Effect of Rotation and Stiffness on Surface Wave Propagation in a Elastic Layer Lying Over a Generalized Thermodiffusive Elastic Half-Space with Imperfect Boundary

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ABSTRACT

The present investigation is to study the surface waves propagation with imperfect boundary between an isotropic elastic layer of finite thickness and a homogenous isotropic thermodiffusive elastic half- space with rotation in the context of Green-Lindsay (G-L model) theory. The secular equation for surface waves in compact form is derived after developing the mathematical model. The phase velocity and attenuation coefficient are obtained for stiffness and then deduced for normal stiffness, tangential stiffness and welded contact. The dispersion curves for these quantities are illustrated to depict the effect of stiffness and thermal relaxation times. The amplitudes of displacements, temperature and concentration are computed at the free plane boundary. Specific loss of energy is obtained and presented graphically. The effects of rotation on phase velocity, attenuation coefficient and amplitudes of displacements, temperature change and concentration are depicted graphically. Some Special cases of interest are also deduced and compared with known results.

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Keywords: Isotropic; Generalized thermoelastic diffusion; Stiffness; Phase velocity; Attenuation coefficient; Amplitude; Rotation; Specific loss

1 INTRODUCTION

INTERFACE modeling has been the subject of numerous studies in material science and composite structure. The importance of researches in this topic can not be overemphasized as it is directly related to the prediction of the overall materials properties, delamination, transmission of force, etc (Benveniste [5], Achenbach and Zhu [1], Hashin [12-13], Zhong and Meguid [33], Pan [24]). The most ideal interface model, as it known, is called perfect bond interface where the displacement and traction are continuous across the interface. However, interfaces are seldom perfect. Therefore, various imperfect models such as three phase and linear like spring models have been introduced by Yu [32], Yu et al. [31] Benveniste [6]. Perhaps the most frequently studied imperfect interface model is the smooth bond interface, where the normal components of the displacements and traction are continuous across the interface, while the shear traction components are zero on the interface.

The Generalized theory of thermoelasticity is one of the modified versions of classical uncoupled and coupled theory of thermoelasticity and has been developed in order to remove the paradox of physical impossible phenomena of infinite velocity of thermal signals in the classical coupled thermoelasticity. Lord and Shulman [18] formulated a generalized theory of thermoelasticity with one thermal relaxation time, who obtained a wave-type equation by postulating a new law of heat conduction instead of classical Fourier's law. Green and Lindsay [11] developed a temperature rate- dependent thermoelasticity that includes two thermal relaxation times and does not violate the classical Fourier's law of heat conduction, when the body under consideration has a center of symmetry.

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One can refer to Hetnarski and Ignaczak [14] for a review and presentation of generalized theories of thermoelasticity. Diffusion can be defined as the random walk of an assemble of particles from regions of high concentration to that of low concentration. Nowadays, there is a great deals of interest in the study of phenomena due to its application in geophysics and electronic industry. In integrated circuit fabrication, diffusion is used to introduce "depants" in controlled amounts into semiconductor substance. In particular, diffusion is used to form the base and emitter in bipolar transistors, intregrated resistors and the source/drain in Metal Oxide Semiconductor (MOS) transistors and poly-silicon gates in MOS transistors. In most of the applications, the concentration is calculated using Fick's Law. This is simple law which does not take into consideration the mutual interaction between the introduced substance and the medium into which it is introduced. Study of this phenomenon diffusion is used to improve the conditions of oil extractions. These days' oil companies are interested in the process thermoelastic diffusion for more efficient extraction of oil from oil deposits.

Until recently, thermodiffusion in solids, especially in metals, was considered as a quantity that is independent of body deformation. Practice, however, indicates that the process of thermodiffusion could have a very considerable influence on the deformation of the body. Thermodiffusion in elastic solid is due to the coupling of temperature, mass diffusion and strain in addition to the exchange of heat and mass with the environment. Nowacki [19-22] developed the theory of thermoelastic diffusion by using coupled thermoelastic model. This implies infinite speed of propagation of thermoelastic waves .Dudziak and Kowalski [9] and Olesiak and Pyryev [23], investigated the theory of thermoelastic diffusion and coupled quasi-stationary problems of thermal diffusion for an elastic layer. They studied the influence of cross effects arising from the coupling of the fields of temperature, mass diffusion and strain due to which the thermal excitation results in additional mass concentration and generates additional fields of temperature. Sherief et al. [27] developed the generalized theory of thermoelastic diffusion with one relaxation time which allows finite speeds of propagation of waves. Recently, Sherief and Saleh [28] investigated the problem of a thermoelastic half space in the context of the theory of generalized thermoelastic diffusion with one relaxation time. Singh [29] discussed the reflection phenomena of waves from free surface of a thermoelastic diffusion with one relaxation time and with two relaxation time in [30]. Aouadi [2-4] investigated different problems in thermoelastic diffusion. Sharma et al. [25-26] discussed the effect of rotation on Rayleigh waves in the piezothermoelastic half-space. Kumar et al. [16] discussed the propagation of Rayleigh waves on free surface in transversely isotropic thermoelastic diffusion. Recently, Kumar et al. [17] derived the basic equations for generalized thermoelastic diffusion (GL model) and discussed the Lamb waves.

In this paper, linear model is adopted to represent the imperfectly bonded interface conditions. The linear model is simplified and idealized situation of imperfectly bonded interface, where the discontinuities in displacements at interfaces have a linear relationship with the interface stresses. Taking these applications into account, the surface waves propagation at imperfect boundary between a isotropic elastic layer and isotropic thermodiffusive elastic half-space with rotation in the context of Green-Lindsay theory is investigated. The phase velocity and attenuation coefficients of wave propagation have been computed from the secular equations. The amplitudes of displacements, temperature, concentration and specific loss are computed and depicted graphically.

