Journal of Solid Mechanics Vol. 5, No. 2 (2013) pp. 206-215

Nonlocal Vibration of Embedded Coupled CNTs Conveying Fluid Under Thermo-Magnetic Fields Via Ritz Method

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Received 8 April 2013; accepted 30 May 2013

ABSTRACT

In this work, nonlocal vibration of double of carbon nanotubes (CNTs) system conveying fluid coupled by visco-Pasternak medium is carried out based on nonlocal elasticity theory where CNTs are placed in uniform temperature change and magnetic field. Considering Euler-Bernoulli beam (EBB) model and Knudsen number, the governing equations of motion are discretized and Ritz method is applied to obtain the frequency of coupled CNTs system. The detailed parametric study is conducted, focusing on the remarkable effects of the Knudsen number, aspect ratio, small scale, thermo-magnetic fields, velocity of conveying fluid and visco-Pasternak medium on the stability of coupled system. The results indicate that magnetic field has significant effect on stability of coupled system. Also, it is found that trend of figures have good agreement with the previous researches. Results of this investigation could be applied for optimum design of nano/micro mechanical devices for controlling stability of coupled systems conveying fluid under thermo-magnetic fields.

Keywords: Vibration; Coupled system; Conveying fluid; Knudsen number; Magnetic field; Visco-Pasternak medium

1 INTRODUCTION

THEORETICAL and experimental studies on one-dimensional nanostructures, such as nanowires and nanotubes, have received much attention since identification of CNTs. Nanotubes have become remarkable because of their great importance in the development of nanodevices. CNTs have demonstrated exceptional mechanical, thermal and electrical properties and the most important features of CNTs are their extremely high stiffness combined with excellent resilience. It has been reported that CNTs possess very high elastic modulus and sustain large elastic strain up to 5% [1].

The dynamical behaviors of micro/nano structures with and without conveying fluid have been widely reported in the literature. It is noted that most nanodevices can be modeled as a beam [2], where the investigating mechanical behaviors of these structures is important in the design of the nanodevices. Ghorbanpour Arani et al. [3] investigated the free transverse vibrations of single walled carbon nanotube (SWCNT) and double walled carbon nanotube (DWCNT) under axial load using the EBB model. Their results showed that for the DWCNTs, the nonlocal theory predictions for the natural frequency are lower than that of the classical theory. In another study, the thermal effect on the buckling of DWCNT resting on a Pasternak foundation using Eringen's nonlocal elasticity theory was studied by Ghorbanpour Arani et al. [4]. They concluded that the strength of a DWCNT was directly related to the Winkler and shear modules of elastic medium. Zhen et al. [5] represented transverse vibration of fluid-conveying DWCNT

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embedded in biological soft tissue based on thermal elasticity theory and nonlocal EBB model. Their results show that the damping parameter of the visco-elastic foundation causes an obvious reduction of the critical flow velocity. Kuang et al. [6] investigated the nonlinear vibrations of DWCNT conveying fluid with considering the nonlinearities of geometry and Van der Waals force. They showed that the effect of geometric nonlinearity on the amplitude frequency properties can be neglected if two types of nonlinearities are simultaneously considered. Surface effects on the free vibration of fluid-conveying nanotubes was presented by Wang [7], who observed that the surface effects with positive elastic constant or positive residual surface tension tend to increase the natural frequency and critical flow velocity. Computational modelling of the fluid flow in CNTs using the EBB theory was studied by Khosravian and Rafii-Tabar [8]. They reported the flow-induced vibrational frequencies in the nanotube were significantly affected by the flow velocity and this, in turn, affected the structural stability of the nanotube, especially at higher flow velocities. Wang et al. [9] analyzed the thermal effect on vibration and instability of CNTs conveying fluid using the same model as [8]. Their results are demonstrated for the dependence of natural frequencies on the flow velocity as well as temperature change.

