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Rayleigh Surface Wave Propagation in Transversely Isotropic Medium with Three-Phase-Lag Model

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ABSTRACT

The present paper is dealing with the propagation of Rayleigh surface waves in a homogeneous transversely isotropic medium .This thermo-dynamical analysis is carried out in the context of three-phase-lags thermoelasticity model. Three phase lag model is very much useful in the problems of nuclear boiling, exothermic catalytic reactions, phonon-electron interactions, phonon scattering etc. The normal mode analysis is employed to obtain the exact expressions of the considered variables. The frequency equations for thermally insulated and isothermal surface in the closed form are derived. Some special cases of frequency equation are also discussed. In order to illustrate the analytical developments, the numerical solution is carried out and the computer simulated results in respect of phase velocity and attenuation coefficient are presented graphically. It is found that the results obtained in the present problem agree with that of the existing results obtained by various researchers. This study may find its applications in the design of surface acoustic waves (SAW) devices, structural health monitoring and damage characterization of materials.

Keywords: Rayleigh waves; Transversely isotropic material; Three-phase-lag model, Frequency equation.

1 INTRODUCTION

RAYLEIGH waves travel near the surface of solids that decrease exponentially as distance from the surface increases. Lord Rayleigh [1] predicted the existence of Rayleigh waves in 1885 after whom they were named. In elastic solids these waves cause the surface particles to move in ellipses in planes normal to the surface and parallel to the direction of propagation where the major axis of the ellipse is along vertical direction. The in-plane motion of a particle is counter-clockwise at the surface and at shallow depths when the wave travels from left to right. Various researchers have shown different applications of Rayleigh waves. The generalized theory of thermoelasticity is one of the modified versions of classical thermoelasticity and has been developed in order to eliminate the shortcomings of classical thermoelasticity, such as infinite velocity of thermoelastic disturbances, unsatisfactory thermoelastic response of a solid to short laser pulses and poor description of thermoelastic behavior at low temperatures. Hetnarski and Ignaczak [2] examined five generalizations of the coupled theory of thermoelasticity. The first generalization is done by Lord and Shulman [3] and is referred to as L-S model. The second generalization is due to Green and Lindsay [4] which is known as G-L model. The G-L model is

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characterized a system of partial differential equations in which, in comparison to the classical system, the constitutive relations for the stress tensor and the entropy are generalized by introducing two different relaxation times into considerations. The third generalization is done by Hetnarski and Ignaczak [5] and is known as H-I model. The next generalization to the coupled theory of thermoelasticity introduced by Green and Naghdi [6] and this theory is concerned with the thermoelasticity theory without energy dissipation, referred to as G-N theory of type-II. The fifth generalization of the coupled theory of thermoelasticity is developed by Tzou [7] and is referred to dual phase lag model. It is an extension of the classical thermoelastic model in which the Fourier law is replaced by an approximation to a modified Fourier law with two different time translations: a phase lag of the heat flux and a phase lag of the temperature gradient. Roychoudhuri [8] has recently introduced three phase lag heat conduction equation in which the Fourier law of heat conduction is replaced by an approximation of three phase lags for the heat flux vector, the temperature gradient and the thermal displacement gradient.

