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Free Vibration of Sandwich Panels with Smart Magneto-Rheological Layers and Flexible Cores

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ABSTRACT

This is the first study on the free vibrational behavior of sandwich panels with flexible core in the presence of smart sheets of oil which is capable of the excitation of magnetic field. In order to model the core, the improved high order theory of sandwich sheets was used by a polynomial with unknown coefficients first degree shear theory was used for the sheets. The derived equations based on Hamilton principle with simple support boundary condition for upper and lower sheets were solved using Navier technique. Accuracy and precision of the theory were investigated by comparing the results of this study with those of analytical and numerical works. In the conclusion section, effect of the intensity of magnetic field and other physical parameters including ratio of sheet's length to width, ratio of sheet's length to thickness, ratio of core thickness to sheet's overall thickness, and ratio of oil layer thickness to sheet's overall thickness on natural frequency was investigated.

Keywords : Sandwich plates ; Flexible cores ; Free vibration ; Improved high order theory.

1 INTRODUCTION

IN recent years, magneto-rheological materials (smart oil with a variable behavior with magnetic field) have played a significant role in the construction of smart structures and materials. A magneto-rheological material is a type of smart fluid which is composed of micro-magnetic particles with a fluid that is generally oil. When applying a magnetic field, viscosity of the liquid is fairly increased to form a viscoelastic solid. The surprising point is that, when the liquid is at its active state, yielding stress of the liquid can be precisely controlled by changing the intensity of the magnetic field. Jacob Rubino is the discoverer and explorer of the primary studies on these fluids [1].

Fluids with rheological-controlling property, like magneto-rheological property, have numerous applications in automobile industry, suspension system and structures [4-2].

Sun et al. [5] used oscillation rheometry technique to obtain the relationship of magnetic field and complex shear module of magneto-rheological materials before yielding stress region. Both theoretically and experimentally, they studied dynamic properties of a sandwich beam with magneto-rheological core for different values of field intensity. Yalsintas and Day [6] investigated response of a sandwich beam with magneto-rheological core using energy method and compared the results with other experimental results. Rajamohan et al. [7] modeled a sandwich beam with a magneto-rheological core by considering the shearing effects of the sheet restraining the magneto-rheological

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inside the core and also the equivalent shear module. They used finite element analysis for solving the problem and investigated effects of the intensity of magnetic field on vibrational properties for different boundary conditions and forced loadings. Also, they estimated the magneto-rheological liquid complex module using the model introduced by Choy et al. [8] and presented relationships as second degree polynomial which depended on the intensity of magnetic field. Nayak et al. [9] examined the response of sandwich beam with magneto-rheological core under an axial load and used Galerkin and Hamilton methods to derive the equations. Also, they could define the instability region of the sandwich beam for different boundary conditions. Yeh [10] investigated vibrational behavior of a sandwich plate with magneto-rheological core using finite element analysis. Moreover, they studied the effects of magnetic field and geometrical parameters. Manorahan et al. [11] investigated the dynamic behavior of a composite sandwich plate with magneto-rheological core and examined effects of magnetic field, fiber angles, and various supports and geometrical parameters.

Rao et al. [12] developed an analytical method for calculating natural frequencies in multi-layered composites and sandwich beams based on higher order theories. Each layer was assumed to have a two dimensional orthotropic property and be based on plane stress. Hamilton principle was used to derive the equilibrium equations. Kant and Swamnatan [13] presented a formulation and analytical solution for analyzing the free vibration of composite multi-layer sandwich sheet based on higher order improved theory. In the presented theory, deformation of the multi-layer was calculated by considering effects of transversal shear deformation, stress and transversal normal strains, and non-linear variables for the in-plane displacement. Muniear and Shenwi [14] investigated shear deformation and effects of damping on dynamic response of sandwich structure-based improved theory. Nayak et al. [15] introduced a new method based on higher order theory and finite element analysis for calculating natural frequencies in composite multilayer sheets and also composite sandwich sheets. They studied effects of various parameters including geometry and material properties on natural frequencies of the structure. Frostig and Thompson [16] used the improved higher order theory in sandwich panels for the analysis of free vibration of sandwich panels with a flexible core. Malekzadeh et al. [17] presented the improved higher order theory in sandwich panels based on higher order theory introduced by Frostig and Thompson [16] to analyze free vibration in sandwich panels. In this theory, effect of plane forces acting on upper and lower sheets of the sandwich sheet and equivalent damping factor for the sandwich panel were calculated and damping of the system was studied in the analysis of the vibration. Setkovic and Vaksanoic [18] presented bending, free vibration, and buckling of the sandwich sheets using layered displacement model. In the presented model, in-plane and linear variations of displacement components and transversal displacement were considered to be fixed. Using the assumed displacement, ratio of strain-displacement, single layered 3D constitutive equations, and motion equations were derived using Hamilton principle. Buckling and vibration of multilayer composite sheets were investigated by Yaoko et al. [19] using finite element analysis for different spacing of the fibers. Formulation of location- dependent stiffness matrix was derived using the properties of non-homogeneous materials.

In this article, for the first time, free vibrational behavior of a sandwich panel with flexible core and composite sheets with the addition of smart magneto-rheological between the layers was investigated. Frostig second order theory in the form of a polynomial with unknown coefficients for the displacement of core and first shear deformation theory for the sheets was used in this analysis. The derived equations were solved based on Hamilton principle with the assumption of a simple support in upper and lower sheets as boundary conditions using Navier technique. Accuracy and precision of the results were compared with the analytical and numerical results of other articles. In the results section, effects of magnetic field and physical parameters like ratio of sheet's length to width, ratio of sheet's length to thickness, ratio of core thickness to sheet's overall thickness, and ratio of oil thickness to sheet's overall thickness on the system frequency were also investigated.

2 MATHEMATICAL FORMULATIONS

Studied sandwich panel was assumed to be rectangular with multi-layer composite sheets with layers of magneto-rheological between the sheets and flexible core. Layers of magneto-rheological and layers of composite sheets maintaining the oil is denoted by indices 2 and 1, respectively, and indices t and b which represent top and bottom sheets respectively. Composite sheet denoted by index 3 along with indices t and b is base composite sheets of sandwich panel connected to the core. Length and width of the panel are "a" and "b", respectively, and coordinate axis is shown in Fig.1. For the dynamic analysis of the panel, small-scale deformation and elastic analysis were studied. Sheets and core were completely attached to each other and strain functions were continuous at the attachment surface of the sheets.

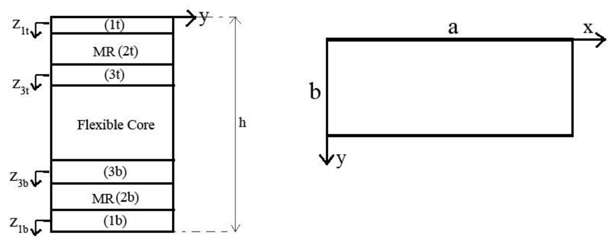

Fig.1
Geometry and dimensions of the sandwich plates.

2.1 Kinematic relations

In this article, first order shear theory was used to model displacement field in sheets:

$$\begin{aligned} & u_i(x,y,z,t)=u_0^i(x,y,t)+z_i\varphi_x^i(x,y,t) \\ & v_i(x,y,z,t)=v_0^i(x,y,t)+z_i\varphi_y^i(x,y,t) \\ & w_i(x,y,z,t)=w_0^i(x,y,t) \\ & i=1t,3t,3b,1b \end{aligned} \tag{1}$$

In the above relations, $u_0^i(x,y,t)$, $v_0^i(x,y,t)$, and $w_0^i(x,y,t)$ are unknown components of the displacement of sheets' middle plane at x, y, and z directions, respectively, and φ_x^i,φ_y^i are unknown components of rotation of sheet's transverse part around x and y axes, respectively. Z_i is the normal axis directed downward with respect to the upper and lower sheets.