2   BASIC EQUATIONS

Following Kumar et al [17], the basic governing equations for homogenous generalized thermodiffusive solid in the absence of body forces, heat and mass diffusion sources are
(i) Constitutive relations

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \delta_{ij}[\lambda\varepsilon_{kk} - \beta_1(T + \tau_1\dot{T}) - \beta_2(C + \tau^1\dot{C})] \tag{1}$$

(ii) Equations of motion in the rotating frame of reference are

$$\mu u_{i,jj} + (\lambda + \mu)u_{j,ij} - \beta_1(T + \tau_1\dot{T})_{,i} - \beta_2(C + \tau^1\dot{C})_{,i} = \rho[\ddot{u}_i + \mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{u})_{,i} + (2\mathbf{\Omega}\times\mathbf{u})_i] \tag{2}$$

(iii) Equation of heat conduction

$$\rho C_E(\dot{T} + \tau_0\ddot{T}) + \beta_1 T_0\dot{\varepsilon}_{kk} + aT_0(\dot{C} + \tau^0\ddot{C}) = KT_{,ii} \tag{3}$$

(iv) Equation of mass diffusion

$$D\beta_2 \varepsilon_{kk,ii} + Da(T + \tau_1 \dot{T})_{,ii} + \dot{C} - Db(C + \tau^1 \dot{C})_{,ii} = 0, \qquad i,j,k = 1,2,3 \tag{4}$$

Here, the medium is rotating with angular velocity $\mathbf{\Omega} = \Omega\hat{v}$, where $\hat{v}$ the unit vector along the axis of rotation and this equation of motion includes two additional terms namely:

(i) The centripetal acceleration $\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{u})$ due to time- varying motion

(ii) The Carioles acceleration $(2\mathbf{\Omega} \times \dot{\mathbf{u}})$

where $\beta_1 = (3\lambda + 2\mu)\alpha_t$ and $\beta_2 = (3\lambda + 2\mu)\alpha_c$; λ, μ are Lame's constants, α_t is the coefficient of linear thermal expansion. ρ, C_E are, respectively, the density and specific heat at constant strain, a,b are respectively , coefficient describing the measure of thermoelastic diffusion effects and of diffusion effects, T_0 is the reference temperature assumed to be such that $|T/T_0| << 1$. τ_0, τ_1 are thermal relaxation times with $\tau_1 \geq \tau_0 \geq 0$ and τ^0, τ^1 are diffusion relaxation times with $\tau^1 \geq \tau^0 \geq 0$. u_i are components of displacement vector. $T(x_1, x_2, x_3)$ is the temperature change and C is the concentration , $\sigma_{ij}(= \sigma_{ji})$, $\varepsilon_{ij} = \dfrac{u_{i,j} + u_{j,i}}{2}$ are respectively , the components of stress and strain tensor. The symbols ","and "." correspond to partial derivative and time derivative, respectively.

Following Bullen [7], the equations of motion and constitutive relations in isotropic elastic medium are given by

$$(\lambda^e + \mu^e)\, u^e_{j,ij} + \mu^e u^e_{i,jj} = \rho^e \ddot{u}^e_i \tag{5}$$

$$\sigma^e_{ij} = \lambda^e \theta^e \delta_{ij} + 2\mu^e \varepsilon^e_{ij}, \qquad i,j = 1,2,3 \tag{6}$$

where

$$\theta^e = u^e_{k,k} \tag{7}$$

$$\varepsilon^e_{ij} = (u^e_{i,j} + u^e_{j,i})/2, \qquad i,j = 1,2,3 \tag{8}$$

and $u^e = (u^e_1, u^e_2, u^e_3)$ is the displacement vector, ρ^e is the density of the isotropic medium and λ^e, μ^e are the Lame's constants, $\sigma^e_{ij}(= \sigma^e_{ji})$ are components of stress tensor, and δ_{ij} is the Kronecker delta.

3 FORMULATION OF THE PROBLEM

As shown in Fig. 1, we consider an isotropic elastic layer (Medium M_1) of thickness H overlaying a homogeneous, isotropic, generalized thermodiffusive elastic half-space in rotating frame of reference (Medium M_2). The origin of the co-ordinate system $(x_1, x_2\ x_3)$ taken at any point on the horizontal surface and x_1-axis in the direction of wave propagation and x_3- axis taking vertically downward into half-space, so that all particles on a line parallel to x_2-axis are equally displaced. Therefore, all the field quantities will be independent of x_2- axis co-ordinate. The interface between isotropic elastic layer and thermodiffusive elastic half-space with rotation has been taken at an imperfect boundary. The displacement vector $\vec{u}$, temperature T, concentration C and rotation for medium M_2 are taken as

$$\vec{u} = (u_1, 0, u_3), \qquad T(x_1, x_2, t), \qquad C(x_1, x_2, t), \qquad \mathbf{\Omega} = (0, \Omega, 0) \tag{9}$$

and displacement vector $\vec{u}^e$ for the layer (Medium M_1) is taken as

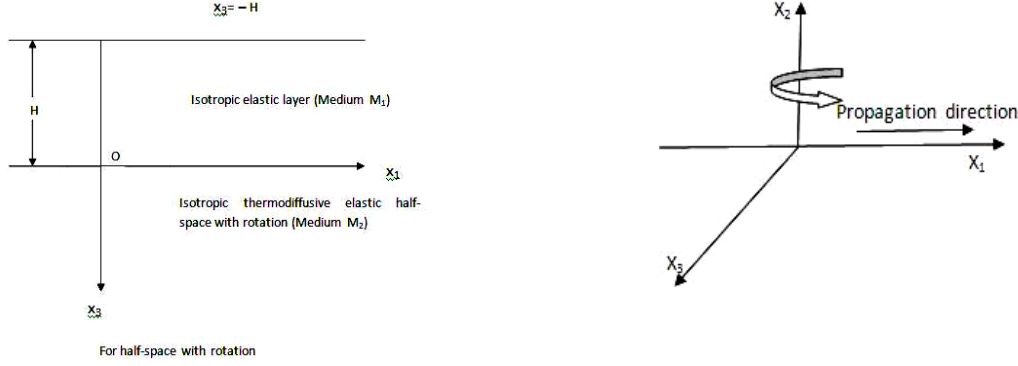$$\vec{u}^e = (u^e_1, 0, u^e_3) \tag{10}$$

Fig. 1
Geometry of the problem.

We define the dimensionless quantities

$$x_i' = \frac{\omega_1^* x_i}{v_1}, \qquad t' = \omega_1^* t, \qquad u_i' = \frac{\omega_1^* u_i}{v_1}, \qquad T' = \frac{\beta_1 T}{\rho v_1^2}, \qquad C' = \frac{\beta_2 C}{\rho v_1^2},$$

$$\tau_0' = \omega_1^* \tau_0, \qquad \tau_1' = \omega_1^* \tau_1, \qquad \tau^{0'} = \omega_1^* \tau^0, \qquad \tau^{1'} = \omega_1^* \tau^1, \qquad \sigma_{ij}' = \frac{\sigma_{ij}}{a_1 T_0}, \tag{11}$$

$$k_n' = \frac{v_1}{\beta_1 T_0 \omega_1^*} k_n, \qquad k_t' = \frac{v_1}{\beta_1 T_0 \omega_1^*} k_t, \qquad \text{where} \qquad v_1^2 = \frac{\lambda + 2\mu}{\rho}, \qquad \omega_1^* = \frac{\rho C_E v_1^2}{K_1}$$

