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Vibration Response of an Elastically Connected Double-Smart Nanobeam-System Based Nano-Electro-Mechanical Sensor

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ABSTRACT

Nonlocal vibration of double-smart nanobeam-systems (DSNBSs) under a moving nanoparticle is investigated in the present study based on Timoshenko beam model. The two smart nanobeams (SNB) are coupled by an enclosing elastic medium which is simulated by Pasternak foundation. The energy method and Hamilton's principle are used to establish the equations of motion. The detailed parametric study is conducted, focusing on the combined effects of the nonlocal parameter, elastic medium coefficients, external voltage, length of SNB and the mass of attached nanoparticle on the frequency of piezoelectric nanobeam. The results depict that the imposed external voltage is an effective controlling parameter for vibration of the piezoelectric nanobeam. Also increase in the mass of attached nanoparticle gives rise to a decrease in the natural frequency. This study might be useful for the design and smart control of nano-devices.

Keywords : DSNBSs; Nonlocal vibration; Pasternak foundation; Timoshenko beam model; Exact solution.

1 INTRODUCTION

APPLICATION of piezoelectric materials in smart structures has been the subject of intense research in the last two decades. The specific characteristic of piezoelectric materials is its ability to produce an electric field when subjected to deformation and vice versa. The direct piezoelectric effect describes the electrostatic reaction to a mechanical load such as sensors, while the converse piezoelectric effect describes the mechanical reaction to an electrostatic load such as actuators. Devices based on piezoelectric elements are commonly used in industry and laboratories. These materials are finding a wide range of applications in electro-mechanical and electric devices, such as actuators, sensors and transducers.

The local theory assumes that the stress at a defined point depends uniquely on the strain at the same point which it is scale independent theory, because it cannot explain size-dependent manner. But there are theories that are capable of account and statement of the size-dependent behavior such as nonlocal elasticity theory, the strain gradient theory, couple stress theory and micropolar theory. Many studies have been carried out on the basis of the nonlocal elasticity theory which is used in the papers of Eringen [1,2]. He considered the stress state at a given point as a function of the strain states of all points in the body, while the local continuum mechanics assumes that the stress state at a given point depends uniquely on the strain state at the same point. There are many works that have used this theory. For example, Ghorbanpour Arani et al. [3] employed nonlocal elasticity theory for nonlinear

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vibration of embedded single walled boron nitride nanotubes (SWBNNTs). Also several researchers have suggested that the nonlocal parameter should be less than 2 nm based on the molecular mechanics and molecular dynamic simulations [4–6]. It should be pointed out that most nanodevices with piezoelectric nanowires or nano-belts as fundamental elements are beam-based [7] structures.

There are a number of beam theories that are used to represent the kinematics of deformation, that include: The Euler–Bernoulli beam theory, Timoshenko beam theory , Reddy beam theory and Levinson beam theory [8] use of all the above mentioned theories to analyze bending, buckling and vibration of nonlocal beams. Basis of our work in this paper is the Timoshenko beam theory.So far, only a few works were reported for the piezoelectric nanostructures based on the surface elasticity theory. Huang and Yu [9] studied effect of the surface piezoelectricity on the electromechanical behavior of a piezoelectric ring. This study shows that the surface piezoelectricity may play an important role in the electromechanical behavior of piezoelectric nanostructures. Yan and Jiang [10] studied the vibration and buckling behaviors of piezoelectric nanobeams with the surface effect. This study shows that the resonant frequencies can be tuned by adjusting the applied electrical load.

However, to the best of the authors' knowledge, very few studies have been reported on nanobeam based mass sensor via nonlocal elasticity theory. Simsek [11] studied nonlocal effects in the forced vibration of an elastically connected double-carbon nanotube system under a moving nanoparticle. He showed that the velocity of the nanoparticle and the stiffness of the elastic layer have significant effects on the dynamic behavior of double carbon nanotube systems.

This paper aims to study the vibration of the piezoelectric nanobeam embedded in Pasternak foundation based on the theory of Eringen's piezoelasticity and Timoshenko beam theory. Also piezoelectric nanobeam is subjected to an applied voltage. The governing linear equations are derived using the Hamilton's principle.

