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A Computational Wear Model of the Oblique Impact of a Ball on a Flat Plate

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ABSTRACT

Many wearing processes are a result of the oblique impacts. Knowing the effective impact parameters on the wear mechanism would be helpful to have the more reliable designs. The H-DD (Hertz-Di Maio Di Renzo) nonlinear model of impact followed by the time increment procedure is used to simulate the impact process of a ball on a flat plate. Restitution parameters are extracted and compared with the experimental data to ensure the accuracy of the impact model. The constant parameters of a wear equation are determined by comparing the results with the experimental data. The results obtained suggest that this simulation method could be used as a predictive way to study the practical design problems and to explain some phenomena associated with impact erosion.

Keywords: Contact; Impact wear; Wear modeling; Steel; Indentation

1 INTRODUCTION

THE wear phenomena have an undeniable role on defecting the mechanical equipments. Studies [1, 2, 3] show that the normal force and the relative velocity of the contact points are the common causes in the mechanical wear. Erosion may be due to the sliding of elements under the steady state of force and velocity or may be the result of the repetitive impacts in which the force and velocity vary during the contact. Steady state wear mechanisms have been solved theoretically for the different wearing geometries [4]. A very complicated case of wear, compound impact wear, is created by the impact of the mechanical elements. The impact is itself a complicated phenomenon and different theories have been presented for its analyzing. The most convenient model of contact includes the Hertz relations which is limited in the elastic deformations. Mindlin [5] improved the Hertz theory for the case that the plastic deformation occurs. Researchers [6, 7, 8] theoretically and experimentally analyzed the variation of the tangential and normal forces due to impact of a ball on the flat plates. The combination of the contact and wear theories could be used to derive the relations for the impact wear in order to manage the impact parameters to have the minimum wear scatters.

Based on the some experimental techniques, many empirical models have been proposed to evaluate the wear rate in different modes of erosion due to solid particle interactions [9–13]. Talia et al. [14] established a theoretical analysis based on a new laboratory technique for solid particle erosion. Using this technique, he could evaluate the effect of particle velocity components (normal and tangential) on the wear rate, separately.

At the present work, the impact of a ball on a flat plate is simulated using the H-DD model [15]. H-DD model is named because of the name of Hertz- Di Maio Di Renzo. It is a combination of Hertz relation improved by Di Maio and Di Renzo. The impact duration is divided into the time intervals and the wear at each contact point is evaluated

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at each interval then is summed to obtain the wear at each point due to one impact. The results are in a good accordance with the experimental data in this field. The results could be used to study the interaction of the impact and material parameters on the impact wear mechanisms.

2 IMPACT

Impact is a phenomenon in which the contact of elements causes the high level forces due to the small period of time.Materials impact behavior depends on the impact initial conditions and the material properties.As schematically is illustrated in Fig.1, the ball impacts the plate at the initial velocity of v_i and angle θ_i and left it at the final velocity of v_f, angular velocity of ω_f and angle θ_f. Impact studies often include in determining the final impact parameters and several ways have been presented for this aim. A simple one is the rigid body analysis. Many aspects of this approach are valid for practical impacts when the small elastic deformations are involved [8].

At the high impact velocities, however, the plastic deformations are involved and the rigid body theories have the notable deviations from the practical observations. Hence, the more accurate analyses would be obtained using the theories which include the variation of the force and displacement during the impact process.

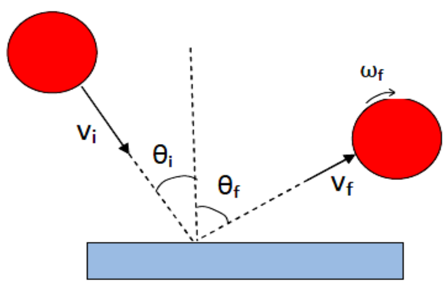


Fig. 1
Schematic of the ball impact on a flat plate.

3 CONTACT MODEL

Contact is generally simulated as the equivalent spring-damper combinations at the normal and tangential directions as illustrated in Fig.2. These models are classified as the linear and nonlinear models based on the relations that explain the spring-damper coefficients.