The increasing use of anisotropic media demands that the study of elastic problems need to be extended to anisotropic medium also. Anisotropy creates also qualitatively new properties of elastic waves and acoustic phenomena that have not got close analogous in isotropic media. Some of them have already found their practical applications in real devices. A theoretical description of elastic waves in anisotropic material is a very non-trivial problem. The study of wave propagation in anisotropic materials has been a subject of extensive investigation in modern days. It has great applications in aircraft, spacecraft, or other engineering industries. Abd-Alla et al. [9] studied Rayleigh waves in an orthotropic thermoelastic medium under gravity, magnetic field and initial stress. The effect of micro polarity, micro stretch and relaxation times on Rayleigh surface waves were discussed by Sharma et al. [10]. Effect of viscosity in anisotropic thermoelastic medium with three phase lag model was investigated by Rajneesh Kumar et al [11]. Sharma and Singh [12] discussed about thermoelastic waves in transversely isotropic media. Rayleigh surface wave propagation in transversely isotropic media under dual phase lag effect and effect of initial stress were examined by Baljeet Singh et al [13]. Shaw and Mukhopadhyay [14] analyzed Rayleigh surface wave propagation in isotropic micro polar solid under three phase lag model of thermoelasticity. Othman et al. [15] examined the influence of gravitational field on generalized thermoelasticity with two-temperature under three-phase-lag model. Othman et al. [16] studied the effect of rotation and initial stress on generalized micro-polar thermoelastic medium with three-phase-lag. Othman and Zidan [17] considered the effect of two temperature and gravity on the 2-D problem of thermoviscoelastic material under three-phase-lag model. Othman and Said [18] proposed 2-D problem of magneto-thermoelasticity fiber-reinforced medium under temperature-dependent properties with three-phase-lag theory. Recently, Biswas et al. [19] investigated Rayleigh surface wave propagation in orthotropic thermoelastic solids under three-phase-lag model.

Nowadays wave motion in the context of thermoelasticity with second sound effects has a great role in various fields of science and technology. The present paper is concerned with the investigations related to the effect three-phase-lags on the Rayleigh wave propagation in transversely isotropic medium. Various aspects of this wave propagation are discussed in the section of numerical results and discussion and the effects of phase lags are pointed out.

2 FORMULATION OF THE PROBLEM

Let us consider a plane strain problem parallel to xz plane of a transversely isotropic thermoelastic solid medium, the boundary of which $z=0$ is free of stresses. We consider Rayleigh surface wave propagation along the direction of x - axis. The medium is assumed to be unstrained and unstressed initially and has uniform temperature.

The displacement components have the following form:

$$u = u(x,z,t), v = 0, w = w(x,z,t) \tag{1}$$

We need to investigate the propagation of Rayleigh surface wave in transversely isotropic thermoelastic half-space in the context of three phase lag model.

The stress displacement relations are given as the following:

$$\tau_{xx} = c_{11}u_{,x} + c_{13}w_{,z} - \beta T \tag{2}$$

$$\tau_{zz} = c_{13}u_{,x} + c_{11}w_{,z} - \beta T \tag{3}$$

$$\tau_{xz} = c_{44}\left(u_{,z} + w_{,x}\right) \tag{4}$$

where $\tau_{xx}, \tau_{zz}, \tau_{xz}$ are the components of the stress tensor , c_{ij} are elastic constants, β is the thermal moduli and $c_{44} = \dfrac{c_{11} - c_{13}}{2}$.

The equations of motion in terms of displacements are given by the following equations:

$$c_{11}u_{,xx} + c_{44}u_{,zz} + (c_{13} + c_{44})w_{,xz} - \beta\, T_{,x} = \rho \frac{\partial^2 u}{\partial t^2} \tag{5}$$

$$(c_{44} + c_{13})u_{,xz} + c_{44}w_{,xx} + c_{11}w_{,zz} - \beta T_{,z} = \rho \frac{\partial^2 w}{\partial t^2} \tag{6}$$

where ρ is the mass density. Equation of three-phase-lag model in transversely isotropic thermoelastic half space is finally obtained as:

$$K_1(1+\tau_T \frac{\partial}{\partial t})\frac{\partial T_{,xx}}{\partial t} + K_3(1+\tau_T \frac{\partial}{\partial t})\frac{\partial T_{,zz}}{\partial t} + K_1^*(1+\tau_v \frac{\partial}{\partial t})T_{,xx} + K_3^*(1+\tau_v \frac{\partial}{\partial t})T_{,zz} = (1+\tau_q \frac{\partial}{\partial t} + \frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2})\frac{\partial^2}{\partial t^2} [\rho C_e T + \beta T_0\left(u_{,x} + w_{,z}\right)] \tag{7}$$

