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Evaluation of Buckling and Post Buckling of Variable Thickness Shell Subjected to External Hydrostatic Pressure

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ABSTRACT

Buckling and post buckling of cylindrical shells under hydrostatic pressure is regarded as important issue in structure of submarines. These cylindrical shells have variable thickness due to construction process which effected by pressure of buckling and its destruction. In this paper, effects of changing thickness on buckling and destruction pressure under external hydrostatic pressure of a shell are studied. Results of buckling pressure of cylindrical shell have been obtained with theoretical relations and finite element method. Then, using machining process a sample of cylindrical shell with variable thickness has been produced. Buckling pressure and post buckling of the constructed sample have been obtained with the reservoir under closed-ended hydrostatic pressure. Changes of the test sample size have been considered with closed-ended testing apparatuses which are used for new evaluation of buckling. In this research, results of the pressure have been obtained in terms of the volume change. At the end, results of the finite element method have been compared with results of the analytical solutions and experimental data. Results show that the shell with variable thickness has buckling pressure close to shell bucking pressure with mean thickness.

Keywords : External pressure; Buckling, Cylindrical shell; Variable thickness; Post buckling.

1 INTRODUCTION

BUCKLING of the shells has been tested up to now with different methods. Goal of all methods is to establish more suitable relationship between analytical and experimental results. However, there is difference between test results and predictions [1-6]. This difference is due to different factors such as geometry of shell, border conditions and construction faults which lead to difference between buckling behavior of the mathematical and test model [7,8]. Regarding hydrostatic pressure, different methods have been performed up to now. Ross and Sadler [9] used open-ended testing apparatus to test buckling. In such apparatus, it is possible to observe buckling or install displacement measure, strain gage or camera but it is difficult to control released energy at time of collapse. To solve this problem, Blashot [10] used closed-ended testing apparatus to reduce sudden destructive effects by filling the fluid inside the test sample. However, deformations in experimental results will be from the stiffness effects of the test sample and fluid inside it. Also, McKee [11] presented control volume testing apparatus. Control volume apparatus allows user to study border of buckling more carefully. The fluid inside the test is connected to fluid of the testing apparatus with help of a tube and the connection can be broken with help of a control valve. A relief valve

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has been embedded in this tube to discharge fluid inside the test sample. In the test method, pressure of the sample and apparatus increases and relative pressure will be created when water of the test sample is discharged. However, it is necessary to observe some cases for performing test with the control volume apparatus. For example, pressure of the apparatus should be higher than the expected pressure for buckling. This additional pressure is dependent on volume and stiffness of the sample and apparatus. In this method, controllable parameter is volume of the outlet water from the test sample relative to volume of inlet water to the apparatus. High volume difference between the sample and apparatus will cause better control, so using this method is not suggested when volume of the test sample and testing apparatus is close to each other. Analytical evaluation of the buckling of cylindrical shells under external hydrostatic pressure is one of the issues which have been considered by researchers. Gosick et al. [12] studied effect of changing peripheral thickness on buckling of cylindrical shell under hydrostatic pressure. Jouve and Fat [13] studied analytical solution of elastic buckling of a non-uniform long cylindrical shell under external hydrostatic pressure. Their studied sample had fixed thickness in longitudinal direction and had two thicknesses in peripheral direction. They proved in their research that buckling pressure will decrease when the area of low thickness or low thickness to high thickness ratio increases. Eksogan et al. [14] analytically studied dynamical buckling of the cylindrical shell with variable thickness under time–dependent uniform external pressure. At first, they used Dannel's equations for dynamical buckling and then the equations were reduced to time-dependent differential equations with Galerkin Method. Also static critical load, dynamical load and the number of corresponding wave were obtained with Ritz method. At last, the obtained results were compared in case of fixed thickness. They had not any experimental study on their results. Aghajari et al. [15] studied buckling and post buckling behavior of the thin-walled cylindrical shell with variable thickness under hydrostatic pressure and evaluated their results with finite element method (FEM). In their research, thickness of shell changed discretely and had two defined values. Deformation of the sample was evaluated with radial displacement measure and point method. However, one of the cases which attracted attention of researchers in recent years is study of buckling behavior of shells with variable thickness. Novin et al. [16] studied cylindrical shell with variable thickness under external pressure. They used theory of thin shells and Bubnov–Galerkin method to solve equations. The major difference between their method and other methods is extraction of new relation for the buckling load and numerical results for shell with simple supported under external pressure. However, the experimental evaluation of the obtained results has not been done and the results have been validated in fixed thickness. Lapatin and Morozov [17] studied analytical relations of a cantilever composite cylindrical shell under lateral uniform pressure. They used Galerkin method to solve their equations and validated their results with FEM. However, their studied sample had fixed thickness in peripheral, radial and longitudinal directions. Chenet al. [18] studied buckling of cylindrical shell with variable wall thickness in each cross section under uniform external pressure. They presented a new method for evaluation of buckling of such shells. In their method, buckling of cylindrical shell with fixed thickness replaces buckling of shell with variable thickness to evaluate buckling of such shells, equivalent thickness and equivalent length. However, change of thickness in their research is stepwise and is not continuous.