Murmu et al. [10] reported an analytical approach to study the effect of a longitudinal magnetic field on the transverse vibration of a magnetically sensitive DWCNT based on nonlocal elasticity theory. Results revealed that presence of a longitudinal magnetic field increases the natural frequencies of the DWCNT. In another study Murmu and Pradhan [11] investigated the thermal vibration of SWCNT based on thermal elasticity mechanics, and nonlocal elasticity theory. They studied the influence of small scale effects, temperature change, Winkler constant and vibration modes of CNT on the natural frequency. Recently, Murmu and Adhikari [12] investigated nonlocal vibration analysis of double nanobeam systems and the governing equations of motion for EBB model in terms of displacements. They solved its coupled equations by the new analytical method to decouple the set of partial differential equations and they showed that small scale parameters and stiffness of the coupling springs have important role in stability of double nanobeam system.

However, to date, no report has been found in the literature on the vibration analysis of coupled system of CNTs conveying fluid embedded in visco-elastic medium subjected to thermo-magnetic fields. Motivated by these considerations, this study aims to study the vibration analysis of double CNTs system based on EBB theory, where one of the CNTs is considered conveying fluid and coupled system is placed in uniform temperature. CNTs are simulated by EBB model and they have been coupled together with visco-Pasternak medium. Ritz method is applied to obtain characteristic parameters of coupled system. The results of this study is hoped to be use to design this kind of nano devices.

2 FUNDAMENTAL EQUATION

Fig. 1 illustrates two CNTs are coupled with visco-Pasternak foundation and subjected to uniform longitudinal magnetic field. The upper nanotube is conveying fluid and vibration of coupled system is investigated using EBB model, where the displacement fields are expressed as [13]:

$$\begin{cases} \overline{u}_i(x,z,t) = u_i(x,t) - z\dfrac{\partial w_i}{\partial x}, \\ \overline{v}_i(x,z,t) = 0, \qquad\qquad i = 1,2 \\ \overline{w}_i(x,z,t) = w_i(x,t), \end{cases} \tag{1}$$

where t is time, u_i and w_i are the total displacements of the i^{th} CNT along the x and z coordinate directions, respectively, $\overline{u}_i$ and $\overline{w}_i$ denote the axial and transverse displacements of the i^{th} CNT. It is also noted that $i = 1,2$ represent the upper and lower CNTs, respectively.

Using Eq. (1), the strain–displacement relation can be written as:

$$\varepsilon_{xx}(x,z) = \frac{\partial \tilde{U}}{\partial x} + \frac{1}{2}\left(\frac{\partial W}{\partial x}\right)^2 \quad , \quad \varepsilon_{zz}(x,z) = 0 \quad , \quad \gamma_{xz}(x,z) = 0, \tag{2}$$

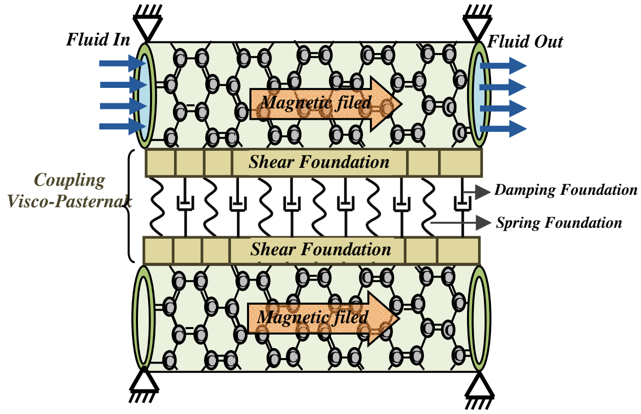

Fig. 1
The schematic of double CNTs which is coupled by visco-Pasternak medium under magnetic field..