where T_0 is the reference uniform temperature of the body, T is the temperature above reference temperature, $K_i\ (i = 1,3)$ are the components of the thermal conductivity tensor, $K_i^*(i = 1,3)$ are the material constant characteristic of the theory, C_e is the specific heat at the constant strain, τ_q, τ_T and τ_v are the phase lags of heat flux, temperature gradient and thermal displacement gradient respectively where $0 \le \tau_T \le \tau_q$ and t denotes time.

In the above equations the comma notation is used for spatial derivatives.

3 BOUNDARY CONDITIONS

The mechanical and thermal boundary conditions at the thermally stress free surface $z = 0$ are

Vanishing of the normal stress component: $\tau_{zz} = 0$ (8)

Vanishing of the tangential stress component: $\tau_{xz} = 0$ (9)

Thermal conditions: $q_z + mT = 0$ (10)

where $m \to 0$ corresponds to the thermally insulated surface and $m \to \infty$ corresponds to the isothermal surface. The normal component of heat flux vector q_z is related to the temperature gradient $T_{,z}$ by the following relation:

$$q_z = \left[\frac{-K_3 D'(1+\tau_T D') - K_3^*(1+\tau_v D')}{D'(1+\tau_q D' + \frac{\tau_q^2}{2}D'^2)}\right]T_{,z} \tag{11}$$

where $D' \equiv \frac{\partial}{\partial t}$.

4 SOLUTION OF THE PROBLEM

By Helmholtz's theorem the displacement vector $\vec{u}$ can be written in terms of the displacement potential ϕ and ψ in the following form:

$$\vec{u} = \vec{\nabla}\phi + \vec{\nabla} \times \vec{\psi} \tag{12}$$

where the scalar potential ϕ and the vector potential $\vec{\psi}$ are Lame's potentials. Eq. (12) reduces to

$$u = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial z}, w = \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x} \tag{13}$$

Substituting from Eq. (13) into the Eqs. (5), (6) and (7), we get

$$c_{11}\left(\phi_{,xx} + \phi_{,zz}\right) - \beta\, T = \rho\ddot{\phi} \tag{14}$$

$$c_{44}\left(\psi_{,xx} + \psi_{,zz}\right) = \rho\ddot{\psi} \tag{15}$$

$$K_1(1+\tau_T\frac{\partial}{\partial t})\dot{T}_{,xx} + K_3(1+\tau_T\frac{\partial}{\partial t})\dot{T}_{,zz} + K_1^*(1+\tau_\nu\frac{\partial}{\partial t})T_{,xx} + K_3^*(1+\tau_\nu\frac{\partial}{\partial t})T_{,zz} = (1+\tau_q\frac{\partial}{\partial t} + \frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}) [\rho C_e\ddot{T} + \beta T_0(\ddot{\phi}_{,xx} + \ddot{\phi}_{,zz})] \tag{16}$$

We take

$$\phi(x,z,t) = \sum_{n=1}^{2} A_n \exp(-\lambda_n z)\exp(ik(x-ct)) \tag{17}$$

$$\psi(x,z,t) = B\exp(-\lambda_3 z)\exp(ik(x-ct)) \tag{18}$$

$$T(x,z,t) = \sum_{n=1}^{2} f_n A_n \exp(-\lambda_n z)\exp(ik(x-ct)) \tag{19}$$

where

$$\lambda_3 = \sqrt{[1-\frac{\rho c^2}{c_{44}}]k^2},\ f_n = \frac{c_{11}\lambda_n^2 - \left[c_{11} - \rho c^2\right]k^2}{\beta};\ n = 1,2.$$