Upon introducing the quantities in Eqs. (2)-(4) and (5), after suppressing the primes, with the aid of (9) and (10), we obtain

$$u_{1,11} + \delta_1 u_{1,33} + \delta_2 u_{3,13} - \tau_\theta^1 T_{,1} - \tau_c^1 C_{,1} = \ddot{u}_1 - \Omega^2 u_1 + 2\Omega \dot{u}_3 \tag{12}$$

$$\delta_2 u_{1,31} + \delta_1 u_{3,11} + u_{3,33} - \tau_\theta^1 T_{,3} - \tau_c^1 C_{,3} = \ddot{u}_3 - \Omega^2 u_3 + 2\Omega \dot{u}_1 \tag{13}$$

$$\nabla^2 T = \tau_\theta^0 \dot{T} + \chi_1 \tau_f^0 \dot{C} + \chi_2 \dot{e} \tag{14}$$

$$q_1^* \nabla^2 e + q_2^* \tau_\theta^1 \nabla^2 T - q_3^* \tau_c^1 \nabla^2 C + \dot{C} = 0 \tag{15}$$

$$\frac{(u_{1,11}^e + u_{3,13}^e)}{\delta_3^2} + \frac{\nabla^2 u_1^e}{\delta_4^2} = \ddot{u}_1^e \tag{16}$$

$$\frac{(u_{1,13}^e + u_{3,33}^e)}{\delta_3^2} + \frac{\nabla^2 u_3^e}{\delta_4^2} = \ddot{u}_3^e \tag{17}$$

where

$$\delta_1 = \frac{\mu}{\lambda + 2\mu}, \qquad \delta_2 = \frac{\lambda + \mu}{\lambda + 2\mu}, \qquad \chi_1 = \frac{a T_0 v_1^2 \beta_1}{\omega_1^* K \beta_2}, \qquad \chi_2 = \frac{\beta_1^2 T_0}{\rho K \omega_1^*}$$

$$q_1^* = \frac{D \omega_1^* \beta_2^2}{\rho v_1^4}, \qquad q_2^* = \frac{D \omega_1^* \beta_2 a}{\beta_1 v_1^2}, \qquad q_3^* = \frac{D \omega_1^* b}{v_1^2}, \qquad \nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2},$$

$$\tau_\theta^1 = 1 + \tau_1 \frac{\partial}{\partial t}, \qquad \tau_c^1 = 1 + \tau^1 \frac{\partial}{\partial t}, \qquad \tau_\theta^0 = 1 + \tau_0 \frac{\partial}{\partial t}, \qquad \tau_f^0 = 1 + \tau^0 \frac{\partial}{\partial t},$$

$$\delta_3^2 = \frac{\rho^e v_1^2}{\lambda^e + \mu^e}, \qquad \delta_4^2 = \frac{\rho^e v_1^2}{\mu^e}, \qquad \delta_5^2 = \frac{\rho^e v_1^2}{\lambda^e + 2\mu^e}, \qquad e = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}, \tag{18}$$

For an isotropic elastic layer, we introduce potential function Φ and Ψ through the relations

$$u_1^e = \frac{\partial \Phi}{\partial x_1} - \frac{\partial \Psi}{\partial x_3}, \qquad u_3^e = \frac{\partial \Phi}{\partial x_3} + \frac{\partial \Psi}{\partial x_1} \tag{19}$$

From the Eqs. (16), (17) and (19), we have

$$\Phi_{,11} + \Phi_{,33} - \delta_5^2 \ddot{\Phi} = 0 \tag{20}$$

$$\Psi_{,11} + \Psi_{,33} - \delta_4^2 \ddot{\Psi} = 0 \tag{21}$$

4 SOLUTION OF THE PROBLEM

To solve the Eqs. (12)-(15) and (20)-(21), we assume the solution in the form

$$(u_1, u_3, T, C, \Phi, \Psi) = (1, W, S, R, P_1, P_2) \, U \, \exp\left[i\xi(x_1 + mx_3 - ct)\right] \tag{22}$$

where $c = \omega / \xi$ is the non dimensional phase velocity, ω is the frequency; m is still parameter, $1, W, S, R$ are respectively, the amplitude ratio of u_1, u_3, T, C w.r.t. u_1. Substituting the values of u_1, u_3, T, C from (22) in Eqs. (12)-(15), we obtain

$$\delta_1 m^2 + 1 - c^2(1 + \Lambda^2) + (\delta_2 m - 2i\Lambda c^2)W + i\omega^{-1}c(\tau_{11}S + \tau_{21}R) = 0 \tag{23}$$

$$\delta_2 m + 2i\Lambda c^2 + (\delta_1 + m^2 - c^2(1 + \Lambda^2))W + i\omega^{-1}cm(\tau_{11}S + \tau_{21}R) = 0 \tag{24}$$

$$\chi_2 \tau_{10} c + \chi_2 \tau_{10} mcW - i\omega^{-1}(i\omega(1 + m^2) + c^2 \tau_{20})S - i\omega^{-1}c^2 \chi_1 R = 0 \tag{25}$$

$$q_1^*(1 + m^2) + mq_1^*(1 + m^2)W - i\omega^{-1}c(1 + m^2)\tau_{11}q_2^*S + (ic\omega^{-1}(1 + m^2)q_3^*\tau_{21} + \omega^{-2}c^3)R = 0 \tag{26}$$

where

$$\Lambda = \Omega\omega^{-1}, \qquad \tau_{11} = 1 - i\omega\tau_1, \qquad \tau_{21} = 1 - i\omega\tau^1, \qquad \tau_{10} = 1 - i\omega\tau_0, \qquad \tau_{20} = 1 - i\omega\tau^0 \tag{27}$$

The system of Eqs. (23)-(26) have a non-trivial solution if the determinant of the coefficient $[1, W, S, R]^{\mathrm{T}}$ vanishes, which yield to the following polynomial characteristic equation

$$m^8 + A^* m^6 + B^* m^4 + C^* m^2 + D^* = 0 \tag{28}$$

The constants A^*, B^*, C^*, D^* are given in Appendix A. The characteristic Eq. (28) is biquadratic in m^2 and hence possesses four roots m_p^2; $p = 1, 2, 3, 4$ corresponding to four roots; there exist three types of quasi-longitudinal waves and one quasi-transverse wave. The formal expression for displacement, temperature and concentration satisfying the radiation condition Re $(m_p) \geq 0$ can be written as

$$(u_1, u_3 T, C) = \sum_{p=1}^{4} A_p(1, n_{1p}, n_{2p}, n_{3p}) \, \exp\left[i\xi(x_1 + im_p x_3 - ct)\right] \tag{29}$$

