Journal of Solid Mechanics Vol. 7, No. 3 (2015) pp. 255-267

Rayleigh Waves in a Homogeneous Magneto-Thermo Voigt-Type Viscoelastic Half-Space under Initial Surface Stresses

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Received 26 March 2015; accepted 27 May 2015

ABSTRACT

This paper deals with the propagation of magneto-thermo Rayleigh waves in a homogeneous viscoelastic half-space under initial stress. It has been observed that velocity of Rayleigh waves depends on viscosity, magnetic field, temperature and initial stress of the half-space. The frequency equation for Rayleigh waves under the effect of magnetic field, stress and temperature for both viscoelastic and elastic medium is first obtained by using classical theory of thermoelasticity and then computed numerically. The variation of phase velocity of Rayleigh waves with respect to initial hydrostatic stress in viscoelastic and elastic half-space is shown graphically. In the absence of various parameters of the medium, the obtained results are in agreement with classical results given by Caloi and Lockett.

Keywords : Initial stress; Temperature; Magnetic field; Rayleigh waves; Voigt-type; Viscoelasticity.

1 INTRODUCTION

THE study of the propagation of Rayleigh waves in the presence of earth's magnetic field, temperature and initial stress is of some importance. On seismograms, there are two types of surface waves which are recognized during distant earthquakes. Both of these waves show dispersion and their speed depends on wavelength. The first wave which is transverse in nature is called Love wave and the second group of surface waves are called Rayleigh waves. Although several studies have been made of Rayleigh waves in homogeneous and non-homogeneous media in presence of temperature and stress but less literature is available to show the effect of magnetic field on Rayleigh waves. Many authors have studied propagation of Rayleigh waves in various media. Addy and Chakraborty [5] have studied a problem in which they have shown the effect of temperature and initial stress on Rayleigh waves in a viscoelastic medium but they did not showed the effect of magnetic field. Sethi et al. [6] has considered a homogeneous viscoelastic medium to study surface wave under surface stresses without taking temperature and magnetic field of earth. Singh and Bala [7] have discussed a problem on Rayleigh wave in temperature field using theory of thermoelasticity. Vinh [8] has given a complete solution of Rayleigh waves in elastic media under the effect of gravity and initial stress. Kakar and Kakar [9] and Abd-Alla et al. [10] have investigated magneto-thermo-viscoelastic Rayleigh waves in granular medium under various parameters. In this work, we have investigated the

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Rayleigh waves in a magneto-thermo-viscoelastic solid half-space under initial stress. The effect of magnetic field, thermal field and initial hydrostatic stress on Rayleigh waves in an elastic and viscoelastic solid is examined at different coupling coefficients of temperature and magnetic field. Biot's equations are modified in context of classical dynamical theory of thermoelasticity with uniform magnetic field. The frequency equation is approximated and analyzed numerically to study the phase velocity of Rayleigh waves under the coupled parameters with the help of MATLAB. In the absence of magnetic field, temperature, viscosity and initial stress of the medium, the derived dispersion relation for Rayleigh waves in magneto-thermo-viscoelastic solid half-space under initial stress satisfies classical results given by the researchers Lockett [1] and Caloi [2].

2 FORMULATION OF THE PROBLEM

We consider Rayleigh wave is propagating along the direction of *X*-axis, *Y*-axis is taken vertically downward and $y=0$ is the surface of the Voigt-type viscoelastic half space. The half space is under an initial stress *P* magnetic field $\vec{\mathrm{H}}(0,0,\mathrm{H})=\vec{\mathrm{H}}_0+\vec{h}$, where, $\vec{h}$ is the perturbed magnetic field over $\vec{\mathrm{H}}_0$ and initial temperature T_0. The boundary of the half -space is traction free and it allows heat exchange with the surroundings.

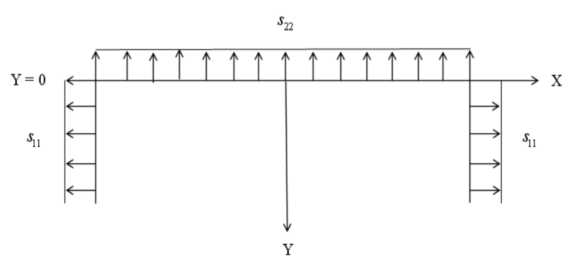

Fig.1
Pre-stressed viscoelastic half-space in presence of magnetic and temperature field.

3 GOVERNING EQUATIONS

We consider linearized equation of electromagnetism, valid for slowly moving media, therefore the governing equations of linear, isotropic and homogenous magneto-thermoelastic solid with initial stress are

$$\vec{\nabla}\times\vec{\mathrm{h}}=\vec{\mathrm{J}}+\varepsilon_e\frac{\partial\vec{\mathrm{E}}}{\partial t} \tag{1}$$

$$\vec{\nabla}\cdot\vec{\mathrm{h}}=0 \tag{2}$$

$$\vec{\nabla}\times\vec{\mathrm{E}}=-\mu_e\frac{\partial\vec{\mathrm{h}}}{\partial t} \tag{3}$$

$$\vec{\mathrm{E}}=-\mu_e\left(\frac{\partial\vec{u}}{\partial t}\times\vec{\mathrm{H}}\right) \tag{4}$$

where, $\vec{\mathrm{E}}$, $\vec{\mathrm{J}}$, μ_e and ε_e are electric field, current density, permeability and permittivity of the medium.