2 REVIEW OF NONLOCAL PIEZOELASTICITY THEORY

Based on the theory of nonlocal piezoelasticity, the stress tensor and the electric displacement at a reference point depend not only on the strain components and electric-field components at the same position but also on all other points of the body. The nonlocal constitutive behaviour for the piezoelectric material can be given as follows [12]:

$$\sigma_{ij}^{nl}(x)=\int_{v}\alpha\left(\left|x-x'\right|,\tau\right)\sigma_{ij}^{l}dV(x'),\quad \forall x\in V \tag{1}$$

$$D_{k}^{nl}(x)=\int_{v}\alpha\left(\left|x-x'\right|,\tau\right)D_{k}^{l}dV(x'),\quad \forall x\in V \tag{2}$$

where $\sigma_{ij}^{nl}(x)$ and σ_{ij}^{l} are, respectively, the nonlocal stress tensor and local stress tensor, and D_k^{nl} and D_k^{l} are the components of the nonlocal and local electric displacement, respectively. $\alpha\left(\left|x-x'\right|,\tau\right)$ is the nonlocal modulus, $\left|x-x'\right|$ is the Euclidean distance, and $\tau=e_0a/l$ defines that e_0 is a material constant determined experimentally or approximated by matching the dispersion curves of the plane waves with those of the atomic lattice dynamics, and *a* and *l* are the internal (e.g. lattice parameter, granular size) and external characteristic lengths (e.g. crack length, wavelength) of the nanostructures, respectively. Consequently, e_0a is a constant parameter which is obtained with molecular dynamics, experimental results, experimental studies and molecular structure mechanics. The constitutive equation of the nonlocal elasticity can be written as: [13]

$$\sigma_{xx}-\left(e_0a\right)^2\frac{\partial^2\sigma_{xx}}{\partial x^2}=c_{11}\varepsilon_{xx}-h_{31}E_z \tag{3}$$

$$\sigma_{xz}-\left(e_0a\right)^2\frac{\partial^2\sigma_{xz}}{\partial x^2}=c_{44}\varepsilon_{xz}-h_{15}E_x \tag{4}$$

where the parameter $\left(e_0a\right)^2$ denotes the small scale effect on the response of structures in nanosize. Similarly, Eq. (2) can be written as: [12]

$$D_x - (e_0 a)^2 \frac{\partial^2 D_x}{\partial x^2} = h_{15}\varepsilon_{xz} + \in_{11} E_x \tag{5}$$

$$D_z - (e_0 a)^2 \frac{\partial^2 D_z}{\partial x^2} = h_{31}\varepsilon_{xx} + \in_{33} E_z \tag{6}$$

where $\sigma_{ij}, \varepsilon_{ij}, D_i$ and E_i are stress, strain, electric displacement and electric field, respectively. Also, c_{ij}, h_{ij} and $\in_{ij}$ denote elastic, piezoelectric and dielectric coefficients, respectively. The electric field can be written as: [14]

$$E = -\nabla\Phi \tag{7}$$

where (Φ) is electric potential which in the thickness direction of the piezoelastic nanobeam can be assumed as follows [15]

$$\Phi(x,z,t) = -\cos\left(\frac{\pi}{h}z\right)\phi(x,t) + \frac{2zV_0}{h}e^{i\Omega t} \tag{8}$$

According to Eq. (7), the electric field in *x*- and *z*- directions can be expressed as:

$$E_x = \cos\left(\frac{\pi z}{h}\right)\left(\frac{\partial}{\partial x}\phi(x,t)\right) \tag{9}$$

$$E_z = -\frac{\sin\left(\frac{\pi z}{h}\right)\pi\phi(x,t)}{h} - \frac{2V_0}{h} \tag{10}$$

3 BASIC FORMULATION

A schematic diagram of a DSNBS coupled by Pasternak foundation is illustrated in Fig. 1 in which geometrical parameters of length, *L* and thickness, *h* are also indicated. The Pasternak foundation model is simulated by spring constants of Winkler type (k_w) and transverse shear constants (G_P).The two SNB is subjected to external electric voltage (ϕ) in thickness direction which is used for wave propagation smart control of the coupled system . Based on the Timoshenko beam theory, the displacement field can be expressed as: [8]

$$U(x,z,t) = u(x,t) + z.\psi(x,t) \tag{11}$$

$$W(x,z,t) = w(x,t) \tag{12}$$

where $u(x,t)$ and $w(x,t)$ are the displacements of the mid-axis about *x*-and *z*-directions, respectively.