Displacement field regarding the core for in-plane displacements and normal displacement is in the form of 3[rd] order polynomial and 2[nd] order polynomial, respectively [16].

$$\begin{aligned} & u_c(x,y,z,t)=u_0(x,y,t)+z_cu_1(x,y,t)+z_c^2u_2(x,y,t)+z_c^3u_3(x,y,t) \\ & v_c(x,y,z,t)=v_0(x,y,t)+z_cv_1(x,y,t)+z_c^2v_2(x,y,t)+z_c^3v_3(x,y,t) \\ & w_c(x,y,z,t)=w_0(x,y,t)+z_cw_1(x,y,t)+z_c^2w_2(x,y,t) \end{aligned} \tag{2}$$

Coefficients of Z_c are unknown and must be determined.

2.2 Compatibility conditions of the displacements

In this article, base sheets were ideally attached to the core. Therefore, all the three displacement components of the base of upper and lower sheets as well as the core were equal at their intersections. So, displacement compatibility conditions at the intersection of each sheet with core can be defined as:

$$\begin{aligned} & u_{3t}(z_{3t}=\frac{h_{3t}}{2})=u_c(z_c=\frac{-h_c}{2}) \qquad v_{3t}(z_{3t}=\frac{h_{3t}}{2})=v_c(z_c=\frac{-h_c}{2}) \qquad w_{3t}(z_{3t}=\frac{h_{3t}}{2})=w_c(z_c=\frac{-h_c}{2}) \\ & u_{3b}(z_{3b}=\frac{-h_{3b}}{2})=u_c(z_c=\frac{h_c}{2}) \qquad v_{3b}(z_{3b}=\frac{-h_{3b}}{2})=v_c(z_c=\frac{h_c}{2}) \qquad w_{3b}(z_{3b}=\frac{-h_{3b}}{2})=w_c(z_c=\frac{h_c}{2}) \end{aligned} \tag{3}$$

By substituting Relations (1) and (2) in Relation (3), compatibility equations can be written as:

$$u_0+u_1\frac{h_c}{2}+u_2\frac{h_c^2}{4}+u_3\frac{h_c^3}{8}=u_0^{3t}-\varphi_x^{3t}\frac{h_{3t}}{2},\quad v_0+v_1\frac{h_c}{2}+v_2\frac{h_c^2}{4}+v_3\frac{h_c^3}{8}=v_0^{3t}-\varphi_y^{3t}\frac{h_{3t}}{2}$$
$$w_0+w_1\frac{h_c}{2}+w_2\frac{h_c^2}{4}=w_0^{3t}$$
$$u_0-u_1\frac{h_c}{2}+u_2\frac{h_c^2}{4}-u_3\frac{h_c^3}{8}=u_0^{3b}+\varphi_x^{3b}\frac{h_{3b}}{2},\quad v_0-v_1\frac{h_c}{2}+v_2\frac{h_c^2}{4}-v_3\frac{h_c^3}{8}=v_0^{3b}+\varphi_y^{3b}\frac{h_{3b}}{2}\tag{4}$$
$$w_0-w_1\frac{h_c}{2}+w_2\frac{h_c^2}{4}=w_0^{3b}$$

Using compatibility Eq. (4), the relationship between displacement-dependent parameters of the middle core can be derived as:

$$u_2=(2(u_0^{3t}+u_0^{3b})-h_{3t}\varphi_x^{3t}+h_{1b}\varphi_x^{3b}-4u_0)/h_c^2,\quad u_3=(4(u_0^{3t}-u_0^{3b})-2(h_{3t}\varphi_x^{3t}+h_{1b}\varphi_x^{3b}-4h_0u_1)/h_c^3$$
$$v_2=(2(v_0^{3t}+v_0^{3b})-h_{3t}\varphi_y^{3t}+h_{3b}\varphi_y^{3b}-4v_0)/h_c^2,\quad v_3=(4(v_0^{3t}-v_0^{3b})-2(h_{3t}\varphi_y^{3t}+h_{3b}\varphi_y^{3b}-4h_cv_1)/h_c^3\tag{5}$$
$$w_1=(w_0^{3t}-w_0^{3b})/h_c,\qquad w_2=2(w_0^{3t}+w_0^{3b}-2w_0)/h_c^2$$

Using Relation (5), unknown values of the problem can be decreased.

2.3 Strains

Strains in upper and lower sheets were obtained using the following equations:

$$\varepsilon_{xx}^i=u_{0,x}^i+z_ik_{xx}^i\qquad k_{xx}^i=\varphi_{x,x}^i$$
$$\varepsilon_{yy}^i=v_{0,y}^i+z_ik_{yy}^i\qquad k_{yy}^i=\varphi_{y,y}^i$$
$$\gamma_{xy}^i=u_{0,y}^i+v_{0,x}^i+z_ik_{xy}^i\qquad k_{xy}^i=\varphi_{x,y}^i+\varphi_{y,x}^i\tag{6}$$
$$\gamma_{xz}^i=\varphi_x^i+w_{0,x}^i,\qquad \gamma_{yz}^i=\varphi_y^i+w_{0,y}^i$$
$$i=1t,3t,3b,1b$$

And the relationship regarding the core strain can be written as:

$$\varepsilon_{xx}^c=u_{0,x}+z_cu_{1,x}+z_c^2u_{2,x}+z_c^3u_{3,x},\quad \varepsilon_{yy}^c=v_{0,y}+z_cv_{1,y}+z_c^2v_{2,y}+z_c^3v_{3,y},\quad \varepsilon_{zz}^c=w_1+2z_cw_2$$
$$\gamma_{xy}^c=u_{0,y}+z_cu_{1,y}+z_c^2u_{2,y}+z_c^3u_{3,y}+v_{0,x}+z_cv_{1,x}+z_c^2v_{2,x}+z_c^3v_{3,x}$$
$$\gamma_{xz}^c=u_1+2z_cu_2+3z_c^2u_3+w_{0,x}+z_cw_{1,x}+z_c^2w_{2,x}\tag{7}$$
$$\gamma_{yz}^c=v_1+2z_cv_2+3z_c^2v_3+w_{0,y}+z_cw_{1,y}+z_c^2w_{2,y}$$

2.4 Strain-stress relationship of magneto-rheological layer

In order to model the magneto-rheological layer, the following cases were assumed:

1. It was assumed that a no-slip condition existed between the layers and magneto-rheological layers.
2. Transverse displacement (w) and normal rotational degrees (φ_y and φ_x) were considered to be the equal on a hypothetical cross-section on the sandwich panel's sheets.

$$\varphi_x^{3t}=\varphi_x^{1t}=\varphi_x^t,\quad \varphi_y^{3t}=\varphi_y^{1t}=\varphi_y^t,\quad w_0^{3t}=\varphi_0^{1t}=\varphi_0^t$$
$$\varphi_x^{3b}=\varphi_x^{1b}=\varphi_x^b,\quad \varphi_y^{3b}=\varphi_y^{1b}=\varphi_y^b,\quad w_0^{3b}=\varphi_0^{1b}=\varphi_0^b$$

3. No normal stress existed in magneto-rheological layer.
4. Magnetorheological material was modeled as a linear viscoelastic material in the pre-yield condition.

