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Two New Non-AFR Criteria for Depicting Strength Differential Effect (SDE) in Anisotropic Sheet Metals

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ABSTRACT

The issue of pressure sensitivity of anisotropic sheet metals is investigated with introducing two new non-AFR criteria which are called here linear and non-Linear pressure sensitive criteria. The yield and plastic potential functions of these criteria are calibrated with directional tensile/compressive yield stresses and directional tensile Lankford coefficients, respectively. To determine unknown coefficients of yield and plastic potential functions of these criteria two error functions are presented which are minimized by Downhill Simplex Method. Three anisotropic materials are considered as case studies such as Al 2008-T4 (BCC), Al 2090-T3 (FCC) and AZ31 (HCP). It is shown that the non-Linear pressure sensitive criterion is more accurate than the linear one and other existed criteria compared to experimental results in calculating the directional mechanical properties of anisotropic sheet metals.

Keywords : Linear pressure sensitive criterion; Non-linear pressure sensitive criterion; Asymmetric anisotropic sheet metals; Non-AFR;Tensile/compressive yield stresses; Lankford coefficients.

1 INTRODUCTION

NEW experimental tests on anisotropic materials showed that yield stresses and Lankford coefficients of some materials were different in tension and compression in different orientations from the rolling direction. Several investigations have been presented to model this phenomenon mathematically. Spitzig and Richmond [1] experimentally showed that in both iron-based materials and aluminum the flow stress was linearly rooted on hydrostatic pressure. Liu et al. [2] extended Hill criterion to contain orthotropic plastic materials with different yield stresses in tension and compression. Barlat et al. [3] introduced a plane stress yield function named Yld2000-2d and confirmed it with experimental and polycrystal data achieved on a binary Al-2.5 *wt. % Mg* alloy sheet. Stoughton and Yoon [4] suggested a non-AFR based on a pressure dependent yield criterion with isotropic hardening which was consistent with Spitzig and Richmond results [1]. Hu and Wang [5] proposed a yield function to anticipate the strength differential effect in tension and compression of materials. Hu [6] offered a yield function which defined the yield state by considering the influence of both loading force and loading direction for anisotropic materials. Artez [7] modified a plane stress yield function based on Hanford non-quadratic yield function called ‘Yld2003’ which was nearly as flexible as Balart yield function ‘Yld2000-2D’ but had a simpler mathematical form. Lee et al. [8] with considering the high directional differences in initial yield stress and also high asymmetry in tension and

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compression developed a yield function with pressure dependent term. Stoughton and Yoon [9] presented a model for proportional loading for any biaxial stress conditions. The model was demonstrated to lead a reduction in error of prediction of anisotropic stress-strain relationship in uniaxial and equal biaxial tension. Hu and Wang [10] defined a plastic potential function to depict the feature of plastic flow of material to build a suitable constitutive model. Huh et al. [11] computed the accuracy of anisotropic yield functions contain of Hill48, Yld89, Yld91, Yld96, Yld2000-2d, BBC2000 and Yld2000-18p based on root-mean square error (RMSE) of yield stresses and Lankford coefficients. Moayyedian and Kadkhodayan [12] were studied derivation of the second differentiation of a general yield surface by implicit time stepping method along with its consistent elastic-plastic modulus. Moreover, the explicit, trapezoidal implicit and fully implicit time stepping schemes were compared in rate-dependent plasticity. It was shown that implementing fully implicit time stepping scheme in rate-dependent plasticity predicts more accurate experimental results than other schemes. Lou et al. [13] proposed an approach to extend symmetric yield functions to consider the SD effect for incompressible sheet metals with associated flow rule. Safaei et al. [14] presented a non-associated plane stress anisotropic constitutive model with mixed isotropic-kinematic hardening. The quadratic Hill 1948 and non-quadratic Yld-2000-2d yield criteria were considered in the non-associated flow rule (non-AFR) model. Yoon et al. [15] proposed a yield function based on the first, second and third stress modified invariants of the stress tensor to depict strength differential effects of anisotropic materials. Safaei et al. [16] presented an approach to describe the evolution of anisotropy during plastic deformation. A non-AFR based on Yld2000-2d anisotropic yield model was employed. They described two simplified methods for the relationship between equivalent plastic strain and compliance factor in a non-AFR model. It was shown that if the non-AFR was simplified without any scaling of the plastic potential function, this resulted in a wrong definition of equivalent plastic strain. However, it was confirmed that this could be correct if the plastic potential function was scaled based on the data at uniaxial stress state. Moayyedian and Kadkhodayan [17] combined von Mises and Tresca surfaces in place of yield and plastic potential functions and vice versa. They showed that taking von Mises and Tresca surfaces as yield and plastic potential functions predicted experimental results more accurate than the associated von Mises. Moreover, taking Tresca and von Mises surfaces as yield and plastic potential functions predicted experimental results more precise than associated Tresca. Oya et al. [18] proposed a new expression for the plastic constitutive model for materials with initial anisotropy. For this purpose, a non-associated normality model, in which the plastic potential function was defined independently of the yield function, had been adopted. An explicit expression for the equivalent plastic strain rate, which was plastic-work-conjugated with the defined equivalent stress corresponding to the proposed yield function, was also presented. Moayyedian and Kadkhodayan [19] introduced Modified Yld2000-2d II with inserting modified Yld2000-2d and Yld2000-2d in place of yield and plastic potential functions respectively to depict the behavior of anisotropic pressure sensitive sheet metals more accurately. Moayyedian and Kadkhodayan [20] modified Burzynski criterion used for pressure sensitive isotropic materials for anisotropic pressure sensitive sheet metals based on non-AFR to better describe the asymmetric anisotropic sheet metals behavior. Ghaei and Taherizadeh [21] presented a model to describe the anisotropic behavior of sheet metals in both yield stresses and plastic strain ratios by using the non-AFR and quadratic yield and potential functions. Additionally, to reproduce an accurate prediction of cyclic plastic deformation phenomena, a two-surface mixed isotropic-nonlinear kinematic hardening model was combined with the quadratic non-AFR anisotropic formulation.

In the current study, two new non-AFR criteria are introduced entitled 'linear pressure sensitive criterion' and 'non-linear pressure sensitive criterion'. Two important aims are investigated here: 1.Investigating the effect of non-linear hydrostatic pressure on yielding of anisotropic sheet metals. 2. Studying the use of non-AFR in anisotropic sheet metals. To calibrate these criteria some experimental data points are needed for the yield and plastic potential functions. It is preferred to use directional tensile and compressive yield stresses for calibrating the yield functions and directional tensile Lankford coefficients for calibrating the plastic potential functions. Three anisotropic materials with different structures are used as case studies, i.e. Al 2008-T4 (BCC), Al 2090-T3 (FCC) and AZ31 (HCP). Directional tensile, compressive and Lankford coefficients of these materials are computed by these criteria and compared with Lou et al. [13], Yoon et al. [15] ones and experimental results. Finally, it is observed that the non-linear pressure sensitive criterion is more successful than the others in predicting experimental directional yield stresses and Lankford coefficients.

2 LINEAR AND NON-LINEAR PRESSURE SENSITIVE CRITERIA

To define the linear and non-linear pressure sensitive criteria, first the yield function of Yoon et al. [15] is improved to consider the influence of non-linear hydrostatic pressure on yielding of asymmetric anisotropic sheet metals with a new calibration approach. Then a new pressure insensitive plastic potential function is added to compute directional Lankford coefficients. Yoon introduced the criterion by the aid of two modified deviatoric stress tensors (s'_{ij} and s''_{ij}) in three dimensional space are:

$$\begin{Bmatrix} s'_{xx} \\ s'_{yy} \\ s'_{zz} \\ s'_{yz} \\ s'_{xz} \\ s'_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{c'_2+c'_3}{3} & -\frac{c'_3}{3} & -\frac{c'_2}{3} & 0 & 0 & 0 \\ -\frac{c'_3}{3} & \frac{c'_3+c'_1}{3} & -\frac{c'_1}{3} & 0 & 0 & 0 \\ -\frac{c'_2}{3} & -\frac{c'_1}{3} & \frac{c'_1+c'_2}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & c'_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & c'_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & c'_6 \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix}$$
$$\begin{Bmatrix} s''_{xx} \\ s''_{yy} \\ s''_{zz} \\ s''_{yz} \\ s''_{xz} \\ s''_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{c''_2+c''_3}{3} & -\frac{c''_3}{3} & -\frac{c''_2}{3} & 0 & 0 & 0 \\ -\frac{c''_3}{3} & \frac{c''_3+c''_1}{3} & -\frac{c''_1}{3} & 0 & 0 & 0 \\ -\frac{c''_2}{3} & -\frac{c''_1}{3} & \frac{c''_1+c''_2}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & c''_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & c''_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & c''_6 \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} \tag{1}$$