The basic equation for electro-magneto-thermoelastic in a homogeneous isotropic solid in the context of coupled dynamical theory may be taken in a unified form in the absence of body force and heat source

a. The stress-strain-temperature relation:

$$s_{ij} = -P(\delta_{ij} + \omega_{ij}) + \bar{\lambda}\, \mathrm{e}_{PP}\, \delta_{ij} + 2\bar{\mu} e_{ij} - \frac{\alpha}{k_T}(\mathrm{T} + \alpha\dot{\mathrm{T}})\delta_{ij}, \tag{5}$$

where, s_{ij} are the components of stress tensor, P is initial pressure, δ_{ij} is the Kronecker delta, ω_{ij} are the components of small rotation tensor, $\bar{\lambda}$, $\bar{\mu}$ are the counterparts of Lame parameters, e_{ij} are the components of the strain tensor, α is the volume coefficient of thermal expansion, k_T is the isothermal compressibility, $\mathrm{T} = \mathrm{T}_a - \mathrm{T}_0$ is small temperature increment, T_a is the absolute temperature of the medium, T_0 is the reference uniform temperature of the body chosen such that $\left|\frac{\mathrm{T}}{\mathrm{T}_0}\right| \ll 1$.

b. The displacement-strain relation:

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{6}$$

where, u_i are the components of the displacement vector.

c. Maxwell stress components:

$$T_{ij} = \mu_e\left[H_i e_i + H_j e_j - \left(H_k e_k\right)\delta_{ij}\right] \qquad \text{(where } i, j, k = 1, 2, 3\text{)} \tag{7}$$

where, H_i , H_j , H_k are the components of primary magnetic field, e_i , e_j , e_k are the stress components acting along *X*-axis, *Y*-axis, Z-axis respectively and δ_{ij} is the Kronecker delta. With the help of Eq. (7), we have the components of Maxwell stress components

$$T_{22} = \mu_e H_0^2\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \text{ and } T_{12} = 0 \tag{8}$$

The dynamical equations of motion for the propagation of wave have been derived by Biot [3] and in two dimensions. These are given by

$$\frac{\partial s_{11}}{\partial x} + \frac{\partial s_{12}}{\partial y} - P\frac{\partial \omega}{\partial y} + B_x = \rho\frac{\partial^2 u}{\partial t^2} \tag{9}$$

$$\frac{\partial s_{12}}{\partial x} + \frac{\partial s_{22}}{\partial y} - P\frac{\partial \omega}{\partial x} + B_y = \rho\frac{\partial^2 v}{\partial t^2} \tag{10}$$

where, s_{11}, s_{22} and s_{12} are incremental thermal stress components. The first two are principal stress components along x- and y-axes, respectively and the last one is shear stress component in the *X*-*Y* plane, ρ is the density of the medium and $u, v.$ are the displacement components along *X* and *Y* directions respectively, B is body force and its components along *X* and *Y* axis are B_x and B_y respectively . ω is the rotational component i.e.

$$\omega = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \text{ and } P = s_{22} - s_{11} \tag{11}$$

We consider a homogeneous solid incompressible half space under constant primary magnetic field H_0 parallel to *Z*-axis. Therefore, the body forces along *X* and *Y* axis are given by

$$B_x = \mu_e H_0^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} \right) \tag{12}$$

$$B_y = \mu_e H_0^2 \left(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial x^2} \right) \tag{13}$$

where, μ_e is permittivity of the medium.

Following Biot [3], the stress-strain relations with Voigt-type viscoelastic half-space under thermal condition are given by

$$s_{11} = \left[\lambda + 2\mu + \left(\lambda' + 2\mu' \right) \frac{\partial}{\partial t} \right] e_{xx} + \left[\lambda + \lambda' \frac{\partial}{\partial t} \right] e_{yy} - \gamma \mathrm{T} \tag{14}$$

$$s_{22} = \left[\lambda + \lambda' \frac{\partial}{\partial t} \right] e_{xx} + \left[\lambda + 2\mu + \left(\lambda' + 2\mu' \right) \frac{\partial}{\partial t} \right] e_{yy} - \gamma \mathrm{T} \tag{15}$$

$$s_{12} = 2 \left[\mu + \mu' \frac{\partial}{\partial t} \right] e_{xy} \tag{16}$$

where,

$$e_{xx} = \frac{\partial u}{\partial x}, \qquad e_{yy} = \frac{\partial v}{\partial x}, \qquad e_{xy} = \frac{1}{2} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \tag{17}$$

where, e_{xx} and e_{yy} are the principle strain components and e_{xy} is the shear strain component, $\gamma = (3\lambda + 2\mu)\alpha_t$, α_t is the coefficient of linear expansion of the material, λ μ are Lame's constants, λ' μ' are viscoelastic parameters, T is the incremental change of temperature from the initial state and τ is second relaxation time.