This research studies buckling and post buckling of the cylindrical shell with variable thickness in longitudinal direction under external hydrostatic pressure. First, the results obtained from the analytical relations have been studied and buckling pressure of the sample model is obtained with finite element analysis. Then, buckling and post buckling experiments have been performed on cylindrical shell with variable thickness. These experiments conclude evaluation of sample size changes and external pressure which have been conducted with closed-ended testing apparatus. At the end, the results obtained from buckling test are compared with analytical results and finite element method.

2 ANALYTICAL SOLUTION

Ratio of the buckling pressure of model with variable thickness to the model with nominal thickness is shown with λ and according to relation (1), it is equal to [17]:

$$\begin{aligned}\lambda = 1-\varepsilon[8(\pi^2+n^{10}(-1+n^2)x_1^{12}+n^2\pi^{10}x_1^2(5+\nu)+n^8\pi^2x_1^{10}(-4+n^2(5+\nu))+2n^2\pi^6x_1^6\\ \times(-2n^2+2y_1^2(-1+\nu)^2(1+\nu)+n^4(7+3\nu))+n^4\pi^4x_1^8(-6n^2+n^4(11+4\nu)-4y_1^2(-1+\nu^2))\\ +\pi^8x_1^4(-n^2+n^4(11+4\nu)-4y_1^2(-1+\nu^2)))]/\times(\pi(\pi^2+n^2x_1^2)^2(\pi^8+4n^2\pi^6x_1^2+2n^4\\ \times(-1+2n^2)\pi^2x_1^6n^6(-1+n^2)x_1^8+\pi^4x_1^4(-n^2+6n^4-12y_1^2(-1+\nu^2))))\end{aligned} \tag{1}$$

Parameters x_1 and y_1 are equal to:

$$x_1 = \frac{L}{R}\ ;\ y_1 = \frac{R}{h_0} \tag{2}$$

In Eqs. (1) and (2), R is radius, L is length and h_0 is nominal thickness of the shell and v is Poisson' ratio, ε is dimensionless parameter of the thickness change and n is the number of peripheral buckling waves. In this section, the test shell sample is introduced to calculate ratio of λ and compare the value obtained from Eq. (1) with results of finite element method. Geometry and dimensions of the cross section of this model are shown in Fig. 1.

It is necessary to note that two flat knobs with thickness of 15 *mm* have been used to fasten the reservoir. Considering dimensions of the geometry, value λ is obtained equal to 0.69 according to Fig. 1. In this model, the thickness varies from 1.53 *mm* to 2 *mm* according to experimental model.

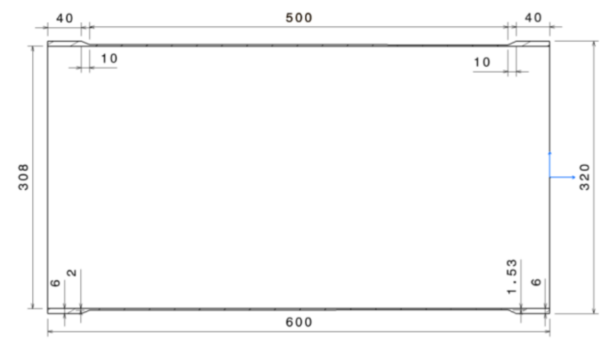

Fig.1
Geometry and dimensions of the model cross section.