Substituting the values of Φ and Ψ from (22) in (20) - (21) and with the aid of (19), we obtain

$$u_1^e = i\xi(B_1 c_5 + B_2 s_5) + \xi m_6(D_1 s_6 - D_2 c_6) \, \exp \, i\xi(x_1 - ct) \tag{30}$$

$$\left[u_3^e = \xi m_5(B_2 c_5 - B_1 s_5) + i\xi(D_1 c_6 + D_2 s_6)\right] \exp \, i\xi(x_1 - ct) \tag{31}$$

where

$$m_5 = \sqrt{c^2 \delta_5^2 - 1}, \qquad m_6 = \sqrt{c^2 \delta_4^2 - 1}, \qquad E_1^2 = \frac{\lambda^e + 2\mu^e}{\rho^e v_1^2}, \qquad E_2^2 = \frac{\mu^e}{\rho^e v_1^2},$$

$$c_5 = \cos(\xi m_5 x_3), \qquad c_6 = \cos(\xi m_6 x_3), \qquad s_5 = \sin(\xi m_5 x_3), \qquad s_6 = \sin(\xi m_6 x_3) \qquad (32)$$

where $A_p (p = 1, 2, 3, 4)$, B_1, B_2, D_1, D_2 are arbitrary constants. The coupling constants n_{1p}, n_{2p}, n_{3p} $(p = 1, 2, 3, 4)$ are given in appendix B.

5 BOUNDARY CONDITIONS

In this paper, linear model is adopted to represent the imperfectly bonded interface conditions. The boundary conditions are the vanishing of the normal stress and tangential stress at free surface. The discontinuities in displacements have linear relations with stresses, continuity of normal and tangential stress, vanishing of the gradient of temperature and concentration at the interface between the isotropic elastic layer and isotropic thermodiffusive elastic half-space. Mathematically, these can be written as
(i) Mechanical conditions

$$\begin{cases} (\sigma_{33}^e)_{M_1} = 0 \\ (\sigma_{31}^e)_{M_1} = 0 \end{cases}, \qquad x_3 = -H \qquad (33)$$

$$\begin{cases} (\sigma_{33})_{M_1} = k_n \left[(u_3)_{M_2} - (u_3)_{M_1} \right] \\ (\sigma_{31})_{M_1} = k_t \left[(u_1)_{M_2} - (u_1)_{M_1} \right] \\ \quad (\sigma_{33}^e)_{M_1} = (\sigma_{33})_{M_2} \\ \quad (\sigma_{31}^e)_{M_1} = (\sigma_{31})_{M_2} \end{cases}, \qquad x_3 = 0 \qquad (34)$$

$$\frac{\partial T}{\partial x_3} = 0, \qquad \text{at} \qquad x_3 = 0 \qquad (35)$$

(iii) Concentration condition

$$\frac{\partial C}{\partial x_3} = 0, \qquad \text{at} \qquad x_3 = 0 \qquad (36)$$

where k_n, k_t normal and transverse stiffness of layer have dimension N m^{-3}.

6 DERIVATION OF THE SECULAR EQUATIONS

Substituting the value of $u_1, u_3, T, C, u_1^e, u_3^e$ from Eqs. (29), (30), (31) in (33)-(36), with the aid of (1), (6) and (9)-(11), after simplification we obtain

$$b_5 \tan(\xi m_5 H) \Delta_1 + b_5 \Delta_2 + \frac{b_6 s_6}{c_5} \Delta_3 - \frac{b_6 c_6}{c_5} \Delta_4 = 0 \qquad (37)$$

where $\Delta_1 = \left| R_{ij} \right|_{7 \times 7}$ the entries R_{ij} of the determinant are given in Appendix C and Δ_2 is obtained by replacing the first column of Δ_1 by $\begin{bmatrix} R_{11}^* & 0 & 0 & R_{41}^* & R_{51}^* & R_{61}^* & R_{71}^* \end{bmatrix}^T$, Δ_3 is obtained by replacing the second column of Δ_2 by $\begin{bmatrix} R_{12}^* & 0 & 0 & R_{42}^* & R_{52}^* & R_{62}^* & R_{72}^* \end{bmatrix}^T$, and Δ_4 is obtained by replacing the third column of Δ_3 by $\begin{bmatrix} R_{13}^* & 0 & 0 & R_{43}^* & R_{53}^* & R_{63}^* & R_{73}^* \end{bmatrix}^T$. The entries of $\Delta_p \, (p=1,2,3,4)$ are given in Appendix C. If we write

$$c^{-1} = v^{-1} + \mathrm{i}\omega^{-1}G \tag{38}$$

Then, $\xi = F + \mathrm{i}G$, where $F = \omega / v$ and G are real numbers. Also the roots of characteristic equations are in general complex. Hence, assume that $m_p = p_p + \mathrm{i}q_p$, so that exponent in the plane wave solutions in (22) becomes

$$\mathrm{i}F(x_1 - m_p^A x_3 - vt) - F\left(\frac{G}{F}x_1 + m_p^R x_3\right) \tag{39}$$

where

$$m_p^R = p_p - \mathrm{i}q_p \frac{G}{F}, \qquad m_p^A = q_p + p_p \frac{G}{F} \tag{40}$$

This shows that v is the propagation velocity and G is the attenuation coefficient of the wave. Upon using the representation (38) in secular Eq. (37), the values of propagation speed v and attenuation coefficient G of wave propagation can be obtained.

7   PARTICULAR CASES
(i) Normal stiffness

In this case, $K_n \neq 0$, $K_t \to \infty$ and the secular Eq. (37) remain the same. But the following will be replaced in the values of $\Delta_p (p=1,2,3,4)$

$$
\begin{aligned}
& R_{71} = -\mathrm{i}\xi, \qquad R_{72} = 0, \qquad R_{73} = \xi m_6, \qquad R_{74} = 1, \qquad R_{75} = 1, \\
& R_{76} = 1, \qquad R_{77} = 1, \qquad R_{71}^* = 0, \qquad R_{72}^* = -\mathrm{i}\xi, \qquad R_{73}^* = 0
\end{aligned}
\tag{41}
$$

(ii) Tangential stiffness

In this case, $K_t \neq 0$, $K_n \to \infty$ and the secular Eq. (37) remain the same with the change values of $\Delta_p (p=1,2,3,4)$ by taking

$$
\begin{aligned}
& R_{61} = 0, \qquad R_{62} = -\mathrm{i}\xi, \qquad R_{63} = 0, \qquad R_{64} = n_{12}, \qquad R_{65} = n_{11}, \\
& R_{66} = n_{13}, \qquad R_{67} = n_{14}, \qquad R_{61}^* = -\xi m_5, \qquad R_{62}^* = 0, \qquad R_{63}^* = -\mathrm{i}\xi
\end{aligned}
\tag{42}
$$

(iii) Welded contact

In this case $K_n \to \infty$, $K_t \to \infty$ and the secular Eq. (37) remain the same, but the value of $\Delta_p (p=1,2,3,4)$. are given by replacing

$$R_{71} = -i\xi, \qquad R_{72} = 0, \qquad R_{73} = \xi m_6, \qquad R_{74} = 1, \qquad R_{75} = 1,$$
$$R_{76} = 1, \qquad R_{77} = 1, \qquad R_{71}^* = 0, \qquad R_{72}^* = -i\xi, \qquad R_{73}^* = 0,$$
$$R_{61} = 0, \qquad R_{62} = -i\xi, \qquad R_{63} = 0, \qquad R_{64} = n_{12}, \qquad R_{65} = n_{11},$$
$$R_{66} = n_{13}, \qquad R_{67} = n_{14}, \qquad R_{61}^* = -\xi m_5, \qquad R_{62}^* = 0, \qquad R_{63}^* = -i\xi \qquad (43)$$

8   SPECIAL CASES
Case (i)

If we take $\Omega = 0$, i.e. in the absence of rotation effect, the frequency equation (37) will reduce to the frequency equation for an isotropic elastic layer and a homogenous isotropic thermodiffusive elastic half-space without rotation.