The von kármán strains associated with the above displacement field can be expressed as:

$$\varepsilon_{xx} = \frac{\partial U}{\partial x} = \frac{\partial u}{\partial x} + z\frac{\partial \psi}{\partial x}, \tag{13}$$

$$\gamma_{xz} = \frac{\partial U}{\partial z} + \frac{\partial W}{\partial x} = \psi(x,t) + \frac{\partial w}{\partial x}, \tag{14}$$

The strain energy of the SNB can be expressed as:

$$U_S = \frac{1}{2}\int_0^L \int_{-h/2}^{h/2} \left(\sigma_{xx}\varepsilon_{xx} + \sigma_{xz}\gamma_{xz} - D_x E_x - D_z E_z\right) dz dx \tag{15}$$

The kinetic energy of the SNB is given by

$$U_K = \frac{1}{2}\int_0^L \left[m_0 \left(\frac{\partial u}{\partial t}\right)^2 + m_0 \left(\frac{\partial w}{\partial t}\right)^2 + m_2 \left(\frac{\partial \psi}{\partial t}\right)^2 \right].dx \tag{16}$$

where $m_0 = \rho h$, $m_2 = \rho h^3/12$ are the mass moments of inertia.

The external work due to surrounding elastic medium and distributed transverse load can be written as:

$$W_v = -\frac{1}{2}\int_0^l \left[q(x,t) - \bar{N}.\left(\frac{\partial w(x,t)}{\partial x}\right)^2 \right].w(x,t)\,dx \tag{17}$$

where $\bar{N} = N_E^b = (2V_0 h_{31})$ is the normal force induced by the external electric voltage V_0 and $q(x,t)$ is the transverse distributed load :

$$q(x,t) = q_1(x,t) - \left(\frac{1}{b}\right).q_2(x,t) \tag{18}$$

where $q_1(x,t)$ is the distributed transverse load due to Pasternak foundation

$$q_1(x,t) = \left\{ k_w.w(x,t) - G_p.\frac{\partial^2 w(x,t)}{\partial x^2} \right\} \tag{19}$$

And $q_2(x,t)$ can be represented for a moving load as follows:

$$q_2(x,t) = \left\{ m_c.\delta(x - x_P).\frac{\partial^2 w(x,t)}{\partial t^2} \right\} \tag{20}$$

where m_c is mass nanoparticle, x_P is location nanoparticle and $\delta(x)$ is the impulse function. also using the following general property of Dirac-delta function for the moving load term

$$\int_{x_1}^{x_2} g(x)\delta^{(n)}(x - x_P)\,dx = \begin{cases} (-1)^n g^n(x_P) & if\ x_1 \langle x_P \langle x_2 \\ 0 & otherwise \end{cases} \tag{21}$$

where $\delta^{(n)}(x)$ represents nth derivative of Dirac-delta function.

Using Hamilton's principle ($\int_0^t \delta(U_K - (U_S - W_v))dt = 0$), the following motion equations can be derived:

$$\delta u : c_{11} h \frac{\partial^2 u}{\partial x^2} + (e_0 a)^2\, m_0 \frac{\partial^4 u}{\partial x^2 \partial t^2} - m_0 \frac{\partial^2 u}{\partial t^2} = 0. \tag{22}$$

$$\delta w : \frac{\partial^2}{\partial x^2} Q_{xx} - q(x,t) - \frac{\partial}{\partial x}\left\{ (2V_0 h_{31}) \frac{\partial w}{\partial x} \right\} - m_0 \frac{\partial^2 w}{\partial t^2} = 0 \tag{23}$$

$$\delta \psi : \frac{\partial}{\partial x} M_{xx} - Q_{xx} - m_2 \frac{\partial^2 \psi}{\partial t^2} = 0 \tag{24}$$

$$\delta \phi : \int_{-h/2}^{h/2} \left[\cos\left(\frac{\pi}{h} z\right).\frac{\partial D_x}{\partial x} + \frac{\pi}{h}.\sin\left(\frac{\pi}{h} z\right).D_z \right] dz = 0. \tag{25}$$

where the normal resultant force N_{xx} and the bending moment M_{xx} and the transverse shear force Q_{xx} can be calculated from the following relations:

$$N_{xx}=\int_{-h/2}^{h/2}\sigma_{xx}dz \qquad M_{xx}=\int_{-h/2}^{h/2}\sigma_{xx}zdz \qquad Q_{xx}=\int_{-h/2}^{h/2}\sigma_{xz}dz$$