According to the Eringen's nonlocal elasticity model [14], the stress state at a reference point in the body is regarded to be dependent not only on the strain state at this point but also on the strain states at all of the points throughout the body. On the other contract, at the local elasticity theory, the stress state at any point corresponds to the strain state at this point. The constitutive equations of the nonlocal elasticity can be considered as:

$$(1-(e_0 a)^2 \nabla^2)\sigma = \tau, \tag{3}$$

where the right hand of Eq. (3) denote the classical stress and $e_0 a$ is a constant parameter showing the small scale effect. In the present model, the normal stress σ_z, the corresponding strain ε_z and the shear strains ε_{xz} and $\varepsilon_{\theta Z}$ are assumed to be negligible. Using Eq. (3) and applying the above assumptions, the following equation is obtained:

$$\sigma_x - (e_0 a)^2 \nabla^2 \sigma_x = E\varepsilon_x - \alpha_x T. \tag{4}$$

where E, α_x denotes Young modulus and thermal expansion coefficient in x direction, respectively. The total potential energy of the coupled system considering the thermal field is defined as:

$$U = \frac{1}{2}\int_0^L \int_{A_t} \sigma_{xx} \varepsilon_x \, dx dA_i = \frac{1}{2}\int_0^L \int_{A_t} (E\varepsilon_x - \alpha_x T)\, \varepsilon_x \, dx dA_i . \tag{5}$$

where A_i represents the cross section area of CNTs. The kinetic energy of the CNTs and fluid flow are [15]:

$$K_{tube} = \frac{1}{2}\rho_t \int_0^L \left\{ \int_{A_i} \left[(\frac{\partial \tilde{U}_i}{\partial x})^2 + (\frac{\partial \tilde{W}_i}{\partial x})^2 \right] \right\} dA_i \, dx \qquad i = 1,2$$

$$K_{fluid} = \frac{1}{2}\rho_f \int_0^L \left\{ \int_{A_f} \left[(\frac{\partial W_1}{\partial t} + u_f \frac{\partial W_1}{\partial x})^2 + {u_f}^2 \right] \right\} dA_f \, dx, \tag{6}$$

where ρ_t, ρ_f denote the density of CNTs and fluid, respectively.

Based on the visco-Pasternak foundations, the effects of the surrounding elastic medium on the nanotubes are considered as follows:

$$F_1 = k_w(W_2 - W_1) - k_G \frac{\partial^2}{\partial x^2}(W_2 - W_1) + C_d \frac{\partial}{\partial t}(W_2 - W_1),$$
$$F_2 = k_w(W_1 - W_2) - k_G \frac{\partial^2}{\partial x^2}(W_1 - W_2) + C_d \frac{\partial}{\partial t}(W_1 - W_2), \tag{7}$$

where k_w, k_G and C_d are spring, shear and damping modulus, respectively. F_1 denotes the external forces applied on upper nanotube and F_2 is applied force on lower nanotube. The external work due to surrounding elastic medium can be written as:

$$W_i^{Elastic\,medium} = \frac{1}{2}\int_0^L F_i W_i \, dx, \tag{8}$$

To achieve the effect of magnetic field on couple CNTs system, the Maxwell's relations can be used as [10]:

$$\vec{J} = \nabla \times \vec{h}, \quad \nabla \times \vec{e} = -\mu \frac{\partial \vec{h}}{\partial t}, \quad div\, \vec{h} = 0,$$
$$\vec{e} = -\mu\left(\frac{\partial \vec{U}}{\partial t} \times \vec{H}\right), \quad \vec{h} = \nabla \times (\vec{U} \times \vec{H}) \tag{9}$$

where $\vec{U}$ is the displacement vector as $\vec{U} = (u, v, w)$, $\vec{J}$ as current density, $\vec{h}$ as distributing vector of the magnetic field, and $\vec{e}$ as strength vectors of the electric field. Applying a uniform longitudinal magnetic field vector $\vec{H} = (H_x, 0, 0)$ on coupled system, the Lorentz force can be expressed as [10]:

$$f_x = 0$$
$$f_y = \mu H_x^2 \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial y \partial z}\right)$$
$$f_z = \mu H_x^2 \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 v}{\partial y \partial z}\right) \tag{10}$$

For the present vibrational analysis in coupled CNTs, assumed that $w = w(x,t)$ only, so that the Lorentz force in the z direction is written as:

$$f_{zi} = \mu H_x^2 \frac{\partial^2 w_i}{\partial x^2} \tag{11}$$

It should be noted that in the present study the effective Lorentz force is a function of magnetic permeability (μ) and (H_x) also. For solving the problem of micro-nano flows a parameter which is called Knudsen number (Kn) is introduced. Knudsen number is a parameter defined as the ratio of the mean-free-path of the molecules to a characteristic length scale which is used for identifying the various flow regimes. For micro and nanotubes, the radius of the tube is assumed as the characteristic length scale.