Components of transverse strain in the core were as follows:

$$\gamma_{xz}^{(2j)} = \frac{\partial w_j}{\partial x} + \frac{\partial u_{2j}}{\partial z} \qquad \gamma_{yz}^{(2j)} = \frac{\partial w_j}{\partial y} + \frac{\partial v_{2j}}{\partial z} \qquad (j = t, b) \tag{8}$$

According to the geometrical relationships between $u_j, v_j, w_j, \varphi_x^j$ and φ_y^j (j=1t,3t,1b,3b) in layers 1 and 3 and also according to Fig. 2, Relations $\frac{\partial u_{2j}}{\partial z}$ and $\frac{\partial v_{2j}}{\partial z}$ can be obtained as:

$$\frac{\partial u_{2j}}{\partial z} = \frac{1}{h_{2j}}\left[(\mathrm{u}_0^{1j} - \mathrm{u}_0^{3j}) - \varphi_x^j\left(\frac{h_{1j} + h_{3j}}{2}\right)\right], \qquad \frac{\partial v_{2j}}{\partial z} = \frac{1}{h_{2j}}\left[(v_0^{1j} - v_0^{3j}) - \varphi_y^j\left(\frac{h_{1j} + h_{3j}}{2}\right)\right] \qquad (\mathrm{j} = \mathrm{t}, \mathrm{b}) \tag{9}$$

By substituting Relations (9) in (8), components of strain in megnetorheological layer can be obtained:

$$\gamma_{xz}^{(2\mathrm{j})} = \frac{\partial w_j}{\partial x} + \frac{u_0^{1j} - u_0^{3j}}{h_{2j}} - \frac{\varphi_x^j}{2h_{2j}}(\mathrm{h}_{1j} + \mathrm{h}_{3j}), \quad \gamma_{yz}^{(2\mathrm{j})} = \frac{\partial w_j}{\partial y} + \frac{v_0^{1j} - v_0^{3j}}{h_{2j}} - \frac{\varphi_y^j}{2h_{2j}}(\mathrm{h}_{1j} + \mathrm{h}_{3j}) \qquad (\mathrm{j} = \mathrm{t}, \mathrm{b}) \tag{10}$$

Finally, the relationship between transverse strains and stresses in megnetorheological layer can be expressed as:

$$\tau_{xz}^{2j} = G_2\gamma_{xz}^{2j}, \tau_{yz}^{2j} = G_2\gamma_{yz}^{2j} \qquad (j = t, b) \tag{11}$$

where G_2 is viscoelastic shear modulus related to megnetorheological layer.

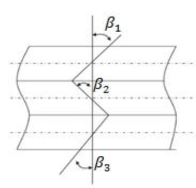

Fig.2
Compatibility relation of systems.

Since the magneto-rheological material behaved like a viscoelastic material in the pre-yield region, thus shear modulus was complex and depended on the intensity of the magnetic field. For this, the relation proposed by Rajamohan et al. [20] for defining the relationship between complex shear modulus of a magneto-rheological fluid and intensity of the magnetic field was used. Complex shear modulus for a viscoelastic material was as follows [20]:

$$G^* = G' + iG'' \tag{12}$$

where G' and G'' are storage and loss modulus, respectively, and described as a polynomial function of magnetic field, B (in Gauss), for a magneto-rheological material [20]:

$$\begin{aligned} G' &= -3.3691B^2 + 4.9975\times10^3 B + 0.873\times10^6 \\ G'' &= -0.9B^2 + 0.8124\times10^3 B + 0.1855\times10^6 \end{aligned} \tag{13}$$

2.5 Motion equations

Minimum potential energy principle was used to calculate the governing motion equations. Based on this principle:

$$\delta \int_{t_1}^{t_2} \left(U - T\right) dt = 0 \tag{14}$$

where U and T are potential and kinetic energies, respectively, δ is first order variation operator, and t_2-t_1 is the integration interval.

Equations for the first order variation of potential energy were defined as:

$$\begin{aligned}\delta U = &\sum_{i=1t,3t,3b,1b} \left(\int_{V_i} (\sigma_{xx}^i\,\delta\varepsilon_{xx}^i + \sigma_{yy}^i\,\delta\varepsilon_{yy}^i + \tau_{xy}^i\,\delta\gamma_{xy}^i + \tau_{xz}^i\,\delta\gamma_{xz}^i + \tau_{yz}^i\,\delta\gamma_{yz}^i)dV_i\right) \\ &+ \sum_{i=2t,2b} \left(\int_{V_i} (\tau_{xz}^i\,\delta\gamma_{xz}^i + \tau_{yz}^i\,\delta\gamma_{yz}^i)dV_i\right) + \int_{V_c} (\sigma_{zz}^c\,\delta\varepsilon_{zz}^c + \tau_{xz}^c\,\delta\gamma_{xz}^c + \tau_{yz}^c\,\delta\gamma_{yz}^c + \sigma_{xx}^c\,\delta\varepsilon_{xx}^c + \sigma_{yy}^c\,\delta\varepsilon_{yy}^c + \tau_{xy}^c\,\delta\gamma_{xy}^c)dV_c\end{aligned} \tag{15}$$

Indices t and b denote upper and lower sheets made of composite material, indices $2t$ and $2b$ demonstrate magneto-rheological layer in upper and lower parts of the panel, and index c shows the core. Equations for the first order variations of kinetic energy were described as:

$$\delta T = -\sum_{i=1t,3t,1b,3b} \left(\int_0^a\int_0^b \rho_i h_i \begin{pmatrix} \dot{u}_i\,\delta\dot{u}_i + \dot{v}_i\,\delta\dot{v}_i \\ +\dot{w}_i\,\delta\dot{w}_i \end{pmatrix} dxdy\right) - \sum_{j=2t,2b} \left(\int_0^a\int_0^b \begin{pmatrix} \rho_j h_j \dot{w}_j\,\delta\dot{w}_j + \\ I_{MR}(\dot{\gamma}_{xz}^j\,\delta\dot{\gamma}_{xz}^j + \dot{\gamma}_{yz}^j\,\delta\dot{\gamma}_{yz}^j) \end{pmatrix} dxdy\right) \tag{16}$$

In the above equations, ρ is the mass density and $I_{MR} = \rho_{MR} h_{MR}^3 / 12$ is inertia moment of magnetorheological layer. Following integrals were regarded as stress results within sheets (layers) and core:

$$\begin{aligned}
&\left\{N_{xx}^i, M_{xx}^i\right\} = \int_{-\frac{h_i}{2}}^{\frac{h_i}{2}} (1, z_i)\sigma_{xx}^i dz_i, \quad \left\{N_{yy}^i, M_{yy}^i\right\} = \int_{-\frac{h_i}{2}}^{\frac{h_i}{2}} \left(1, z_i\right)\sigma_{yy}^i dz_i, \quad \left\{N_{xy}^i, M_{xy}^i\right\} = \int_{-\frac{h_i}{2}}^{\frac{h_i}{2}} (1, z_i)\tau_{xy}^i dz_i \\
&Q_{xz}^i = \int_{-\frac{h_i}{2}}^{\frac{h_i}{2}} \tau_{xz}^i dz_i, \quad Q_{yz}^i = \int_{-\frac{h_i}{2}}^{\frac{h_i}{2}} \tau_{yz}^i dz_i \qquad (\mathrm{i = 1t, 3t, 1b, 3b}) \\
&\left\{N_{xx}^c, M_{nxx}^c\right\} = \int_{-\frac{h_c}{2}}^{\frac{h_c}{2}} (1, z_c^n)\sigma_{xx}^c dz_c, \quad \left\{N_{yy}^c, M_{nyy}^c\right\} = \int_{-\frac{h_c}{2}}^{\frac{h_c}{2}} \left(1, z_c^n\right)\sigma_{yy}^c dz_c, \quad \left\{N_{xy}^c, M_{nxy}^c\right\} = \int_{-\frac{h_c}{2}}^{\frac{h_c}{2}} (1, z_c^n)\tau_{xy}^c dz_c \\
&\left\{Q_{xz}^c, M_{nxz}^c\right\} = \int_{-\frac{h_c}{2}}^{\frac{h_c}{2}} \left(1, z_c^n\right)\tau_{xz}^c dz_c, \quad \left\{Q_{yz}^c, M_{nyz}^c\right\} = \int_{-\frac{h_c}{2}}^{\frac{h_c}{2}} \left(1, z_c^n\right)\tau_{yz}^c dz_c, \quad \left\{R_z^c, M_z^c\right\} = \int_{-\frac{h_c}{2}}^{\frac{h_c}{2}} (1, z_c)\sigma_{zz}^c dz_c \\
&Q_{xz}^j = \int_{-\frac{h_j}{2}}^{\frac{h_j}{2}} \tau_{xz}^j dz_j, \quad Q_{yz}^j = \int_{-\frac{h_j}{2}}^{\frac{h_j}{2}} \tau_{yz}^j dz_j, \quad j = 2t, 2b
\end{aligned} \tag{17}$$

