

Dynamics Analysis of the Steady and Transient States of a Nonlinear Piezoelectric Beam by a Finite Element Method

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ABSTRACT

This paper presents a finite element formulation for the dynamics analysis of the steady and transient states of a nonlinear piezoelectric beam. A piezoelectric beam with damping is studied under harmonic excitation. A numerical method is used for this analysis. In the paper, the central difference formula of four order is used and compared with the central difference formula of two order in the time response of the structure. The NPBDA program is developed with Matlab software. In this program, the Newmark technique for dynamic analysis is used, the Newton-Raphson iterative and Simpson methods are used for the nonlinear solution. To verify the NPBDA results, the experimental results of Malatkar are used for the nonlinear vibration analysis of a beam without piezoelectric properties. Then, the piezoelectric effect on the frequency mode values and the time response are obtained. Afterwards, the modulation frequency in the nonlinear beam and the piezoelectric effect in this parameter are verified.

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Keywords : Piezoelectric beam; Nonlinear function; Dynamic behavior; Transient; Steady state; Finite element.

1 INTRODUCTION

THE dynamic loads of structures have an important effect on strength and control. Vibration analysis can be obtained through harmonic excitation. There are transient and steady states in the time response during harmonic excitation. The behavior of piezoelectric structures is extracted by the coupling of electrical and mechanical parameters. This model can be in the form of a flexible beam and can be used for large deformations. Large deformation and nonlinear material behavior can cause improper results when using linear theory. In this case nonlinear theory must be used. In order to study this model, the finite element method can be applied. Finite element formulations have been used for the modeling of piezoelectric structures in many studies.

Ha used the 3D element for a multi-layer model and applied this element to the actuator and sensor [1]. The results were compared with the Crawley research and the results were appropriate [2]. Rao used the finite element formulation in regards to thermo-piezoelectric problems [3]. In this work, the linear equations from Mindlin were used and applied to the sensor and actuator. Moetakef presented a tetrahedron element, which had 10 nodes for linear strain and 20 nodes for parabolic strain, and the experimental results were acceptable [5],[6]. Suleman and Venkayya [7],[8] used a bilinear shape function with a 4 node quadratic element. Zemcik developed a piezoelectric shell element and implemented it with ANSYS software [9]. Lazarus presented a finite element model for the nonlinear vibrations of piezoelectric layered beams with application in NEMS [10]. Ghayour and Jabbari presented

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the effect of support and concentrated mass on the performance of a piezoelectric beam actuator and frequencies through finite element method [11]. The element for the modeling of smart structures was studied by Kogl and Bucalem [12]. This element was then used by Crawley and Lazarus [2]. Piefort and Preumont used the Mindlin shell elements for piezoelectric materials [13],[4]. The response can respond in the low amplitude solution to harmonic excitation; Sebald et al. suggested a method to excite the system to jump to the high amplitude solution for broadband piezoelectric energy harvesting[17]. Erturk and Inman investigated the dynamic response, including the chaotic response on high-energy orbits of the bistable Duffing oscillator with electromechanical coupling[18]. Friswell et al.[12] proposed a cantilever beam with a tip mass that is mounted vertically and excited in the transverse direction at its base[19]. This device had highly non-linear with two potential wells for large tip masses, when the beam was buckled. Bendigeri et al. developed finite element for the dynamic analysis of a structure with piezoelectric property[20]. An eight noded isoparametric three dimensional hexahedral element was improved to model the coupled electro-mechanical behavior. In this work, the effects due to piezoelectric for the developed finite element explained.

This research focuses on the study of the structural response of a nonlinear piezoelectric beam, under the influence of a harmonic excitation load. In this paper, the NPBDA program is developed and implemented for the dynamics analysis of the steady and transient states of a nonlinear piezoelectric beam. The Newmark technique for dynamic analysis, and the Newton-Raphson iterative and Simpson methods for nonlinear solution are applied. The central difference formula of four order is used in the time response of the structure. The piezoelectric effect on the frequency mode values and the time response are obtained. The nonlinear state and the damping effect in the frequency results are presented. The voltage response of the piezoelectric beam is shown. The modulation frequency in the nonlinear beam and the piezoelectric effect in this parameter are verified.

2 NUMERICAL SOLUTION

2.1 Finite element model of a nonlinear piezoelectric beam

The beam model is a piezoelectric bimorph which can be used as an actuator and a sensor. The beam element is based on Euler-Bernoulli theory. It is supposed that the length of the beam is large compared with the thickness, in order that the shear deformation and rotary inertia effects into the model can be neglected.

The proposed element contains two nodes, and each node has two structural degrees of freedom (u, θ) and two electrical degrees of freedom ϕ and ψ (Fig.1). The deflection function $u(x)$ and electrical potential ϕ across the beam length and thickness are evaluated by Eq. (1) [9].

$$\begin{aligned} u(x) &= [N^u] \{\hat{u}\} & \phi &= [N^\phi] \{\hat{\phi}\} & \{\hat{u}\} &= \{u_n \ \theta_n \ u_{n+1} \ \theta_{n+1}\}^T & \{\hat{\phi}\} &= \{\phi_n \ \psi_n \ \phi_{n+1} \ \psi_{n+1}\}^T \\ [N^u] &= [N^{1u} \ N^{1\theta} \ N^{2u} \ N^{2\theta}] & [N^\phi] &= [N^{1\phi} \ N^{1\psi} \ N^{2\phi} \ N^{2\psi}] \end{aligned} \quad (1)$$

where $\{\hat{u}\}$, is the displacement vector of the nodes, $\{\hat{\phi}\}$, nodes potential, $[N^u]$ the shape functions of structural degrees of freedom (u, θ) and $[N^\phi]$, the shape functions of electrical degrees of freedom (ϕ, ψ).