3 FINITE ELEMENT ANALYSIS

The initial model studied in laboratory test process of cylindrical shells buckling has been implemented in ANSYS commercial finite element software and analyzed. The reservoir has been loaded in FEM for analysis based on sample loading process to simulate loading steps statically. In order to considering static analysis, loading with time step of 30 s has been considered according to Table 1.

Table 1
Hydrostatic loading steps on test model.

row	Time (s)	Hydrostatic pressure load (bar)
1	0	0
2	30	1
3	60	2
4	90	3
5	120	4
6	150	5
7	180	6
8	210	7
9	240	7.2
10	270	7.4
11	300	7.6
12	330	7.8
13	360	8
14	390	8.2
15	420	8.4

Loading conditions in all analyses have been considered the same to study for this sample. Support conditions in finite element analysis are three nodes with angles of 0, 90 and 270 degrees are restrained completely from one end of the model. It is evident that complete restraint means deletion of all displacement and rotation degrees of freedom for these points. Since models with low thickness are usually analyzed with shell elements, this sample has been modeled with Shell181 elements and considering variable thickness in axial direction. Results of analysis have been obtained and compared with solid element. They also show that there are no obvious differences between them. To do this modeling, a cylindrical shell has been modeled according to dimensions of Fig.1 and fastened with two plates in both sides. Considering that the constructed model has variable thickness, a model with variable thickness has been implemented in finite element analysis.

In analysis of buckling load, it is very difficult to predict buckling mode of the shell because buckling mode is highly dependent on border conditions and geometrical faults. When a structure is under pressure or lateral load, it becomes unstable near buckling load. In shell structures, there are many possible conditions near buckling load. The condition of which structure has been deformed is determined with many parameters which can include geometrical faults and material faults. Therefore, this is not a true conclusion that if the predicted buckling mode doesn't match with the experimental results, the predicted buckling load will be meaningless. Of course, this mismatch of mode shape has been studied completely in papers. For example, McKee et al. completely studied buckling test process and its different methods. They mentioned that buckling mode shape is based on finite element and has four waves but ultimate deformation remaining in the model shows only one buckling wave with permanent deformation after lifting [12].

Buckling of the model under hydrostatic loading has been analyzed considering border conditions mentioned above. Considering loading according to Table 1., pressure of buckling is equal to 9.7108 bars and a deformation is equal to 1.05 *mm*. Distribution of total deformation caused by buckling has given in Figs. 2 to 4 in the first three modes. Finite element analyses were done in different modes of solid and shell element analysis and restrained clamped– free boundary condition. Its results have been given in Table 2.

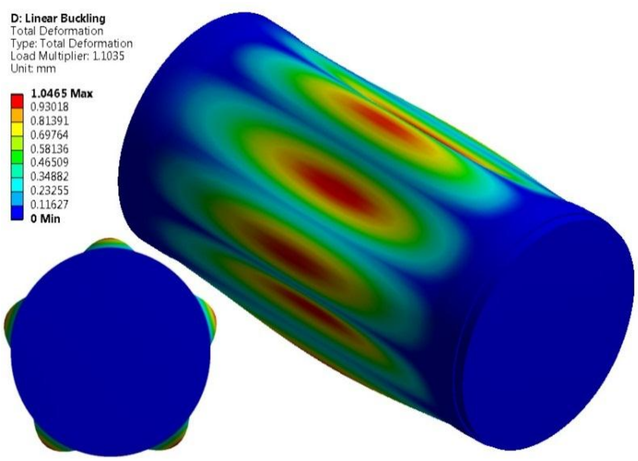

Fig.2
Distribution of total deformation in first mode of buckling.