And density integrals in layers and middle core were as follows:

$$I_n^i = \int_{-\frac{h_i}{2}}^{\frac{h_i}{2}} (z_i^n \rho_i) dz_i \qquad i = c, 3t, 1t, 3b, 1b \quad n = 0, 1, 2 \tag{18}$$

After all the above mathematical processes and substituting resulting stress, density integrals and also integration by parts and factoring unknown values of displacement field, motion equations can be developed as:

δu_0^{1t} :

$$N_{xx,x}^{1t}+N_{xy,y}^{1t}-\frac{1}{h_{2t}}Q_{xz}^{2t}=I_0^{1t}\ddot{u}_0^{1t}+I_1^{1t}\ddot{\varphi}_x^t+\frac{1}{h_{2t}}\ddot{w}_{0,x}^t+\frac{1}{h_{2t}^2}(\ddot{u}_0^{1t}-\ddot{u}_0^{3t})-\frac{h_{1t}+h_{3t}}{2h_{2t}^2}\ddot{\varphi}_x^t \tag{19a}$$

δu_0^{3t} :

$$\begin{aligned}&N_{xx,x}^{3t}+N_{xy,y}^{3t}+\frac{1}{h_{2t}}Q_{xz}^{2t}+\frac{2}{h_c^2}\left(M_{2xx,x}^c+M_{2xy,y}^c-2M_{Q1xz}^c\right)-\frac{4}{h_c^3}\left(M_{3xx,x}^c+M_{3xy,y}^c-3M_{Q2xz}^c\right)=(I_0^{3t}+\frac{1}{h_{2t}^2})\ddot{u}_0^{3t}\\&+(I_1^{3t}+\frac{h_{1t}+h_{3t}}{2h_{2t}^2})\ddot{\varphi}_x^t-\frac{1}{h_{2t}}\ddot{w}_{0,x}^t-\frac{1}{h_{2t}^2}\ddot{u}_0^{1t}+\frac{2}{h_c^2}\left(I_2^c\ddot{u}_0+I_3^c\ddot{u}_1\right)+\frac{2I_4^c}{h_c^4}\left(2\ddot{u}_0^c+2\ddot{u}_0^{3b}+h_{3t}\ddot{\varphi}_x^t-h_{3b}\ddot{\varphi}_x^b-4\ddot{u}_0\right)\\&+\frac{8}{h_c^4}I_5^c\ddot{u}_1+\frac{8}{h_c^5}\left(-2I_c^5\ddot{u}_0^{3t}-h_{3t}I_c^5\ddot{\varphi}_x^t+2I_c^6\ddot{u}_1\right)+\frac{4}{h_c^3}\left(-I_3^c\ddot{u}_0-I_4^c\ddot{u}_1+4I_5^c\ddot{u}_0\right)+\frac{8I_6^c}{h_c^6}\left(-2\ddot{u}_0^{3b}+2\ddot{u}_0^{3t}+h_{3t}\ddot{\varphi}_x^t+h_{3b}\ddot{\varphi}_x^b\right)\end{aligned} \tag{19b}$$

δu_0^{1b} :

$$N_{xx,x}^{1b}+N_{xy,y}^{1b}+\frac{1}{h_{2b}}Q_{xz}^{2b}=I_0^{1b}\ddot{u}_0^{1b}+I_1^{1b}\ddot{\varphi}_x^b-\frac{1}{h_{2b}}\ddot{w}_{0,x}^b-\frac{1}{h_{2b}^2}(\ddot{u}_0^{1b}-\ddot{u}_0^{3b})+\frac{h_{1b}+h_{3b}}{2h_{2b}^2}\ddot{\varphi}_x^b \tag{19c}$$

δu_0^{3b}

$$\begin{aligned}&N_{xx,x}^{3b}+N_{xy,y}^{3b}-\frac{1}{h_{2b}}Q_{xz}^{2b}+\frac{2}{h_c^2}\left(M_{2xx,x}^c+M_{2xy,y}^c-2M_{Q1xz}^c\right)+\frac{4}{h_c^3}\left(M_{3xx,x}^c+M_{3xy,y}^c-3M_{Q2xz}^c\right)=(I_0^{3b}+\frac{1}{h_{2b}^2})\ddot{u}_0^{3b}\\&+(I_1^{3b}-\frac{h_{1b}+h_{3b}}{2h_{2b}^2})\ddot{\varphi}_x^b-\frac{1}{h_{2b}^2}\ddot{u}_0^{1b}+\frac{1}{h_{2b}}\ddot{w}_{0,x}^b+\frac{2}{h_c^2}\left(I_2^c\ddot{u}_0+I_3^c\ddot{u}_1\right)+\frac{2I_4^c}{h_c^4}\left(2\ddot{u}_0^{3t}+2\ddot{u}_0^{3b}+h_{3t}\ddot{\varphi}_x^t-h_{3b}\ddot{\varphi}_x^b-4\ddot{u}_0\right)\\&-\frac{8}{h_c^4}I_5^c\ddot{u}_1+\frac{8}{h_c^5}\left(2I_c^5\ddot{u}_0^{3b}-h_{3b}I_c^5\ddot{\varphi}_x^b-2I_c^6\ddot{u}_1\right)+\frac{4}{h_c^3}\left(I_3^c\ddot{u}_0+I_4^c\ddot{u}_1-4I_5^c\ddot{u}_0\right)+\frac{8I_6^c}{h_c^6}\left(2\ddot{u}_0^{3b}-2\ddot{u}_0^{3t}-h_{3t}\ddot{\varphi}_x^t-h_{3b}\ddot{\varphi}_x^b\right)\end{aligned} \tag{19d}$$

δv_0^{1t} :

$$N_{yy,y}^{1t}+N_{xy,x}^{1t}-\frac{1}{h_{2t}}Q_{yz}^{2t}=I_0^{1t}\ddot{v}_0^{1t}+I_1^{1t}\ddot{\varphi}_y^t+\frac{1}{h_{2t}}\ddot{w}_{0,\mathrm{y}}^t+\frac{1}{h_{2t}^2}(\ddot{v}_0^{1t}-\ddot{v}_0^{3t})-\frac{h_{1t}+h_{3t}}{2h_{2t}^2}\ddot{\varphi}_y^t \tag{19e}$$

δv_0^{1b} :

$$N_{yy,y}^{1b}+N_{xy,x}^{1b}+\frac{1}{h_{2b}}Q_{yz}^{2b}=I_0^{1b}\ddot{v}_0^{1b}+I_1^{1b}\ddot{\varphi}_y^b-\frac{1}{h_{2b}}\ddot{w}_{0,\mathrm{y}}^b-\frac{1}{h_{2b}^2}(\ddot{v}_0^{1b}-\ddot{v}_0^{3b})+\frac{h_{1b}+h_{3b}}{2h_{2b}^2}\ddot{\varphi}_y^b \tag{19f}$$

δv_0^{3t} :

$$\begin{aligned}&N_{yy,y}^t+N_{xy,x}^t+\frac{1}{h_{2t}}Q_{yz}^{2t}+\frac{2}{h_c^2}\left(M_{2yy,y}^c+M_{2xy,x}^c-2M_{Q1yz}^c\right)-\frac{4}{h_c^3}\left(M_{3yy,y}^c+M_{3xy,x}^c-3M_{Q2yz}^c\right)=(I_0^t+\frac{1}{h_{2t}^2})\ddot{v}_0^{3t}+\\&(I_1^t+\frac{h_{1t}+h_{3t}}{2h_{2t}^2})\ddot{\varphi}_y^t-\frac{1}{h_{2t}^2}\ddot{v}_0^{1t}+\frac{2}{h_c^2}\left(I_2^c\ddot{v}_0+I_3^c\ddot{v}_1\right)-\frac{1}{h_{2t}}\ddot{w}_{0,\mathrm{y}}^t+\frac{2I_4^c}{h_c^4}\left(2\ddot{v}_0^c+2\ddot{v}_0^{3b}+h_{3t}\ddot{\varphi}_y^t-h_{3b}\ddot{\varphi}_y^b-4\ddot{v}_0\right)\\&+\frac{8}{h_c^4}I_5^c\ddot{v}_1+\frac{8}{h_c^5}\left(-2I_c^5\ddot{v}_0^{3t}-h_tI_c^5\ddot{\varphi}_y^t+2I_c^6\ddot{v}_1\right)+\frac{4}{h_c^3}\left(-I_3^c\ddot{v}_0-I_4^c\ddot{v}_1+4I_5^c\ddot{v}_0\right)+\frac{8I_6^c}{h_c^6}\left(-2\ddot{v}_0^{3b}+2\ddot{v}_0^{3t}+h_{3t}\ddot{\varphi}_y^t+h_{3b}\ddot{\varphi}_y^b\right)\end{aligned} \tag{19g}$$