The force displacement relations at the normal and tangential directions are as follow:

$$F_n = K_n U_n + C_n v_n \tag{1}$$

$$F_t = K_t U_t + C_t v_t \tag{2}$$

where K and C are the stiffness and damping coefficients, U and v are the particle displacement and velocity and the subscripts n and t denote the normal and tangential directions, respectively Eqs. (1) and (2) describe the calculation procedure in case of surface adhesion. The classical criterion to distinguish adhesion from sliding derives from Coulomb's law of attrition and applies to the modulus of the normal and tangential forces:

$$F_t \le \mu F_n \tag{3}$$

A non-linear model is derived by the combination of Hertz's theory in normal direction and improvement to Mindlin's no-slip model in tangential direction [15]. The resulting elastic force is:

$$F_n^e = \frac{4}{3} E^* \sqrt{R U_n} U_n \tag{4}$$

In which R is the ball radius, U_n is the normal overlap and E^* is the equivalent modulus of elasticity and is determined by the following relation.

$$\frac{1}{E^*}=\frac{1-\theta_1^2}{E_1}+\frac{1-\theta_2^2}{E_2} \tag{5}$$

where, E and v are the modulus of elasticity and Poisson's ratio of the contacting bodies , respectively. The contribution of damping force on the normal force is as follow: [15]

$$F_n^d=\sqrt{\frac{4}{3}E^*\sqrt{RU_n}v_n} \tag{6}$$

In which v_n is the ball normal velocity. The resulting normal force is then

$$F_n=F_n^e+F_n^d \tag{7}$$

In tangential direction, the possible force-displacement configuration depends on both the normal and tangential loading history. In fact, micro-slip develops on part of the contact area which can be in similar or opposite direction with respect to the applied tangential load. The presence of the micro-slip arises from the application of Coulomb's law of attrition on a microscopic scale, i.e., relating the maximum tangential to normal stress ratio. Only when the slip condition applies to the entire contact area (gross sliding) the integration of the microscopic law reduces to Eq. (3). For all non-gross sliding cases it is impossible to obtain an integral formulation of the force-displacement relation and an incremental approach must be used. Di Renzo and Di Maio (2005) [15] proposed a correction to Mindlin's no-slip solution capable to increase the degree of accuracy of the solution while maintaining an integral formulation. The resulting formulation for the H-DD (Hertz-Di Maio, Di Renzo) model is expressed as:

$$F_t=\frac{2}{3}\left(8G^*\sqrt{RU_n}\right)U_t \tag{8}$$

where U_n and U_t are the interference of bodies in the normal and tangential directions (and are determined by Eq. (12)) and G^* is the equivalent shear modulus of elasticity of materials which is given as [15]

$$\frac{1}{G^*}=\frac{2-\theta_1}{G_1}+\frac{2-\theta_2}{G_2} \tag{9}$$

A key parameter in collision simulations is the contact duration, and the best relation for its determination would be as follow [15]

$$\tau_C^H=\frac{4\Gamma(0.4)\sqrt{\pi}C^2R}{5\Gamma(0.9)v_{in}} \tag{10}$$

where $\Gamma(.)$ is the gamma function and C indicates a constant defined by the following relation.

$$C=\left(\frac{15mv_{im}^2}{16E^*R^3}\right)^{0.2} \tag{11}$$

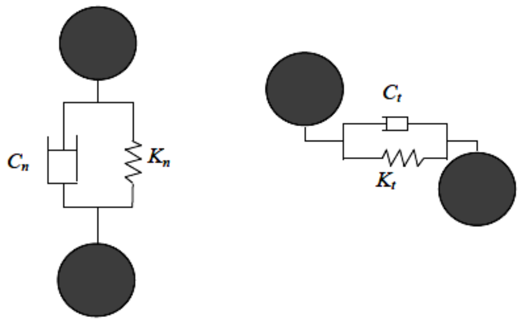


Fig. 2
The spring-damper combination as the equivalent of the contact forces.

4 MODELING

The single impact of a ball on a flat plate is modeled using the H-DD nonlinear relations. The impact duration is divided into the time intervals and the position, velocity and acceleration of ball along with the normal and tangential forces are calculated over these intervals. By this procedure, the ball path due to the which is schematically illustrated in Fig. (3) can be determined. The constant acceleration over each time interval is assumed the value at the beginning of the interval. So the displacement and the velocity at the end of an interval can be determined using the following equations:

$$U[i] = \frac{1}{2} a[i](\Delta t)^2 + v[i-1]\Delta t + U[i-1] \tag{12}$$

$$v[i] = a[i]\Delta t + v[i-1] \tag{13}$$

where $U[i]$ and $v[i]$ are respectively the displacement and velocity at the end of interval i and $a[i]$ is the acceleration at the beginning of the interval for the both normal and tangential directions. The accelerations are calculated by the Newton's formulation in normal and tangential direction as follow:

$$a_n[i] = \frac{f_n[i]}{m} \quad , \quad a_t[i] = \frac{f_t[i]}{m} \tag{14}$$

The coordinate systems shown in Fig. (3) are used to determine the points of contact on each body over each time interval. The x_1y_1 system is a non-rotational system attached to the ball and static xy system is attached to the flat plate. The points of plate that are in contact with the ball are determined at each time interval and so the normal pressure and relative velocity could be evaluated at those points.

4.1 Validation of contact model

The presented procedure for contact is validated to rely it for the wear evaluation. The results are compared with the FEM analyses [16] and the experimental data [8, 17]. Fig. (4) shows the comparison between the experimental [17] and the model results for the variation of the peak normal load with respect to the impact angle at the different initial velocities.

The comparison of the model predictions and experimental data [8] for the rebound angle of the center of mass and the contact point is illustrated in Fig. (5). The negative value of the contact point rebound angle means that the tangential part of its final velocity is negative as a result of the clockwise ball rotational velocity. There are many other parameters which may be considered for the validation purpose but all of them couldn't be discussed here. Based on what illustrated in Figs. (4-6) it can be inferred that the model is able to predict the impact kinematic properly and its results can be used for the wear evaluations.

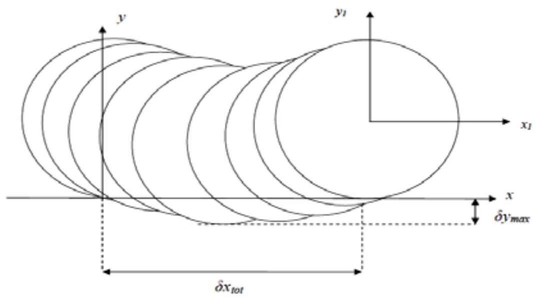


Fig. 3
Instantaneous ball position due to impact.

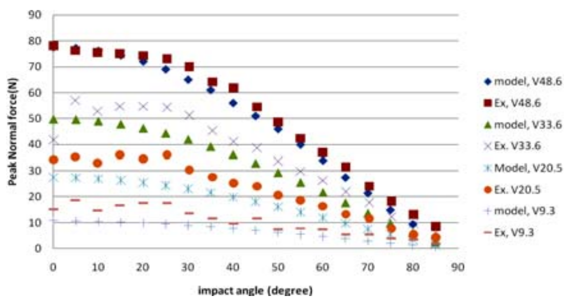


Fig. 4
variation of the maximum normal force [4] with respect to the impact angle for the different initial velocities, velocities are in mm/s.

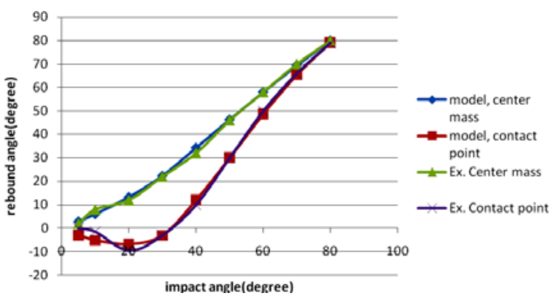


Fig. 5
Model and experimental [8] results of rebound angle of the contact point and center of mass of ball.

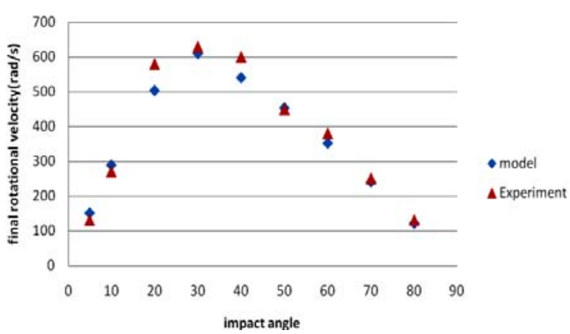


Fig. 6
Model and experimental [8] results of the ball final rotational velocity.

5 WEAR

The after being assumed the validity of the contact model, the impact wear mechanism could be analyzed. The ball track on the flat plate in a plastic deformation and a worn layer as illustrated exaggeratedly in Fig. (7) was included.