λ_1^2 and λ_2^2 are the roots of the biquadratic equation

$$\eta^4 + [\frac{(ik^3cK_1\tau_1 - k^2\tau_2K_1^* + \rho C_e k^2c^2)c_{11} - (c_{11}-\rho c^2)(K_3^*\tau_2 - ikc\tau_1K_3) + T_0\beta^2k^2c^2}{c_{11}(K_3^*\tau_2 - ikc\tau_1K_3)}]\eta^2 - [\frac{(c_{11}-\rho c^2)\{ik^3cK_1\tau_1 - k^2\tau_2K_1^* + \rho C_e k^2c^2\} + T_0\beta^2k^2c^2}{c_{11}(K_3^*\tau_2 - ikc\tau_1K_3)}]k^2 = 0$$

where $\tau_1 = \frac{\tau_3}{\tau_5}, \tau_2 = \frac{\tau_4}{\tau_5}, \tau_3 = 1 - ikc\tau_T, \tau_4 = 1 - ikc\tau_v, \tau_5 = 1 - ikc\tau_q - \frac{k^2c^2}{2}\tau_q^2$.
where k is the wave number and c is the phase velocity of Rayleigh surface waves.

5 DERIVATION OF FREQUANCY EQUATION

Now the stress components in terms of thermoelastic displacement potentials are given by

$$\tau_{zz} = c_{13}\phi_{,xx} + c_{11}\phi_{,zz} - (c_{13} - c_{11})\psi_{,xz} - \beta T \tag{20}$$

$$\tau_{xz} = c_{44}(2\phi_{,xz} - \psi_{,zz} + \psi_{,xx}) \tag{21}$$

If we take $\lambda_1 = \sqrt{k^2 - \xi_1^2}, \lambda_2 = \sqrt{k^2 - \xi_2^2}, \lambda_3 = \sqrt{k^2 - \zeta^2}$, then frequency equation for thermally insulated boundary under three-phase-lag model can be obtained as:

$$[2 - \frac{\rho c^2}{c_{44}}]^2[\gamma_1^2 + \gamma_1\gamma_2 + \gamma_2^2 - 1 + \frac{\rho c^2}{c_{11}}] - 4\gamma_1\gamma_2\gamma_3(\gamma_1 + \gamma_2) = 0 \tag{22}$$

The frequency equation for isothermal boundary under three-phase-lag model is obtained as:

$$[(2 - \frac{\rho c^2}{c_{44}})^2(\gamma_1 + \gamma_2) - 4\gamma_3(\gamma_1\gamma_2 + 1 - \frac{\rho k^2c^2}{c_{11}})] = 0 \tag{23}$$

where $\gamma_1^2 = 1 - \frac{\xi_1^2}{k^2}, \gamma_2^2 = 1 - \frac{\xi_2^2}{k^2}, \gamma_3^2 = 1 - \frac{\zeta^2}{k^2}, \zeta^2 = \frac{\rho k^2 c^2}{c_{44}}$, and ξ_1^2 and ξ_2^2 are the roots of the biquadratic equation

$$\xi^4 - [k^2 + \frac{(ik^3cK_1\tau_1 - k^2\tau_2K_1^* + \rho C_e k^2c^2)c_{11} + T_0\beta^2k^2c^2}{(K_3^*\tau_2 - ikc\tau_1K_3)c_{11}} + \frac{\rho k^2c^2}{c_{11}}]\xi^2 + [\frac{ik^3cK_1\tau_1 - k^2\tau_2K_1^* + \rho C_e k^2c^2}{(K_3^*\tau_2 - ikc\tau_1K_3)} + k^2]\frac{\rho k^2c^2}{c_{11}} = 0 \tag{24}$$

6 DISCUSSION OF FREQUENCY EQUATION

The frequency Eq. (22) reduces for the case of theory of classical coupled thermoelasticity when we put $\tau_v = \tau_q = \tau_T = 0$ and $K_i^* = 0 (i = 1,3)$ in the Eq. (22).When we put $\tau_v = \tau_T = 0$, $\tau_q^2 = 0$, $K_i^* = 0 (i = 1,3), \tau_q \neq 0$ in Eq. (22), the result we obtain, agrees with the frequency equation of Lord-Shulman model.