δv_0^{3b} :

$$\begin{aligned}&N_{yy,y}^{3b}+N_{xy,x}^{3b}-\frac{1}{h_{2b}}Q_{yz}^{2b}+\frac{2}{h_c^2}\left(M_{2yy,y}^c+M_{2xy,x}^c-2M_{Q1yz}^c\right)+\frac{4}{h_c^3}\left(M_{3yy,y}^c+M_{3xy,x}^c-3M_{Q2yz}^c\right)=(I_0^{3b}+\frac{1}{h_{2b}^2})\ddot{v}_0^{3b}+\\&(I_1^{3b}-\frac{h_{1b}+h_{3b}}{2h_{2b}^2})\ddot{\varphi}_y^b+\frac{1}{h_{2b}}\ddot{w}_{0,\mathrm{y}}^b-\frac{1}{h_{2b}^2}\ddot{v}_0^{1b}+\frac{2}{h_c^2}\left(I_2^c\ddot{v}_0+I_3^c\ddot{v}_1\right)+\frac{2I_4^c}{h_c^4}\left(2\ddot{v}_0^{3t}+2\ddot{v}_0^{3b}+h_{3t}\ddot{\varphi}_y^t-h_{3b}\ddot{\varphi}_y^b-4\ddot{v}_0\right)\\&-\frac{8}{h_c^4}I_5^c\ddot{v}_1+\frac{8}{h_c^5}\left(2I_c^5\ddot{v}_0^{3b}-h_bI_c^5\ddot{\varphi}_y^b-2I_c^6\ddot{v}_1\right)+\frac{4}{h_c^3}\left(I_3^c\ddot{v}_0+I_4^c\ddot{v}_1-4I_5^c\ddot{v}_0\right)+\frac{8I_6^c}{h_c^6}\left(2\ddot{v}_0^{3b}-2\ddot{v}_0^{3t}-h_{3t}\ddot{\varphi}_y^t-h_{3b}\ddot{\varphi}_y^b\right)\end{aligned} \tag{19h}$$

δw_0^b :

$$Q_{xz,x}^{1b}+Q_{yz,y}^{1b}+Q_{xz,x}^{3b}+Q_{yz,y}^{3b}+Q_{xz,x}^{2b}+Q_{yz,y}^{2b}+\frac{1}{h_c}\left(M_{Q1xz,x}^{c}+M_{Q1yz,y}^{c}-R_z^c\right)+\frac{2}{h_c^2}\left(-2M_z^c+M_{Q2xz,x}^{c}+M_{Q2yz,y}^{c}\right)$$
$$=(I_0^{1b}+I_0^{2b}+I_0^{3b})\ddot{w}_0^b-\ddot{w}_{0,xx}^b-\frac{1}{h_{2b}}(\ddot{v}_{0,y}^{1b}-\ddot{v}_{0,y}^{3b})+\frac{h_{1b}+h_{3b}}{2h_{2b}}\ddot{\varphi}_{y,y}^{b}+\frac{1}{h_c}I_1^c\ddot{w}_0+\frac{I_2^c}{h_c^2}\left(\ddot{w}_0^b-\ddot{w}_0^t+2\ddot{w}_0\right)+\frac{4I_3^c}{h_c^3}\left(\ddot{w}_0^b-\ddot{w}_0\right) \quad (19i)$$
$$+\frac{4I_4^c}{h_c^4}\left(\ddot{w}_0^t+\ddot{w}_0^b-2\ddot{w}_0\right)$$

δw_0^t :

$$Q_{xz,x}^{3t}+Q_{yz,y}^{3t}+Q_{xz,x}^{1t}+Q_{yz,y}^{1t}+Q_{xz,x}^{2t}+Q_{yz,y}^{2t}+\frac{1}{h_c}\left(-M_{Q1xz,x}^{c}-M_{Q1yz,y}^{c}+R_z^c\right)+\frac{2}{h_c^2}\left(-2M_z^c+M_{Q2xz,x}^{c}+M_{Q2yz,y}^{c}\right)$$
$$=(I_0^{1t}+I_0^{2t}+I_0^{3t})\ddot{w}_0^t-w_{0,xx}^t-\frac{1}{h_{2t}}(\ddot{v}_{0,y}^{1t}-\ddot{v}_{0,y}^{3t})+\frac{h_{1t}+h_{3t}}{2h_{2t}}\ddot{\varphi}_{y,y}^{t}-\frac{1}{h_c}I_1^c\ddot{w}_0+\frac{I_2^c}{h_c^2}\left(-\ddot{w}_0^b+\ddot{w}_0^t+2\ddot{w}_0\right)+\frac{4I_3^c}{h_c^3}\left(-\ddot{w}_0^t+\ddot{w}_0\right) \quad (19j)$$
$$+\frac{4I_4^c}{h_c^4}\left(\ddot{w}_0^t+\ddot{w}_0^b-2\ddot{w}_0\right)$$

$\delta\varphi_x^t$:

$$M_{xx,x}^{1t}+M_{xy,y}^{1t}+M_{xx,x}^{3t}+M_{xy,y}^{3t}-Q_{xz}^{1t}-Q_{xz}^{3t}+\frac{h_{1t}+h_{3t}}{2h_{2t}}Q_{xz}^{2t}+\frac{h_t}{h_c^2}\left(M_{2xx,x}^{c}+M_{2xy,y}^{c}-2M_{Q1xz}^{c}\right)$$
$$+\frac{2h_{3t}}{h_c^3}\left(M_{3xx,x}^{c}+M_{3xy,y}^{c}-3M_{Q2xz}^{c}\right)=I_1^{1t}\ddot{u}_0^{1t}+I_1^{3t}\ddot{u}_0^{3t}+(I_2^{1t}+I_2^{3t})\ddot{\varphi}_x^t-\frac{h_{1t}+h_{3t}}{2h_{2t}}\ddot{w}_{0,x}^t- \quad (19k)$$
$$\frac{h_{1t}+h_{3t}}{2h_{2t}^2}(\ddot{u}_0^{1t}-\ddot{u}_0^{3t})+\frac{(h_{1t}+h_{3t})^2}{4h_{2t}^2}\ddot{\varphi}_x^t+\frac{h_{3t}}{h_c^2}\left(I_2^c\ddot{u}_0+I_3^c\ddot{u}_1\right)+\frac{h_{3t}I_4^c}{h_c^4}\left(2\ddot{u}_0^t+2\ddot{u}_0^b+h_{3t}\ddot{\varphi}_x^t-h_{1b}\ddot{\varphi}_x^b-4\ddot{u}_0\right)-\frac{4h_{3t}I_5^c}{h_c^4}\ddot{u}_1+$$
$$\frac{4h_{3t}I_5^c}{h_c^5}\left(-2\ddot{u}_0^{3t}+2\ddot{u}_0-h_t\ddot{\varphi}_x^t\right)+\frac{8h_{3t}I_6^c}{h_c^5}\ddot{u}_1-\frac{2h_{3t}}{h_c^3}\left(I_3^c\ddot{u}_0+I_4^c\ddot{u}_1\right)+\frac{4h_{3t}I_6^c}{h_c^6}\left(2\ddot{u}_0^{3t}-2\ddot{u}_0^{1b}+h_{3t}\ddot{\varphi}_x^t+h_{1b}\ddot{\varphi}_x^b\right)$$