A view on the wear equations shows that the wear is a result of the sliding surfaces under the effect of normal force. The wear equations are generally a relation defining one of the wear parameter (mass, volume or depth of the worn area) with respect to the normal force, relative tangential velocity and material properties of the contacting surfaces. Wear may be due to the steady state of the force and velocity or be due to an impact process in which there are the variable force and velocity over the contact region. However, at this case also the normal force and the relative velocity have the essential role on the wear mechanism. The wear evaluation would be a complicated process. Analysis of a number of wear equations obtained theoretically and experimentally shows that in many cases the wear rate at a contact point can be presented in the form: [4]

$$\frac{dw_*}{dt} = K_w P^{\alpha} v_c^{\beta} \tag{15}$$

where w_* is a wear parameter (volume, mass or depth of the worn material), K_w is the wear coefficient, and α and β are parameters which depend on the material properties, friction conditions, temperature, and etc.

Eq. (15) is used to evaluate the wear of a flat plate due to a single impact. So the contacting points of the xy coordinate system and the corresponding points from the x_1y_1 coordinate system should be determined at each instant of the impact duration. The contact force and relative velocity have the different values on the various points of the contact area and also vary due to the impact duration. The elastic normal load at any contact point could be obtained by using the Hertz contact theory. It should be noted that the contact area is different in size and position for the different time intervals, so the normal load and the radius of the contact area vary over the impact time. Based on the Hertz relations, the load distribution over the contact area is:

$$P_i(r) = P_{i\ \max}(1 - \frac{r^2}{a_i^2}) \tag{16}$$

where $P_i(r)$ is the normal load at each contact point of distance r from the center of contact circle, $P_{i\ max}$ is the maximum normal load and a_i is the radius of the contact area and the subscript i denotes the time interval i. $P_{i\ max}$ could be obtained by the Hertz theory of contact as: [16]

$$P_{i_{\max}} = \frac{0.605(1-2\theta_2)}{\pi}\left(\frac{P_{ic}E^{*2}}{R^2}\right)^{\frac{2}{3}} \tag{17}$$

where, P_{ic} is the static equivalent contact load.

The elastic deformation develops until the yielding initiates when $P_{i\ max}$ reaches the $1.6S_y$ [16] then the plastic deformation alters the pressure distribution. Fig. (8) shows how the pressure distribution changes and becomes significantly flattened when the plastic zone extends to the contact surface.[16]

Based on Eq. (17) and $P_{i\ max}=1.6S_y$ at onset of yielding, the normal force at the yielding instant can then be derived as: [16]

$$F_y = \frac{\pi^3(1.65y)^3 R^2}{6E^{*2}} \tag{18}$$

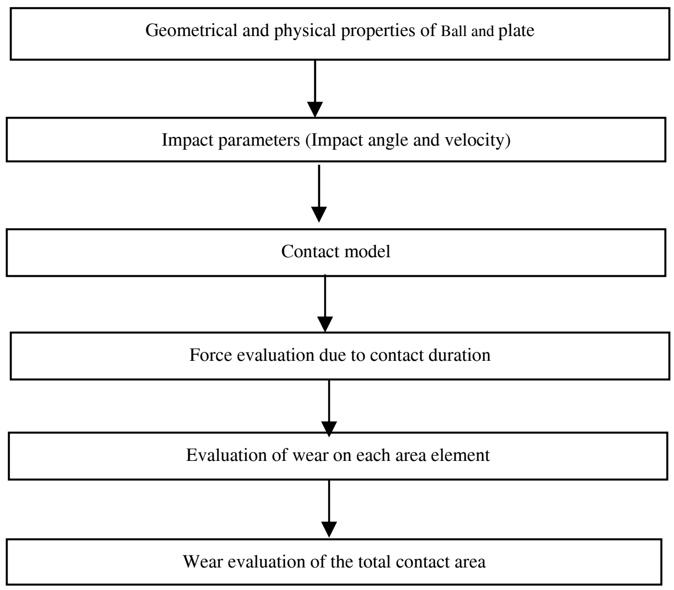
when the plastic deformation occurs, the pressure distribution is given by the following equation.[18]

$$P_i(r) = \frac{2p^*}{\pi}\sin^{-1}(\frac{a_i - r^2}{a^{*2} - r^2}) \tag{19}$$

In which p^* and a^* are respectively the pressure and radius of the contact area at the end of loading (overlap) or the start of unloading (restitution). By substitution the Eq. (19) into the Eq. (15) and determination of α, β and K_w the impact wear would be derived.