The frequency Eq. (22) reduces for the case of GN model type-III when we put $\tau_v = \tau_q = \tau_T = 0$.

If we put $K_i^* = 0 (i = 1,3)$ in Eq. (22), then we obtain the frequency equation for dual phase lag model of thermoelasticity.

7 SOLUTION OF FREQUENCY EQUATION

Generally wave number (k) and the phase velocity (c) of the wave are complex quantities. If we take

$$c^{-1} = V^{-1} + i\omega^{-1}Q \tag{25}$$

So that, $k = R + iQ, R = \dfrac{\omega}{V}$ where V and Q are real.

The exponent in the surface wave solution becomes $iR(x - Vt) - Qx$. This shows that V is the propagation speed, Q is the attenuation coefficient and ω is the angular frequency of the waves.

8 SPECIAL CASES

Case 1

Frequency equation of classical coupled thermoelasticity in transversely isotropic half- space is obtained by taking $\tau_q = \tau_T = \tau_\nu = 0, K_i^* = 0 (i = 1,3),$
which is

$$[2 - \frac{\rho c^2}{c_{44}}]^2 [\gamma_1^2 + \gamma_1\gamma_2 + \gamma_2^2 - 1 + \frac{\rho c^2}{c_{11}}] - 4\gamma_1\gamma_2\gamma_3(\gamma_1 + \gamma_2) - \frac{m}{k}[(2 - \frac{\rho c^2}{c_{44}})^2 (\gamma_1 + \gamma_2) - 4\gamma_3(\gamma_1\gamma_2 + 1 - \frac{\rho k^2 c^2}{c_{11}})] = 0 \tag{26}$$

where $m \to 0$ corresponds to the thermally insulated surface and $m \to \infty$ corresponds to the isothermal surface.

Here $\gamma_1^2 = 1 - \dfrac{\xi_1^2}{k^2}, \gamma_2^2 = 1 - \dfrac{\xi_2^2}{k^2}, \gamma_3^2 = 1 - \dfrac{\zeta^2}{k^2}, \zeta^2 = \dfrac{\rho k^2 c^2}{c_{44}}$ and ξ_1^2 and ξ_2^2 are the roots of the biquadratic equation

$$\xi^4 - [(k^2 - \frac{K_1 k^2}{K_3}) + \frac{\rho k^2 c^2}{c_{11}} + (1+\kappa)\frac{ikc\rho C_e}{K_3}]\xi^2 + [\frac{\rho k^2 c^2}{c_{11}} - \frac{K_1}{K_3}\frac{\rho k^2 c^2}{c_{11}}]k^2 + \frac{ik^3 c^3 \rho^2 C_e}{c_{11} K_3} = 0 \tag{27}$$

In which $k = \dfrac{T_0 \beta^2}{\rho C_e c_{11}}$.

Now if we take $c_{11} = \lambda + 2\mu, c_{13} = \lambda, K_1 = K_3 = K$, then we get the frequency equation of Rayleigh waves for isotropic half space in case of classical coupled thermoelasticity as follows:

$$(2 - \frac{c^2}{c_2^2})^2 [\gamma_1^2 + \gamma_1\gamma_2 + \gamma_2^2 - 1 + \frac{c^2}{c_1^2}] - 4\gamma_1\gamma_2\gamma_3(\gamma_1 + \gamma_2) - \frac{m}{k}[(2 - \frac{c^2}{c_2^2})^2 (\gamma_1 + \gamma_2) - 4\gamma_3(\gamma_1\gamma_2 + 1 - \frac{k^2 c^2}{c_1^2})] = 0 \tag{28}$$