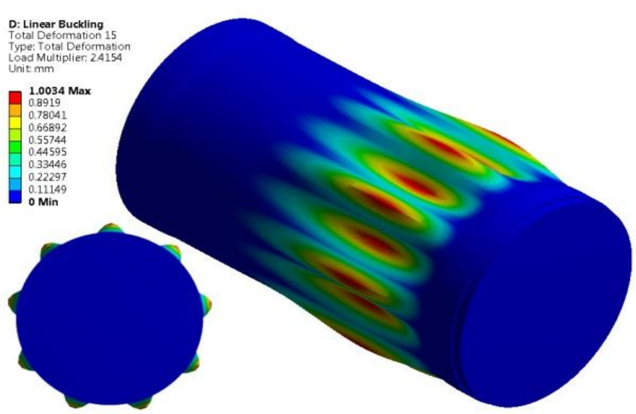

Fig.3
Distribution of total deformation in second mode of buckling.

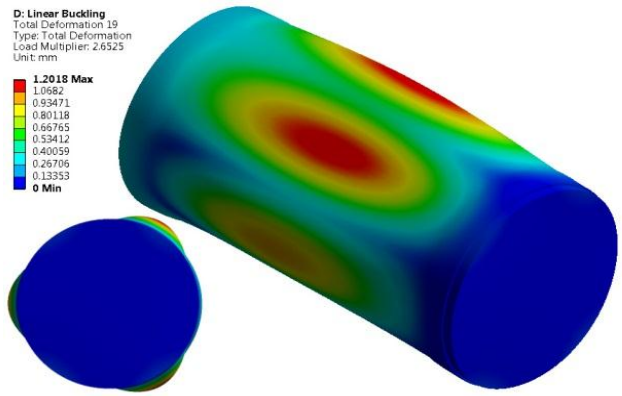

Fig.4
Distribution of total deformation in third mode of buckling.

Table 2
Buckling pressure in shell and solid element with various boundary conditions.

Boundary condition	Solid element	Shell element
Clamped-free	10.5556	10.6242
Clamped-free in three node	9.8076	9.7108

Considering that thickness of shell is variable, buckling has been analyzed in maximum, minimum and mean thicknesses of shell to evaluate buckling of shells with fixed thickness with results of variable thickness shell and its results are given in Table 3.

Table 3
Comparison of buckling pressure in constant and variable thickness.

Thickness (*mm*)	1.53	1.765	2	Variable thickness (from 1.53 to 2)
Buckling pressure (bar)	7.4351	10.1598	13.5828	9.7108
Difference percent in comparison with variable thickness	23.4%	4.6%	39.9%	-

As results shown in Table 3., shell with variable thickness is buckled under pressure which is close to buckling pressure for fixed thickness shell with mean thickness of the shell by variable thickness.

Considering the results of finite element analysis, λ (ratio of buckling pressure of the models with variable to nominal thicknesses of 2 *mm*) is obtained according to Table 4. As this Table shows, difference between result of finite element analysis and the results obtained from Eq. 1 is insignificant which indicates suitable accuracy of the finite element results. Considering coefficient λ, it can be concluded that buckling pressure of the sample with variable thickness is 31% lower than buckling pressure of the sample with fixed thickness (with the maximum thickness of the sample with variable thickness).

Table 4
Buckling coefficient (λ) in theoretical relation and finite element method.

Analytical method	FEM	Error (%)
0.69	0.71	2.8

For post buckling modeling, first the initial imperfection credited by buckling mode shape are implemented in the finite element software, that shown in Fig. 5. After defining boundary conditions, the deformed model has been under pressure and buckling analysis has been conducted again as shown in Fig. 6. The specification of finite element analysis has been defined as a nonlinear analysis. The post buckling manner of the cylinder has been investigated in experimental study and compared with FEM.

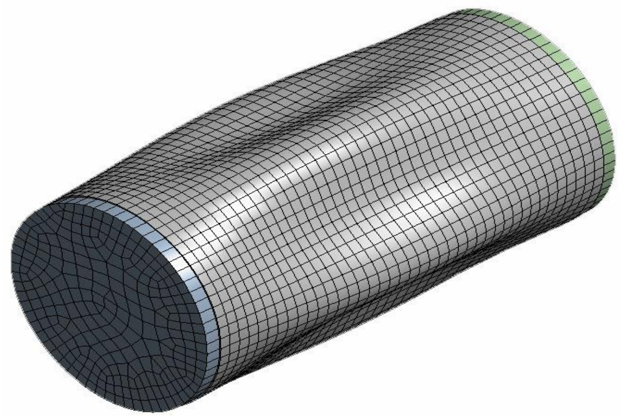

Fig.5
Buckled model for post buckling analysis.