The motion equations can be written as:
Equations of motion for SNB-1:

$$\delta u_1: c_{11}h\frac{\partial^2 u_1}{\partial x^2}+(e_0a)^2 m_0\frac{\partial^4 u_1}{\partial x^2\partial t^2}-m_0\frac{\partial^2 u_1}{\partial t^2}=0. \tag{26}$$

$$\begin{aligned}\delta w_1: &\left\{-\frac{2h_{15}h}{\pi}\right\}\frac{\partial^2\phi_1}{\partial x^2}+\{hc_{44}-2h_{31}V_0\}\frac{\partial^2 w_1}{\partial x^2}+hc_{44}\frac{\partial\psi_1}{\partial x}+(e_0a)^2 m_0\frac{\partial^4 w_1}{\partial x^2\partial t^2}+2(e_0a)^2h_{31}V_0\frac{\partial^4 w_1}{\partial x^4}\\&-\left(1-(e_0a)^2\frac{\partial^2}{\partial x^2}\right).\left\{+k_w(w_1-w_2)-G_p\frac{\partial^2}{\partial x^2}(w_1-w_2)-\frac{m_c}{b}\delta(x-x_p)\frac{\partial^2 w_1}{\partial t^2}\right\}-m_0\frac{\partial^2 w_1}{\partial t^2}=0.\end{aligned} \tag{27}$$

$$\delta\psi_1: \left\{\frac{2h_{31}h}{\pi}+\frac{2h_{15}h}{\pi}\right\}.\frac{\partial\phi_1}{\partial x}+\frac{c_{11}h^3}{12}\frac{\partial^2\psi_1}{\partial x^2}+(e_0a)^2 m_2\frac{\partial^4\psi_1}{\partial x^2\partial t^2}-c_{44}h\frac{\partial w_1}{\partial x}-c_{44}h\psi_1-m_2\frac{\partial^2\psi_1}{\partial t^2}=0. \tag{28}$$

$$\delta\phi_1: \frac{2h_{15}h}{\pi}\frac{\partial^2 w_1}{\partial x^2}-\frac{\pi^2\in_{33}}{2h}\phi_1+\left\{\frac{2h_{31}h}{\pi}+\frac{2h_{15}h}{\pi}\right\}.\frac{\partial\psi_1}{\partial x}+\frac{\in_{11}h}{2}\frac{\partial^2\phi_1}{\partial t^2}=0. \tag{29}$$

Equations of motion for SNB-2:

$$\delta u_1: c_{11}h\frac{\partial^2 u_2}{\partial x^2}+(e_0a)^2 m_0\frac{\partial^4 u_2}{\partial x^2\partial t^2}-m_0\frac{\partial^2 u_2}{\partial t^2}=0. \tag{30}$$

$$\begin{aligned}\delta w_2: &\left\{-\frac{2h_{15}h}{\pi}\right\}\frac{\partial^2\phi_2}{\partial x^2}+\{hc_{44}-2h_{31}V_0\}\frac{\partial^2 w_2}{\partial x^2}+hc_{44}\frac{\partial\psi_2}{\partial x}+(e_0a)^2 m_0\frac{\partial^4 w_2}{\partial x^2\partial t^2}+2(e_0a)^2h_{31}V_0\frac{\partial^4 w_2}{\partial x^4}\\&-\left(1-(e_0a)^2\frac{\partial^2}{\partial x^2}\right).\left\{+k_w(w_2-w_1)-G_p\frac{\partial^2}{\partial x^2}(w_2-w_1)\right\}-m_0\frac{\partial^2 w_2}{\partial t^2}=0.\end{aligned} \tag{31}$$

$$\delta\psi_2: \left\{\frac{2h_{31}h}{\pi}+\frac{2h_{15}h}{\pi}\right\}.\frac{\partial\phi_2}{\partial x}+\frac{c_{11}h^3}{12}\frac{\partial^2\psi_2}{\partial x^2}+(e_0a)^2 m_2\frac{\partial^4\psi_2}{\partial x^2\partial t^2}-c_{44}h\frac{\partial w_2}{\partial x}-c_{44}h\psi_1-m_2\frac{\partial^2\psi_2}{\partial t^2}=0. \tag{32}$$