Case (ii)
If we take $\Omega = 0, H = 0, \tau_1 > 0, \tau^1 > 0$ and in the absence of diffusion effect i.e. $b_1 = b_3 = a = b = 0$, the equation (37) will reduce to the frequency equation for Rayleigh wave

$$\left(2 - \frac{c^2}{\delta_1^2}\right)^2 (\overline{m}_1^2 + \overline{m}_2^2 + \overline{m}_1\overline{m}_2 - 1 + c^2) - 4\overline{m}_1\overline{m}_2\overline{m}_3(\overline{m}_1 + \overline{m}_2) = 0 \qquad (44)$$

Here $\overline{m}_3 = \sqrt{1 - \dfrac{c^2}{\delta_1^2}}$, and $\overline{m}_p(p = 1, 2)$ are the roots of the equation (28), obtained by taking $b_1 = b_3 = a = b = 0$

and $\tau_1 > 0, \tau^1 > 0$. The resulting equation (44) is similar to the equation (20) as given by Dawn et al. [8].

Case (iii)
In the absence of isotropic elastic layer, thermal and diffusion effects, we obtain the frequency equation corresponding to isotropic elastic half-space by changing the dimensionless quantities into physical quantities as

$$\left(2 - \frac{c^2}{c_2^2}\right)^2 = 4\left(1 - \frac{c^2}{c_1^2}\right)^{1/2}\left(1 - \frac{c^2}{c_2^2}\right)^{1/2} \qquad (45)$$

where $c_1^2 = \dfrac{\lambda + 2\mu}{\rho}, c_2^2 = \dfrac{\mu}{\rho}$. The frequency Eq. (45) is same as derived in Ewing et al. [10]

9   SURFACE DISPLACEMENTS, TEMPERATURE CHANGE AND CONCENTRATION

The amplitude of surface displacements, temperature change and concentration at the surface $x_3 = 0$ during Rayleigh wave propagation in the cases of stress free, vanishing of the gradient of temperature and concentration of the half space are

$$u_1 = \overline{E}A\exp[iF(x_1 - vt), \qquad u_3 = \overline{R}A\exp[iF(x_1 - vt),$$
$$T = \overline{W}A\exp[iF(x_1 - vt), \qquad C = \overline{L}A\exp[iF(x_1 - vt) \qquad (46)$$

where

$$A = A_1\exp(-Fx_1), \qquad (47)$$

$$\bar{E} = \frac{(F_1^* - F_2^* + F_3^* - F_4^*)}{F_1^*}, \qquad \bar{R} = \frac{(n_{11}F_1^* - n_{12}F_2^* + n_{13}F_3^* - n_{14}F_4^*)}{F_1^*},$$

$$\bar{W} = \frac{(n_{21}F_1^* - n_{22}F_2^* + n_{23}F_3^* - n_{24}F_4^*)}{F_1^*}, \qquad \bar{L} = \frac{(n_{31}F_1^* - n_{32}F_2^* + n_{33}F_3^* - n_{34}F_4^*)}{F_1^*}$$

and $F_p^*(p = 1, 2, 3, 4)$ are given in Appendix D

10   SPECIFIC LOSS

The Specific loss is the ratio of energy (ΔW) dissipated in taking a specimen through a stress cycle, to the elastic energy (W) stored in the specimen when the strain is maximum. The Specific loss is the most direct method of defining internal friction for a material. For a sinusoidal plane wave of small amplitude, Kolsky [15], shows the specific loss $\Delta W / W$ equals 4π times the absolute value of imaginary part of ξ to the real part of ξ, i.e.

$$\frac{\Delta W}{W} = 4\pi \left| \frac{\mathrm{Im}(\xi)}{\mathrm{Re}(\xi)} \right| = 4\pi \left| \frac{vG}{\omega} \right| = 4\pi \left| \frac{G}{F} \right| \tag{48}$$

11   NUMERICAL RESULTS AND DISCUSSION

Following Sherief and Saleh [21], we take the following values of relevant parameter for the copper material as

$$\lambda = 7.76 \times 10^{10} \, \mathrm{Kgm^{-1}s^{-2}}, \qquad \mu = 3.86 \times 10^{10} \, \mathrm{Kgm^{-1}s^{-2}}, \qquad T_0 = 0.293 \times 10^3 \, \mathrm{K},$$

$$C_E = .3831 \times 10^3 \, \mathrm{JKg^{-1}K^{-1}}, \; \alpha_t = 1.78 \times 10^{-5} \, \mathrm{K^{-1}}, \qquad \alpha_c = 1.98 \times 10^{-4} \, \mathrm{m^3Kg^{-1}},$$

$$a = 1.2 \times 10^4 \, \mathrm{m^2s^{-2}K^{-1}}, \quad b = 9 \times 10^5 \, \mathrm{Kg^{-1}m^5s^{-2}}, \qquad D = 0.85 \times 10^{-8} \, \mathrm{Kgsm^{-3}},$$

$$\rho = 8.954 \times 10^3 \, \mathrm{Kgm^{-3}}, \qquad K = 0.383 \times 10^3 \, \mathrm{Wm^{-1}K^{-1}}, \qquad \tau_0 = 0.07 \, \mathrm{s},$$

$$\tau_1 = 0.06 \, \mathrm{s}, \qquad \tau^0 = 0.04, \qquad \tau^1 = 0.03$$

The elastic parameters for Granite are given by Bullen [32]

$$\lambda^e = 2.238 \times 10^3 \, \mathrm{J.Kg^{-1}K^{-1}}, \qquad \mu^e = 2.238 \times 10^3 \, \mathrm{J.Kg^{-1}K^{-1}}, \qquad \rho^e = 2.65 \times 10^3 \, \mathrm{J.Kgm^{-3}}$$

The non- dimensional phase velocity and attenuation coefficient of wave propagation in the context of Green Lindsay theory (G-L) of thermoelastic diffusion have been computed for various values of non dimensional wave number from Eq. (38) and represented graphically. The solid line, small dashes line, big dashes line, big dashes line with dotted respectively refer to stiffness(WS), normal stiffness(NS), tangential stiffness(TS),welded contact(WC) in case of without rotation(WTR) and centre symbols on these lines correspond to thermoelastic with rotation(WR).