The strain S , and the electric field vector $\{E\}$, can be expressed as Eq. (2).

$$S = [B^u] \{\hat{u}\} \quad \{E\} = -[B^\phi] \{\hat{\phi}\} \quad (2)$$

$$[B^u] = \frac{d^2}{dx^2} [N^u] = \left[\frac{6}{h^2} \left(\frac{2x}{h} - 1 \right) \quad \frac{-2}{h} \left(\frac{3x}{h} - 2 \right) \quad \frac{6}{h^2} \left(-\frac{2x}{h} + 1 \right) \quad \frac{-2}{h} \left(\frac{3x}{h} - 1 \right) \right] \quad (3)$$

$$[B^\phi] = -\nabla [N^\phi] = - \left\{ \begin{array}{l} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial z} \end{array} \right\} [N^\phi] = \left[\begin{array}{cccc} \frac{1}{2h} + \frac{1}{2ht} z & \frac{1}{2h} - \frac{1}{2ht} z & -\frac{1}{2h} - \frac{1}{2ht} z & -\frac{1}{2h} + \frac{1}{2ht} z \\ \frac{1}{2t} + \frac{1}{2ht} x & \frac{1}{2t} - \frac{1}{2ht} x & -\frac{1}{2ht} x & \frac{1}{2ht} x \end{array} \right] \quad (4)$$

where $[B^u]$ and $[B^\phi]$ are the shape function derivatives, t , thickness, h , element length.

In this article, the element shape function uses the properties of the beam and shell elements. The motion equations of a piezoelectric structure are obtained through the Hamilton principle. Nonlinear behavior must be considered for highly flexible structures. The motion equation of a piezoelectric nonlinear structure with damping is presented by Eq. (5) [10,11].

$$\begin{bmatrix} [M] & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \hat{u} \\ \hat{\phi} \end{Bmatrix} + \begin{bmatrix} [C] & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \hat{u} \\ \hat{\phi} \end{Bmatrix} + \left(\begin{bmatrix} [K^{uu}] & [K^{u\phi}] \\ [K^{\phi u}] & [K^{\phi\phi}] \end{bmatrix} + \begin{bmatrix} [K_N^{uu}] & [K_N^{u\phi}] \\ [K_N^{\phi u}] & [0] \end{bmatrix} \right) \begin{Bmatrix} \hat{u} \\ \hat{\phi} \end{Bmatrix} = \begin{Bmatrix} \{f^u\} \\ \{f^\phi\} \end{Bmatrix} \quad (5)$$

where $[M]$, is mass matrix, $[C]$, damping matrix, $[K^{uu}]$, linear stiffness matrix, $[K^{u\phi}]$, linear piezoelectric matrix, $[K^{\phi\phi}]$, dielectric matrix, $[K_N^{uu}]$, nonlinear stiffness matrix, $[K_N^{u\phi}]$, nonlinear piezoelectric matrix, $\{f^u\}$, mechanical force vector, $\{f^\phi\}$, electrical load vector,

$$[K^{uu}] = \int_V [B^u]^T [c^E] [B^u] dV \quad (6)$$

$$[K^{u\phi}] = \int_V [B^u]^T [e] [B^\phi] dV = [K^{\phi u}]^T \quad (7)$$

$[K^{u\phi}]$ for the series connection of the piezoelectric layers is presented in Eq.(8).

$$[K^{u\phi}] = 2eb \begin{bmatrix} \frac{t}{4h} & \frac{t}{4h} & \frac{t}{4h} & \frac{-t}{4h} \\ \frac{t}{4} + \frac{5t^2}{24h} & \frac{-t}{4} + \frac{5t^2}{24h} & \frac{-t^2}{24h} & \frac{-5t^2}{24h} \\ \frac{-t}{4h} & \frac{-t}{4h} & \frac{-t}{4h} & \frac{t}{4h} \\ \frac{-t}{2} + \frac{5t^2}{24h} & \frac{-5t^2}{24h} & \frac{t}{2} + \frac{t^2}{24h} & \frac{t}{4} + \frac{5t^2}{24h} \end{bmatrix} \quad (8)$$

$[K^{\phi\phi}]$ for the series connection of the piezoelectric layers is presented in Eq.(9).