In this equation σ_{ij} is stress tensor while $c'_i\left(i=1,6\right)$ and $c''_i\left(i=1,6\right)$ are unknown anisotropic coefficients that are determined by different experimental tests. Assuming plane stress conditions in $\sigma_{xx}-\sigma_{yy}$ plane for sheet metals, the modified deviatoric stress components can be achieved as:

$$\begin{cases} \begin{cases} s'_{xx} = \frac{c'_2+c'_3}{3}\sigma_{xx} - \frac{c'_3}{3}\sigma_{yy} \\ s'_{yy} = -\frac{c'_3}{3}\sigma_{xx} + \frac{c'_3+c'_1}{3}\sigma_{yy} \\ s'_{zz} = -\frac{c'_2}{3}\sigma_{xx} - \frac{c'_1}{3}\sigma_{yy} \\ s'_{xy} = c'_6\tau_{xy} \end{cases} \\ \begin{cases} s''_{xx} = \frac{c''_2+c''_3}{3}\sigma_{xx} - \frac{c''_3}{3}\sigma_{yy} \\ s''_{yy} = -\frac{c''_3}{3}\sigma_{xx} + \frac{c''_3+c''_1}{3}\sigma_{yy} \\ s''_{zz} = -\frac{c''_2}{3}\sigma_{xx} - \frac{c''_1}{3}\sigma_{yy} \\ s''_{xy} = c''_6\tau_{xy} \end{cases} \end{cases} \tag{2}$$

Hence, the modified invariants are defined for anisotropic sheet metals are defined as:

$$\begin{cases} \bar{I}_1 = h_x \sigma_{xx} + h_y \sigma_{yy} \\ J_2' = -s'_{xx} s'_{yy} - s'_{yy} s'_{zz} - s'_{xx} s'_{zz} + s'^2_{xy} \\ J_3'' = s''_{xx} s''_{yy} s''_{zz} - s''_{zz} s''^2_{xy} \end{cases} \tag{3}$$

In this equation $\bar{I}_1$ is the first modified invariant of stress tensor while J_2' and J_3'' are the second and the third modified invariants of modified deviatoric stress tensors. Inserting Eq. (2) into Eq. (3), the modified invariants are obtained in terms of stress components as follows:

$$\begin{cases} \bar{I}_1 = h_x \sigma_{xx} + h_y \sigma_{yy} \\ J_2' = a_1' \sigma_{xx}^2 + a_2' \sigma_{xx} \sigma_{yy} + a_3' \sigma_{yy}^2 + a_4' \tau_{xy}^2 \\ J_3'' = a_1'' \sigma_{xx}^3 + a_2'' \sigma_{xx}^2 \sigma_{yy} + a_3'' \sigma_{xx} \sigma_{yy}^2 + a_4'' \sigma_{yy}^3 \\ + a_5'' \sigma_{xx} \tau_{xy}^2 + a_6'' \sigma_{yy} \tau_{xy}^2 \end{cases} \tag{4}$$

As it is seen in Eq. (4), $\bar{I}_1$ is a linear, J_2' is a quadratic and J_3'' is a cubic function of stress components. In Eq. (4), h_x and h_y are unknown coefficients for computing the first modified invariant of stress tensor (modified hydrostatic pressure) which can be computed with experimental tests and $a_i'\left(i=1,4\right)$ and $a_i''\left(i=1,6\right)$ are determined in terms of unknown coefficients c_i' and c_i'' as:

$$\begin{cases} a_1' = \dfrac{c_2'^2 + c_3'^2 + c_2' c_3'}{9} \\ a_2' = \dfrac{c_1' c_2' - c_1' c_3' - c_2' c_3' - 2 c_3'^2}{9} \\ a_3' = \dfrac{c_1'^2 + c_3'^2 + c_1' c_3'}{9} \\ a_4' = c_6'^2 \end{cases} \qquad \begin{cases} a_1'' = \dfrac{c_3'' c_2''^2 + c_2'' c_3''^2}{27} \\ a_2'' = \dfrac{c_1'' c_3''^2 - c_1'' c_2''^2 - c_3'' c_2''^2 - 2 c_2'' c_3''^2}{27} \\ a_3'' = \dfrac{c_2'' c_3''^2 - c_2'' c_1''^2 - c_3'' c_1''^2 - 2 c_1'' c_3''^2}{27} \\ a_4'' = \dfrac{c_3'' c_1''^2 + c_1'' c_3''^2}{27} \\ a_5'' = \dfrac{c_2'' c_6''^2}{3} \\ a_6'' = \dfrac{c_1'' c_6''^2}{3} \end{cases} \tag{5}$$

Now the yield functions of linear and non-linear pressure sensitive criteria are respectively presented as:

$$\begin{cases} F\left(\sigma_{ij}\right) = \bar{I}_1 + \left(J_2'^{\frac{3}{2}} + J_3''\right)^{\frac{1}{3}} = \sigma\left(\bar{\varepsilon}^p\right) \\ F\left(\sigma_{ij}\right) = \left(\bar{I}_1^3 + J_2'^{\frac{3}{2}} + J_3''\right)^{\frac{1}{3}} = \sigma\left(\bar{\varepsilon}^p\right) \end{cases} \tag{6}$$

It is observed that the yield function of non-linear pressure sensitive criterion is nonlinearly depended on modified hydrostatic pressure while the yield function of Yoon was linearly depended. In Eq. (6), $\bar{\varepsilon}^p$ is the effective plastic strain and $\sigma\left(\bar{\varepsilon}^p\right)$ defines the isotropic hardening for different anisotropic materials. These two yield functions are asymmetric in $\sigma_{xx} - \sigma_{yy}$ plane due to $\bar{I}_1$ and J_3'' .

In the current research, to compute the directional Lankford coefficients, a pressure insensitive plastic potential function is considered for two criteria. Two modified deviatoric stress tensors ($\bar{s}_{ij}'$ and $\bar{s}_{ij}''$) are introduced as in Eq.

(7) where $\bar{c}_i'\left(i=1,6\right)$ and $\bar{c}_i''\left(i=1,6\right)$ are unknown coefficients and are determined with different directional experimental tests. The modified deviatoric stress tensors are as shown in Eq. (8). Hence, two modified second and third stress invariants of modified deviatoric stress tensors are introduced, Eq. (9).

$$\begin{Bmatrix} \bar{s}_{xx}' \\ \bar{s}_{yy}' \\ \bar{s}_{zz}' \\ \bar{s}_{yz}' \\ \bar{s}_{xz}' \\ \bar{s}_{xy}' \end{Bmatrix} = \begin{bmatrix} \frac{\bar{c}_2'+\bar{c}_3'}{3} & -\frac{\bar{c}_3'}{3} & -\frac{\bar{c}_2'}{3} & 0 & 0 & 0 \\ -\frac{\bar{c}_3'}{3} & \frac{\bar{c}_3'+\bar{c}_1'}{3} & -\frac{\bar{c}_1'}{3} & 0 & 0 & 0 \\ -\frac{\bar{c}_2'}{3} & -\frac{\bar{c}_1'}{3} & \frac{\bar{c}_1'+\bar{c}_2'}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{c}_4' & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{c}_5' & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{c}_6' \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \bar{s}_{xx}'' \\ \bar{s}_{yy}'' \\ \bar{s}_{zz}'' \\ \bar{s}_{yz}'' \\ \bar{s}_{xz}'' \\ \bar{s}_{xy}'' \end{Bmatrix} = \begin{bmatrix} \frac{\bar{c}_2''+\bar{c}_3''}{3} & -\frac{\bar{c}_3''}{3} & -\frac{\bar{c}_2''}{3} & 0 & 0 & 0 \\ -\frac{\bar{c}_3''}{3} & \frac{\bar{c}_3''+c_1''}{3} & -\frac{\bar{c}_1''}{3} & 0 & 0 & 0 \\ -\frac{\bar{c}_2''}{3} & -\frac{\bar{c}_1''}{3} & \frac{\bar{c}_1''+\bar{c}_2''}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{c}_4'' & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{c}_5'' & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{c}_6'' \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} \tag{7}$$