4 SOLUTION OF THE PROBLEM

From Eq. (12), Eq. (13), Eq. (14), Eq. (15), Eq. (16) and Eq. (17), we get

$$\begin{aligned}
&\left(\lambda + 2\mu + \left(\lambda' + 2\mu' \right) \frac{\partial}{\partial t} \right) \frac{\partial^2 u}{\partial x^2} + \left(\left(\lambda + \mu \right) + \left(\lambda' + \mu' \right) \frac{\partial}{\partial t} \right) \frac{\partial^2 v}{\partial x \partial y} \\
&+ \left(\mu + \mu' \frac{\partial}{\partial t} \right) \frac{\partial^2 u}{\partial^2 y} + \mu_e H_0^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} \right) = \rho \frac{\partial^2 u}{\partial t^2} + \gamma \left(\frac{\partial \mathrm{T}}{\partial x} \right) \\
&\left(\lambda + 2\mu + \left(\lambda' + 2\mu' \right) \frac{\partial}{\partial t} \right) \frac{\partial^2 v}{\partial y^2} + \left(\lambda + \mu + \left(\lambda' + \mu' \right) \frac{\partial}{\partial t} \right) \frac{\partial^2 u}{\partial x \partial y} \\
&+ \left(\mu + \mu' \frac{\partial}{\partial t} \right) \frac{\partial^2 v}{\partial^2 x} + \mu_e H_0^2 \left(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2} \right) = \rho \frac{\partial^2 v}{\partial t^2} + \gamma \left(\frac{\partial \mathrm{T}}{\partial y} \right)
\end{aligned} \tag{18}$$

Eq. (18) can be solved by choosing potential functions ϕ and ψ as:

$$u = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial y} \text{ and } v = \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y} \tag{19}$$

From Eqs. (18) and (19), we get

$$\nabla^2\phi+\frac{\left(\lambda^{/}+2\mu^{/}\right)}{(\lambda+2\mu+\mu_e H_0^2)}\frac{\partial}{\partial t}\left(\nabla^2\phi\right)=\frac{\rho}{(\lambda+2\mu+\mu_e H_0^2)}\frac{\partial^2\phi}{\partial t^2}+\frac{\gamma \mathrm{T}}{(\lambda+2\mu+\mu_e H_0^2)} \tag{20}$$

$$\nabla^2\psi+\frac{\mu^{/}}{\mu}\left(\frac{\partial}{\partial t}\left(\nabla^2\psi\right)\right)=\frac{\rho}{\mu}\left(\frac{\partial^2\psi}{\partial t^2}\right) \tag{21}$$

Now, from Eqs. (5) and (19), using Classical Dynamical theory: $\tau=\tau_0=0, \delta_{ij}=0,$ we get

$$K\nabla^2\mathrm{T}=\rho c_p\left(\frac{\partial \mathrm{T}}{\partial t}\right)+\gamma \mathrm{T}_0\left[\frac{\partial}{\partial t}\left(\nabla^2\phi\right)\right] \tag{22}$$

where, $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$

The solution of ϕ, ψ and T can be obtained in the following form

$$\phi=\mathrm{A}(y)\exp[ik\{x-ct\}] \tag{23}$$

$$\psi=\mathrm{B}(y)\exp[ik\{x-ct\}] \tag{24}$$

$$\mathrm{T}=C(y)\exp[ik\{x-ct\}] \tag{25}$$

where, k is wave number, ω is angular frequency and $c=\frac{\omega}{k}$ phase velocity.

Using Eq. (23) and Eq. (25), T is eliminated from Eq. (20) and Eq. (22), and we get

$$\left[\nabla^2-\frac{c_p\rho}{\mathrm{K}}\frac{\partial}{\partial t}\right]\left[\nabla^2-\frac{1}{c_1^2-i\omega c_1^{/2}}\frac{\partial^2}{\partial t^2}\right]\phi-\frac{\gamma^2\mathrm{T}_0}{\left[(\lambda+2\mu+\mu_e H_0^2)-i\omega\left(\lambda^{/}+2\mu^{/}\right)\right]\mathrm{K}}\nabla^2\left(\frac{\partial\phi}{\partial t}\right)=0 \tag{26}$$

Eq. (21) with the help of Eq. (24) can be written as:

$$\left[\nabla^2-\frac{1}{c_2^2-i\omega c_2^{/2}}\frac{\partial^2}{\partial t^2}\right]\psi=0 \tag{27}$$

where, $c_1^2=\frac{(\lambda+2\mu+\mu_e H_0^2)}{\rho}$, $c_1^{/2}=\frac{\left(\lambda^{/}+2\mu^{/}\right)}{\rho}$, $c_2^2=\frac{\mu}{\rho}$, $c_2^{/2}=\frac{\mu^{/}}{\rho}$