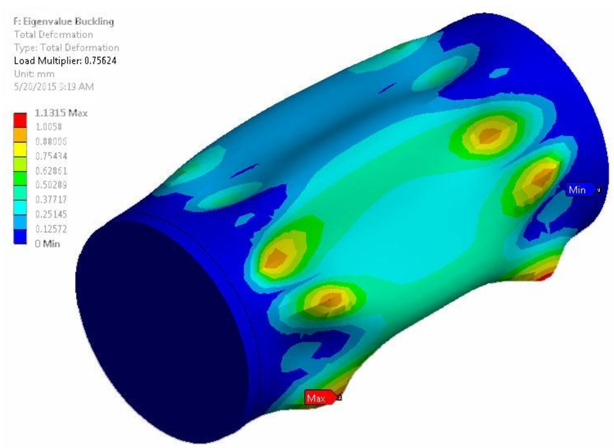

Fig.6
Distribution of total deformation in the post buckling analysis.

4 EXPERIMENTAL STUDY

4.1 Test apparatus

Here consider that there is no significant difference between diameter of the test sample and apparatus, closed-ended buckling method with measurement of the outlet fluid volume from the test sample has been used. To evaluate the buckling moment, the outlet fluid can be controlled by controlling the relief valve in pressure close to buckling pressure. In this method, changes of pressure can be measured and evaluated in terms of change of test sample volume (the discharged fluid volume). In Figs. 7 and 8, external hydrostatic pressure testing apparatus, reservoir pressure gauge, the test sample and relief valve are shown.

Fig.7
Pressure gauge and relief valve.

Fig.8
External hydrostatic pressure testing apparatus.

4.2 Test results
4.2.1 Buckling

In this section, buckling resistance of a cylindrical shell against external hydrostatic pressure was studied. Goal of this test is to compare the results obtained from finite element method with real conditions of cylindrical shells. The test sample has dimensions shown in Fig. 1 and according to Fig. 9. The thickness of shell in experimental model has been measured by ultrasonic thickness meter with $\pm 0.1\ mm$ accuracy. The initial imperfection of the model has effect on buckling analysis. So the variable thickness can be considered as an initial imperfection in such model. Also, out of roundness of manufacturing process in this sample was less than 0.2 *mm*, and can be neglected.

Fig.9
Cylindrical shell before test.

This sample has been tested in two stages of buckling and post buckling. In the first stage which buckling test has been performed, loading is done based on Table 1. and pressure diagram has been obtained in terms of volume change according to Fig. 10. As this Figure shows, volume change of the test sample to pressure of 8.4 bars is linear (elastic deformation) and buckling has occurred in the test sample in pressure of 8.6 bars. Fig.11 shows the test sample after buckling and its buckling mode is observed clearly.

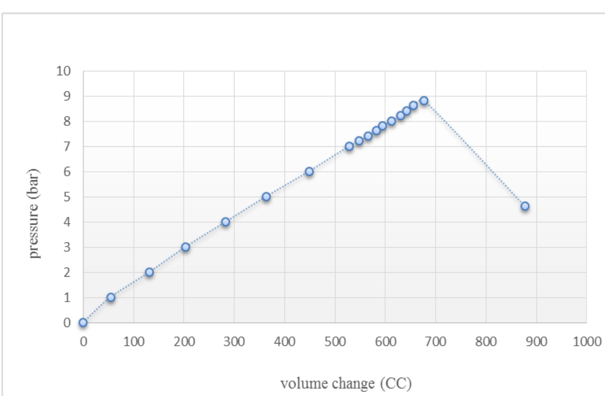

Fig.10
Pressure according to volume change for buckling of model test.