$$[K^{\phi\phi}] = \frac{ebt^2}{3h} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ -1 & 1 & 1 & -1 \end{bmatrix} \quad (9)$$

$$[K^{\phi\phi}] = -\int_V [B^\phi]^T [\epsilon^s] [B^\phi] dV = 2 \frac{\epsilon}{t} \begin{bmatrix} \frac{2t}{3h} + \frac{7h}{12t} & \frac{2t}{3h} + \frac{h}{12t} & \frac{-2t}{3h} - \frac{5h}{12t} & \frac{5t}{12h} + \frac{5h}{12t} \\ \frac{2t}{3h} + \frac{h}{12t} & \frac{2t}{3h} + \frac{h}{6t} & \frac{-2t}{3h} - \frac{h}{12t} & \frac{-t}{12h} - \frac{h}{12t} \\ \frac{-2t}{3h} - \frac{5h}{12t} & \frac{-2t}{3h} - \frac{h}{12t} & \frac{2t}{3h} + \frac{h}{6t} & \frac{t}{3h} - \frac{h}{6t} \\ \frac{5t}{12h} + \frac{5h}{12t} & \frac{5t}{12h} - \frac{h}{12t} & \frac{t}{3h} - \frac{h}{6t} & \frac{2t}{3h} + \frac{h}{6t} \end{bmatrix} \quad (10)$$

$$[M] = \int_V [N^u]^T \rho [N^u] dV \quad (11)$$

where $[c^E]$, is elasticity tensor under a constant electric field, $[e]$, piezoelectric stress matrix, $[\varepsilon^s]$, dielectric matrix, ε , permittivity factor, ρ , mass density.

In this equation, nonlinear geometry is applied in $[K_N^{u\varphi}]$ and $[K_N^{uu}]$. Nonlinear stiffness matrix is the combination of $[K_{NE}^{uu}]$ and $[K_{NM}^{uu}]$. Matrix $[K_{NE}^{uu}]$ represents the curvature nonlinearity and $[K_{NM}^{uu}]$ shows the inertia nonlinearity [14].

$$[K_N^{uu}] = [K_{NE}^{uu}] + [K_{NM}^{uu}] \quad (12)$$

$$[K_{NM}^{uu}] = \frac{1}{2} \rho A \int_0^h [B^u]^T [B^u] \mathfrak{G}_2 ds \quad (13)$$

$$[K_{NE}^{uu}] = EI \int_0^h [B^u]^T [B^u] \mathfrak{G}_2 ds \quad (14)$$

where E , is the young module, I , inertia moment, A , section area, h , length element, g_1 and g_2 , nonlinear functions.

The piezoelectric nonlinear matrix is defined as:

$$[K_N^{u\varphi}] = \int_V [B_N^u]^T [e]^T [B^{\varphi}] dV \quad (15)$$

$$[B_N^u] = \frac{du}{dx} [B^u] \quad (16)$$

where u , is transverse displacement in the y direction.

The nonlinear damping matrix is calculated using proportional damping[21]. The damping matrix is the combination of the mass and the total stiffness matrix, especially in dynamic analysis [22].

$$[C] = \alpha_1 [M] + \alpha_2 [K] \quad (17)$$

$$[K] = [K^{uu}] + [K_N^{uu}] \quad (18)$$

α_1 and α_2 can be simplified by using natural frequency (ω_n) and damping ratio (ξ_n) [14].

$$\xi_n = \frac{\alpha_1}{2\omega_n} + \frac{\alpha_2 \omega_n}{2} \quad (19)$$

The vibration behavior of the nonlinear beam is studied by using the finite element method. The Galerkin weighted method is used in the finite element model. The Newmark procedure is applied for time response and Newton-Raphson iteration is implemented to obtain the nonlinear stiffness matrix and nonlinear response of piezoelectric beam.

2.2 Nonlinear function g_1

The calculation method of function g_1 influences the results. Function g_1 is obtained by the Newton-Raphson iteration technique for time step, and is then used in analysis. Function g_1 is defined by Eq. (20) [14].

$$g_1 = \int_L \int_0^x \frac{\partial^2 v'^2}{\partial t^2} dx dx \quad (20)$$

where L , is beam length (Fig.2).

In this article, two methods are presented to solve function g_1 .

Method 1

In method 1, function g_1 is solved by using the central difference formula of two order for time step j .

$$J_j(x) = \left. \frac{\partial^2 v'^2}{\partial t^2} \right|_j = \frac{v'^{j+1} - 2v'^j + v'^{j-1}}{\Delta t^2} \quad (21)$$

Method 2

In method 2, function g_1 is solved by using the central difference formula of four order for time step j .

$$J_j(x) = \left. \frac{\partial^2 v'^2}{\partial t^2} \right|_j = \frac{-v'^{j+2} + 16v'^{j+1} - 30v'^j + 16v'^{j-1} - v'^{j-2}}{12\Delta t^2} \quad (22)$$

$$g_1 = \int_L \int_0^x J(x) dx dx \quad (23)$$

The integral of Eq. (23) can be calculated using Simpson's rule [16].

$$I_1(x_n) = \int_0^{x_n} J(x) dx = \frac{x_n}{3(k-1)} \left[J(0) + 4 \sum_{\substack{i=2 \\ \text{even } i}}^{n-1} J(x_i) + 2 \sum_{\substack{i=3 \\ \text{odd } i}}^{n-2} J(x_i) + J(x_n) \right] \quad (24)$$

$$g_1(x_n) = \int_L I_1(x) dx = \frac{x_n - 1}{3(a-n)} \left[I_1(L) + 4 \sum_{\substack{i=1 \\ \text{even } i}}^{a-n-1} I_1(x_{n+i}) + 2 \sum_{\substack{i=2 \\ \text{odd } i}}^{a-n-2} I_1(x_{n+i}) + I_1(x_n) \right] \quad (25)$$

where a , is the total number of calculated steps, n , the number of step, x_n , the point position n .

Function g_1 is referred to as g_{old} in method 1 and g_{new} in method 2. The function g_{old} was used in previous papers[11,14].