Substituting Eq. (23) into Eq. (26) and Eq. (24) into Eq. (27), we obtain the following differential equations:

$$\left(\frac{\partial^2}{\partial y^2}-\alpha_1^2\right)\left(\frac{\partial^2}{\partial y^2}-\alpha_2^2\right)\mathrm{A}(y)=0 \tag{28}$$

$$\left(\frac{\partial^2}{\partial y^2}-\delta^2\right)\mathrm{B}(y)=0 \tag{29}$$

where

$$\alpha_1^2 = k^2 - \alpha^2, \alpha_2^2 = k^2 - \beta^2, \ \delta^2 = k^2 - \tau^2, \text{ and } \tau = \frac{\omega^2}{\left(c_2^2 - i\omega c_2^{/2}\right)} \tag{30}$$

Here, α^2 and β^2 are the roots of the following biquadratic equation

$$\Lambda^4 - \Lambda^2[\sigma^2 + \mathrm{q}(1+\varepsilon)] + \sigma^2\,\mathrm{q} = 0 \tag{31}$$

where, $\Lambda^2 = -\nabla^2$. and the roots α^2, β^2 are

$$\alpha^2 = \mathrm{q}\left[1 - \frac{q\varepsilon}{\sigma^2 - q}\right] \text{ and } \beta^2 = \sigma^2\left[1 + \frac{q\varepsilon}{\sigma^2 - q}\right] \tag{32}$$

Here, $\sigma^2 = \dfrac{\omega^2}{c_1^2 - i\omega c_1^{/2}}$, $q = \dfrac{i\omega c_p \rho}{K}$ and $\varepsilon = \dfrac{\gamma^2 T_0}{K\rho\left[\lambda + 2\mu + \mu_e H_0^2 - i\omega(\lambda^{/} + 2\mu^{/})\right]}$ are magneto-thermoelastic coupling parameters.

The requirement that the stresses and hence the functions ϕ and ψ vanish as $(\mathrm{x}^2 + \mathrm{y}^2) \to \infty$ leads to the following solutions of Eq. (28) and Eq. (29)

$$\mathrm{A}(y) = Ee^{-\alpha_1 y} + Fe^{-\alpha_2 y} \tag{33}$$

$$\mathrm{B}(y) = Ge^{-\delta y} \tag{34}$$

Introducing Eq. (33) and Eq. (34) in Eq. (23) and Eq. (24), we get

$$\phi(x,y,t) = [Ee^{-\alpha_1 y} + Fe^{-\alpha_2 y}]\mathrm{e}^{i(kx - \omega t)} \tag{35}$$

$$\psi(x,y,t) = [Ge^{-\delta y}]\mathrm{e}^{i(kx - \omega t)} \tag{36}$$

Using Eq. (20), Eq. (25) and Eq. (35), we get

$$T = \frac{\rho m^2}{\gamma}[E\vartheta_1 e^{-\alpha_1 y} + F\vartheta_2 e^{-\alpha_2 y}]\mathrm{e}^{i(kx - \omega t)} \tag{37}$$

where

$$m^2 = c_1^2 - i\omega c_1^{/2}, \vartheta_1 = \sigma^2 - \alpha^2, \vartheta_2 = \sigma^2 - \beta^2 \tag{38}$$

5 BOUNDARY CONDITIONS AND FREQUENCY EQUATION

The initial conditions are supplemented by the following boundary conditions:

1. Continuity condition for normal initial stress at $Y = 0$

$$\nabla f_y = s_{22} + P\frac{\partial u}{\partial x} = 0 \tag{39a}$$

2. Continuity condition for tangential initial stress at $Y = 0$

$$\nabla f_x = s_{12} - P\frac{\partial u}{\partial x} = 0 \tag{39b}$$

3. Continuity condition for temperature at $Y = 0$

$$\frac{\partial T}{\partial y} + \Theta T = 0 \tag{39c}$$

where ∇f_x and ∇f_y are incremental boundary forces per unit initial area and is Θ is the ratio of heat transfer coefficient and thermal conductivity.

From Eqs. (14), (15), (16), (9), (35) and (36), the first boundary condition (38a) becomes

$$\begin{aligned} &E\left[k\alpha_1\left\{i(2\mu - P) + 2\omega\mu^{/}\right\}\right] + F\left[k\alpha_2\left\{i(2\mu - P) + 2\omega\mu^{/}\right\}\right] \\ &+G\left[k^2\left\{(2\mu - P) - 2i\omega\mu^{/}\right\} - \tau^2(\mu - i\omega\mu^{/})\right] = 0 \end{aligned} \tag{40}$$

From Eqs. (14), (15), (16), (9), (35), (36) and (37), the second boundary condition (38b) becomes

$$\begin{aligned} &E\left[\rho m^2(k^2 - \sigma^2) - k^2\left\{(\lambda + P) - i\omega\lambda^{/}\right\}\right] + F\left[\rho m^2(k^2 - \sigma^2) - k^2\left\{(\lambda + P) - i\omega\lambda^{/}\right\}\right] \\ &+G\left[-k\delta\left\{i(2\mu - P) + 2\omega\mu^{/}\right\}\right] = 0 \end{aligned} \tag{41}$$

From Eq. (37) and $\Theta = 0$ (thermal insulation), the third boundary condition (39c) becomes

$$E\left[\vartheta_1(\Theta - \alpha_1)\right] + F\left[\vartheta_2(\Theta - \alpha_2)\right] = 0 \tag{42}$$