Eq. (28) is similar as the result obtained in Nowinski [20], where $\gamma_1^2 = 1 - \dfrac{\xi_1^2}{k^2}, \gamma_2^2 = 1 - \dfrac{\xi_2^2}{k^2}, \gamma_3^2 = 1 - \dfrac{\zeta^2}{k^2}, \zeta^2 = \dfrac{k^2 c^2}{c_2^2}$

and ξ_1^2 and ξ_2^2 are the roots of the biquadratic equation

$$\xi^4 - [\frac{k^2 c^2}{c_1^2} + (1+\kappa)\frac{ikc\rho C_e}{K}]\xi^2 + \frac{ik^3 c^3 \rho C_e}{K c_1^2} = 0 \tag{29}$$

In which $k = \dfrac{T_0 \beta^2}{\rho^2 C_1^2 c_e}$.

Case 2

Neglecting thermal parameters and taking $c_{11} = c_{33} = \lambda + 2\mu, c_{13} = \lambda, c_{44} = \mu$, the frequency equation of Rayleigh waves in isotropic elastic half space is obtained as:

$$\left(2-\frac{c^2}{c_2^2}\right)^2 = 4\left(1-\frac{c^2}{c_1^2}\right)^{\frac{1}{2}}\left(1-\frac{c^2}{c_2^2}\right)^{\frac{1}{2}} \tag{30}$$

where $c_1^2 = \dfrac{\lambda+2\mu}{\rho}, c_2^2 = \dfrac{\mu}{\rho}$.

9 NUMERICAL RESULTS AND DISCUSSION

In order to illustrate the theoretical results obtained in the preceding section, we take the following values of the relevant parameters for cobalt material as follows:

$$c_{11} = 3.071\times10^{11} N/m^2, c_{13} = 1.650\times10^{11} N/m^2, c_{44} = 1.51\times10^{11} N/m^2,$$
$$T_0 = 298K, C_e = 4.27\times10^2 J/KgK, \beta = 7.04\times10^6 N/m^2K, \rho = 8.836\times10^3 Kg/m^3,$$
$$K_1 = 6.90\times10^2 W/mK, K_3 = 7.01\times10^2 W/mK, K_1^* = 1.313\times10^2 W/s, K_3^* = 1.54\times10^2 W/s,$$
$$\tau_q = 2.0\times10^{-7}\sec, \tau_T = 1.5\times10^{-7}\sec, \tau_\nu = 1.0\times10^{-8}\sec.$$

The phase velocity, attenuation coefficient and specific loss factor for different values of time lags in the context of three-phase-lag (TPL), Two-phase-lag, G-N theory type-III and Lord- Shulman (L-S) models of thermoelasticity have been computed for various values of frequency as well as wave number from the frequency equations for stress free thermally insulated and isothermal boundaries.

The variation of phase velocity of Rayleigh waves with respect to frequency for thermally insulated boundaries is shown in Fig. 1. It is observed that the value of phase velocity increases with the increase of frequency in case of both theories (three phase lag model, G-N theory type-III and L-S theory) for both thermally insulated and isothermal boundaries. The phase velocity is very closer for all the thermoelastic models.

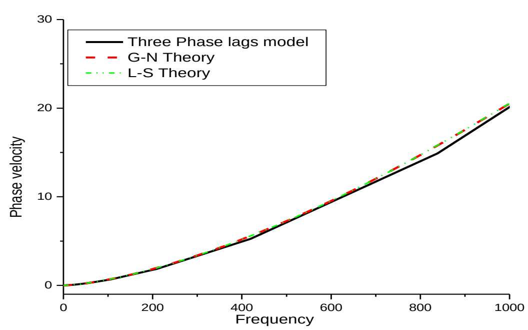

Fig.1
Comparison of phase velocity with respect to frequency (for various thermoelastic models).