$$\begin{cases} \bar{s}_{xx}' = \frac{\bar{c}_2'+\bar{c}_3'}{3}\sigma_{xx} - \frac{\bar{c}_3'}{3}\sigma_{yy} \\ \bar{s}_{yy}' = -\frac{\bar{c}_3'}{3}\sigma_{xx} + \frac{\bar{c}_3'+\bar{c}_1'}{3}\sigma_{yy} \\ \bar{s}_{zz}' = -\frac{\bar{c}_2'}{3}\sigma_{xx} - \frac{\bar{c}_1'}{3}\sigma_{yy} \\ \bar{s}_{xy}' = \bar{c}_6'\tau_{xy} \end{cases} \qquad \begin{cases} \bar{s}_{xx}'' = \frac{\bar{c}_2''+\bar{c}_3''}{3}\sigma_{xx} - \frac{\bar{c}_3''}{3}\sigma_{yy} \\ \bar{s}_{yy}'' = -\frac{\bar{c}_3''}{3}\sigma_{xx} + \frac{\bar{c}_3''+\bar{c}_1''}{3}\sigma_{yy} \\ \bar{s}_{zz}'' = -\frac{\bar{c}_2''}{3}\sigma_{xx} - \frac{\bar{c}_1''}{3}\sigma_{yy} \\ \bar{s}_{xy}'' = \bar{c}_6''\tau_{xy} \end{cases} \tag{8}$$

$$\begin{cases} \bar{J}_2' = -\bar{s}_{xx}'\bar{s}_{yy}' - \bar{s}_{yy}'\bar{s}_{zz}' - \bar{s}_{xx}'\bar{s}_{zz}' + \bar{s}_{xy}'^2 \\ \bar{J}_3'' = \bar{s}_{xx}''\bar{s}_{yy}''\bar{s}_{zz}'' - \bar{s}_{zz}''\bar{s}_{xy}''^2 \end{cases} \tag{9}$$

Substituting Eq. (8) into (9), $\bar{J}_2'$ and $\bar{J}_3''$ are obtained in terms of stress components and $a_i'\left(i=1,4\right)$ and $a_i''\left(i=1,6\right)$ are computed, Eqs. (10) and (11).

$$\begin{cases} \bar{J}_2' = \bar{a}_1'\sigma_{xx}^2 + \bar{a}_2'\sigma_{xx}\sigma_{yy} + \bar{a}_3'\sigma_{yy}^2 + \bar{a}_4'\tau_{xy}^2 \\ \bar{J}_3'' = \bar{a}_1''\sigma_{xx}^3 + \bar{a}_2''\sigma_{xx}^2\sigma_{yy} + \bar{a}_3''\sigma_{xx}\sigma_{yy}^2 + \bar{a}_4''\sigma_{yy}^3 \\ +\bar{a}_5''\sigma_{xx}\tau_{xy}^2 + \bar{a}_6''\sigma_{yy}\tau_{xy}^2 \end{cases} \tag{10}$$

$$\begin{cases} \bar{a}'_1 = \dfrac{\bar{c}'^2_2 + \bar{c}'^2_3 + \bar{c}'_2\bar{c}'_3}{9} \\ \bar{a}'_2 = \dfrac{\bar{c}'_1\bar{c}'_2 - \bar{c}'_1\bar{c}'_3 - \bar{c}'_2\bar{c}'_3 - 2\bar{c}'^2_3}{9} \\ \bar{a}'_3 = \dfrac{\bar{c}'^2_1 + \bar{c}'^2_3 + \bar{c}'_1\bar{c}'_3}{9} \\ \bar{a}'_4 = \bar{c}'^2_6 \end{cases} \qquad \begin{cases} \bar{a}''_1 = \dfrac{\bar{c}''_3\bar{c}''^2_2 + \bar{c}''_2\bar{c}''^2_3}{27} \\ \bar{a}''_2 = \dfrac{\bar{c}''_1\bar{c}''^2_3 - \bar{c}''_1\bar{c}''^2_2 - \bar{c}''_3\bar{c}''^2_2 - 2\bar{c}''_2\bar{c}''^2_3}{27} \\ \bar{a}''_3 = \dfrac{\bar{c}''_2\bar{c}''^2_3 - \bar{c}''_2\bar{c}''^2_1 - \bar{c}''_3\bar{c}''^2_1 - 2\bar{c}''_1\bar{c}''^2_3}{27} \\ \bar{a}''_4 = \dfrac{\bar{c}''_3\bar{c}''^2_1 + \bar{c}''_1\bar{c}''^2_3}{27} \\ \bar{a}''_5 = \dfrac{\bar{c}''_2\bar{c}''^2_6}{3} \\ \bar{a}''_6 = \dfrac{\bar{c}''_1\bar{c}''^2_6}{3} \end{cases} \tag{11}$$

Thus, plastic potential function of linear and non-linear pressure sensitive criteria is proposed as follows:

$$G\left(\sigma_{ij}\right) = \left(\bar{J}'^{\frac{3}{2}}_2 + \bar{J}''_3\right)^{\frac{1}{3}} \tag{12}$$

The pressure insensitivity of the plastic potential function of these criteria is due to satisfying the incompressibility of the plastic flow rule. It should be noted that Yoon et al. [15] have not considered a plastic potential function to compute Lankford coefficients. The proposed plastic potential function of linear and non-linear pressure sensitive criteria is generally asymmetric in $\sigma_{xx} - \sigma_{yy}$ plane because of $\bar{J}''_3$. To find the function in terms of stress components, Eq. (11) can be inserted in Eq. (10) and its result can be substituted into Eq. (12). To calibrate the function, its first differentiation with respect to stress tensor is required as in the following:

$$\begin{cases} \dfrac{\partial G}{\partial \sigma_{xx}} = G^{-2}\left(\dfrac{1}{2}\bar{J}'^{\frac{1}{2}}_2 \dfrac{\partial \bar{J}'_2}{\partial \sigma_{xx}} + \dfrac{1}{3}\dfrac{\partial \bar{J}''_3}{\partial \sigma_{xx}}\right) \\ \dfrac{\partial G}{\partial \sigma_{yy}} = G^{-2}\left(\dfrac{1}{2}\bar{J}'^{\frac{1}{2}}_2 \dfrac{\partial \bar{J}'_2}{\partial \sigma_{yy}} + \dfrac{1}{3}\dfrac{\partial \bar{J}''_3}{\partial \sigma_{yy}}\right) \\ \dfrac{\partial G}{\partial \tau_{xy}} = G^{-2}\left(\dfrac{1}{2}\bar{J}'^{\frac{1}{2}}_2 \dfrac{\partial \bar{J}'_2}{\partial \tau_{xy}} + \dfrac{1}{3}\dfrac{\partial \bar{J}''_3}{\partial \tau_{xy}}\right) \end{cases} \tag{13}$$

In the Eq. (13), $\dfrac{\partial \bar{J}'_2}{\partial \sigma_{ij}}$ and $\dfrac{\partial \bar{J}''_3}{\partial \sigma_{ij}}$ are determined in terms of stress components in the bellow:

$$\begin{cases} \dfrac{\partial \bar{J}'_2}{\partial \sigma_{xx}} = 2\bar{a}'_1\sigma_{xx} + \bar{a}'_2\sigma_{yy} \\ \dfrac{\partial \bar{J}'_2}{\partial \sigma_{yy}} = \bar{a}'_2\sigma_{xx} + 2\bar{a}'_3\sigma_{yy} \\ \dfrac{\partial \bar{J}'_2}{\partial \tau_{xy}} = 2\bar{a}'_4\tau_{xy} \end{cases} \qquad \begin{cases} \dfrac{\partial \bar{J}''_3}{\partial \sigma_{xx}} = 3\bar{a}''_1\sigma^2_{xx} + 2\bar{a}''_2\sigma_{xx}\sigma_{yy} + \bar{a}''_3\sigma^2_{yy} + \bar{a}''_5\tau^2_{xy} \\ \dfrac{\partial \bar{J}''_3}{\partial \sigma_{yy}} = \bar{a}''_2\sigma^2_{xx} + 2\bar{a}''_3\sigma_{xx}\sigma_{yy} + 3\bar{a}''_4\sigma^2_{yy} + \bar{a}''_6\tau^2_{xy} \\ \dfrac{\partial \bar{J}''_3}{\partial \tau_{xy}} = 2\left(\bar{a}''_5\sigma_{xx} + \bar{a}''_6\sigma_{yy}\right)\tau_{xy} \end{cases} \tag{14}$$