Now eliminating *E, F, G* from Eq. (40), Eq. (41) and Eq. (42), we get

$$\begin{vmatrix} \left[k\alpha_1\begin{Bmatrix} i(2\mu - P) \\ +2\omega\mu^{/} \end{Bmatrix}\right] & \left[k\alpha_2\begin{Bmatrix} i(2\mu - P) \\ +2\omega\mu^{/} \end{Bmatrix}\right] & \begin{bmatrix} k^2\begin{Bmatrix} (2\mu - P) \\ -2i\omega\mu^{/} \end{Bmatrix} \\ -\tau^2(\mu - i\omega\mu^{/}) \end{bmatrix} \\ \begin{bmatrix} \rho m^2(k^2 - \sigma^2) \\ -k^2\left\{(\lambda + P) - i\omega\lambda^{/}\right\} \end{bmatrix} & \begin{bmatrix} \rho m^2(k^2 - \sigma^2) \\ -k^2\left\{(\lambda + P) - i\omega\lambda^{/}\right\} \end{bmatrix} & \left[-k\delta\left\{i(2\mu - P) + 2\omega\mu^{/}\right\}\right] \\ \left[\vartheta_1(\Theta - \alpha_1)\right] & \left[\vartheta_2(\Theta - \alpha_2)\right] & 0 \end{vmatrix} = 0 \tag{43}$$

Case.1

If we neglect initial stress, magnetic field and temperature in the half-space, then P and q both will be zero, $\Theta = 0$ and $\alpha_1 = k$ and Eq. (43) reduces to

$$\begin{vmatrix} \left[k^2 \dfrac{2i\rho\omega^2}{k_\beta^2}\right] & \left[k\sqrt{k^2-k_\alpha^2}\dfrac{2i\rho\omega^2}{k_\beta^2}\right] & \left[\dfrac{2k^2\rho\omega^2}{k_\beta^2}-\rho\omega^2\right] \\ \left[\begin{matrix}\dfrac{\rho\omega^2}{k_\alpha^2}(k^2-k_\alpha^2) \\ -k^2\rho\omega^2\left(\dfrac{1}{k_\alpha^2}-\dfrac{2}{k_\beta^2}\right)\end{matrix}\right] & \left[\begin{matrix}\dfrac{\rho\omega^2}{k_\alpha^2}(k^2-k_\alpha^2) \\ -k^2\rho\omega^2\left(\dfrac{1}{k_\alpha^2}-\dfrac{2}{k_\beta^2}\right)\end{matrix}\right] & \left[-\dfrac{2i\rho\omega^2}{k_\beta^2}k\sqrt{k^2-k_\beta^2}\right] \\ -kk_\alpha^2 & 0 & 0 \end{vmatrix} = 0 \tag{44}$$

where

$$k_\alpha^2 = \frac{\rho\omega^2}{\lambda+2\mu+\mu_e H_0^2 - i\omega(\lambda^/ + 2\mu^/)} \text{ and } k_\beta^2 = \frac{\rho\omega^2}{\mu - i\omega\mu^/} \tag{45}$$

Expanding Eq. (44) and after simplification we get

$$1-8\frac{k^2}{k_\beta^2}+\left[24-16\frac{k_\alpha^2}{k_\beta^2}\right]\frac{k^4}{k_\beta^4}-16\left[1-\frac{k_\alpha^2}{k_\beta^2}\right]\frac{k^6}{k_\beta^6}=0 \tag{46}$$

Eq. (46) matches with the classical equation given by Caloi.

Case.2

If we consider initial stress, magnetic field and temperature in the half-space and by assuming $\lambda = \mu$ and $\lambda^/ = \left(\frac{4}{3}\right)\mu^/ : \alpha, \beta, \varepsilon, \sigma, \sigma$ will now be changed to $\alpha^/, \beta^/, \varepsilon^/$ and $\sigma^/$ are given as:

$$\alpha^{/2} = \mathrm{q}\left[1-\frac{q\varepsilon^/}{\sigma^{/2}-q}\right],\ \beta^{/2} = \sigma^{/2}\left[1+\frac{q\varepsilon^/}{\sigma^{/2}-q}\right],\ \varepsilon^/ = \frac{3\gamma^2 T_0}{K\rho\left[9\mu+3\mu_e H_0^2 - i\omega 10\mu^/\right]},\ \sigma^2 = \frac{\omega^2}{c_{01}^2 - i\omega c_{01}^{/2}}$$
$$c_{01}^2 = \frac{(3\mu+\mu_e H_0^2)}{\rho},\ c_{01}^{/2} = \frac{10\mu^/}{3\rho} \tag{47}$$