According to the Knudsen number the classification of the various flow regimes is given as: continuum flow regime ($0 < Kn < 10^{-2}$), slip flow regime ($10^{-2} < Kn < 10^{-1}$), transition flow regime ($10^{-1} < Kn < 10$), free molecular flow regime ($Kn > 10$) [16]. For CNT conveying fluid, Knudsen number may be larger than 10^{-2}. Therefore, the assumption of no-slip boundary conditions is no longer credible, and a modified model should be used. So $V_{avg,slip}$ is replaced by $VCF \times V_{avg,(no-slip)}$ in the basic equations where it is determined as follows [17]:

$$VCF \equiv \frac{V_{avg,slip}}{V_{avg,(no-slip)}} = (1+aKn)\left[1+4\left(\frac{2-\sigma_v}{\sigma_v}\right)\left(\frac{Kn}{1+Kn}\right)\right] \tag{12}$$

where σ_v is tangential momentum accommodation coefficient. For most practical applications σ_v is chosen to be 0.7 and a can be expressed as the following relation:

$$a = a_0 \frac{2}{\pi}\left[\tan^{-1}\left(a_1 Kn^B\right)\right] \tag{13}$$

In which, $a_1 = 4$ and $B = 0.04$, are some experimental parameters. The coefficient a_0 is formulated as [17]:

$$\lim_{Kn\to\infty} a = a_0 = \frac{64}{3\pi\left(1-\frac{4}{b}\right)} \tag{14}$$

where $b = -1$.

3 SOLUTION PROCEDURE

The geometric boundary conditions of simply supported CNTs at two ends are:

$$u(x,t) = w(x,t) = 0, \quad at \quad x = 0, L \tag{15}$$

According to basic Ritz method, the displacement component u and w can be expanded by following expression (trial functions), which satisfy the geometric boundary conditions of the coupled system at both ends [18]:

$$\begin{aligned} u &= \sum_{i=1}^{N} A_i(t)\, x^i (1-x), \\ w &= \sum_{i=1}^{N} B_i(t)\, x^i (1-x). \end{aligned} \tag{16}$$

where $A_i(t)$ and $B_i(t)$ are unknown functions of time and N is the summation integer that can be increased to obtained maximum accuracy and acceptable results.

The vector $\{q\}$, which is containing the displacement vector ($\{q_d\}$), is introduced as $\{q\} = \left[\{q_d\}\right]^T$, where $\{q_d\} = \left[\{A_i\},\{B_i\}\right]$. Hence, the dimension of $\{q_d\}$ is $2N \times 1$ and may be explained as the number of degrees of freedom (DOF) of the beam used in the modal expansions. In this study, the discretized equations of motion are directly obtained by minimizing the energy of the system. Hence, the Lagrange equations of motion are expressed as: [19]

$$\frac{d}{dt}\left[\frac{\partial(K_t + K_f)}{\partial \dot{q}_k}\right] - \frac{\partial(K_t + K_f)}{\partial q_k} + \frac{\partial U_t}{\partial q_k} = Q_k, \; k = 1, ..., 2N \tag{17}$$

where Q are the generalized forces obtained by differentiation of the virtual work W done by external forces:

$$Q_i = -\frac{\partial W}{\partial q_i}. \tag{18}$$

Using mode expansion (16), all the terms in Eq. (17) are evaluated and a system of discretized governing equations is obtained in matrix form as follows:

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = \{0\} \tag{19}$$

where $[M]$ and $[C]$ are matrices of mass and damping, respectively and $[K]$ is stiffness matrix that contains linear and nonlinear terms as:

$$[K] = [K]^{Linear} + [K]^{Nonlinear} \tag{20}$$

where $[K]^{nonlinear}$ have small value and can be neglected. The general solution of Eq. (19) can be written as:

$$\{q_d\} = \{\hat{q}_d\}\exp(\Omega t) \tag{21}$$

where Ω is complex circular frequency containing imaginary and real parts denoting natural and damping frequencies, respectively and $\{\hat{q}_d\}$ is amplitude of the displacements vector. Substituting Eq. (21) in Eq. (19), yields

$$\left([M_{dd}]\Omega^2 + [D_{dd}]\Omega + [K_m]\right)\{\hat{q}_d\} = \{0\} \tag{22}$$

In order to obtain complex frequency of coupled conveying fluid system, should be used an eigenvalue procedure from Eq. (22).

4 NUMERICAL RESULTS AND DISCUSSION

In this section, effects of parameters such as aspect ratios (L/d), temperature gradient (ΔT), Knudsen number, elastic medium on frequency versus fluid velocity (u_f) of the simply support double CNTs are shown in Figs. 2 to 9. It is noted that $\mathrm{Im}(\omega)$ represents the resonance frequencies of the double CNTs. Mechanical and geometrical properties of the CNTs are considered as [21]:

$$r = 11.43\,nm \quad h = 0.075\,nm \quad L/r = 10 \quad \upsilon = 0.34 \quad \alpha_x = 1.2\times10^{-6} \quad E = 70Gpa$$
$$\rho = 2700\,k\,gr/m^3 \tag{23}$$

Fig.2 illustrates fundamental frequency changes ($\mathrm{Im}(\omega)$) of CNTs versus flow velocity (u_f) in three different mode. The curves demonstrate the frequency changes of upper CNT conveying fluid and the straight lines related to lower CNT which is without fluid. This figure physically implies that due to eliminating visco-Pasternak foundation that is coupled CNTs, both CNTs vibrate separately. As can be seen, $\mathrm{Im}(\omega)$ decreases with increasing velocity of fluid. For zero resonance frequency, coupled system becomes unstable due to the divergence via a pitchfork bifurcation and the corresponding fluid velocity is called the critical flow velocity. Therefore, with increasing flow velocity, system stability decreases and became susceptible to buckling.Fig.3 shows the fundamental frequency changes versus flow velocity for coupled and uncoupled system. It is found from this figure that existence of visco-Pasternak medium which linked two CNTs, enlarge the stability region of system and increase the resonance frequency. This point is valid for all modes of system. The effect of elastic medium on fundamental frequency versus fluid velocity presented at Fig. 4. It is obvious that existence of spring and shear foundations enlarge the stability region of coupled CNTs system and increase the resonance frequency. Also, the frequency of Winkler and Pasternak mediums are maximum and minimum, respectively and visco-Pasternak case located between. As can be seen, $\mathrm{Im}(\omega)$ increases by increasing the elastic foundation stiffness and decrease as u_f increases.Fig.5 depicts fundamental frequency versus flow velocity for different values of small scale. It is obvious that nonlocal parameter is significant parameter in vibration of coupled system. As can be seen increasing the nonlocal parameter decreases the frequency and critical flow velocity. It is need to point out that, the zero value for nonlocal parameter (i.e. $e_0a = 0$) denotes the result obtained by the classic EBB model which has the highest frequency and critical fluid velocity. Fig. 6 shows the effect of magnetic field on fundamental frequency of coupledsystem. As already has been mentioned applying magnetic field in axial direction generate the force in radial direction call Lorentz force. It is found that increasing magnetic intensity (H_z) increase frequency and critical flow velocity. Regarding Lorentz force effect, it has been concluded that the magnetic field is basically an effective factor on increasing resonance frequency leading to stability of coupled CNTs system.Fig.7 demonstrates the effect of aspect ratio (L/d) on the imaginary components of fundamental frequency versus fundamental fluid velocity. It is evident that resonance frequency of the coupled CNTs system increase with decreasing L/d. In addition, as L/d decreases, the critical flow velocity increases. Therefore the low aspect ratio should be taken into account for CNT in optimum design of nano/micro devices. Temperature is one of the topics discussed in this study in which Fig. 8 illustrates the imaginary component of fundamental frequency versus flow velocity for different values of temperature change. It is found from this figure that the frequency and critical flow velocity decrease with increasing temperature change because a larger temperature change reduce the coupled system stiffness. Knudsen number is defined based on various flow regimes. Here, the slip flow regime is considered, where Fig. 9 illustrates the imaginary parts of the fundamental frequency versus flow velocity for three values of Knudsen number. As can be observed from this figure, $\mathrm{Im}(\omega)$ and critical flow velocity of coupled CNTs system decrease with increasing Knudsen number, where the small Knudsen number can be has remarkable effect on the stability region of system.