3 CALIBRATION OF LINEAR AND NON-LINEAR PRESSURE SENSITIVE CRITERIA

To calibrate the linear and non-linear pressure sensitive criteria, biaxial and uniaxial tensile and compressive yield stresses for its yield function and biaxial and uniaxial tensile Lankford coefficients for their plastic potential functions are needed in different orientations. In tensile test (in θ direction from rolling direction) the stress components can be found as:

$$\begin{cases} \sigma_{xx} = \sigma_\theta^T cos^2\theta \\ \sigma_{yy} = \sigma_\theta^T sin^2\theta \\ \tau_{xy} = \sigma_\theta^T sin\theta cos\theta \end{cases} \tag{15}$$

And similarly in compression test it is found that:

$$\begin{cases} \sigma_{xx} = -\sigma_\theta^C cos^2\theta \\ \sigma_{yy} = -\sigma_\theta^C sin^2\theta \\ \tau_{xy} = -\sigma_\theta^C sin\theta cos\theta \end{cases} \tag{16}$$

For tensile biaxial test the stress components are:

$$\begin{cases} \sigma_{xx} = \sigma_b^T \\ \sigma_{yy} = \sigma_b^T \\ \tau_{xy} = 0 \end{cases} \tag{17}$$

And for compressive biaxial test the stress components are:

$$\begin{cases} \sigma_{xx} = -\sigma_b^C \\ \sigma_{yy} = -\sigma_b^C \\ \tau_{xy} = 0 \end{cases} \tag{18}$$

In the current study, due to the pressure dependency of yield function of linear and non-linear pressure sensitive criteria, a non-AFR in plasticity theory is employed. Then the increment of the plastic strain tensor $\left(d\varepsilon_{ij}^p\right)$ components is introduced as:

$$\begin{cases} d\varepsilon_{xx}^p = d\lambda \dfrac{\partial G}{\partial \sigma_{xx}} \\ d\varepsilon_{yy}^p = d\lambda \dfrac{\partial G}{\partial \sigma_{yy}} \\ d\varepsilon_{xy}^p = d\lambda \dfrac{\partial G}{\partial \tau_{xy}} \end{cases} \tag{19}$$

where $d\lambda$ plastic multiplier and G is plastic potential function. In Eq. (19), the incompressibility condition of increment of plastic strain tensor stated as:

$$d\varepsilon_{zz}^p = -d\varepsilon_{xx}^p - d\varepsilon_{yy}^p \tag{20}$$

Due to incompressibility condition of plastic strain in Eq. (20), the proposed plastic potential function is pressure insensitive. Moreover, tensile uniaxial $\left(R_\theta^T\right)$ and biaxial $\left(R_b^T\right)$ Lankford coefficients (*R*-values) are defined as Eq. (21). By inserting Eqs. (15) to (18) into the first relation in Eq. (6) for linear pressure sensitive criterion and into the second relation of Eq. (6) for the non-linear pressure sensitive criterion the directional uniaxial and biaxial tensile and compressive yield stresses can be achieved respectively as Eq. (22).

$$\begin{cases} R_\theta^T = \dfrac{d\,\varepsilon_{yy}^p}{d\,\varepsilon_{zz}^p} = -\dfrac{\dfrac{\partial G}{\partial \sigma_{xx}} sin^2\theta + \dfrac{\partial G}{\partial \sigma_{yy}} cos^2\theta - \dfrac{\partial G}{\partial \tau_{xy}} sin\theta cos\theta}{\dfrac{\partial G}{\partial \sigma_{xx}} + \dfrac{\partial G}{\partial \sigma_{yy}}} \\ R_b^T = \dfrac{d\,\varepsilon_{yy}^p}{d\,\varepsilon_{xx}^p} = \dfrac{\dfrac{\partial G}{\partial \sigma_{yy}}}{\dfrac{\partial G}{\partial \sigma_{xx}}} \end{cases} \tag{21}$$

$$\begin{cases} \begin{cases} \sigma_\theta^T = \dfrac{\sigma\left(\bar{\varepsilon}^p\right)}{A + \left(B^{\frac{3}{2}} + C\right)^{\frac{1}{3}}} \\ \sigma_\theta^C = \dfrac{\sigma\left(\bar{\varepsilon}^p\right)}{-A + \left(B^{\frac{3}{2}} + C\right)^{\frac{1}{3}}} \\ \sigma_b^T = \dfrac{\sigma\left(\bar{\varepsilon}^p\right)}{\left(h_x + h_y\right) + \left[\left(a_1' + a_2' + a_3'\right)^{\frac{3}{2}} + \left(a_1'' + a_2'' + a_3'' + a_4''\right)\right]^{\frac{1}{3}}} \\ \sigma_b^C = \dfrac{\sigma\left(\bar{\varepsilon}^p\right)}{-\left(h_x + h_y\right) + \left[\left(a_1' + a_2' + a_3'\right)^{\frac{3}{2}} - \left(a_1'' + a_2'' + a_3'' + a_4''\right)\right]^{\frac{1}{3}}} \end{cases} \\ \begin{cases} \sigma_\theta^T = \dfrac{\sigma\left(\bar{\varepsilon}^p\right)}{\left[A^3 + B^{\frac{3}{2}} + C\right]^{\frac{1}{3}}} \\ \sigma_\theta^C = \dfrac{\sigma\left(\bar{\varepsilon}^p\right)}{\left[-A^3 + B^{\frac{3}{2}} - C\right]^{\frac{1}{3}}} \\ \sigma_b^T = \dfrac{\sigma\left(\bar{\varepsilon}^p\right)}{\left[\left(h_x + h_y\right)^3 + \left(a_1' + a_2' + a_3'\right)^{\frac{3}{2}} + \left(a_1'' + a_2'' + a_3'' + a_4''\right)\right]^{\frac{1}{3}}} \\ \sigma_b^C = \dfrac{\sigma\left(\bar{\varepsilon}^p\right)}{\left[-\left(h_x + h_y\right)^3 + \left(a_1' + a_2' + a_3'\right)^{\frac{3}{2}} - \left(a_1'' + a_2'' + a_3'' + a_4''\right)\right]^{\frac{1}{3}}} \end{cases} \end{cases} \tag{22}$$

Moreover, by inserting Eq. (15) and Eq. (17) into Eq. (12) and the results into Eq. (21), the uniaxial and biaxial tensile Lankford coefficients are obtained as:

$$\begin{cases} R_\theta^T = -\dfrac{D}{E} \\ R_b^T = \dfrac{H}{I} \end{cases} \tag{23}$$

where A,B,C,D,E,H,I are as follows:

$$\begin{cases} A = h_x \cos^2\theta + h_y \sin^2\theta \\ B = a_1' \cos^4\theta + \left(a_2' + a_4'\right)\cos^2\theta \sin^2\theta + a_3' \sin^4\theta \\ C = a_1''\cos^6\theta + \left(a_2'' + a_5''\right)\cos^4\theta \sin^2\theta + \left(a_3'' + a_6''\right)\cos^2\theta \sin^4\theta + a_4'' \sin^6\theta \\ D = \dfrac{1}{2}\begin{bmatrix} \bar{a}_1' \cos^4\theta + \left(\bar{a}_2' + \bar{a}_4'\right)\cos^2\theta \sin^2\theta + \\ \bar{a}_3' \sin^4\theta \end{bmatrix}^{\frac{1}{2}} \\ \begin{pmatrix} \bar{a}_2' \cos^4\theta + \left(2\bar{a}_1' + 2\bar{a}_3' - 2\bar{a}_4'\right)\cos^2\theta \sin^2\theta \\ + \bar{a}_2' \sin^4\theta \end{pmatrix} + \dfrac{1}{3}\begin{bmatrix} \bar{a}_2'' \cos^6\theta + \left(3\bar{a}_1'' + 2\bar{a}_3'' - 2\bar{a}_5'' + \bar{a}_6''\right) \\ \cos^4\theta \sin^2\theta + \left(2\bar{a}_2'' + 3\bar{a}_4'' + \bar{a}_5'' - 2\bar{a}_6''\right) \\ \cos^2\theta \sin^4\theta + \bar{a}_3'' \sin^6\theta \end{bmatrix} \\ E = \dfrac{1}{2}\begin{pmatrix} \bar{a}_1' \cos^4\theta + \left(\bar{a}_2' + \bar{a}_4'\right)\cos^2\theta \sin^2\theta + \\ \bar{a}_3' \sin^4\theta \end{pmatrix}^{\frac{1}{2}} \left[\left(2\bar{a}_1' + \bar{a}_2'\right)\cos^2\theta + \left(\bar{a}_2' + 2\bar{a}_3'\right)\sin^2\theta\right] + \\ \dfrac{1}{3}\begin{bmatrix} \left(3\bar{a}_1'' + \bar{a}_2''\right)\cos^4\theta + \\ \left(2\bar{a}_2'' + 2\bar{a}_3'' + \bar{a}_5'' + \bar{a}_6''\right)\cos^2\theta \sin^2\theta + \\ \left(\bar{a}_3'' + 3\bar{a}_4''\right)\sin^4\theta \end{bmatrix} \\ H = \dfrac{1}{2}\left(\bar{a}_1' + \bar{a}_2' + \bar{a}_3'\right)^{\frac{1}{2}}\left(\bar{a}_2' + 2\bar{a}_3'\right) + \dfrac{1}{3}\left(\bar{a}_2'' + 2\bar{a}_3'' + 3\bar{a}_4''\right) \\ I = \dfrac{1}{2}\left(\bar{a}_1' + \bar{a}_2' + \bar{a}_3'\right)^{\frac{1}{2}}\left(2\bar{a}_1' + \bar{a}_2'\right) + \dfrac{1}{3}\left(3\bar{a}_1'' + 2\bar{a}_2'' + \bar{a}_3''\right) \end{cases} \tag{24}$$