3 VIBRATION ANALYSIS RESULTS

The developed finite element program has been named Nonlinear Piezoelectric Beam Dynamic Analysis (NPBDA). The NPBDA program is applied for the nonlinear piezoelectric beam.

To verify the NPBDA results, the experimental results of Malatkar and the numerical results of Delgado[14] are used for the nonlinear vibration analysis of a beam without piezoelectric properties. First, the experimental results of Malatkar[15] for the steady state response of a beam without piezoelectric properties are shown. The corresponding numerical results from the NPBDA program are compared with the experimental results of Malatkar for a beam without piezoelectric properties. Then, the NPBDA program is applied to compute transient and steady time response of piezoelectric beam under the influence of a harmonic excitation load (Fig. 1). The harmonic excitation load is P by Eq. (26) [15].

$$P = P_d \cos \omega t \tag{26}$$

where P_d , is the amplitude of load, ω , excitation frequency, t , time. The amplitude of load is considered 0.15g [14].

This piezoelectric beam has two layers. The series connection of the piezoelectric layers represents opposite polarities. The piezoelectric layer is taken to be PZT-5A[18]. The properties of the piezoelectric material are shown in Tables 1, 2. In this study, the condition of the series connection is used in the model (Fig. 2). The Newton-Raphson iterative and Simpson methods for nonlinear solution are used. Time step is applied for the Newton-Raphson iterative method. Number of element and time step are important for convergence of nonlinear vibration results, so the results of frequency analysis are shown with a change in number of element and the time step in Figs. 3 , 4. The time step of 0.001sec and number of element of 20 obtain the converged results, so a time step of 0.001sec is considered.

The nonlinear displacement vector is calculated using iterative method. This method obtains the converged results by Eq.(27) [14].

$$\theta = \sum_{i=1}^N |v_{Lj}^i - v_{Nj}^i| \quad \theta \leq Tolerance \tag{27}$$

where v_{Lj}^i , is the linear displacement vector of element i , v_{Nj}^i , the nonlinear displacement vector of element i , N , the total number of elements. If the error θ exceeds tolerance, v_{Nj} is assigned to v_{Lj} and a new v_{Nj} is calculated. This procedure is repeated until convergence is obtained.

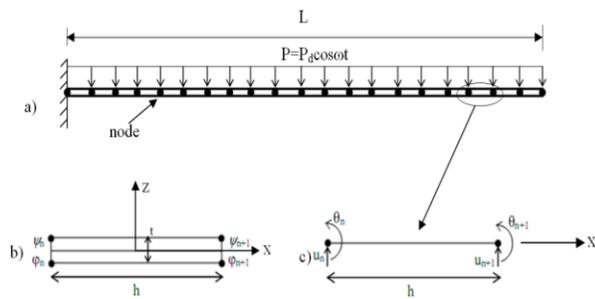


Fig.1
a)The finite element model of piezoelectric beam b) Nodes potential degrees of freedom c) Nodes displacement degrees of freedom.

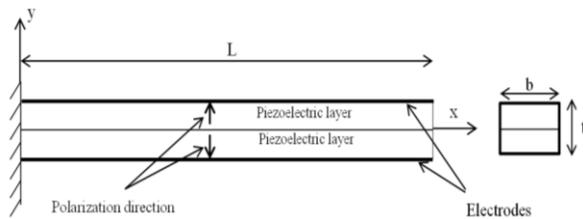


Fig.2
Series connection of the piezoelectric layers.

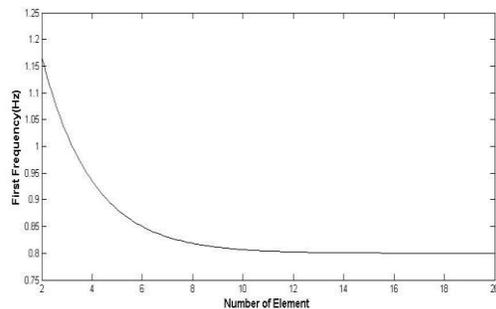
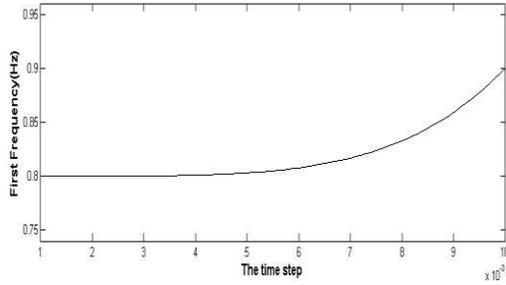


Fig.3
Change in the first frequency with change in number of element.

**Fig.4**

Change in the first frequency with change in the time step.

Table 1

The properties of the piezoelectric beam dimensions.

t(m)	b(m)	L(m)
$0.55 \cdot 10^{-3}$	$12.71 \cdot 10^{-3}$	$662 \cdot 10^{-3}$

Table 2

The properties of the piezoelectric material.

Piezoelectric stress matrix $[c/m^2] \bar{e}$	Dielectric permittivity matrix $\varepsilon [F/m]$	Elasticity constant $E [GPa]$	Poisson ratio $\nu [-]$	Mass density $\rho [kg/m^3]$
$\begin{bmatrix} 0 & 0 & 0 \\ -10.4 & 10.4 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 13.3 \cdot 10^{-9} \end{bmatrix}$	165.5	0.3	7400

3.1 Numerical results of natural frequencies

The stiffness matrix of the piezoelectric model is dependent on the electrical boundary conditions, and the piezoelectric coupling parameter can affect Eigen frequencies. Numerical results showed that the presence of piezoelectric properties affected the increase of the first and second frequency of a nonlinear beam. However, the effect on later modes is less, which can be stated as a 72% increase in the first frequency and a 57% increase in the second frequency.