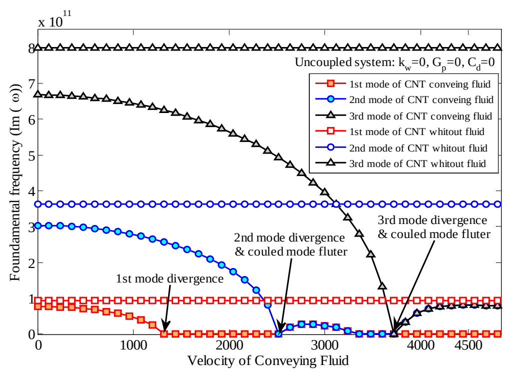

Fig. 2
Fundamental frequency for CNTs with or without fluid for different vibration modes.

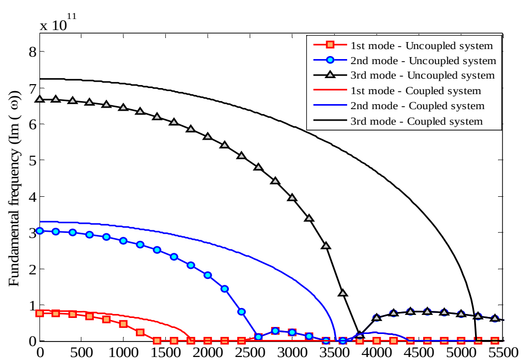

Fig. 3
Effect of vibration modes on fundamental frequency of coupled and uncoupled system.

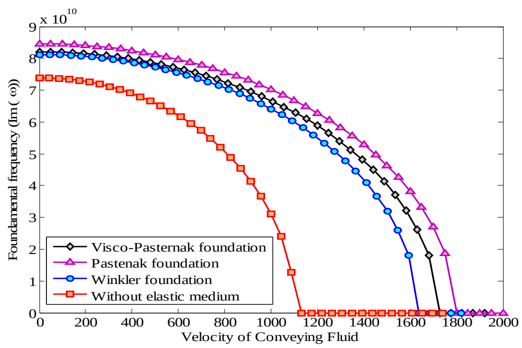

Fig. 4
Effect of elastic medium on the fundamental frequency for different values of flow velocity.

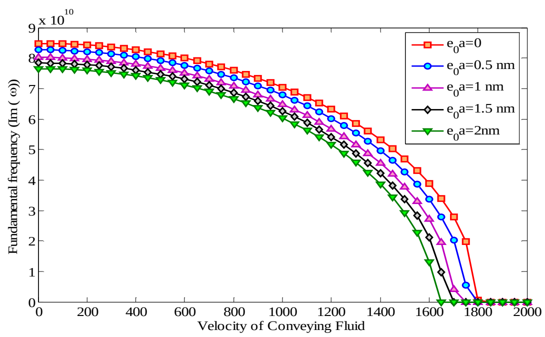

Fig. 5
Fundamental frequency (Im(ω)) versus flow velocity for different values of small scale parameter e_0a .