$$\delta\phi_2: \frac{2h_{15}h}{\pi}\frac{\partial^2 w_2}{\partial x^2}-\frac{\pi^2\in_{33}}{2h}\phi_2+\left\{\frac{2h_{31}h}{\pi}+\frac{2h_{15}h}{\pi}\right\}.\frac{\partial\psi_2}{\partial x}+\frac{\in_{11}h}{2}\frac{\partial^2\phi_2}{\partial t^2}=0. \tag{33}$$

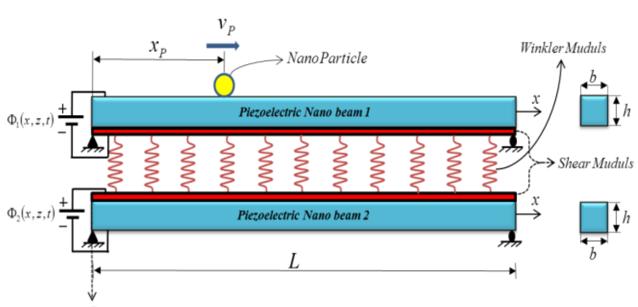

Fig.1
Schematic figure of the double-smart nanobeam-systems under a moving nanoparticle.

4 SOLUTION PROCEDURE

A proper solution for the vibration of the coupled system can be expressed in the following form of [11]

$$d_j(x,t)=\sum_{m=1}^{\infty} d_{0j}\sin\left(\frac{m\pi}{L}x\right).e^{i\omega t} \quad , \quad d=u,w,\psi,\phi \tag{34}$$

where (ω) is frequency.

$$\begin{bmatrix} M_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & M_{22} & 0 & M_{24} & 0 & M_{26} & 0 & 0 \\ 0 & 0 & M_{33} & 0 & 0 & 0 & 0 & 0 \\ 0 & M_{42} & 0 & M_{44} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{55} & 0 & 0 & 0 \\ 0 & M_{62} & 0 & 0 & 0 & M_{66} & 0 & M_{68} \\ 0 & 0 & 0 & 0 & 0 & 0 & M_{77} & 0 \\ 0 & 0 & 0 & 0 & 0 & M_{86} & 0 & M_{88} \end{bmatrix} \begin{bmatrix} u_{01} \\ w_{01} \\ \psi_{01} \\ \phi_{01} \\ u_{02} \\ w_{02} \\ \psi_{02} \\ \phi_{02} \end{bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{35}$$

where $M_{ij}\,(i,j=1,2,\ldots,8)$ for nonlocal piezoelasticity theory are given by:

$$M_{11}=\left\{(e_0a)^2 m_0\left(\frac{m\pi}{L}\right)^2\left(\frac{L}{2}\right)+m_0\left(\frac{L}{2}\right)\right\}.\omega^2-c_{11}h\left(\frac{m\pi}{L}\right)^2\left(\frac{L}{2}\right). \tag{36}$$

$$\begin{aligned} M_{22}=&\left\{(e_0a)^2 m_0\left(\frac{m\pi}{L}\right)^2\left(\frac{L}{2}\right)+m_0\left(\frac{L}{2}\right)-\left[1+(e_0a)^2\left(\frac{m\pi}{L}\right)^2\right].\left(\frac{m_c}{b}\sin^2\left(\frac{m\pi}{L}x_p\right)\right)\right\}\omega^2 \\ &-(c_{44}h-2h_{31}V_0)\left(\frac{m\pi}{L}\right)^2\left(\frac{L}{2}\right)+2(e_0a)^2 h_{31}V_0\left(\frac{m\pi}{L}\right)^4\left(\frac{L}{2}\right)-(e_0a)^2 k_w\left(\frac{m\pi}{L}\right)^2\left(\frac{L}{2}\right)-k_w\left(\frac{L}{2}\right) \\ &-(e_0a)^2 G_p\left(\frac{m\pi}{L}\right)^4\left(\frac{L}{2}\right)-G_p\left(\frac{m\pi}{L}\right)^2\left(\frac{L}{2}\right). \end{aligned} \tag{37}$$