Then, Eq. (43) reduces to

$$\begin{vmatrix} \left[2i\chi_1\dfrac{(\xi\mu-i\omega\mu^/)}{(\mu-i\omega\mu^/)}\right] & \left[2i\chi_2\dfrac{(\xi\mu-i\omega\mu^/)}{(\mu-i\omega\mu^/)}\right] & \left[\dfrac{\tau^2}{k^2}-\dfrac{2(\xi\mu-i\omega\mu^/)}{(\mu-i\omega\mu^/)}\right] \\ \left[\dfrac{2(\xi\mu-i\omega\mu^/)}{(\mu-i\omega\mu^/)}-\dfrac{\tau^2}{k^2}\right] & \left[\dfrac{2(\xi\mu-i\omega\mu^/)}{(\mu-i\omega\mu^/)}-\dfrac{\tau^2}{k^2}\right] & \left[2i\chi_3\dfrac{(\xi\mu-i\omega\mu^/)}{(\mu-i\omega\mu^/)}\right] \\ \left[\vartheta_1(\Theta-k\chi_1)\right] & \left[\vartheta_2(\Theta-k\chi_2)\right] & 0 \end{vmatrix} = 0 \tag{48}$$

where

$$\chi_1=\sqrt{1-\frac{\alpha'^2}{k^2}},\chi_2=\sqrt{1-\frac{\beta'^2}{k^2}},\chi_3=\sqrt{1-\frac{\tau^2}{k^2}} \text{ and } \xi=1-\frac{P}{2\mu}. \tag{49}$$

Solving Eq. (48), we get

$$\left[\frac{2(\xi\mu-i\omega\mu')}{(\mu-i\omega\mu')}-\frac{\tau^2}{k^2}\right]^2=4\chi_3\left[\frac{(\xi\mu-i\omega\mu')}{(\mu-i\omega\mu')}\right]^2\left[\frac{k\chi_1\chi_2(\vartheta_1-\vartheta_2)+\Theta(\chi_1\vartheta_2-\chi_2\vartheta_1)}{\Theta(\vartheta_1-\vartheta_2)+k(\chi_1\vartheta_1-\vartheta_2\chi_2)}\right] \tag{50}$$

For $\Theta\to 0$, it is the case of convection i.e. no heat transfer (thermal isolation), Eq. (50) reduces to

$$\left[\frac{2(\xi\mu-i\omega\mu')}{(\mu-i\omega\mu')}-\frac{\tau^2}{k^2}\right]^2=4\chi_3\left[\frac{(\xi\mu-i\omega\mu')}{(\mu-i\omega\mu')}\right]^2\left[\frac{\chi_1\chi_2(\vartheta_1-\vartheta_2)}{(\chi_1\vartheta_1-\vartheta_2\chi_2)}\right] \tag{51}$$

For $\Theta\to\infty$, heat transfer is large, Eq. (50) reduces to

$$\left[\frac{2(\xi\mu-i\omega\mu')}{(\mu-i\omega\mu')}-\frac{\tau^2}{k^2}\right]^2=4\chi_3\left[\frac{(\xi\mu-i\omega\mu')}{(\mu-i\omega\mu')}\right]^2\left[\frac{(\chi_1\vartheta_2-\chi_2\vartheta_1)}{(\vartheta_2-\vartheta_1)}\right] \tag{52}$$

The values of $\alpha_1,\alpha_2,\delta,\vartheta_1$ and ϑ_2 can be reduced in terms of χ_1,χ_2 and χ_3 , therefore from Eq. (50), we get

$$\begin{aligned}&\left[\frac{2(\xi\mu-i\omega\mu')}{(\mu-i\omega\mu')}-\frac{c^2}{c_1^2-i\omega c_1'^2}\right]^2\left[\chi_1^2+\chi_2^2+\chi_1\chi_2-1+\frac{c^2}{c_1^2-i\omega c_1'^2}\right]-4\left[\frac{(\xi\mu-i\omega\mu')}{(\mu-i\omega\mu')}\right]^2\chi_1\chi_2\chi_3(\chi_1+\chi_2)\\&=\frac{h}{k}\left[\left\{\frac{2(\xi\mu-i\omega\mu')}{(\mu-i\omega\mu')}-\frac{c^2}{c_1^2-i\omega c_1'^2}\right\}(\chi_1+\chi_2)-4\left[\frac{(\xi\mu-i\omega\mu')}{(\mu-i\omega\mu')}\right]^2\chi_3\left(\chi_1\chi_2+1-\frac{c^2}{c_1^2-i\omega c_1'^2}\right)\right]\end{aligned} \tag{53}$$

where, $c^2=\dfrac{\omega^2}{k^2}$ is the phase velocity of Rayleigh waves.

Using the assumption $\lambda=\mu$ and $\lambda'=\left(\dfrac{4}{3}\right)\mu'$, Eq. (31) changes to

$$\alpha'^2+\beta'^2=\sigma'^2+\mathrm{q}(1+\varepsilon') \text{ and } \alpha'^2\beta'^2=\sigma'^2\,\mathrm{q} \tag{54}$$

From Eq. (49) and Eq. (54), we get

$$\chi_1^2+\chi_2^2=2-\frac{c^2}{c_1^2-i\,\omega c_1'^2}-\frac{ic^2(1+\varepsilon')}{\varpi\left(c_1^2-i\,\omega c_1'^2\right)},\chi_1^2\chi_2^2=1-\frac{c^2}{c_1^2-i\,\omega c_1'^2}-\frac{ic^2}{\varpi\left(c_1^2-i\,\omega c_1'^2\right)}\left(1+\varepsilon'-\frac{c^2}{c_1^2-i\,\omega c_1'^2}\right) \tag{55}$$

where, $\varpi=\dfrac{K\omega}{c_p\rho c_{01}^2-i\omega c_{01}'^2}$, is reduced frequency.