4 PARAMETER EVALUATIONS AND ROOT MEAN SQUARE ERRORS (RMSEs)

The yield functions which are asymmetric functions (pressure sensitive) are required to be calibrated with ten yield stress experimental tests such as uniaxial tensile $\left(\sigma_\theta^T\right)$, compressive $\left(\sigma_\theta^C\right)$ yield stresses in orientations of $0^\circ, 15^\circ, 45^\circ$ and 90° from the rolling direction and also tensile biaxial $\left(\sigma_b^T\right)$ and compressive biaxial $\left(\sigma_b^C\right)$ yield stresses. The new proposed plastic potential function which is an asymmetric function (pressure insensitive), is calibrated with eight experimental results such as uniaxial tensile Lankford coefficients $\left(R_\theta^T = \dfrac{d\,\varepsilon_{yy}^p}{d\,\varepsilon_{zz}^p}\right)$ in $0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ$ and 90°, and also biaxial tensile Lankford coefficient $\left(R_b^T = \dfrac{d\,\varepsilon_{yy}^p}{d\,\varepsilon_{xx}^p}\right)$. With these experimental results for an anisotropic sheet metal, 10 unknown coefficients in yield functions such as $h_x, h_y, c_i'\left(i = 1,2,3,6\right)$ and $c_i''\left(1,2,3,6\right)$, and also 8 parameters in the plastic potential functions such as $\bar{c}_i'\left(i = 1,2,3,6\right)$ and $\bar{c}_i''\left(i = 1,2,3,6\right)$ are determined by minimizing error functions $\left(E_1,E_2\right)$ in Eq. (25) and Eq. (26) respectively with Downhill Simplex Method.

$$
\begin{aligned}
E_1 = & \left[\frac{\left(\sigma_0^T\right)_{\exp.}}{\left(\sigma_0^T\right)_{pred.}}-1\right]^2+\left[\frac{\left(\sigma_{15}^T\right)_{\exp.}}{\left(\sigma_{15}^T\right)_{pred.}}-1\right]^2+\left[\frac{\left(\sigma_{45}^T\right)_{\exp.}}{\left(\sigma_{45}^T\right)_{pred.}}-1\right]^2+ \\
& \left[\frac{\left(\sigma_{90}^T\right)_{\exp.}}{\left(\sigma_{90}^T\right)_{pred.}}-1\right]^2+\left[\frac{\left(\sigma_b^T\right)_{\exp.}}{\left(\sigma_b^T\right)_{pred.}}-1\right]^2+\left[\frac{\left(\sigma_0^C\right)_{\exp.}}{\left(\sigma_0^C\right)_{pred.}}-1\right]^2+ \\
& \left[\frac{\left(\sigma_{15}^C\right)_{\exp.}}{\left(\sigma_{15}^C\right)_{pred.}}-1\right]^2+\left[\frac{\left(\sigma_{45}^C\right)_{\exp.}}{\left(\sigma_{45}^C\right)_{pred.}}-1\right]^2+\left[\frac{\left(\sigma_{90}^C\right)_{\exp.}}{\left(\sigma_{90}^C\right)_{pred.}}-1\right]^2+\left[\frac{\left(\sigma_b^C\right)_{\exp.}}{\left(\sigma_b^C\right)_{pred.}}-1\right]^2
\end{aligned}
\tag{25}
$$

By minimizing E_1 and E_2, the unknown coefficients of yield and plastic potential functions are achieved for an anisotropic sheet metal. To understand the difference between the calibration of modified criterion and Yoon et al. [15], it is mentioned that Yoon constructed an error function for obtaining yield function with eight experimental data points such as $\sigma_0^T,\sigma_{45}^T,\sigma_{90}^T,\sigma_b^T,\sigma_0^C,\sigma_{45}^C,\sigma_{90}^C,\sigma_b^C$ and to obtain h_x and h_y for a pressure sensitive anisotropic material, they proposed uniaxial tensile yield stress tests which should be carried out in a hydrostatic pressure chamber. They have not proposed plastic potential function for predicting Lankford coefficients.

$$
\begin{aligned}
E_2 = & \left[\frac{\left(R_0^T\right)_{pred.}}{\left(R_0^T\right)_{exp.}}-1\right]^2+\left[\frac{\left(R_{15}^T\right)_{pred.}}{\left(R_{15}^T\right)_{exp.}}-1\right]^2+\left[\frac{\left(R_{30}^T\right)_{pred.}}{\left(R_{30}^T\right)_{exp.}}-1\right]^2+\left[\frac{\left(R_{45}^T\right)_{pred.}}{\left(R_{45}^T\right)_{exp.}}-1\right]^2+ \\
& \left[\frac{\left(R_{60}^T\right)_{pred.}}{\left(R_{60}^T\right)_{exp.}}-1\right]^2+\left[\frac{\left(R_{75}^T\right)_{pred.}}{\left(R_{75}^T\right)_{exp.}}-1\right]^2+\left[\frac{\left(R_{90}^T\right)_{pred.}}{\left(R_{90}^T\right)_{exp.}}-1\right]^2+\left[\frac{\left(R_b^T\right)_{pred.}}{\left(R_b^T\right)_{exp.}}-1\right]^2
\end{aligned}
\tag{26}
$$

After finding 18 unknown coefficients of yield and plastic potential functions, the accuracy of two criteria in compared to experimental results can be investigated with introducing root-mean square errors (RMSEs) of the tensile $\left(E_\sigma^T\right)$, compressive $\left(E_\sigma^C\right)$ yield stresses, tensile Lankford coefficients $\left(E_R^T\right)$, biaxial tensile yield stress $\left(E_\sigma^{Tb}\right)$, biaxial compressive yield stress $\left(E_\sigma^{Cb}\right)$, biaxial tensile Lankford coefficient $\left(E_\sigma^{Tb}\right)$ as in Eqs. (27-32). By computing the RMSEs, the accuracy of two criteria and other ones such as Yoon et al. [15] and Lou et al. [13] in compared to experimental results can be investigated.