In this study, nonlinear conditions are considered and frequency results are obtained a variety of conditions. Comparisons of the coupling and uncoupling beams were made in undamping state. The results have been shown in Table 3. The existence of a nonlinear state affects the results and has the maximum of increasing of the frequency value by 23.6% in comparison to the linear state in the second mode.

Table 3

Results of vibration analysis.

Number of mode	Natural frequency (undamping, uncoupling and nonlinear) (Hz)	Natural frequency (undamping, uncoupling and linear) (Hz)	Natural frequency (undamping, coupling and nonlinear) (Hz)	Natural frequency (undamping, coupling and linear) (Hz)
1	0.81	0.96	2.28	2.27
2	5.88	6	13.2	10.68
3	16.72	16.82	16.64	14.77
4	32.85	32.97	35.33	32.86

3.2 Numerical results of steady state

The NPBDA program is applied to compute steady time response of nonlinear piezoelectric beam. Each time step equals 0.001sec in the numerical solution and the time of the steady state response is considered 120sec.

Table 4. presents CPU time for the vibration analysis of the nonlinear beam with piezoelectric and without piezoelectric properties in the time step of 0.001sec.

Fig. 5 shows the FFT for the base and the tip of the beam without piezoelectric properties. The figure shows the jump in the frequency 17.58Hz. This frequency is excitation frequency. The figure does not show other structural

frequencies. The result of the time response of the tip of the nonlinear beam without piezoelectric properties is shown in Fig. 6. The figure shows harmonic behavior and the steady state will be reached in the 80sec. And, the response for the last three seconds is in Fig.7. According to this figure, the steady state is seen.

Fig. 8 shows the FFT for the base and the tip of the piezoelectric beam. In the tip FFT, a single frequency is shown, but in the base FFT, two frequencies are shown. The result of the time response of the tip of the nonlinear piezoelectric beam is presented in Fig. 9 and the response for the last three seconds is in Fig.10. According to this figure, the piezoelectric property in nonlinear beam effects the time period of the steady state and it is greater than that for a beam without piezoelectric property, thus the steady state is not seen even passing 120sec. The figure shows that the presence of piezoelectric properties causes an increase in the global stiffness value. This is the reason for the reduction in the displacement of the piezoelectric beam in comparison to the beam without piezoelectric properties.

Table 4

CPU time for the vibration analysis of nonlinear beam with piezoelectric and without piezoelectric properties.

	Nonlinear piezoelectric beam	Nonlinear beam without piezoelectric properties
CPU time	9.9898×10^4 (sec)=27.78(h)	1.2964×10^5 (sec)=36.11(h)

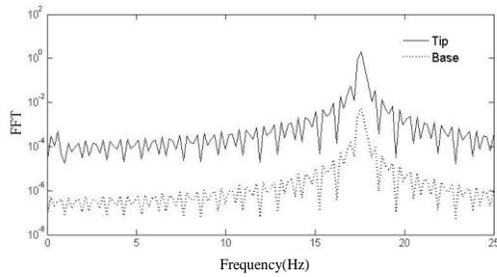


Fig.5
Steady state FFT for the base and the tip of the nonlinear beam without piezoelectric properties.

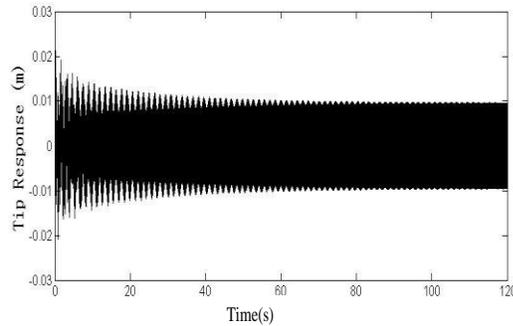


Fig.6
Time response of the tip of the nonlinear beam without piezoelectric properties.

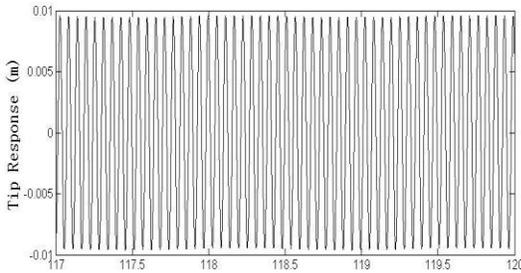
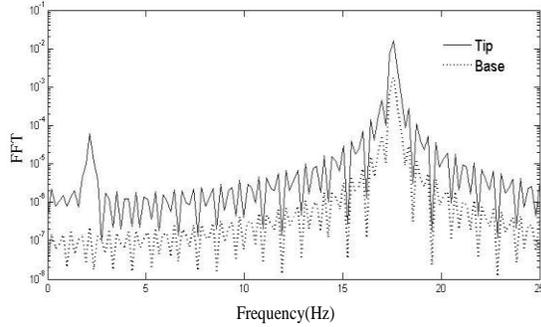
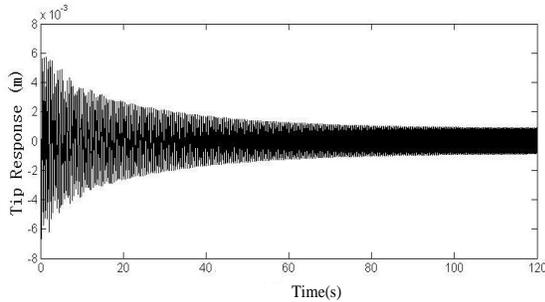