From Eq. (55) and Eq. (53), expending the quantities χ_1,χ_2 in series of ϖ and neglecting the terms of the order $\varpi^{\frac{1}{2}}$ we get

$$\left[\left(\xi\mu-\frac{\rho c^2}{2}\right)-i\,\omega\mu'\right]^2=(\xi\mu-i\,\omega\mu')^2\left(1-\frac{\rho c^2}{\mu-i\,\omega\mu'}\right)^{\frac{1}{2}}\left(1-\frac{3\rho c^2}{\left(9\mu+3\mu_e H_0^2-i\,\omega 10\mu'\right)\left(1+\varepsilon'\right)}\right)^{\frac{1}{2}} \tag{56}$$

For liquid medium $\lambda = \mu = 0$, Eq. (56) reduces to

$$\left[\frac{P}{2}+\frac{\rho c^2}{2}+i\omega\mu^{/}\right]^2=\left(\frac{P}{2}+i\omega\mu^{/}\right)^2\left(1-\frac{i\rho c^2}{\omega\mu^{/}}\right)^{\frac{1}{2}}\left(1-\frac{3\rho c^2}{\left(3\mu_e H_0^2-i\omega 10\mu^{/}\right)\left(1+\varepsilon_0\right)}\right)^{\frac{1}{2}} \tag{57}$$

where, $\varepsilon_0 = \dfrac{3\gamma^2 T_0}{K\rho\left[3\mu_e H_0^2 - i\omega 10\mu^{/}\right]}$

Rationalizing and squaring Eq. (58), we get

$$\left[\frac{P}{2\omega\mu^{/}}+\frac{\rho c^2}{2\omega\mu^{/}}+i\right]^4=\left(\frac{P}{2\omega\mu^{/}}+i\right)^4\left(1-\frac{i\rho c^2}{\omega\mu^{/}}\right)\left(1-\frac{3\rho c^2\varepsilon_0}{\left(3\mu_e H_0^2-i\omega 10\mu^{/}\right)\left(1+\varepsilon_0^2\right)}-\frac{3\rho c^2}{\left(3\mu_e H_0^2-i\omega 10\mu^{/}\right)\left(1+\varepsilon_0^2\right)}\right) \tag{58}$$

Expanding Eq. (58) and taking real parts only

$$\begin{aligned}&\left(S_p+\frac{V^2}{2}\right)^4-6\left(S_p+\frac{V^2}{2}\right)^2+1=\left(S_p^4-6S_p^2+1\right)\left[1-\frac{3V^2c_c}{10\left(1+c_c^2\right)}-\frac{3V^4}{10\left(1+c_c^2\right)}\right]\\&+4V^2S_p\left(S_p^2-1\right)\left[1-\frac{3V^2c_c}{10\left(1+c_c^2\right)}+\frac{3}{10\left(1+c_c^2\right)}\right]\end{aligned} \tag{59}$$

where, $V^2 = \dfrac{c^2\rho}{\omega\mu^{/}}, c_c = \dfrac{3\gamma^2 T_0}{K\rho\left[3\mu_e H_0^2 - i\omega 10\mu^{/}\right]}$ and $S_p = \dfrac{P}{2\omega\mu^{/}}$.

Case.3

If we ignore the viscous properties of the half-space, then $\mu^{/} = 0$ and Eq. (56) reduces to

$$\left[2(1-S)-\nu^2\right]^2 = 4(1-S)^2\left[1-\nu^2\right]^{\frac{1}{2}}\left[1-\nu^2\mathrm{M}_c\right]^{\frac{1}{2}} \tag{60}$$

where, $S = \dfrac{P}{2\mu}, \nu^2 = \dfrac{c^2}{c_2^2}$ and $\mathrm{M}_c = \dfrac{c_2^2}{(1+\varepsilon)c_1^2}$

If we ignore the initial stress of the half-space, then $S = 0$ and Eq. (60) reduces to

$$\left[2-\nu^2\right]^2 = 4\left[1-\nu^2\right]^{\frac{1}{2}}\left[1-\nu^2\mathrm{M}_c\right]^{\frac{1}{2}} \tag{61}$$

Eq. (61) is the dispersion relation for Rayleigh waves in elastic medium and it is in agreement of the classical result given by Lockett.

6 NUMERICAL ANALYSIS AND DISCUSSION

The graphs are plotted separately for real parts for phase velocity against initial stress for magneto-thermo-viscoelastic half-space (Fig. 2) and magneto-thermo-elastic half-space (Fig. 3). In order to show the effect of thermo-magneto coupling parameter on wave number, we have taken following data for elastic and viscoelastic medium in Table 1.