$$M_{24}=\left(\frac{2h_{15}h}{\pi}\right)\left(\frac{m\pi}{L}\right)^2\left(\frac{L}{2}\right). \tag{38}$$

$$M_{26}=(e_0a)^2 k_w\left(\frac{m\pi}{L}\right)^2\left(\frac{L}{2}\right)+k_w\left(\frac{L}{2}\right)+(e_0a)^2 G_p\left(\frac{m\pi}{L}\right)^4\left(\frac{L}{2}\right)+G_p\left(\frac{m\pi}{L}\right)^2\left(\frac{L}{2}\right). \tag{39}$$

$$M_{33}=\left\{(e_0a)^2 m_2\left(\frac{m\pi}{L}\right)^2\left(\frac{L}{2}\right)+m_2\left(\frac{L}{2}\right)\right\}.\omega^2-\left(\frac{c_{11}h^3}{12}\right)\left(\frac{m\pi}{L}\right)^2\left(\frac{L}{2}\right)-c_{44}h\left(\frac{L}{2}\right). \tag{40}$$

$$M_{42}=-\left(\frac{2h_{15}h}{\pi}\right)\left(\frac{m\pi}{L}\right)^2\left(\frac{L}{2}\right). \tag{41}$$

$$M_{44}=-\frac{\pi^2\in_{33}}{2h}\left(\frac{L}{2}\right)-\frac{\in_{11}h}{2}\left(\frac{m\pi}{L}\right)^2\left(\frac{L}{2}\right). \tag{42}$$

$$M_{55}=\left\{(e_0a)^2 m_0\left(\frac{m\pi}{L}\right)^2\left(\frac{L}{2}\right)+m_0\left(\frac{L}{2}\right)\right\}.\omega^2-c_{11}h\left(\frac{m\pi}{L}\right)^2\left(\frac{L}{2}\right). \tag{43}$$

$$M_{62}=(e_0a)^2\,k_w\left(\frac{m\pi}{L}\right)^2\left(\frac{L}{2}\right)+k_w\left(\frac{L}{2}\right)+(e_0a)^2\,G_p\left(\frac{m\pi}{L}\right)^4\left(\frac{L}{2}\right)+G_p\left(\frac{m\pi}{L}\right)^2\left(\frac{L}{2}\right). \tag{44}$$

$$\begin{aligned}M_{66}&=\left\{(e_0a)^2\,m_0\left(\frac{m\pi}{L}\right)^2\left(\frac{L}{2}\right)+m_0\left(\frac{L}{2}\right)\right\}.\omega^2-(c_{44}h-2h_{31}V_0)\left(\frac{m\pi}{L}\right)^2\left(\frac{L}{2}\right)\\&+2(e_0a)^2\,h_{31}V_0\left(\frac{m\pi}{L}\right)^4\left(\frac{L}{2}\right)-(e_0a)^2\,k_w\left(\frac{m\pi}{L}\right)^2\left(\frac{L}{2}\right)-k_w\left(\frac{L}{2}\right)-(e_0a)^2\,G_p\left(\frac{m\pi}{L}\right)^4\left(\frac{L}{2}\right)-G_p\left(\frac{m\pi}{L}\right)^2\left(\frac{L}{2}\right).\end{aligned} \tag{45}$$

$$M_{68}=\left(\frac{2h_{15}h}{\pi}\right)\left(\frac{m\pi}{L}\right)^2\left(\frac{L}{2}\right). \tag{46}$$

$$M_{77}=\left\{(e_0a)^2\,m_2\left(\frac{m\pi}{L}\right)^2\left(\frac{L}{2}\right)+m_2\left(\frac{L}{2}\right)\right\}\omega^2-\left(\frac{c_{11}h^3}{12}\right)\left(\frac{m\pi}{L}\right)^2\left(\frac{L}{2}\right)-c_{44}h\left(\frac{L}{2}\right). \tag{47}$$

$$M_{86}=-\left(\frac{2h_{15}h}{\pi}\right)\left(\frac{m\pi}{L}\right)^2\left(\frac{L}{2}\right). \tag{48}$$

$$M_{88}=-\frac{\pi^2\in_{33}}{2h}\left(\frac{L}{2}\right)-\frac{\in_{11}h}{2}\left(\frac{m\pi}{L}\right)^2\left(\frac{L}{2}\right) \tag{49}$$

In order to obtain a non-trivial solution, it is necessary to set the determinant of the coefficient matrix in Eq. (35) equal to zero which yields the frequency of the system.