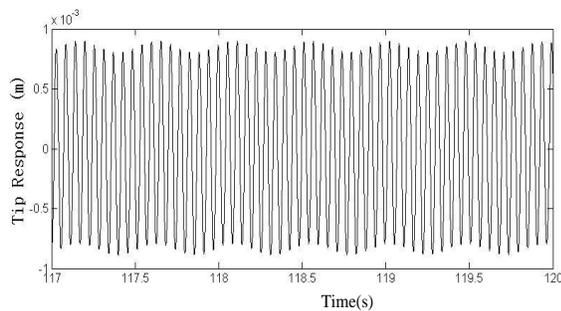
Fig.7
Time response of the tip of the nonlinear beam without piezoelectric properties for the last three seconds.

**Fig.8**

Steady state FFT for the base and the tip of the nonlinear piezoelectric beam.

**Fig.9**

Time response of the tip of the nonlinear piezoelectric beam.

**Fig.10**

Time response of the tip of the nonlinear piezoelectric beam for the last three seconds.

3.3 Numerical results of transient state

The time step equal 0.001sec for the numerical solution. The time of the transient state response is considered 3sec. Fig.11 shows the FFT for the base and the tip of the beam without piezoelectric properties. The figure shows the structural frequencies. Frequency results are presented in Table 5. The result of time response of the nonlinear beam without piezoelectric properties is shown in Fig.12. The greatest displacement of tip of the beam without piezoelectric properties during the 3 second period is 21mm. the figure shows that amplitude modulation is created in the nonlinear beam without piezoelectric property.

Fig.13 shows the FFT for the base and the tip of the piezoelectric beam. Frequency results are presented in Table 6. The result of the time response of the tip of the nonlinear piezoelectric beam is shown in Fig. 14. According to the figure, the greatest displacement of tip of the beam during the 3 second period was 7mm, which shows a decrease in displacement relative to the beam without piezoelectric properties.

Figs.15 and 16 show the voltage response of the base and the tip of the beam. In these figures it is shown that the voltage response in the tip of the piezoelectric beam changes within the limit $[-80, 80]$, while the voltage of the base of the piezoelectric beam changes within the limit $[-60, 60]$. The existing difference could be the result of a change in the slope of the beam.

Fig.17 shows the FFT of the first three seconds and the last three seconds for the piezoelectric beam. Considering the results shown, the natural frequencies of the beam are made clear by the FFT figure for the first three seconds while, in the last three seconds (steady state) only a single frequency is seen.

Table 5

Frequencies results for nonlinear beam without piezoelectric properties.

The first frequency (Hz)	The second frequency (Hz)	The third frequency (Hz)	The fourth frequency (Hz)
0.586	5.47	16.02	32.03

Table 6

Frequencies results for nonlinear piezoelectric beam.

The first frequency (Hz)	The second frequency (Hz)	The third frequency (Hz)	The fourth frequency (Hz)
2.148	12.89	16.21	34.57

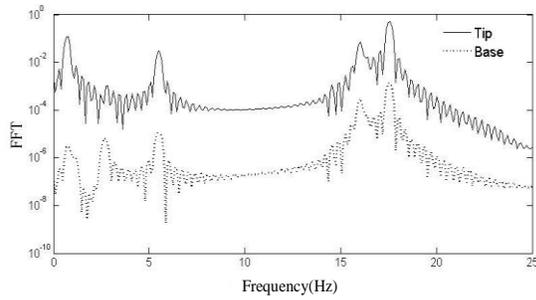


Fig.11
Transient state FFT for the base and the tip of the nonlinear beam without piezoelectric properties.

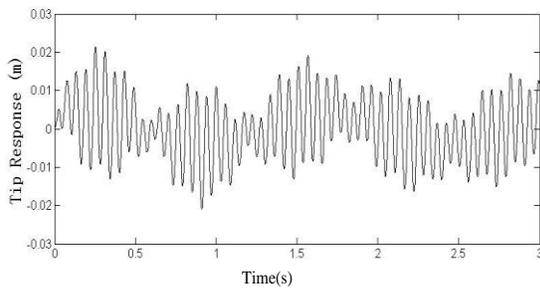


Fig.12
Time response of the tip of the nonlinear beam without piezoelectric properties.

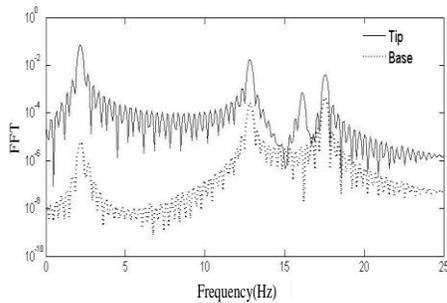


Fig.13
Transient state FFT for the base and the tip of the nonlinear piezoelectric beam.