$\delta\varphi_x^b$:

$$M_{xx,x}^{1b}+M_{xy,y}^{1b}-Q_{xz}^{1b}+M_{xx,x}^{3b}+M_{xy,y}^{3b}-Q_{xz}^{3b}+\frac{h_{1b}+h_{3b}}{2h_{2b}}Q_{xz}^{2b}+\frac{h_{3b}}{h_c^2}\left(M_{2xx,x}^{c}+M_{2xy,y}^{c}-2M_{Q1xz}^{c}\right)$$
$$-\frac{2h_{3b}}{h_c^3}\left(M_{3xx,x}^{c}+M_{3xy,y}^{c}-3M_{Q2xz}^{c}\right)=I_1^{3b}\ddot{u}_0^{3b}+I_1^{1b}\ddot{u}_0^{1b}+(I_2^{1b}+I_2^{3b})\ddot{\varphi}_x^b-\frac{h_{1b}+h_{3b}}{2h_{2b}}\ddot{w}_{0,x}^b-\frac{h_{1b}+h_{3b}}{2h_{2b}^2}(\ddot{u}_0^{1b}-\ddot{u}_0^{3b})+ \quad (19l)$$
$$\frac{(h_{1b}+h_{3b})^2}{4h_{2b}^2}\ddot{\varphi}_x^b+\frac{h_{3b}}{h_c^2}\left(I_2^c\ddot{u}_0+I_3^c\ddot{u}_1\right)+\frac{h_{3b}I_4^c}{h_c^4}\left(2\ddot{u}_0^{3t}+2\ddot{u}_0^{3b}+h_{3t}\ddot{\varphi}_x^t-h_{3b}\ddot{\varphi}_x^b-4\ddot{u}_0\right)-\frac{4h_{3b}I_5^c}{h_c^4}\ddot{u}_1+\frac{4h_{3b}I_5^c}{h_c^5}\left(-2\ddot{u}_0^{3b}+2\ddot{u}_0-h_{3b}\ddot{\varphi}_x^b\right)$$
$$+\frac{8h_{3b}I_6^c}{h_c^5}\ddot{u}_1-\frac{2h_{3b}}{h_c^3}\left(I_3^c\ddot{u}_0+I_4^c\ddot{u}_1\right)+\frac{4h_{3b}I_6^c}{h_c^6}\left(2\ddot{u}_0^{3t}-2\ddot{u}_0^{3b}+h_{3t}\ddot{\varphi}_x^t+h_{3b}\ddot{\varphi}_x^b\right)$$

$\delta\varphi_y^t$:

$$M_{yy,y}^{3t}+M_{xy,x}^{3t}-Q_{yz}^{3t}+M_{yy,y}^{1t}+M_{xy,x}^{1t}-Q_{yz}^{1t}+\frac{h_{1t}+h_{3t}}{2h_{2t}}Q_{yz}^{2t}+\frac{h_{3t}}{h_c^2}\left(M_{2yy,y}^{c}+M_{2xy,x}^{c}-2M_{Q1yz}^{c}\right)$$
$$-\frac{2h_{3t}}{h_c^3}\left(M_{3yy,y}^{c}+M_{3xy,x}^{c}-3M_{Q2yz}^{c}\right)=I_1^{3t}\ddot{v}_0^{3t}+I_1^{1t}\ddot{v}_0^{1t}+(I_2^{3t}+I_2^{1t})\ddot{\varphi}_y^t-\frac{h_{1t}+h_{3t}}{2h_{2t}}\ddot{w}_{0,y}^t-\frac{h_{1t}+h_{3t}}{2h_{2t}^2}(\ddot{v}_0^{1t}-\ddot{v}_0^{3t})+ \quad (19m)$$
$$\frac{(h_{1t}+h_{3t})^2}{4h_{2t}^2}\ddot{\varphi}_y^t+\frac{h_{3t}}{h_c^2}\left(I_2^c\ddot{v}_0+I_3^c\ddot{v}_1\right)+\frac{h_{3t}I_4^c}{h_c^4}\left(2\ddot{v}_0^t+2\ddot{v}_0^b+h_{3t}\ddot{\varphi}_y^t-h_{1b}\ddot{\varphi}_y^b-4\ddot{v}_0\right)-\frac{4h_{3t}I_5^c}{h_c^4}\ddot{v}_1+\frac{4h_{3t}I_5^c}{h_c^5}\left(-2\ddot{v}_0^{3t}+2\ddot{v}_0-h_{3t}\ddot{\varphi}_y^t\right)$$
$$+\frac{8h_{3t}I_6^c}{h_c^5}\ddot{v}_1-\frac{2h_{3t}}{h_c^3}\left(I_3^c\ddot{v}_0+I_4^c\ddot{v}_1\right)+\frac{4h_{3t}I_6^c}{h_c^6}\left(2\ddot{v}_0^{3t}-2\ddot{v}_0^{1b}+h_{3t}\ddot{\varphi}_y^t+h_{1b}\ddot{\varphi}_y^b\right)$$

$\delta\varphi_y^b$:

$$M_{yy,y}^{1b}+M_{xy,x}^{1b}-Q_{yz}^{1b}+M_{yy,y}^{3b}+M_{xy,x}^{3b}-Q_{yz}^{3b}+\frac{h_{1b}+h_{3b}}{2h_{2b}}Q_{yz}^{2b}-\frac{h_{3b}}{h_c^2}\left(M_{2yy,y}^{c}+M_{2xy,x}^{c}-2M_{Q1yz}^{c}\right)$$
$$-\frac{2h_{3b}}{h_c^3}\left(M_{3yy,y}^{c}+M_{3xy,x}^{c}-3M_{Q2yz}^{c}\right)=I_1^{1b}\ddot{v}_0^{1b}+I_1^{3b}\ddot{v}_0^{3b}+(I_2^{1b}+I_2^{3b})\ddot{\varphi}_y^b-\frac{h_{1b}+h_{3b}}{2h_{2b}}\ddot{w}_{0,y}^{b}-\frac{h_{1b}+h_{3b}}{2h_{2b}^2}(\ddot{v}_0^{1b}-\ddot{v}_0^{3b}) \tag{19n}$$
$$+\frac{(h_{1b}+h_{3b})^2}{4h_{2b}^2}\ddot{\varphi}_y^b+\frac{h_{3b}}{h_c^2}\left(I_2^c\ddot{v}_0+I_3^c\ddot{v}_1\right)+\frac{h_{3b}I_4^c}{h_c^4}\left(2\ddot{v}_0^{3t}+2\ddot{v}_0^{3b}+h_{3t}\ddot{\varphi}_y^t-h_{3b}\ddot{\varphi}_y^b-4\ddot{v}_0\right)-\frac{4h_{3b}I_5^c}{h_c^4}\ddot{v}_1$$
$$+\frac{4h_{3b}I_5^c}{h_c^5}\left(-2\ddot{v}_0^{3b}+2\ddot{v}_0-h_{3b}\ddot{\varphi}_y^b\right)+\frac{8h_{3b}I_6^c}{h_c^5}\ddot{v}_1-\frac{2h_{3b}}{h_c^3}\left(I_3^c\ddot{v}_0+I_4^c\ddot{v}_1\right)+\frac{4h_{3b}I_6^c}{h_c^6}\left(2\ddot{v}_0^{3t}-2\ddot{v}_0^{3b}+h_{3t}\ddot{\varphi}_y^t+h_{3b}\ddot{\varphi}_y^b\right)$$

δu_0^c:

$$N_{xx,x}^c+N_{xy,y}^c-\frac{4}{h_c^2}\left(M_{2xx,x}^c+M_{2xy,y}^c-2M_{Q1xz}^c\right)=I_0^c\ddot{u}_0+I_1^c\ddot{u}_1+\frac{I_2^c}{h_c^2}\left(2\ddot{u}_0^{3t}+2\ddot{u}_0^{3b}+h_{3t}\ddot{\varphi}_x^t-h_{3b}\ddot{\varphi}_x^b-4\ddot{u}_0\right)$$
$$-\frac{4}{h_c^2}I_3^c\ddot{u}_0+\frac{2}{h^3}I_3^c\left(-2\ddot{u}_0^{3t}+2\ddot{u}_0^{3b}-h_{3t}\ddot{\varphi}_x^t-h_{3b}\ddot{\varphi}_x^b-2h_c\ddot{u}_1\right)-\frac{4}{h_c^4}I_4^c\left(2\ddot{u}_0^{3t}+2\ddot{u}_0^{3b}+h_{3t}\ddot{\varphi}_x^t-h_{3b}\ddot{\varphi}_x^b-4\ddot{u}_0\right) \tag{19o}$$
$$-\frac{8}{h_c^5}I_5^c\left(-2\ddot{u}_0^{3t}+2\ddot{u}_0^{3b}-h_{3t}\ddot{\varphi}_x^t-h_{3b}\ddot{\varphi}_x^b-2h_c\ddot{u}_1\right)$$