11.1.1 Phase velocity and attenuation coefficient

Fig. 2 shows that the value of phase velocity for with stiffness (WS), normal stiffness (NS), tangential stiffness (TS) and welded contact (WC) decreases in case of WR for lower wave number, but in case of WTR the value of phase velocity for WS, TS, WC increases for lower wave number and for higher wave number, the values of phase velocity in both cases WT and WTR become dispersionless, and for comparison, it is noticed that the value of phase velocity for WS,NS,TS,WC remain more in case of WR more (in comparison with WTR) for higher wave number. Fig. 3 shows that the value of attenuation coefficient for WS, NS, TS,WC in both cases WT and WTR increases with oscillation and, in comparison, it is noticed that the value of attenuation coefficient for WS, NS, TS,WC in case of WR remain more (in comparison with WTR) for higher wave number.

11.1.2 Amplitudes of displacements, temperature change and concentration

The variations of amplitude of displacements (u_1, u_3), temperature Change (T) and concentration with respect to wave number have been computed and are shown in Figs. 4-7. Fig. 4 indicates that for lower wave number, the value of u_1 in case of WR, decreases, and for WTR it increases, but for higher wave number the values of u_1 in both cases WR and WTR become constant and for comparison, it is noticed that the value of u_1 in case of WR remain more (in comparison with WTR) for lower wave number, but for higher wave number reverse behavior occur. Fig. 5 shows that the value of u_3 in case of WR, decreases for lower wave number and it increases for higher wave number. But in case of WTR, reverse behavior occur and for comparison, it is evident that the value of u_3 in case of WTR remain more (in comparison with WR).
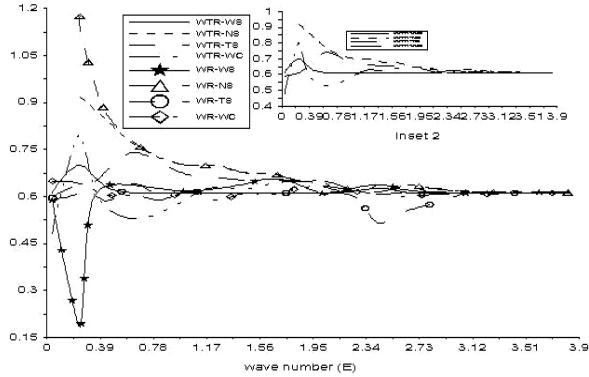

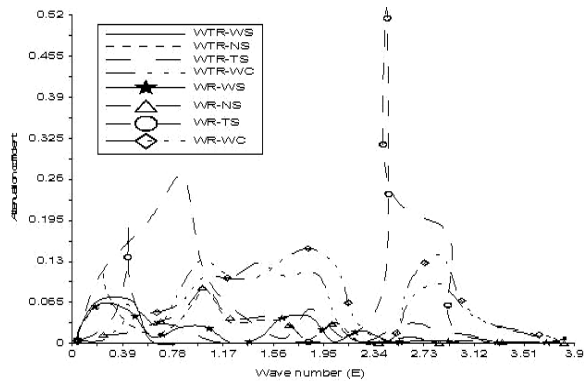**Fig. 2**
Variation of phase velocity w.r.t wave number.

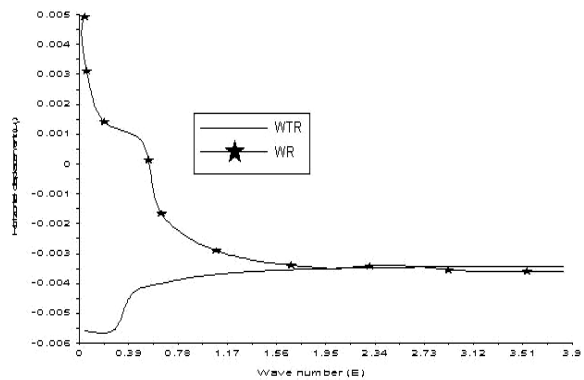
Fig. 3
Variation of attenuation coefficient w.r.t wave number.

Fig. 4
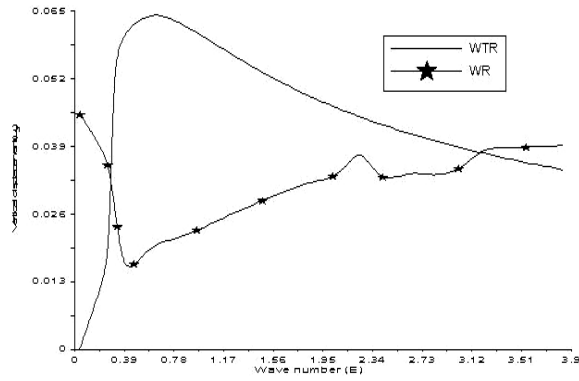Variation of horizontal displacement ($u1$) w.r.t wave number.

Fig. 5
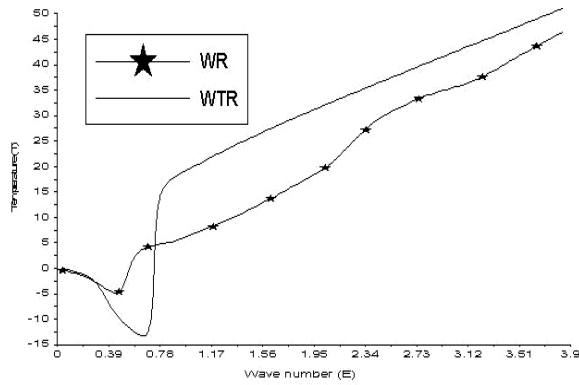Variation of vertical displacement (u_3) w.r.t wave number.

Fig. 6
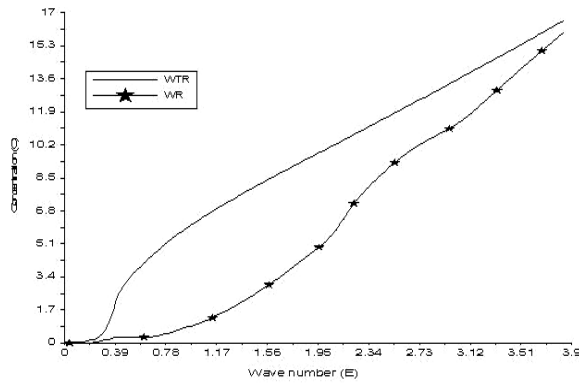Variation of temperature change (T) w.r.t wave number.