5 CASE STUDIES

To calibrate the linear and non-linear pressure sensitive criteria, eighteen experimental results for each one are required as explained in previous section. To the best knowledge of the authors these experimental values are not computed for any anisotropic sheet metal, therefore three proper anisotropic materials are considered, i.e. Al2008-T4, Al2090-T3 and AZ31.

$$E_{\sigma}^{T}=\frac{1}{7}\left(\begin{array}{l}\left[\dfrac{\left(\sigma_{0}^{T}\right)_{exp.}-\left(\sigma_{0}^{T}\right)_{pred.}}{\left(\sigma_{0}^{T}\right)_{exp.}}\right]^{2}+\left[\dfrac{\left(\sigma_{15}^{T}\right)_{exp.}-\left(\sigma_{15}^{T}\right)_{pred.}}{\left(\sigma_{15}^{T}\right)_{exp.}}\right]^{2}+\\ \left[\dfrac{\left(\sigma_{30}^{T}\right)_{exp.}-\left(\sigma_{30}^{T}\right)_{pred.}}{\left(\sigma_{30}^{T}\right)_{exp.}}\right]^{2}+\left[\dfrac{\left(\sigma_{45}^{T}\right)_{exp.}-\left(\sigma_{45}^{T}\right)_{pred.}}{\left(\sigma_{45}^{T}\right)_{exp.}}\right]^{2}+\\ \left[\dfrac{\left(\sigma_{60}^{T}\right)_{exp.}-\left(\sigma_{60}^{T}\right)_{pred.}}{\left(\sigma_{60}^{T}\right)_{exp.}}\right]^{2}+\left[\dfrac{\left(\sigma_{75}^{T}\right)_{exp.}-\left(\sigma_{75}^{T}\right)_{pred.}}{\left(\sigma_{75}^{T}\right)_{exp.}}\right]^{2}+\\ \left[\dfrac{\left(\sigma_{90}^{T}\right)_{exp.}-\left(\sigma_{90}^{T}\right)_{pred.}}{\left(\sigma_{90}^{T}\right)_{exp.}}\right]^{2}\end{array}\right)^{\frac{1}{2}}\times 100 \tag{27}$$

$$E_{\sigma}^{C}=\frac{1}{7}\left(\begin{array}{l}\left[\dfrac{\left(\sigma_{0}^{C}\right)_{exp.}-\left(\sigma_{0}^{C}\right)_{pred.}}{\left(\sigma_{0}^{C}\right)_{exp.}}\right]^{2}+\left[\dfrac{\left(\sigma_{15}^{C}\right)_{exp.}-\left(\sigma_{15}^{C}\right)_{pred.}}{\left(\sigma_{15}^{C}\right)_{exp.}}\right]^{2}+\\ \left[\dfrac{\left(\sigma_{30}^{C}\right)_{exp.}-\left(\sigma_{30}^{C}\right)_{pred.}}{\left(\sigma_{30}^{C}\right)_{exp.}}\right]^{2}+\left[\dfrac{\left(\sigma_{45}^{C}\right)_{exp.}-\left(\sigma_{45}^{C}\right)_{pred.}}{\left(\sigma_{45}^{C}\right)_{exp.}}\right]^{2}+\\ \left[\dfrac{\left(\sigma_{60}^{C}\right)_{exp.}-\left(\sigma_{60}^{C}\right)_{pred.}}{\left(\sigma_{60}^{C}\right)_{exp.}}\right]^{2}+\left[\dfrac{\left(\sigma_{75}^{C}\right)_{exp.}-\left(\sigma_{75}^{C}\right)_{pred.}}{\left(\sigma_{75}^{C}\right)_{exp.}}\right]^{2}+\\ \left[\dfrac{\left(\sigma_{90}^{C}\right)_{exp.}-\left(\sigma_{90}^{C}\right)_{pred.}}{\left(\sigma_{90}^{C}\right)_{exp.}}\right]^{2}\end{array}\right)^{\frac{1}{2}}\times 100 \tag{28}$$

$$E_{R}^{T}=\frac{1}{7}\left(\begin{array}{l}\left[\dfrac{\left(R_{0}^{T}\right)_{exp.}-\left(R_{0}^{T}\right)_{pred.}}{\left(R_{0}^{T}\right)_{exp.}}\right]^{2}+\left[\dfrac{\left(R_{15}^{T}\right)_{exp.}-\left(R_{15}^{T}\right)_{pred.}}{\left(R_{15}^{T}\right)_{exp.}}\right]^{2}+\\ \left[\dfrac{\left(R_{30}^{T}\right)_{exp.}-\left(R_{30}^{T}\right)_{pred.}}{\left(R_{30}^{T}\right)_{exp.}}\right]^{2}+\left[\dfrac{\left(R_{45}^{T}\right)_{exp.}-\left(R_{45}^{T}\right)_{pred.}}{\left(R_{45}^{T}\right)_{exp.}}\right]^{2}+\\ \left[\dfrac{\left(R_{60}^{T}\right)_{exp.}-\left(R_{60}^{T}\right)_{pred.}}{\left(R_{60}^{T}\right)_{exp.}}\right]^{2}+\left[\dfrac{\left(R_{75}^{T}\right)_{exp.}-\left(R_{75}^{T}\right)_{pred.}}{\left(R_{75}^{T}\right)_{exp.}}\right]^{2}+\\ \left[\dfrac{\left(R_{90}^{T}\right)_{exp.}-\left(R_{90}^{T}\right)_{pred.}}{\left(R_{90}^{T}\right)_{exp.}}\right]^{2}\end{array}\right)^{\frac{1}{2}}\times 100 \tag{29}$$

$$E_{\sigma}^{Tb}=\frac{\left|\left(\sigma_{b}^{T}\right)_{exp.}-\left(\sigma_{b}^{T}\right)_{pred.}\right|}{\left(\sigma_{b}^{T}\right)_{exp.}}\times 100 \tag{30}$$

$$E_{\sigma}^{Cb} = \frac{\left| \left(\sigma_b^C \right)_{exp.} - \left(\sigma_b^C \right)_{pred.} \right|}{\left(\sigma_b^C \right)_{exp.}} \times 100 \tag{31}$$

$$E_{R}^{Tb} = \frac{\left| \left(R_b^T \right)_{exp.} - \left(R_b^T \right)_{pred.} \right|}{\left(R_b^T \right)_{exp.}} \times 100 \tag{32}$$

In the following, the mechanical properties of three materials such as Al 2008-T4 (a BCC material), Al 2090-T3 (a FCC material) which are aluminum alloys and also AZ31 (a HCP material) which is a magnesium alloy given by Lou et al. [13] and Yoon et al. [15] are presented in Tables 1 to 3. If the biaxial tensile and compressive yield stresses cannot be determined for an anisotropic material experimentally, they can be determined from uniaxial tensile and compressive yield stresses in $0^{\circ}, 45^{\circ}$, and 90° direction $\left(\sigma_b^T = \frac{\sigma_0^T + 2\sigma_{45}^T + \sigma_{90}^T}{4}, \sigma_b^C = \frac{\sigma_0^C + 2\sigma_{45}^C + \sigma_{90}^C}{4} \right)$. Using these mechanical properties, the material parameters of the yield and plastic potential functions in Eqs. (6, 12) can be achieved by minimizing the error functions in Eqs. (25, 26) with Downhill Simplex Method, see Tables 4-7.

Table 1
Experimental results for Al 2008-T4, Al 2090-T3 and AZ31 in tension.

	Al 2008-T4	Al 2090-T3	AZ31
σ_0^T	211.67	279.62	170.82
σ_{15}^T	211.33	269.72	-
σ_{30}^T	208.50	255.00	-
σ_{45}^T	200.03	226.77	177.13
σ_{60}^T	197.30	227.50	-
σ_{75}^T	194.30	247.20	-
σ_{90}^T	191.56	254.45	191.83
σ_b^T	185.00	289.40	179.23

Table 2
Experimental results for Al 2008-T4, Al 2090-T3 and AZ31 in compression.

	Al 2008-T4	Al 2090-T3	AZ31
σ_0^C	213.79	248.02	96.58
σ_{15}^C	219.15	260.75	-
σ_{30}^C	227.55	255.00	-
σ_{45}^C	229.82	237.75	94.45
σ_{60}^C	222.75	245.75	-
σ_{75}^C	220.65	263.75	-
σ_{90}^C	214.64	266.48	103.38
σ_b^C	222.02	247.50	97.47

Table 3
Experimental results for Al 2008-T4, Al 2090-T3 and AZ31 for Lankford coefficients in tension.

	Al 2008-T4	Al 2090-T3	AZ31
R_0^T	0.870	0.210	-
R_{15}^T	0.814	0.330	-
R_{30}^T	0.634	0.690	-
R_{45}^T	0.500	1.580	-
R_{60}^T	0.508	1.050	-
R_{75}^T	0.506	0.550	-
R_{90}^T	0.530	0.690	-
R_b^T	1.000	0.670	-

Table 4
Coefficients in the yield function of linear pressure sensitive criterion for Al 2008-T4, Al 2090-T3 and AZ31.