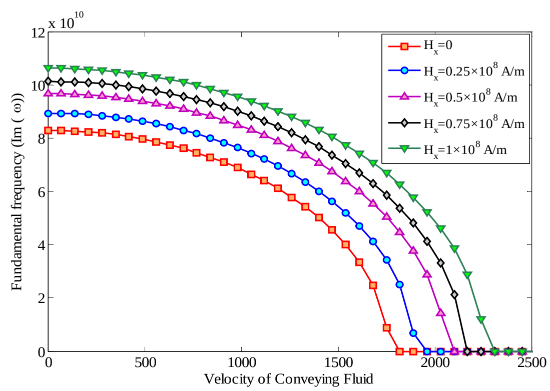

Fig.6
Effect of magnetic field on fundamental frequency of coupled system.

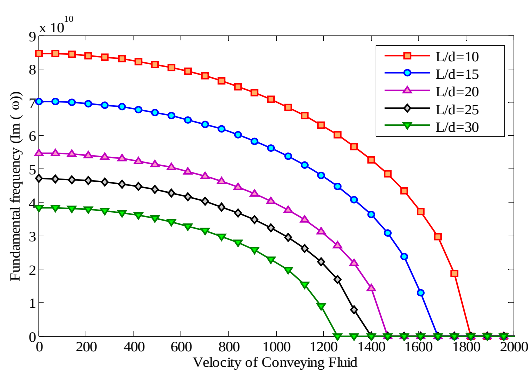

Fig. 7
Fundamental frequency (Im(ω)) versus flow velocity for various aspect ratios (L / d) .

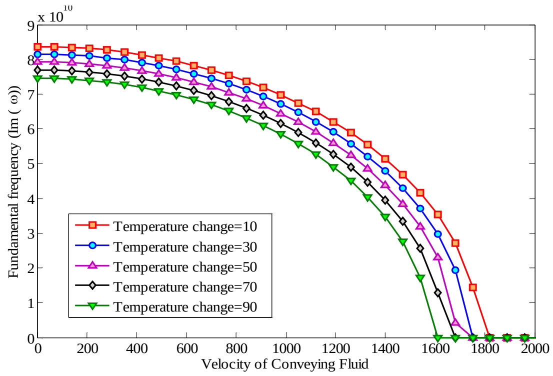

Fig. 8
Effect of temperature changes on fundamental frequency (Im(ω)) versus flow velocity.

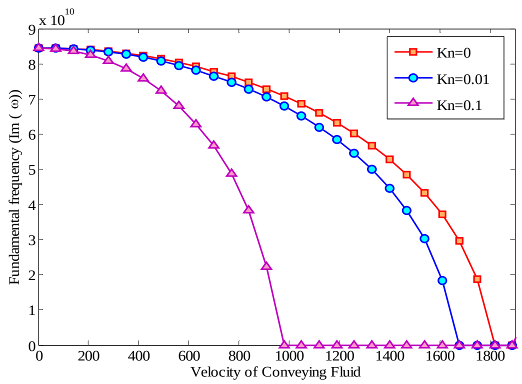

Fig. 9
The effect of Knudsen number on fundamental frequency (Im(ω)) .

5 CONCLUSION

In this study, general theoretical vibration analysis of coupled CNTs system subjected to thermo-magnetic fields was developed, where one of nanotubes is conveying fluid. The CNTs linked by visco-Pasternak medium which is considering damping and shear effects. Based on EBB theory fundamental equations evaluated and were solved by Ritz approach. Regarding CNTs conveying fluid, it can be observed that Knudsen number have significant effects on the mechanical behavior of the system. Also, it has been found that the magnitude of fundamental frequency is strongly dependent on the imposed magnetic field so that increasing the magnetic intensity significantly increses the critical flow velocity. The findings of present study may be used in advanced applications of this kind of nano/micro mechanical devices.

ACKNOWLEDGMENTS

The authors are grateful to University of Kashan for supporting this work by Grant No. 65475/59.

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