Fig. 7
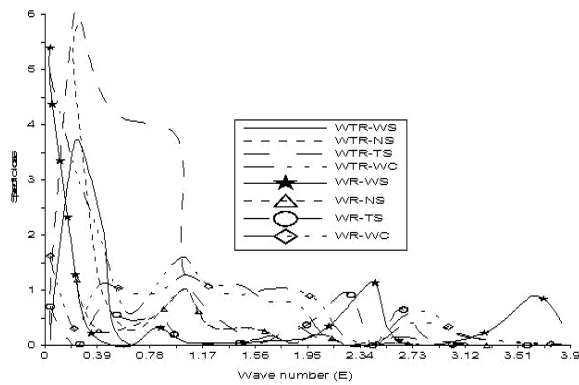Variation of concentration change (E) w.r.t wave number.

Fig. 8
Variation of specific loss w.r.t wave number.

Fig. 6 indicates that the value of T in both cases, WR and WTR, increases monotonically for higher wave number and for comparison, it is noticed that the value of T in case of WTR remain more (in comparison with WR) for higher wave number. Fig. 7 shows that the values of C in both cases WR and WTR increases monotonically and for comparison it is evident that the value of C in case WTR remains more (in comparison with WR).

11.1.3 Specific loss of energy

Fig. 8 shows the variation of Specific Loss with respect to wave number. It is noticed that the value of Specific Loss for WS, NS, TS, WC decreases with oscillation in both cases WR and WTR for lower wave number but become dispersionless for higher wave number and for comparison it is noticed that the values of Specific Loss for WS, NS, TS, WC in case of WR remain more (in comparison with WTR) for higher wave number.

12 CONCLUDING REMARKS

The propagation of surface waves at imperfect boundary between isotropic elastic layer of finite thickness and isotropic thermodiffusive elastic half- space with rotation in the context of Green-Lindsay theory have been discussed. The secular equation in compact form has been derived. The phase velocity and attenuation coefficient are depicted graphically for stiffness, normal stiffness, tangential stiffness and welded contact. The amplitudes of displacements, temperature and concentration are computed at the free plane boundary and presented graphically. Specific loss of energy is obtained and depicted graphically. It is noticed that the value of phase velocity and attenuation coefficient for WS, NS, TS, WC in case of with rotation(WR) remain more (in comparison with without rotation WTR).The value of displacements u_1, u_3 remain more in case of with rotation(in comparison with without rotation) for lower wave number, but for higher wave number reverse behavior occur and the value of temperature change(T) and concentration(C) remain more in case of without rotation (in compare with rotation).

The numerical results are found to be significantly in agreement with the corresponding analytic results. The effects of relaxation times and rotation are observed on phase velocity, attenuation coefficient and amplitudes of displacements, temperature change and concentration. The analysis to be carried will be useful in the design and construction of rotating sensors and surface acoustic wave's devices in addition to possible biosensing applications.

APPENDIX A

$$A^* = \frac{f_1 + f_2 + f_3}{\delta_1 d_7},$$

$$B^* = \frac{f_4 + (1 - h_1)f_1 - (f_5 + f_6 + f_7)}{\delta_1 d_7}$$

$$C^* = \frac{\left[f_4 + (1 - h_1)f_1 - \delta_2 f_9 + h_2 g_1 - h_3 g_2 \right]}{\delta_1 d_7}$$

$$D^* = \frac{d_1 g_3 + g_4 - d_3 g_5 - d_4 g_6}{\delta_1 d_7}$$

$$f_1 = d_2 - (\delta_1 - h_1)l_3 + h_2 d_5 - h_3 d_6$$

$$f_2 = \delta_2 d_9, \qquad f_3 = (1 - h_1)d_7 - h_3(q_1^* - \delta_2 q_1^*)$$

$$f_4 = d_1 + (\delta_1 - h_1)d_2 - h_2 d_3 - h_3 d_4$$

(A.1)

$$f_5 = \delta_2(d_8 - h_2 d_6), \qquad f_6 = l_1 h_2 d_5$$
$$f_7 = h_3[(\delta_1 - h_1)q_1^* + d_6 - \delta_2 d_6]$$
$$f_8 = \delta_1 d_1(\delta_1 - h_1), \qquad f_9 = \delta_2 d_1 - h_2 d_3 - d_4 h_3$$
$$g_1 = \delta_2 d_3 + (\delta_1 - h_1)d_5 - d_3, \qquad g_2 = (\delta_1 - h_1)d_6 + d_4 - \delta_2 d_4$$
$$g_3 = (1 - h_1)(\delta_1 - h_1), \qquad g_4 = l_1^2 d_1$$
$$g_5 = h_2(\delta_1 - h_1), \qquad g_6 = h_3(\delta_1 - h_1)$$
$$h_1 = c^2(1 + \Lambda^2), \qquad h_2 = i\omega^{-1}c\tau_{11}, \qquad h_3 = i\omega^{-1}c\tau_{21}, \qquad h_5 = \chi_2 c\tau_{20} \qquad h_7 = -i\omega^{-1}c^2\chi_1$$
$$l_1 = 2i\Lambda c^2, \qquad l_2 = i\omega^{-1}c\tau_{11}q_2^*, \qquad l_3 = -i\omega^{-1}c\tau_{21}q_3^*, \qquad l_4 = \omega^{-2}c^3, \qquad l_1^* = 1 - i\omega^{-1}c^2\tau_{20}$$
$$d_1 = l_1^*(l_4 - l_3) + h_7 l_2, \qquad d_2 = l_4 + h_7 l_2 - l_1^* l_3 - l_3, \qquad d_3 = l_4 h_5 - h_5 l_3 - h_7 q_1^*, \qquad d_4 = l_2 h_5 + l_1^* q_1^*,$$
$$d_5 = l_3 h_5 + h_7 q_1^*, \qquad d_6 = l_2 h_5 + l_1 q_1^* + q_1^*, \qquad d_7 = l_3 + h_3 q_1^*, \qquad d_8 = \delta_2 d_2 + h_2 d_5, \qquad d_9 = h_3 q_1^* + \delta_2 l_3$$