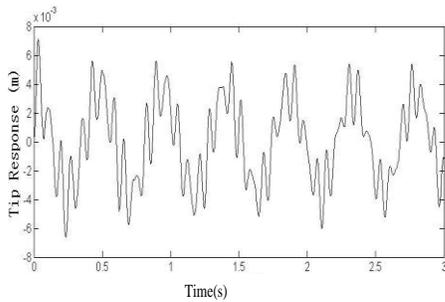


Fig.14
Time response of the tip of the nonlinear piezoelectric beam.

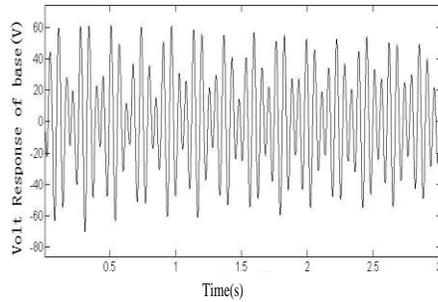


Fig.15
Voltage response for the base of the piezoelectric beam.

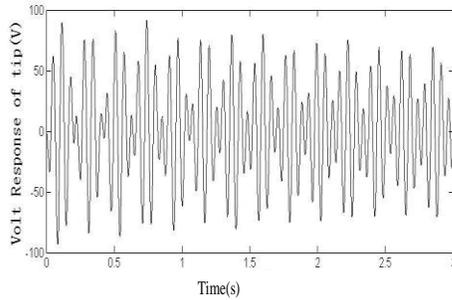


Fig.16
Voltage response of the tip of the piezoelectric beam.

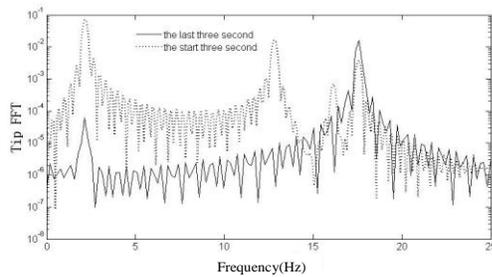


Fig.17
FFT of the first three seconds and the last three seconds for the tip of the piezoelectric beam.

3.4 The effect of function g_1 in the vibration results

The function g_1 is a nonlinear function in the stiffness matrix. The functions g_{new} and g_{old} can be defined for function g_1 . The function g_1 depends on the time, the total number of step and the time step. Comparison of the functions g_{new} and g_{old} is presented in Fig.18. This figure shows that the maximum different between the functions g_{new} and g_{old} is in the base of the beam. The value of functions g_{new} and g_{old} is the same in the tip of the beam.

Fig.19 presents the effect of function g_{new} and g_{old} on the behavior of FFT for a nonlinear piezoelectric beam. Fig. 20 presents the effect of function g_{new} on the behavior of FFT for a nonlinear piezoelectric beam and a beam without piezoelectric properties.

The time response of a piezoelectric beam depends on function g_1 in the numerical method. Fig. 21 presents the effect of functions g_{new} and g_{old} on the time response for a nonlinear beam without piezoelectric properties.

Displacement of the tip of the beam without piezoelectric properties with the function g_{new} represents a 40 percent decrease in comparison to the use of function g_{old} the function g_{new} is the reference solution. The effect of the functions g_{old} and g_{new} has been presented in Table 7. The numerical results in Table 7. show that the function g_1 does not influence frequency results.

Table 7
The results of the natural frequencies for functions g_{old} and g_{new} .

Number of mode	Natural frequency of coupling and undamping beam with function g_{new} (Hz)	Natural frequency of coupling and undamping beam with function g_{old} (Hz)	Natural frequency of coupling and damping beam with function g_{new} (Hz)	Natural frequency of coupling and damping beam with function g_{old} (Hz)
1	2.28	2.29	2.148	2.148
2	13.2	13.21	12.89	12.79
3	16.64	16.65	16.21	16.11
4	35.33	35.34	34.57	34.5

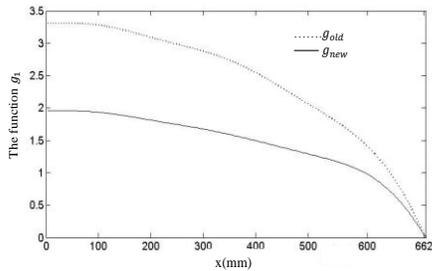


Fig.18
Comparisons of functions g_{new} and g_{old} ($t=3\text{sec}$, time step=0.001, the total number of step=200).

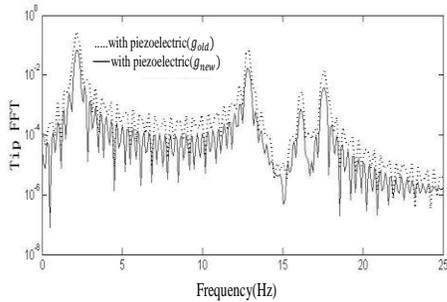


Fig.19
Effect of functions g_{new} and g_{old} on FFT behavior at the tip of the nonlinear piezoelectric beam.

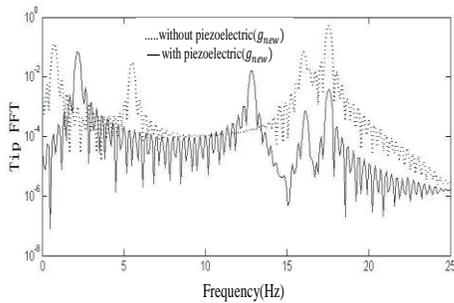


Fig.20
Effect of function g_{new} on FFT behavior at the tip of the nonlinear piezoelectric beam and the beam without piezoelectric properties.