Proposed value of K_w for the steel plates is between 10^{-10}-10^{-13}. [4]. To achieve the best results, the different values of α and β are tried. For $\alpha=1$ and $\beta=0.3$ the model results are in good agreement with the experimental and FEM results [19]. The variation of the worn mass with respect to the impact angle is illustrated in Fig. (9). For the comparison purposes, we refer to the FEM data of impacting a flow of solid particles on a flat plate. [19] The solid particles with concentration of 464 g/m^3 due to 52 minutes impact the plate by the velocity of 20m/s. the wear results for the different impact angle is illustrated in Fig. (10). The sliding duration has the important role on the wear. Analyses indicate that the impact duration is a function of the initial velocity. Moreover, the dimensionless sliding duration (t_s/t^*) is independent of the impact velocity. Variation of the dimensionless sliding duration with respect to the impact angle Fig.(11) shows that the maximum wear occurs at the impact angle of about 60 degrees. The sliding duration is increased as the impact angle increases, so the wear would increase. After the impact angle increases higher than 60 degrees, the normal load decreases and the wear declines subsequently.

The complete process of wear evaluation is given in the following flow chart.



6 RESULTS AND DISCUSSION

The impact wear of a flat plate has been evaluated. The H-DD nonlinear relations are manipulated to model the impact process. The time increment procedure is used to track the ball path and determine the restitution parameters. Some of the impact parameters are compared with the experimental data to validate impact model. The wear is evaluated using a wear equation and the results have a good accordance with the validated FEM data. The presented model could be used to analyze the wear rate of the mechanical elements which are subjected to the impact loadings.

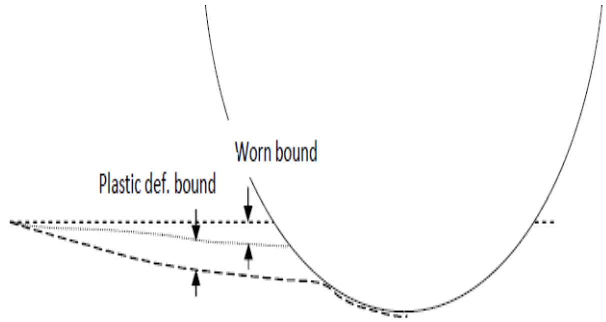


Fig.7
Schematic of deformed surface of the flat plate due to impact, plastic deformation bound(_ _ _), worn layer bound(….).

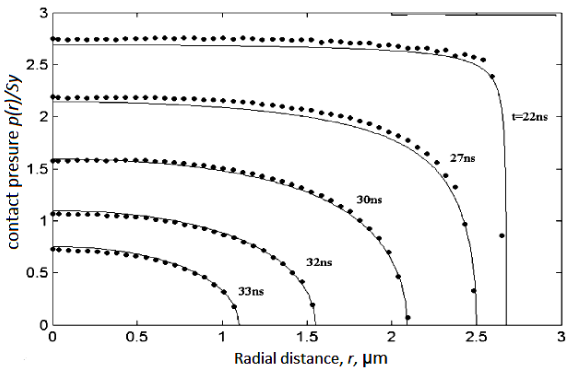


Fig. 8
Pressure distribution under the contact area at different instants of impact (restitution period) [16].

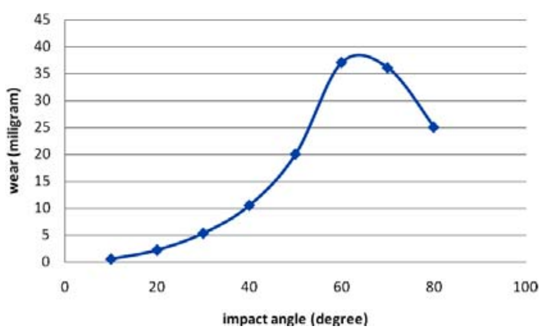


Fig. 9
Variation of the wear mass for the α=1 and β=0.3.