5 RESULTS AND DISCUSSION

The results presented here are based on the following data used for geometry and material properties of SNB [16-19]: the thickness $h=2$ nm, mass density $\rho=7500\ \mathrm{Kg/m^3}$, elastic constant $C_{11}=132$ GPa, piezoelectric constant $h_{31}=-4.1\ \mathrm{C/m^2}$, dielectric constants $\in_{11}=5.841\times10^{-9}\ \mathrm{C/Vm}$ and $\in_{33}=7.124\times10^{-9}\ \mathrm{C/Vm}$ .The elastic medium coefficients are $K_W=8.9995035\times10^{17}\ \mathrm{N/m^3}$ and $G_P=2.071273\ \mathrm{N/m}$. Also, the mass of the nanoparticle is $m_C=1\times10^{-21}$ gr and the location of the additional nanoparticle is $x_P=0.25$nm [20]. In the following subsections, the effects of nonlocal parameter, surrounding elastic medium, external voltage, SNB length and mass of attached nanoparticle on vibration of the DSNBS are studied and discussed in details.

The effect of the external electric voltage (V_0) on the frequency ratio with respect to nonlocal parameter (e_0a) is demonstrated in Fig. 2. In this case, we take $m=4$. It is shown that applying positive electric potential can increase the frequency ratio of the (DSNBSs) and vice versa. This is because the imposed positive and negative voltages generate the compressive and tensile forces in the thickness SNB, respectively. Meanwhile, the effect of external voltage becomes more prominent at higher (e_0a). Hence, the imposed external voltage is an effective controlling parameter for vibration of DSNBSs. It is also concluded that increasing the (e_0a) decreases the frequency ratio. This is due to the fact that the increase of nonlocal parameter decreases the interaction force between SNB atoms, and that leads to a softer structure.

Fig. 3 illustrates the effect of mode number of DSNBS on the variation of the frequency ratio versus (e_0a). As can be seen, frequency ratio decreases with increasing mode numbers. Also, the small scale effects on the frequency ratio become more distinguished at higher modes. Obviously, the difference between the frequency ratio of the DSNBS is larger at higher nonlocal parameters. Furthermore, the frequency ratio for all mode numbers decreases by increasing the (e_0a).

The effect of elastic medium constants on the frequency ratio versus nonlocal parameter (e_0a) is illustrated in Figs. 4 and 5. Noted that the elastic medium in this study is simulated as spring constants of Winkler-type (K_w) and shear constants of Pasternak-type (G_P). In general, frequency ratio increases with increasing elastic medium constants. This is because increasing Winkler and Pasternak coefficients increases the system stiffness. Furthermore, the effect of Pasternak constant on the frequency ration is higher than Winkler constant. It is because Pasternak foundation considers not only the normal stresses but also the transverse shear deformation and continuity among the spring elements.

The effect of the length of the nanobeam on the frequency ratio with respect to nonlocal parameter (e_0a) is demonstrated in Fig. 6. As can be seen, frequency ratio increases with increasing length. This is due to the fact that increasing the length leads to softer structures.

Fig. 7 depicts effect of the attached nanoparticle mass (such as a buckyball and molecular or bacterium) on the frequency ratio versus nonlocal parameter. As can be seen with increasing the attached nanoparticle mass on upper piezoelectric nanobeam, frequency ratio increases.

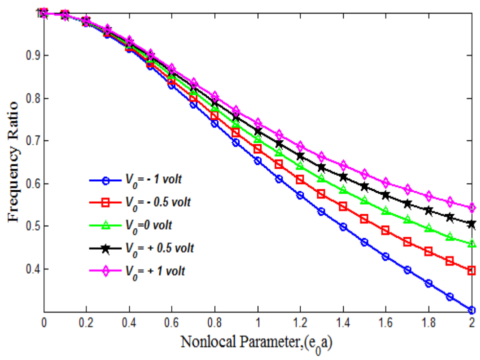

Fig.2
Effect of the external voltage on frequency ratio versus nonlocal parameter.