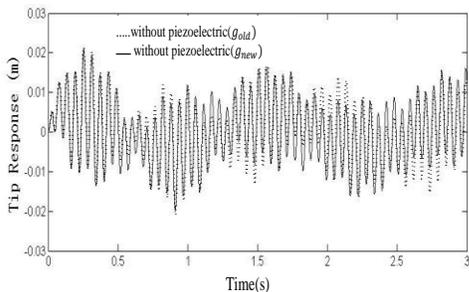


Fig.21
Effect of functions g_{new} and g_{old} on the time response at the tip of the nonlinear beam without piezoelectric properties.

4 THE EXPERIMENTAL RESULTS

Fig.22 shows the structure and dimensions used for the test. The beam was excited by a harmonic load at the base of beam. An accelerometer was installed to measure input excitation at the base. A strain gage was attached near the base of the beam to show time response. The base of beam was excited by Eq.(26).

Malatkar applied an excitation frequency close to the third natural frequency of the structure, so it is assumed to be 17.54Hz [15]. Table 8. shows the experimental natural frequencies and damping ratios for a horizontal beam. The natural frequencies of a horizontal beam are lower than those of a vertical beam (effect of gravity).

Fig.23 presents FFT response, and frequency values can be obtained from this. The high frequency value is at 17.57 (excitation frequency). This diagram shows that the third mode frequency component is modulated [15]. The low frequency value is 1.58Hz.

Table 8

Experimental natural frequencies and damping ratios.

Mode number	Natural frequency(Hz)	damping ratio
1	0.574	0.009
2	5.727	0.00185
3	16.55	0.00225
4	32.67	0.005

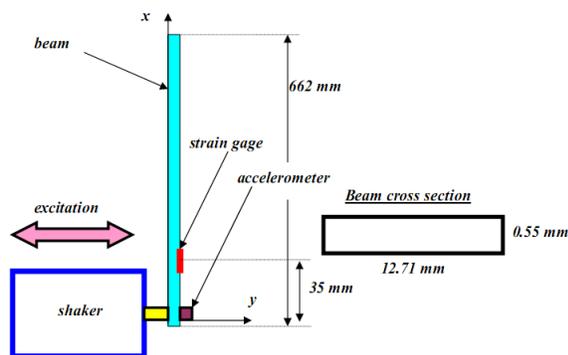


Fig.22
Structure and experimental set up used by Malatkar for test.

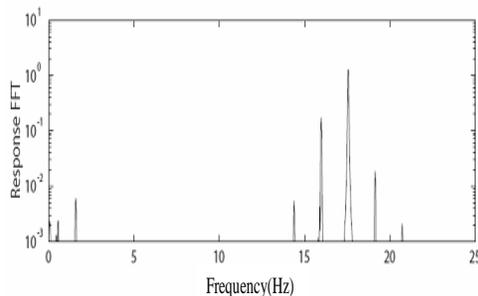


Fig.23
FFT response from experimental result.

5 CONCLUSIONS

A finite element program for analyzing the nonlinear vibration behavior of a piezoelectric beam has been presented in this paper. The time response for a coupling and uncoupling beam is obtained during both steady and transient states. The Newmark technique for dynamic analysis, and the Newton-Raphson iterative and Simpson methods for nonlinear solution were used. The results of the time response and FFT were obtained.

The results were compared to the experimental results of Malatkar and the numerical results of Delgado for a nonlinear beam without piezoelectric properties.

Numerical results showed that the presence of piezoelectric properties affected the increase of the first and second frequency of a nonlinear beam. However, the effect on later modes is less, which can be stated as a 72% increase in the first frequency and a 57% increase in the second frequency.

The existence of a nonlinear state affects the frequency results and has the maximum of increasing of the frequency value by 23.6% in comparison to the linear state in the second mode.

The existence of piezoelectric properties affects the time response of the beam and increases stiffness, therefore decreasing displacement.

Existence of vibration produces periodic voltage. The voltage response in the tip of the piezoelectric beam is different in comparison to the voltage of the base of the piezoelectric beam. The existing difference could be the result of a change in the slope of the beam.

The numerical results of the time response showed that the presence of piezoelectric properties caused an increase in the global stiffness value. This is the reason for the reduction in the displacement of the piezoelectric beam in comparison to the beam without piezoelectric properties.

Comparisons of the time responses of the coupling and uncoupling beams indicate that the presence of piezoelectric properties effected the time period of the steady state. The time it took for the piezoelectric beam to reach a steady state was greater than that for a beam without piezoelectric properties.

Comparisons of the FFT of the first three seconds and the last three seconds show that the frequency results are different. So, the frequency results for a nonlinear beam depend on the amount of time considered.

Function g_1 is one of the parameters that is used for presenting the nonlinear conditions of a beam, so in the NPBDA program, function g_1 is determined by the function four order. The results showed that this function does not affect the frequency results but can influence time response. The function g_{new} is preferred to be used in the solution.

The numerical results showed that the FFT of the nonlinear piezoelectric beam changed. Thus, the frequency results of an uncoupling beam and a piezoelectric beam are different.

The results show that amplitude modulation was created in the nonlinear beam without piezoelectric properties, while the presence of piezoelectric properties eliminated this.

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