APPENDIX B

$$n_{1p} = -\frac{\left[\Lambda_1 + i\Lambda_2 m_p - \Lambda_3 m_p^2 - i\Lambda_4 m_p^3 - \Lambda_5 m_p^4 - i\Lambda_6 m_p^5\right]}{E_p^*},$$

$$n_{2p} = \frac{\left[\Lambda_7 + \Lambda_8 m_p + \Lambda_9 m_p^2 - \Lambda_{10} m_p^3 + \Lambda_{11} m_p^4\right]}{E_p^*},$$

$$n_{3p} = -\frac{\left[\Lambda_{12} - \Lambda_{13} m_p + \Lambda_{14} m_p^2 - \Lambda_{15} m_p^3 + \Lambda_{16} m_p^4 - \Lambda_{17} m_p^5 + \Lambda_{18} m_p^6\right]}{E_p^*}, \qquad p = 1, 2, 3, 4$$

$$E_p^* = G_1 - G_2 m_p^2 - G_3 m_p^6 + G_4 m_p^8 \tag{B.1}$$

where

$$G_1 = (\delta_1 - h_1)d_1, \qquad G_2 = d_1 + (\delta_1 - h_1)d_2 - h_2 d_3 - h_3 d_4, \qquad G_3 = d_2 - (\delta_1 - h_1)l_3 + h_2 d_5 - h_3 d_6,$$
$$G_4 = d_7, \qquad \Lambda_1 = l_1 d_1, \qquad \Lambda_2 = \delta_2 d_1 - h_2 d_3 - h_3 d_4, \qquad \Lambda_3 = l_1 d_2, \qquad \Lambda_4 = d_8 - h_3 d_6, \Lambda_5 = -l_1 l_3,$$
$$\Lambda_6 = d_9, \qquad \Lambda_7 = -(\delta_1 - h_1)d_3, \qquad \Lambda_8 = l_1 d_3, \qquad \Lambda_9 = \delta_2 d_3 + (\delta_1 - h_1)d_5 - d_3, \qquad \Lambda_{10} = l_1 d_5,$$
$$\Lambda_{11} = d_5 - \delta_2 d_5, \qquad \Lambda_{12} = (\delta_1 - h_1)d_4, \qquad \Lambda_{13} = l_1 d_4, \qquad \Lambda_{14} = d_4 + (\delta_1 - h_1)d_6 - \delta_2 d_4,$$
$$\Lambda_{15} = l_1 d_6, \qquad \Lambda_{16} = d_6 + (\delta_1 - h_1)q_1^* - \delta_2 d_6, \qquad \Lambda_{17} = q_1^* l_1, \qquad \Lambda_{18} = q_1^*(1 - \delta_2) \tag{B.2}$$

APPENDIX C

$$R_{14} = R_{15} = R_{16} = R_{17} = R_{21} = R_{22} = R_{23} = R_{31} = R_{32} = R_{33} = R_{41} = R_{42} = R_{51} = R_{53} = 0,$$
$$R_{11} = w_8, \qquad R_{12} = -y_7, \qquad R_{13} = w_7, \qquad R_{24} = b_2, \qquad R_{25} = b_1, \qquad R_{26} = b_3, \qquad R_{27} = b_4,$$
$$R_{34} = q_2, \qquad R_{35} = q_1, \qquad R_{36} = q_3, \qquad R_{37} = q_4, \qquad R_{43} = b_6, \qquad R_{44} = J_2, \qquad R_{45} = J_1,$$
$$R_{46} = J_3, \qquad R_{47} = J_4, \qquad R_{52} = b_7, \qquad R_{54} = s_2^*, \qquad R_{55} = s_1^*, \qquad R_{56} = s_3^*, \qquad R_{57} = s_4^*,$$
$$R_{61} = -b_5, \qquad R_{62} = i\xi, \qquad R_{63} = -b_6, \qquad R_{64} = r_2, \qquad R_{65} = r_1, \qquad R_{66} = r_3, \qquad R_{67} = r_4,$$
$$R_{71} = -i\xi, \qquad R_{72} = b_7, \qquad R_{73} = -\xi m_6, \qquad R_{74} = k_t, \qquad R_{75} = k_t, \qquad R_{76} = k_t, \qquad R_{77} = k_t,$$
$$R_{11}^* = y_8, \qquad R_{41}^* = -b_5, \qquad R_{51}^* = -b_8, \qquad R_{61}^* = -\xi m_5, \qquad R_{71}^* = -b_8, \qquad R_{42}^* = R_{52}^* = 0,$$
$$R_{12}^* = w_8, \qquad R_{62}^* = -b_5, \qquad R_{72}^* = -i\xi, \qquad R_{13}^* = -y_7, \qquad R_{43}^* = 0, \qquad R_{53}^* = b_7, R_{63}^* = i\xi, R_{73}^* = b_7 \tag{C.1}$$

where

$$\Gamma_1 = \frac{\mu}{\beta_1 T_0}, \qquad \Gamma_2 = \frac{\lambda}{\beta_1 T_0}, \qquad \Gamma_3 = \frac{\rho v_1^2 \tau_{11}}{\beta_1 T_0}, \qquad \Gamma_4 = \frac{\rho v_1^2 \tau_{21}}{\beta_1 T_0}, \qquad \Gamma_1^e = \frac{\mu^e}{\beta_1 T_0}$$

$$b_5 = \frac{\lambda^e(1+m_5^2)+2\mu^e}{\beta_1 T_0}, \qquad b_6 = \frac{2\mu^e m_6}{\beta_1 T_0}, \qquad b_7 = (1+m_6^2)\Gamma_1^e$$

$$b_8 = 2im_5\Gamma_1^e, \qquad w_5 = b_5 c_5, \qquad w_6 = b_6 s_6, \qquad y_5 = b_5 s_5,$$

$$y_8 = b_8 c_5, \qquad w_7 = b_7 s_6, \qquad w_8 = b_8 s_5, \qquad y_6 = b_6 c_6, \qquad y_7 = b_7 c_6,$$

$$\begin{cases} b_p = n_{2p} m_p, \qquad q_p = n_{3p} m_p, r_p = k_n n_{1p}, \qquad J_p = [i\Gamma_2 \xi - \Gamma_3 n_{2p} - \Gamma_5 \xi m_p - \Gamma_4 n_{3p}], \\ \qquad\qquad\qquad s_p^* = \xi \Gamma_1 (in_{1p} - m_p) \end{cases} \qquad p = 1,2,3,4 \tag{C.2}$$

APPENDIX D

$$F_1^* = q_2(J_3 s_4^* - J_4 s_3^*) - q_3(J_2 s_4^* - J_4 s_2^*) + q_4(J_2 s_3^* - J_3 s_2^*),$$

$$F_2^* = q_1(J_3 s_4^* - J_4 s_3^*) - q_3(J_1 s_4^* - J_4 s_1^*) + q_4(J_1 s_3^* - J_3 s_1^*),$$

$$F_3^* = q_1(J_2 s_4^* - J_4 s_2^*) - q_2(J_1 s_4^* - J_4 s_1^*) + q_4(J_1 s_2^* - J_2 s_1^*),$$

$$F_4^* = q_1(J_2 s_3^* - J_3 s_2^*) - q_2(J_1 s_3^* - J_3 s_1^*) + q_3(J_1 s_2^* - J_2 s_1^*), \tag{D.1}$$

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