth to mention that the MsMs due to reciprocal nature deformed when subjected to magnetic field. It is clear from the Fig.7(a) that the velocity feedback gain $K_c C(t)$ acts as a controller parameter where the natural frequency of MsNP significantly minimizes with increasing in velocity feedback gain.

The same figure has been also plotted for studying the effect of in-plane forces on the natural frequency on MsNP in x and y directions (Fig.8) where applying these forces has the same result in both directions. It can be found the maximum values of frequency at the end of plotted surface and its minimum at the first.
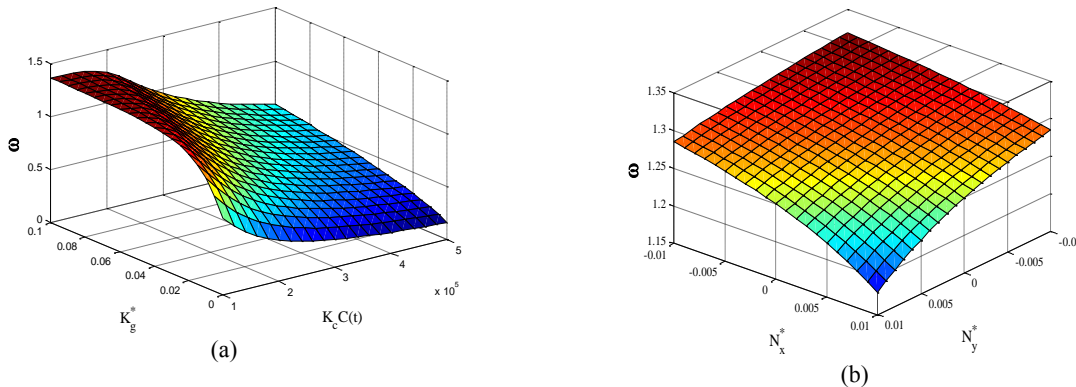

Fig.7
Three dimension plot of dimensionless frequency variation versus:a) velocity feedback gain and Pasternak constant.b) in-plane forces in x and y directions.

Finaly, Fig. 8 compares the results of three plate theories to confirm the results of a new trigonometric higher order shear deformation theory. As can be seen from the Fig. 8 the result of this new theory is very close to first order shear deformation theory by considering shear correction factor and third order shear deformation theory and a little difference is due to the accuracy of this new theory where considers six independent parameters and does not ignored $\sigma_{zz} \neq 0$.
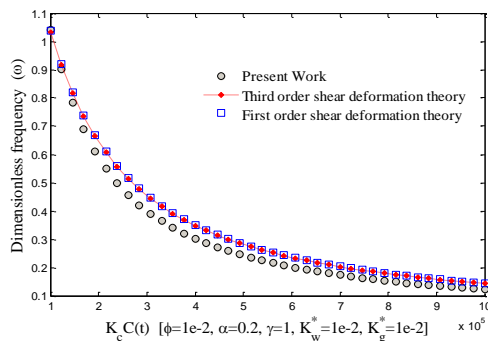
Fig.8
Comparison the results using three theories.

6   CONCLUSIONS

At the first time, a control feedback system is used to investigate the free vibration response of rectangular nanoplate made of MsM. In this regard, a new trigonometric higher order shear deformation theory was utilized in order to enhance the accuracy of results where normal shear stress was considered using six independent parameters. It is worth to mention that nonlocal continuum theory was rewritten for MsNP for the first time in this work while MsNP embedded on Pasternak foundation and the plate simultaneously undergone in-plane forces. The results of this research have been listed as follow:

- Increasing small scale parameter leads to decrease the dimensionless frequency of MsNP.
- Aspect and thickness ratios increases the dimensionless frequency of MsNP, but the frequency rate for increasing aspect ratio is greater than thickness ratio.
- Velocity feedback gain can be controlled by the vibration behavior of MsNP where it minimizes the natural frequency as a controller parameter.
- The dimensionless frequency of MsNP decreased due to in-plane compression forces and increased with applying tension forces.
- Stability of MsNP is significantly affected by elastic medium where Pasternak modulus increased the dimensionless frequency of MsNP.

According to above results, MsNP can be used for smart control vibration of nano-structure especially in sensor and actuators such as wireless linear micro-motor and smart nano-valves in injectors.

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