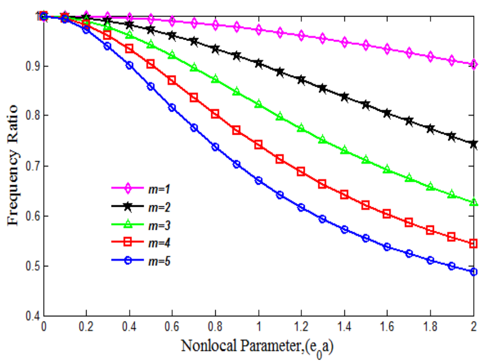

Fig.3
Effect of mode number on the frequency ratio versus nonlocal parameter.

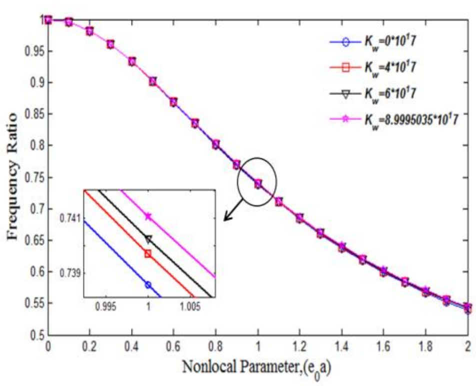

Fig.4
Spring constants of Winkler foundation effect on the frequency ratio versus nonlocal parameter.

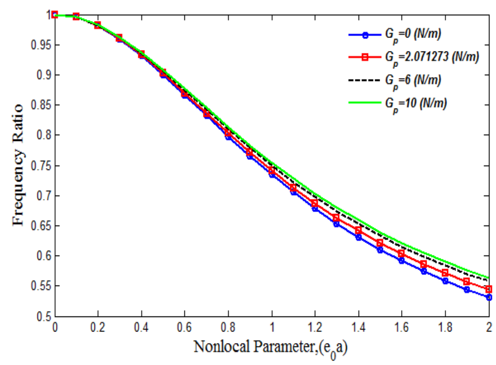

Fig.5
Shear constants of Pasternak foundation effect on the frequency ratio versus nonlocal parameter.

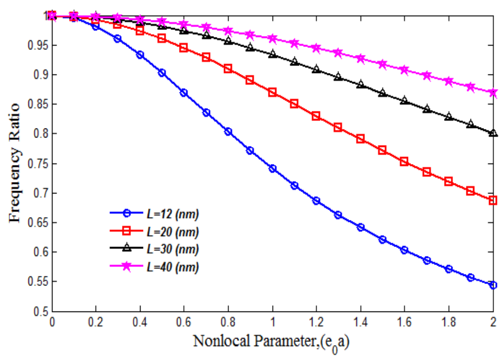

Fig.6
Effect of lenght on the frequency ratio versus nonlocal parameter.

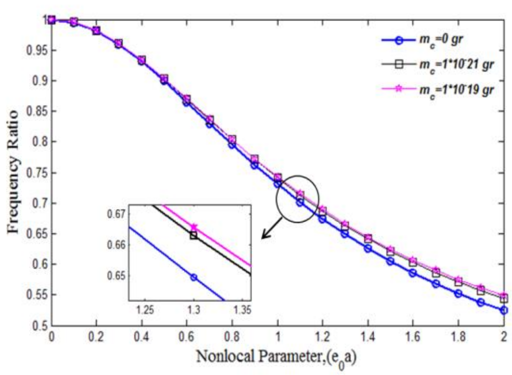

Fig.7
Effect of the attached nanoparticle on the frequency ratio versus nonlocal parameter.

6 CONCLUSIONS

In the present work, vibration of (DSNBSs) under a moving nanoparticle was investigated based on Timoshenko beam model and nonlocal piezoelasticity theory. The two SNB were coupled by an enclosing elastic medium which was simulated by Pasternak foundation. The natural frequency of the piezoelectric nanobeam was obtained using an exact solution so that the effects of the nonlocal parameter, external voltage, elastic medium coefficients and nanoparticle were considered. It can be observed that, applying positive electric potential decreases the frequency ratio of DSNBS and vice versa. Furthermore, with increasing the attached nanoparticle mass on upper piezoelectric nanobeam, frequency ratio increases. Finally, it is hoped that the results presented in this paper be helpful for study and design of bonded systems based on smart control and electromechanical sensors.

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