δu_1^c :

$$M_{1xx,x}^c+M_{1xy,y}^c-Q_{xz}^c-\frac{4}{h_c^2}\left(M_{3xx,x}^c+M_{3xy,y}^c-3M_{Q2xz}^c\right)=I_1^c\ddot{u}_0+I_2^c\ddot{u}_1+\frac{I_3^c}{h_c^2}\left(2\ddot{u}_0^{3t}+2\ddot{u}_0^{1b}+h_{3t}\ddot{\varphi}_x^t-h_{1b}\ddot{\varphi}_x^b-4\ddot{u}_0\right)$$
$$-\frac{4}{h_c^2}I_4^c\ddot{u}_1+\frac{2}{h_c^3}I_4^c\left(-2\ddot{u}_0^{3t}+2\ddot{u}_0^{1b}-h_{3t}\ddot{\varphi}_x^t-h_{1b}\ddot{\varphi}_x^b-2h_c\ddot{u}_1\right)-\frac{4}{h_c^4}I_5^c\left(2\ddot{u}_0^{3t}+2\ddot{u}_0^{1b}+h_{3t}\ddot{\varphi}_x^t-h_{1b}\ddot{\varphi}_x^b-4\ddot{u}_0\right)- \tag{19p}$$
$$\frac{8}{h_c^5}I_6^c\left(-2\ddot{u}_0^{3t}+2\ddot{u}_0^{1b}-h_{3t}\ddot{\varphi}_x^t-h_{1b}\ddot{\varphi}_x^b-2h_c\ddot{u}_1\right)$$

δv_0^c :

$$N_{yy,y}^c+N_{xy,x}^c-\frac{4}{h_c^2}\left(M_{2yy,y}^c+M_{2xy,x}^c-2M_{Q1yz}^c\right)=I_0^c\ddot{v}_0+I_1^c\ddot{v}_1+\frac{I_2^c}{h_c^2}\left(2\ddot{v}_0^{3t}+2\ddot{v}_0^{1b}+h_{3t}\ddot{\varphi}_y^t-h_{1b}\ddot{\varphi}_y^b-4\ddot{v}_0\right)$$
$$-\frac{4}{h_c^2}I_3^c\ddot{v}+\frac{2}{h_c^3}I_3^c\left(-2\ddot{v}_0^{3t}+2\ddot{v}_0^{1b}-h_{3t}\ddot{\varphi}_y^t-h_{1b}\ddot{\varphi}_y^b-2h_c\ddot{v}_1\right)-\frac{4}{h_c^4}I_4^c\left(2\ddot{v}_0^{3t}+2\ddot{v}_0^{1b}+h_{3t}\ddot{\varphi}_y^t-h_{1b}\ddot{\varphi}_y^b-4\ddot{v}_0\right) \tag{19q}$$
$$-\frac{8}{h_c^5}I_5^c\left(-2\ddot{v}_0^{3t}+2\ddot{v}_0^{1b}-h_{3t}\ddot{\varphi}_y^t-h_{1b}\ddot{\varphi}_y^b-2h_c\ddot{v}_1\right)$$

δv_1^c :

$$M_{1yy,y}^c+M_{1xy,x}^c-Q_{yz}^c-\frac{4}{h_c^2}\left(M_{3yy,y}^c+M_{3xy,x}^c-3M_{Q2yz}^c\right)=I_1^c\ddot{v}_0+I_2^c\ddot{v}_1+\frac{I_3^c}{h_c^2}\left(2\ddot{v}_0^{3t}+2\ddot{v}_0^{1b}+h_{3t}\ddot{\varphi}_y^t-h_{1b}\ddot{\varphi}_y^b-4\ddot{v}_0\right)$$
$$-\frac{4}{h_c^2}I_4^c\ddot{v}_1+\frac{2}{h_c^3}I_4^c\left(-2\ddot{v}_0^{3t}+2\ddot{v}_0^{1b}-h_{3t}\ddot{\varphi}_y^t-h_{1b}\ddot{\varphi}_y^b-2h_c\ddot{v}_1\right)-\frac{4}{h_c^4}I_5^c\left(2\ddot{v}_0^{3t}+2\ddot{v}_0^{1b}+h_{3t}\ddot{\varphi}_y^t-h_{1b}\ddot{\varphi}_y^b-4\ddot{v}_0\right)- \tag{19r}$$
$$\frac{8}{h_c^5}I_6^c\left(-2\ddot{v}_0^{3t}+2\ddot{v}_0^{1b}-h_{3t}\ddot{\varphi}_y^t-h_{1b}\ddot{\varphi}_y^b-2h_c\ddot{v}_1\right)$$

δw_0^c :

$$Q^c_{xz,x}+Q^c_{yz,y}-\frac{4}{h_c^2}\left(-2M_z^c+M^c_{Q2xz,x}+M^c_{Q2yz,y}\right)=I_0^c\ddot{w}_0+\frac{I_1^c}{h_c}\left(\ddot{w}_0^b-\ddot{w}_0^t\right)+\frac{2I_2^c}{h_c^2}\left(\ddot{w}_0^b+\ddot{w}_0^t-2\ddot{w}_0\right)$$
$$-\frac{4I_3^c}{h_c^3}\left(\ddot{w}_0^b-\ddot{w}_0^t\right)-\frac{8I_4^c}{h_c^4}\left(\ddot{w}_0^b+\ddot{w}_0^t-2\ddot{w}_0\right) \tag{19s}$$

The number of unknown coefficients was 19 which was the sum of displacements of sheets and middle core. 19 unknown values included:

$$\left(u_0^{1t},u_0^{1b},u_0^{3t},u_0^{3b},v_0^{1t},v_0^{1b},v_0^{3t},v_0^{3b},w_0^t,w_0^b,\varphi_x^t,\varphi_x^b,\varphi_y^t,\varphi_y^b,u_0^c,u_1^c,v_0^c,v_1^c,w_0^c\right)$$

2.6 Fundamental equations for force within the sheets

Since each single layer has isotropic properties with respect to the symmetry axis of its own material and according to Hooks' law, fundamental equations for the sheets can be derived as:

$$\begin{aligned}
N^i_{xx}&=A^i_{11}\varepsilon^i_{0xx}+A^i_{12}\varepsilon^i_{0yy}+A^i_{16}\varepsilon^i_{0xy}+B^i_{11}K^i_{xx}+B^i_{12}K^i_{yy}+B^i_{16}K^i_{xy}\\
N^i_{yy}&=A^i_{12}\varepsilon^i_{0xx}+A^i_{22}\varepsilon^i_{0yy}+A^i_{26}\varepsilon^i_{0xy}+B^i_{12}K^i_{xx}+B^i_{22}K^i_{yy}+B^i_{26}K^i_{xy}\\
N^i_{xy}&=A^i_{16}\varepsilon^i_{0xx}+A^i_{26}\varepsilon^i_{0yy}+A^i_{66}\varepsilon^i_{0xy}+B^i_{16}K^i_{xx}+B^i_{26}K^i_{yy}+B^i_{66}K^i_{xy} \qquad \mathrm{i}=1\mathrm{t},3\mathrm{t},3\mathrm{b},\ 1\mathrm{b}\\
M^i_{xx}&=B^i_{11}\varepsilon^i_{0xx}+B^i_{12}\varepsilon^i_{0yy}+B^i_{16}\varepsilon^i_{0xy}+D^i_{11}K^i_{xx}+D^i_{12}K^i_{yy}+D^i_{16}K^i_{xy}\\
M^i_{yy}&=B^i_{12}\varepsilon^i_{0xx}+B^i_{22}\varepsilon^i_{0yy}+B^i_{26}\varepsilon^i_{0xy}+D^i_{12}K^i_{xx}+D^i_{22}K^i_{yy}+D^i_{26}K^i_{xy}\\
M^i_{xy}&=B^i_{16}\varepsilon^i_{0xx}+B^i_{26}\varepsilon^i_{0yy}+B^i_{66}\varepsilon^i_{0xy}+D^i_{16}K^i_{xx}+D^i_{26}K^i_{yy}+D^i_{66}K^i_{xy}\\
Q^i_{yz}&=k\left[A^i_{44}\left(\varphi^i_y+w^i_{,y}\right)+A^i_{45}\left(\varphi^i_x+w^i_{,x}\right)\right],\quad Q^i_{xz}=k\left[A^i_{45}\left(\varphi^i_y+w^i_{,y}\right)+A^i_{55}\left(\varphi^i_x+w^i_{,x}\right)\right]
\end{aligned} \tag{20}$$