	Al 2008-T4	Al 2090-T3	AZ31
c_1'	1.9095	0.8355	2.1153
c_2'	1.7286	-3.4787	2.5661
c_3'	1.7117	2.7316	2.3311
c_6'	1.6715	-2.1596	2.4751
c_1''	-0.0086	-0.0165	-1.0019
c_2''	-0.1139	0.0324	-1.6883
c_3''	5.1651	16.2643	-3.6159
c_6''	-0.0028	-20.6152	3.2528
h_x	0.0426	-0.1605	-0.1768
h_y	0.0621	0.0758	-0.2536

Table 5
Coefficients in the yield function of non-linear pressure sensitive criterion for Al 2008-T4, Al 2090-T3 and AZ31.

	Al 2008-T4	Al 2090-T3	AZ31
c_1'	1.9541	1.9920	2.3407
c_2'	1.7078	1.6551	2.7187
c_3'	1.8569	1.8120	2.3128
c_6'	1.7925	2.1289	2.5491
c_1''	-0.0626	0.0987	-0.2131
c_2''	-0.1128	8.4847	-0.2709
c_3''	5.3326	-0.0049	-13.2943
c_6''	-0.0009	0.7497	-9.9399
h_x	0.2461	-0.3160	-0.6423
h_y	0.4374	0.1637	-0.6699

Table 6
Coefficients in the plastic potential function of linear pressure sensitive criterion for Al 2008-T4, Al 2090-T3 and AZ31.

	Al 2008-T4	Al 2090-T3	AZ31
c'_1	1.2338	0.7243	-
c'_2	1.4122	1.4675	-
c'_3	0.3153	-0.4850	-
c'_6	0.4779	1.3773	-
c''_1	1.4175	0.9125	-
c''_2	2.2194	0.6933	-
c''_3	-0.1905	5.2155	-
c''_6	-0.0010	-1.0651	-

Table 7
Coefficients in the plastic potential function of non-linear pressure sensitive criterion for Al 2008-T4, Al 2090-T3 and AZ31.

	Al 2008-T4	Al 2090-T3	AZ31
c'_1	1.2338	0.5277	-
c'_2	1.4122	1.0691	-
c'_3	0.3152	-0.3533	-
c'_6	0.4778	1.0034	-
c''_1	1.4176	0.6647	-
c''_2	2.2194	0.5051	-
c''_3	-0.1905	3.7995	-
c''_6	-0.0011	0.7759	-

5.1 Application to Al 2008-T4

Substituting material parameters from Tables 4, 5. into Eq. (6) for Al 2008-T4, the yield functions in $\sigma_{xx} - \sigma_{yy}$ plane can be determined for linear and non-linear criteria. The yield louses predicted by linear and non-linear pressure sensitive criteria are compared with other criteria and experimental data in Fig. 1. It is seen that all criteria could forecast experimental results with proper accuracy for Al 2008-T4. Using Eqs. (22, 24) and coefficients from Table 4 and 5., directional tensile and compressive yield stresses can be achieved. Figs. 2, 3 show the tensile and compressive yield stresses in different orientations compared to other criteria and experimental data. It is seen that new criteria can predict experimental tensile and compressive yield stresses more precisely. Fig. 4 shows the Lankford coefficients in different directions for coefficients of Tables 6, 7. and using Eqs. (23, 24). It is seen that the new criteria predict experimental data with better accuracy compared to Lou et al. [13] and approximately similar to each other.

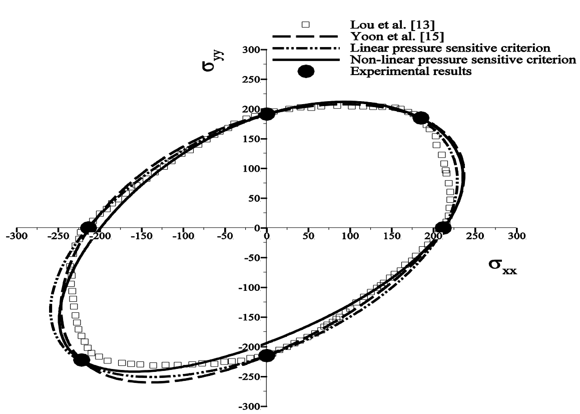

Fig.1
Comparison of yield functions in $\sigma_{xx} - \sigma_{yy}$ plane for Al 2008-T4.

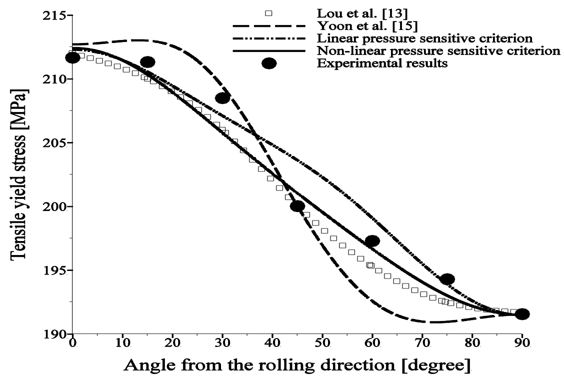

Fig.2
Comparison of the tensile yield stress directionality for Al 2008-T4.

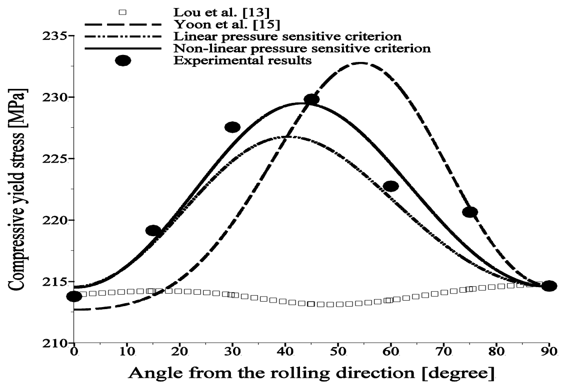

Fig.3
Comparison of the compressive yield stress directionality for Al 2008-T4.

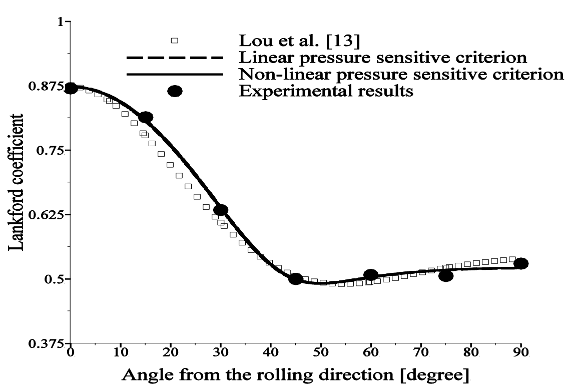

Fig.4
Comparison of Lankford coefficients directionality for Al 2008-T4.

5.2 Application to Al 2090-T3

In this section the yield functions in $\sigma_{xx} - \sigma_{yy}$ plane and also directional tensile and compressive yield stresses along with directional Lankford coefficients of linear and non-linear pressure sensitive criteria are investigated for Al 2090-T3 which is a FCC material. It is observed that the experimental data are predicted with all three criteria in $\sigma_{xx} - \sigma_{yy}$ plane with proper accuracy, Fig. 5. Moreover, although the new criteria are not as successful as the others in predicting experimental tensile yield stresses they are very accurate in predicting the experimental comressive yield stressesin in compare of others, Figs. 6, 7. It is observed that the new criteia are much better than Lou et al. [13] one and has almost the same accuracy to predict the directional Lankford coefficients compared to experimental results, Fig. 8.

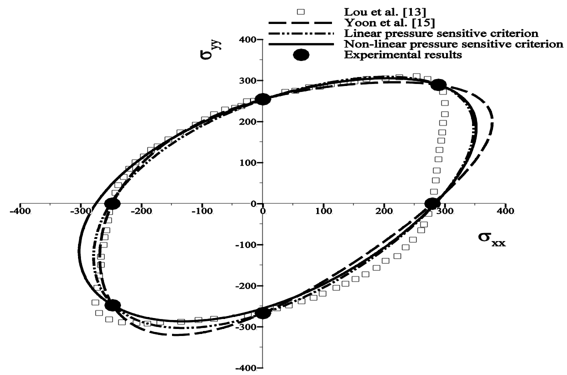

Fig.5
Comparison of yield functions in $\sigma_{xx} - \sigma_{yy}$ plane for Al 2090-T3.