Fig.11
Buckling mode shape after test.

4.2.2 Comparing experimental results and FEM on buckling

By comparing results of finite element analysis with the test results and considering similar loadings, finite element results are confirmed. As shown in Table5., difference between finite element and experiment results is approximately 12% respective of other defects of the model.

Table 5
Buckling pressure in finite element method and model test.

Model test	FEM	Error (%)
8.6	9.7	12

4.2.3 Post buckling

In the next stage, the buckled test sample has been put in the pressure reservoir of the testing apparatus and the external pressure applied on the test sample increases. Diagram of post buckling pressure –volume change of the test sample is shown in Fig. 12.

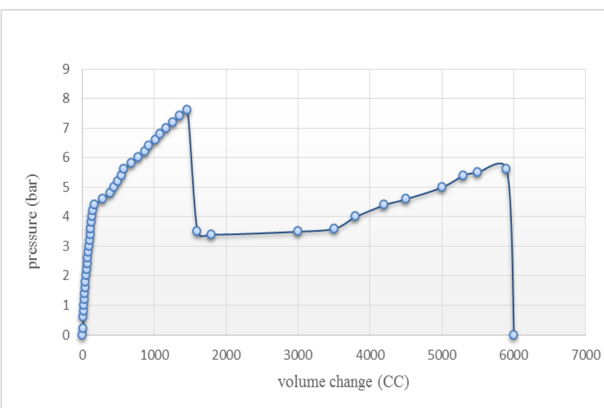

Fig.12
Pressure according to volume change for post bucking of model test.

As shown in Fig. 12, volume change of the test volume up to pressure of 4.4 bars is linear and this pressure is close to the dropped pressure of the testing apparatus reservoir (4.6 bars) and volume changes nonlinearly from pressure of 4.4 bars to 7.6 bars. In this pressure, buckling occurs in the test sample with which other three modes that are created in the test sample and totally four modes are created in environment of the sample. After the second buckling, pressure of the testing apparatus is reduced to 3.3 bars. In this experiment, pressure of the testing apparatus almost remains fixed in this pressure such that plastic strain of the test sample increases with increasing pressure and pressure of the testing apparatus remains fixed. Plastic behavior of the test sample changes almost in pressure of 3.6 bars. Then increasing slope of pressure increases based on strain rate and the test sample is punctured in pressure of 5.6 bars and pressure of the testing apparatus becomes zero. At the end, the test sample is removed from the testing apparatus and shape of the sample after post buckling is shown in Fig. 13.

Fig.13
Model test after collapse.

4.2.4 Comparing experimental results and FEM on post buckling

By comparing results of FEM with the test, and considering similar loadings, finite element results are confirmed. As shown in Table 5., difference between FEM and experiment results is approximately 1.3% respective of other defects of the model. This means that the results of buckling and post buckling in FEM are in good agreement with experiment. Although, the post-buckling behavior is described by the load-deflection diagram and it would not be correct to define it by a certain single value, the result of buckling analysis in the second test step for deformed cylinder of FEM and model test are presented and compared in Table 4.

Table 6
Buckling pressure for deformed cylinder in finite element method and model test.

Model test	FEM	Error (%)
7.6	7.5	1.3

5 CONCLUSIONS

By comparing and studying results obtained from analytical solution, finite element method and experiment the following results are concluded:

1. Considering coefficient λ , it can be concluded that buckling pressure of the sample with variable thickness is 31% lower than buckling pressure of the sample with fixed thickness (with the highest thickness of the sample with variable thickness).
2. Difference between result of finite element analysis and analytical solution in calculation of λ is insignificant which indicates suitable accuracy of the finite element results.
3. By comparing results of buckling test with finite element method, it can be concluded that the support system which has been proposed by ANSYS commercial finite element software gives better results than the common support system (clamp-free supported).
4. Buckling pressure of the sample with variable thickness is close to buckling pressure of the sample with fixed thickness with equivalent thickness with mean of the sample with variable thickness.
5. The volume control test method is not used in the samples with volume close to the testing reservoir volume. In these cases, the closed –ended testing apparatus which is able to control fluid volume of the test sample is suggested.

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