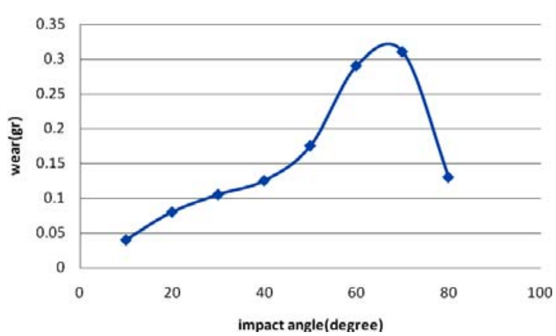


Fig. 10
Variation of the worn mass with respect to the impact angle for impacting a flow of solid particles [19].

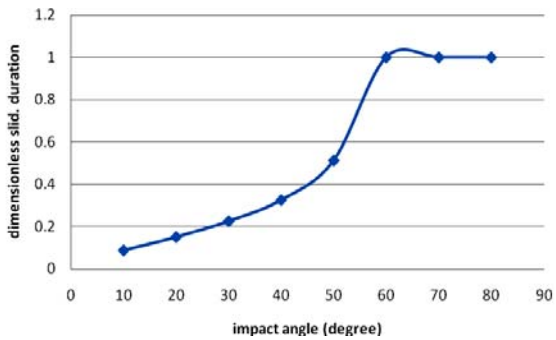


Fig. 11
Variation of the dimensionless sliding duration with respect to the impact angle.

7 CONCLUSIONS

The H-DD impact model followed by the time increment procedure well predicts the impact behavior of a ball on a flat plate.Based on the validated FEM results, the wear Eq. (15) would be an acceptable wear model to evaluate the impact wear.

REFERENCES

[1] Bayer R. G., Engel P. A., Sirico J. L., 1971, Impact wear testing machine, *Wear* **24**:343-354.
[2] Engel P. A., Lyons T. H., Sirico J. L., 1973, Impact wear for steel specimens, *Wear* **23**:185-201.
[3] Engel P. A., Millis D.B., 1982, Study of surface topology in impact wear, *Wear* **75**:423-442.
[4] Goryacheva I.G., *Contact Mechanics in Tribology*, Institute for Problems in Mechanics, Russian Academy of Sciences, Moscow, Russia, Kluwer Academic Publishers.
[5] Mindlin R. D., Deresiewicz H., 1953, Elastic spheres in contact under varying oblique forces, *Applied Mechanics* **16**:259-268.
[6] Maw N., 1975, The oblique impact of elastic spheres, *Wear* **25**: 101-114.
[7] Gorham D. A., Kharaz A. H., 2000, The measurement of particle rebound characteristics, *Powder Technology* **112**:193-202.
[8] Kharaz A.H., Gorham D.A., Salman A.D., 2001, An experimental study of the elastic rebound of spheres, *Powder Technology* **120**(3): 281-291.
[9] Levy A., 1993, The erosion–corrosion of tubing steels in combustion boiler environments, *Corrosion Science* **35**:1035-1056.
[10] Bellman R., Levy A., 1981, Erosion mechanism in ductile metals, *Wear* **70**(1): 1-27.
[11] Lindsley B.A., Marder A.R., 1999, The effect of velocity on the solid particle erosion rate of alloys, *Wear* **225–229**: 510-516.
[12] Head W.J., Harr M.E., 1970, The development of a model to predict the erosion of materials by natural contaminants, *Wear* **15**: 1-46.
[13] Xie Y., Clark H.McI., Hawthorne H.M., 1999, Modelling slurry particle dynamics in the Coriolis erosion tester, *Wear* **225–229**: 405-416.
[14] Talia M., Lankarani H., Talia J.E., 1999, New experimental technique for the study and analysis of solid particle erosion mechanisms, *Wear* **225–229** (2):1070-1077.
[15] Di Maio F. P., Di Renzo A., 2005, Modeling particle contacts in distinct element simulations, *Chemical Engineering Research and Design* **83:**1287-1297.
[16] Chuan-yu W., Long-yuan L., Colin T., 2003, Rebound behaviour of spheres for plastic impacts, *International Journal of Impact Engineering* **28**: 929-946.
[17] Lewis A.D., Rogers R. J., 1988, Experimental and numerical study of forces during oblique impact, *Journal of Sound and* **125**(3): 403-412.
[18] Mesarovic SDJ., Johnson KL., 2000, Adhesive contact of elastic–plastic spheres, *Journal of Mechanical Physic Solids* **24**: 127-138
[19] Ashrafizadeh H., Ashrafizadeh F., 2012, A numerical 3D simulation for prediction of wear caused by solid particle impact, *Wear* **276-277**: 75-84.