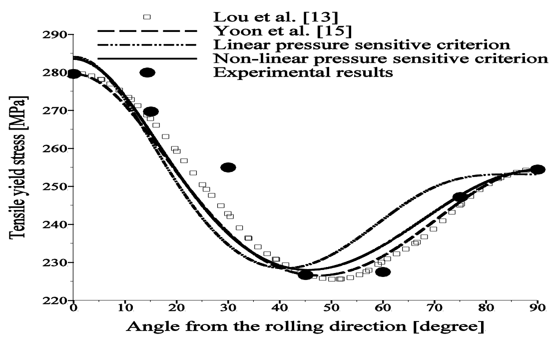

Fig.6
Comparison of the tensile yield stress directionality for Al 2090-T3.

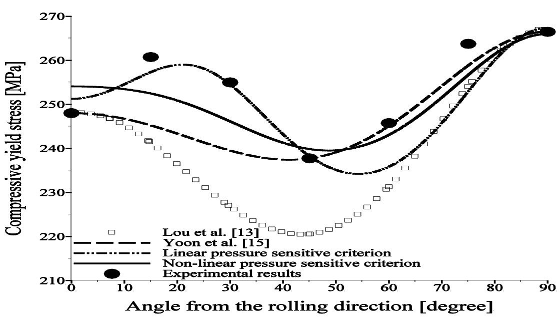

Fig.7
Comparison of the compressive yield stress directionality for Al 2090-T3.

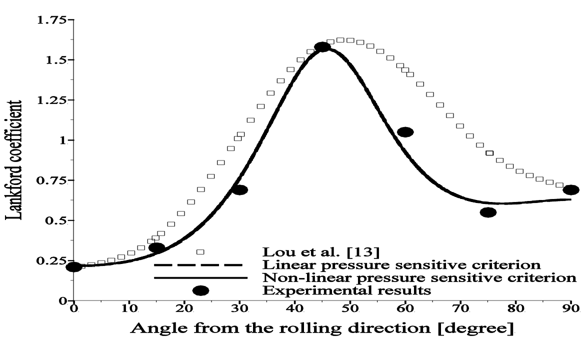

Fig.8
Comparison of Lankford coefficients directionality for Al 2090-T3.

5.3 Application to AZ31

The linear and non-linear pressure sensitive criteria can also be applied for a HCP material, AZ31 at 3% plastic strain. The experimental data for AZ31 are showed in Tables 1, 2.

Fig. 9 shows the yield functions of linear and non-linear pressure sensitive criteria in $\sigma_{xx} - \sigma_{yy}$ plane and its comparison with experimental results. It is observed that although the experimental data are predicted by both new criteria the geometries of loci are different. Moreover, the directional tensile and compressive yield stresses of two criteria are nearly the same.

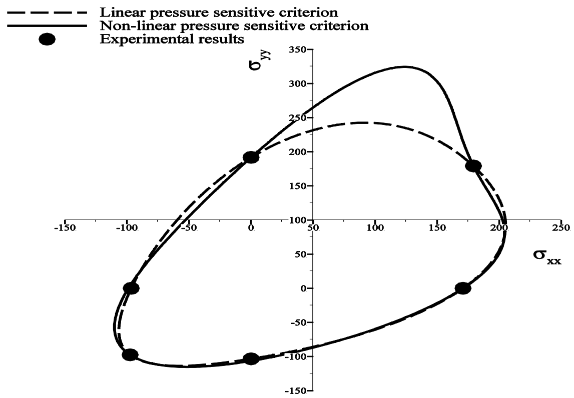

Fig.9
Comparison of yield functions in $\sigma_{xx} - \sigma_{yy}$ plane for AZ31.

6 DISCUSSIONS

In order to compare the accuracy of linear and non-linear pressure sensitive criteria for Al 2008-T4, Al 2090-T3 and AZ31, the RMSEs are employed. The experimental results of these materials are shown in Tables 1-3. The relative errors inserted in Tables 8-10. show the differences between different criteria and experimental data.

Table 8. refers to computing relative errors of Al 2008-T4 compared with experimental results. In computing uniaxial tensile yield stresses, the relative error $\left(E_\sigma^T\right)$ is minimum for non-linear pressure sensitive criterion i.e. this criterion is the most accurate one in computing this item with relative error of $E_\sigma^T = 0.2414\%$ which is a highly acceptable low error. Other criteria are used to compute directional uniaxial tensile yield stresses as Lou et al. [13] with $E_\sigma^T = 0.2704\%$, linear pressure sensitive criterion with $E_\sigma^T = 0.3119\%$ and Yoon et al. [15] with $E_\sigma^T = 0.4460\%$. In regard with computing uniaxial compressive yield stresses the relative error is $\left(E_\sigma^C\right)$. It this case also the non-linear pressure sensitive criterion with relative error with $E_\sigma^C = 0.2607\%$ is the best choice and other criteria can be arranged in order as linear pressure sensitive criterion with $E_\sigma^C = 0.3926\%$, Yoon et al. [15] with $E_\sigma^C = 0.8255\%$ and Lou et al. [13] with $E_\sigma^C = 1.5915\%$ respectively. In predicting the tensile uniaxial Lankford coefficients the relative error is $\left(E_R^T\right)$. Both linear and non-linear pressure sensitive criteria are more accurate ones and have nearly the same relative error with $E_R^T = 0.4507\%$ and Lou et al. [13] has more difference with $E_R^T = 3.9852\%$.

Table 8
The obtained computation errors for Al 2008-T4 compared with experimental results (in percentage).

	Yoon et al. [15]	Lou et al. [13]	linear pressure sensitive criterion	non-linear pressure sensitive criterion
E_σ^T	0.4460	0.2704	0.3119	0.2414
E_σ^{Tb}	0.0694	0.0778	0.9695	0.1213
E_σ^C	0.8255	1.5915	0.3926	0.2607
E_σ^{Cb}	0.1170	6.0219	0.7801	0.0720
E_R^T	-	3.9852	0.4507	0.4507
E_R^{Tb}	-	13.6586	0.0635	0.0768

Table 9. indicates the relative errors for Al 2090-T3 compared with experimental results. In computing uniaxial tensile yield stresses, the lowest and highest errors are related to Lou et al. [13] and linear pressure sensitive criteria with $E_{\sigma}^{T}=0.7350\%$ and $E_{\sigma}^{T}=1.5430\%$, respectively. However, in calculating the uniaxial compressive yield stresses, the lowest and highest errors are the ones of linear pressure sensitive and Lou et al. [13] criteria with $E_{\sigma}^{C}=0.8646\%$ and $E_{\sigma}^{C}=2.4651\%$, respectively. While, in predicting the tensile uniaxial Lankford coefficients the non-linear pressure sensitive and Lou et al. [13] criteria have the minimum and maximum errors, respectively.

Generally, it may be concluded that the non-linear pressure sensitive is the most proper criterion to predict directional mechanical properties in asymmetric anisotropic sheet metals.

Table 9
The obtained computation errors for Al 2090-T3 compared with experimental results (in percentage).

	Yoon et al. [15]	Lou et al. [13]	linear pressure sensitive criterion	non-linear pressure sensitive criterion
E_{σ}^{T}	1.0599	0.7350	1.5430	1.1307
E_{σ}^{Tb}	0.0009	0.0015	0.2563	0.0081
E_{σ}^{C}	1.2441	2.4651	0.8646	0.8728
E_{σ}^{Cb}	0.0005	8.2141	0.2963	0.0070
E_{R}^{T}	-	12.8890	3.3955	3.3953
E_{R}^{Tb}	-	4.1151	1.5543	1.5762

Table 10
The obtained computation errors for AZ31 compared with experimental results (in percentage).

	linear pressure sensitive criterion	non-linear pressure sensitive criterion
E_{σ}^{T}	0.0033	0.0043
E_{σ}^{Tb}	0.0011	0.0018
E_{σ}^{C}	0.0161	0.0149
E_{σ}^{Cb}	0.0021	0.0009

7 CONCLUSIONS

Two new non-AFR criteria with introducing a pressure sensitive function for their yield functions and a pressure insensitive function for their plastic potential functions are proposed entitled linear and non-linear pressure sensitive criteria. The dependency to modified hydrostatic pressure is linear to the former and non-linear to the latter. The yield and plastic potential functions are calibrated with ten and eight experimental data, respectively. To verify the linear and non-linear pressure sensitive criteria, three anisotropic materials are selected contain of Al 2008-T4 (a BCC material), Al 2090-T3 (a FCC material) and AZ31 (a HCP material). Finally, with computing relative errors it is shown that generally the non-linear and linear pressure sensitive criteria can predict directional tensile, compressive yield stresses and Lankford coefficients more successful than Yoon et al. [15] and Lou et al. [13] compared to experimental results.

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