Table 1
Data for elastic and viscoelastic medium**.**

Parameter	Numerical Value
α_t	$16.6\times10^{-6}K^{-1}$
ρ	$7.14\times10^{3}kg/m^{3}$,
$\mu_e H_0^2$	$1.24\times10^{9}N/m^{2}$
λ	$9.5\times10^{10}N/m^{2}$
μ	$4.5\times10^{10}N/m^{2}$
$\lambda^{/}$	$1.25\times10^{10}N/m^{2}$
$\mu^{/}$	$7.15\times10^{10}N/m^{2}$
c_p	$0.39\ KJ/Kg\ K$
K	$401\ W/(m.K)$

Above theory clearly indicates that the phase velocity of Rayleigh waves depend on the initial hydrostatic stress, magnetic field, temperature and viscoelastic parameters of the medium. In order to study in greater detail, the dependence of phase velocity of Rayleigh wave on temperature, stress and magnetic parameter together, the various graphs are plotted. Fig. 2 shows the variation of magneto-thermo-viscoelastic Rayleigh waves in half-space. Variation of phase velocity of Rayleigh waves with initial stress at constant values of coupling coefficients in the presence of viscoelastic parameter is shown in Fig. 2. It is observed that the phase velocity of Rayleigh waves depend upon magnetic field parameter, temperature parameter, viscoelastic parameter and initial stress parameter of the medium. On observing the various curves in Fig. 2, we find that at different real values of c_c i.e. $\left(c_c\right)_{real}=\dfrac{\gamma^2 T_0}{K\rho\mu_e H_0^2}$, the thermo-magneto coupling c_c is 0, 0.2, 0.4, 0.6, 0.8, higher the value of c_c lesser will be the phase velocity of Rayleigh waves in viscoelastic medium. The Fig. 2 reflects that as we increase c_c, the phase velocity increases for all S_p but the nature of curve remains same. Rayleigh waves in viscoelastic liquid decreases as thermo-magneto coupling c_c inceases.

Variation of phase velocity of Rayleigh waves with initial stress at constant values of coupling coefficients in the absence of viscoelastic parameter is shown in Fig. 3. It is observed that the phase velocity of Rayleigh waves depend upon magnetic field parameter, temperature parameter and initial stress parameter of the medium. On observing the various curves in Fig. 3, we find that at different values of $\mathrm{M}_c=\dfrac{c_2^2}{(1+\varepsilon)c_1^2}$, the thermo-magneto coupling M_c is 0.1, 0.2, 0.3, 0.4, there is slight variation in the phase velocity of Rayleigh waves; it means that Rayleigh waves are less dependent on coupling coefficient M_c between magnetic field, temperature field and strain field in elastic medium and more dependent on initial stress of the elastic medium. The Fig. 3 reflects that as we increase M_c, the phase velocity increases for all S but the nature of curve remains same. We conclude that in an elastic solid medium, Rayleigh waves are more dependants on the initial stress and magnetic field of the medium than the coupling coefficient M_c between temperature and strain fields.

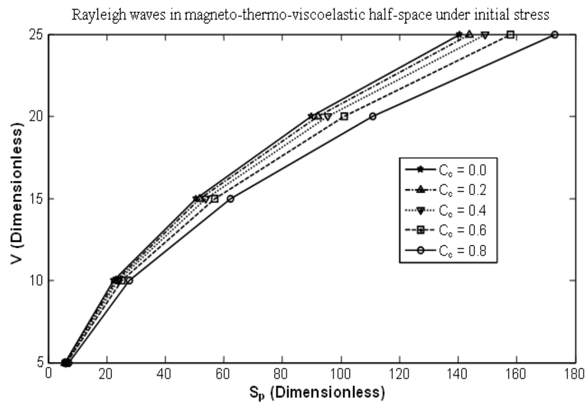

Fig.2
Variation of magneto-thermo-viscoelastic Rayleigh waves in half-space.

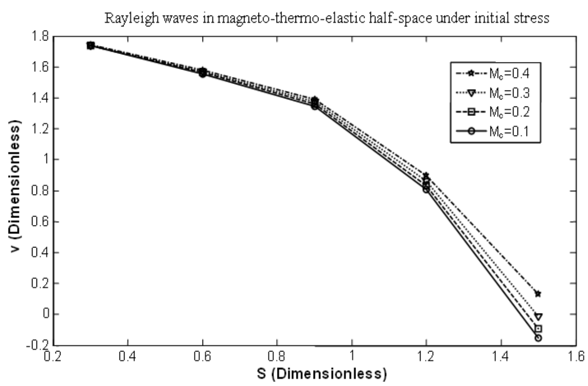

Fig.3
Variation of magneto-thermo-elastic Rayleigh waves in half-space.

7 CONCLUSION

It can be concluded that initial stress plays a remarkable role for both elastic and viscoelastic Rayleigh waves. The variation of magneto-thermo-viscoelastic Rayleigh waves in half-space is contrary with the variation of magneto-thermo-elastic Rayleigh waves in half-space. Various parameter of earth such as magnetic field, temperature field and initial hydrostatic stress influences Rayleigh waves. This problem attracts the attention of geologists and seismologists.

ACKNOWLEDGMENTS

The author is thankful to unknown reviewers for their valuable comments.

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