Variations of attenuation coefficient for different thermoelastic model for thermally insulated boundaries are shown in Fig. 2. It is observed that in comparison with G-N model type -III, L-S model attains low value in the frequency region $25 \le \omega \le 300$ and both converge towards the profiles of phase lag models. For L-S model, attenuation coefficient remains constant in the frequency range $150 \prec \omega \prec 300$. Attenuation coefficient for dual and three phase lag models coincides with zero and remains constant.

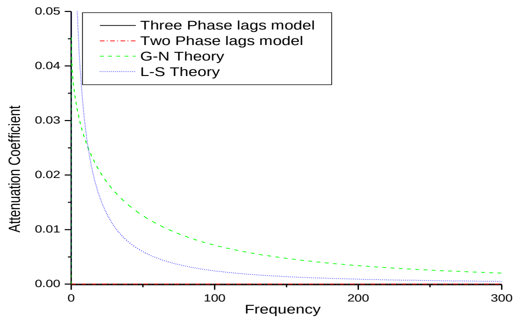

Fig.2
Comparison of attenuation coefficient with respect to frequency for thermally insulated boundaries for various thermoelastic models.

In Fig. 3, variation of attenuation coefficient with respect to wave number for thermally insulated boundaries is presented. The value of attenuation coefficient for L-S model is very high initially but with the increase in wave number, it decreases and converges towards zero. The value of attenuation coefficient for various values of τ_q remains same and coincides with zero with the increase in wave number.

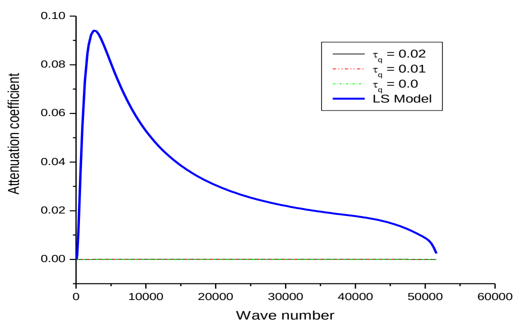

Fig.3
Comparison of attenuation coefficient with respect to wave number for thermally insulated boundaries for various values of τ_q .

In Fig. 4, variation of attenuation coefficient with respect to wave number for thermally insulated boundaries is presented. The value of attenuation coefficient for L-S model is very high initially but with the increase in wave number, it decreases and converges towards zero. The value of attenuation coefficient for various values of τ_v remains same and coincides with zero with the increase in wave number.

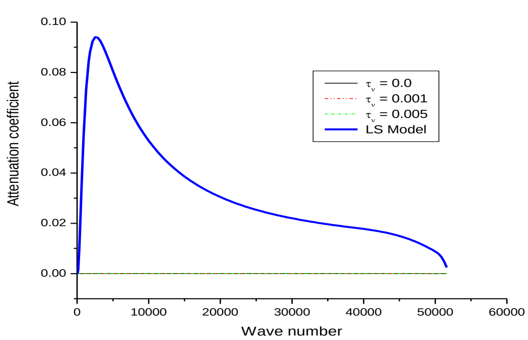

Fig.4
Comparison of attenuation coefficient with respect to wave number for thermally insulated boundaries for various values of τ_v .

In Fig. 5, variation of attenuation coefficient with respect to wave number for isothermal boundaries is presented. For $\tau_q = 0$, attenuation coefficient remains constant and coincides with zero. With the increase of the value of τ_q , the value of attenuation coefficient increases. The value of attenuation coefficient is higher for $\tau_q = 0.02$.

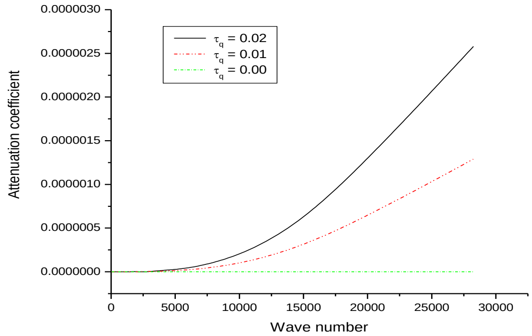

Fig.5
Comparison of attenuation coefficient with respect to wave number for isothermal boundaries for various values of τ_q .

In Fig. 6, variation of attenuation coefficient with respect to wave number for isothermal boundaries is presented. It is noticed in figure-6 that for $\tau_q = 0.01, 0.02$, the attenuation coefficient increases rapidly with the increase of wave number but later it decreases gradually with the increase of wave number and converges towards zero. For $\tau_q = 0$, attenuation coefficient remains constant and coincides with zero. With the increase of the value of τ_q , the value of attenuation coefficient increases. The value of attenuation coefficient is higher for $\tau_q = 0.02$.

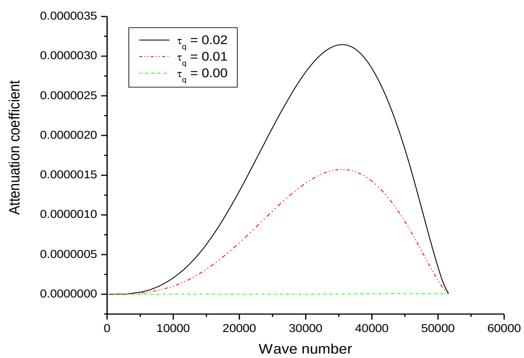

Fig.6
Comparison of attenuation coefficient with respect to wave number for isothermal boundaries for various values of τ_q .

It is observed in Fig.7 that for $\tau_\nu = 0.001, 0.005$, the attenuation coefficient remains constant and coincides with zero with the increase of wave number but in the low region of wave number, attenuation coefficient first increases and then decreases with the increase of wave number and finally converges towards zero for $\tau_\nu = 0$.

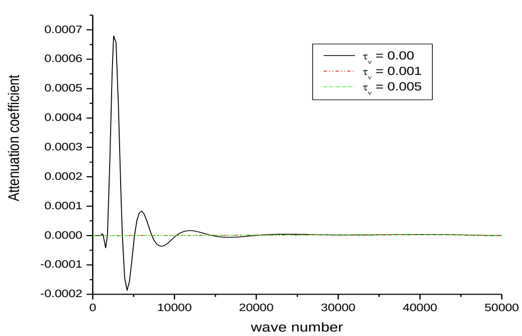

Fig.7
Comparison of attenuation coefficient with respect to wave number for isothermal boundaries for various values of τ_ν .

10 CONCLUSIONS

After analytical discussions and numerical observations, we can conclude the following phenomena:

i. Analytical solutions based upon normal mode analysis of the thermoelastic problem in solids have been developed and utilized. This method is applicable to a wide range of problems in thermodynamics and thermoelasticity.
ii. Three-phase-lag model is a mathematical model that includes the heat flux vector, the temperature gradient and the thermal displacement gradient, which are useful in the problems of heat transfer, heat conduction, nuclear boiling, exothermic catalytic reactions, phonon-electron interactions, phonon-scattering. For this reason, the three phase lag model is the most adequate theory to describe the present problem.
iii. For Rayleigh surface wave propagating in a transversely isotropic thermoelastic solid, the phase velocity remains identical for all the thermoelastic models. It increases with the increasing value of frequency.
iv. Phase lag due to heat flux is much dominating factor in comparison with other phase lags.
v. For all the thermoelastic models, attenuation coefficient attains a very high value when wave number is smaller and low value when wave number takes larger values in case of thermally insulated boundaries but for isothermal boundaries, attenuation coefficient increases with the increasing value of wave number.

The applications range from geophysical problems to quantitive non-destructive evaluation of mechanical structures and acoustic tomography for medical purposes. The problem though is theoretical, but it can provide useful information for experimental researchers working in the field of geophysics, earthquake engineering and seismologist working in the field of mining tremors and drilling into the crust of the earth.

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