where A^i_{mn}, B^i_{mn} and D^i_{mn} are tensile, coupling, and bending stiffness matrices with 3×3 size, the elements of which are obtained by the following relations:

$$(A^i_{mn},B^i_{mn},D^i_{mn})=\sum_{k=1}^{N}\int_{-\frac{h_i}{2}}^{\frac{h_i}{2}}(\bar{\mathrm{Q}}_{mn})^i_k\,(1,\mathrm{z}_i,\mathrm{z}_i^2)\,\mathrm{dz}_i,\quad (\mathrm{m},\ \mathrm{n}=1,2,6),\quad i=1\mathrm{t},3\mathrm{t},1\mathrm{b},3\mathrm{b} \tag{21}$$

N is number of single-layered sheets and $\left(\bar{Q}_{mn}\right)^i$ is stiffness of single-layered sheets.

2.7 Fundamental equations for force within the core

In isotropic cores, similar to foam with flexible properties, elasticity modulus, E^c and shear modulus, G^c are constant value and relations between stresses and strains are defined as:

$$\begin{cases}
\sigma^c_{xx}=E^c\varepsilon^c_{xx} & \tau^c_{xy}=G^c\gamma^c_{xy}\\
\sigma^c_{yy}=E^c\varepsilon^c_{yy} & \tau^c_{xz}=G^c\gamma^c_{xz}\\
\sigma^c_{zz}=E^c\varepsilon^c_{zz} & \tau^c_{yz}=G^c\gamma^c_{yz}
\end{cases} \tag{22}$$

By substituting relations regarding strains in core (7) in Eq. (22) and considering Relations (5) and (17), equations relating to resultant stresses can be described as:

$$N_{xx}^{c}=E^{c}\left\{h_c u_{0,x}+\left(\frac{h_c}{12}\right)\left[2(u_{0,x}^{3b}+u_{0,x}^{3t})-(h_{3b}\varphi_{x,x}^{b}-h_{3t}\varphi_{x,x}^{t})-4u_{0,x}\right]\right\} \tag{23a}$$

$$N_{yy}^{c}=E^{c}\left\{\frac{2}{3}h_c v_{0,x}+\left(\frac{h_c}{12}\right)\left[2(v_{0,x}^{3b}+v_{0,x}^{3t})-(h_{3b}\varphi_{y,y}^{b}-h_{3t}\varphi_{y,y}^{t})\right]\right\} \tag{23b}$$

$$N_{xy}^{c}=\frac{2}{3}E^{c}h_c u_{0,y}+\left\{\left(\frac{G^{c}h_c}{12}\right)\left[2(u_{0,y}^{3b}+u_{0,y}^{3t})-(h_{3b}\varphi_{x,y}^{b}-h_{3t}\varphi_{x,y}^{t})+2(v_{0,x}^{3b}+v_{0,x}^{3t})-(h_{3b}\varphi_{y,x}^{b}-h_{3t}\varphi_{y,x}^{t})\right]\right\} \tag{23c}$$

$$N_{1xx}^{c}=E^{c}\left\{\frac{1}{30}h_c^3 u_{1,x}+\left(\frac{h_c^3}{80}\right)\left[4(u_{0,x}^{3b}+u_{0,x}^{3t})-2(h_{3b}\varphi_{x,x}^{b}+h_{3t}\varphi_{x,x}^{t})\right]\right\} \tag{23d}$$

$$N_{2xx}^{c}=E^{c}\left\{\frac{1}{30}h_c^3 u_{0,x}+\left(\frac{h_c^3}{80}\right)\left[2(u_{0,x}^{3b}+u_{0,x}^{3t})-(h_{3b}\varphi_{x,x}^{b}-h_{3t}\varphi_{x,x}^{t})\right]\right\} \tag{23e}$$

$$M_{3xx}^{c}=E^{c}\left\{\frac{h_c^5}{280}u_{1,x}+\left(\frac{h_c^4}{448}\right)\left[4(u_{0,x}^{3b}+u_{0,x}^{3t})-2(h_{3b}\varphi_{x,x}^{b}+h_{3t}\varphi_{x,x}^{t})\right]\right\} \tag{23f}$$

$$M_{1xy}^{c}$$
$$=G^{c}\left\{\frac{h_c^3}{30}u_{1,y}+\left(\frac{h_c^3}{80}\right)\left[4(u_{0,y}^{3b}+u_{0,y}^{3t})-2(h_{3b}\varphi_{x,y}^{b}+h_{3t}\varphi_{x,y}^{t})\right]+\frac{h_c^3}{30}v_{1,y}+\left(\frac{h_c^3}{80}\right)\left[4(v_{0,y}^{3b}+v_{0,y}^{3t})-2(h_{3b}\varphi_{y,x}^{b}+h_{3t}\varphi_{y,x}^{t})\right]\right\} \tag{23g}$$

$$M_{2xy}^{c}$$
$$=G^{c}\left\{\frac{1}{30}h_c^3 u_{0,y}+\left(\frac{h_c^3}{80}\right)\left[2(u_{0,y}^{3b}+u_{0,y}^{3t})-(h_{3b}\varphi_{x,y}^{b}-h_{3t}\varphi_{x,y}^{t})\right]+\frac{1}{30}h_c^3 v_{0,x}+\left(\frac{h_c^3}{80}\right)\left[2(v_{0,x}^{3b}+v_{0,x}^{3t})-(h_{3b}\varphi_{y,x}^{b}-h_{3t}\varphi_{y,x}^{t})\right]\right\} \tag{23h}$$

$$M_{3xy}^{c}$$
$$=G^{c}\left\{\frac{h_c^5}{280}u_{1,y}+\left(\frac{h_c^4}{448}\right)\left[4(u_{0,y}^{3b}+u_{0,y}^{3t})-2(h_{3b}\varphi_{x,y}^{b}+h_{3t}\varphi_{x,y}^{t})\right]+\frac{h_c^5}{280}v_{1,y}+\left(\frac{h_c^4}{448}\right)\left[4(v_{0,x}^{3b}-v_{0,x}^{3t})-2(h_{3b}\varphi_{y,x}^{b}+h_{3t}\varphi_{y,x}^{t})\right]\right\} \tag{23i}$$

$$M_{Q1xz}^{c}=G^{c}\left\{\frac{1}{12}h_c^3 w_{1,x}+\left(\frac{h_c}{6}\right)\left[2(u_{0}^{3b}+u_{0}^{3t})-(h_{3b}\varphi_{x}^{b}+h_{3t}\varphi_{x}^{t})-4u_{0}\right]\right\} \tag{23j}$$

$$M_{Q2xz}^{c}=G^{c}\left\{\left(\frac{3h_c^3}{80}\right)\left[4(u_{0}^{3b}+u_{0}^{3t})-2(h_{3b}\varphi_{x}^{b}+h_{3t}\varphi_{x}^{t})\right]-6h_c^3u_1+\left(\frac{h_c^3}{80}\right)\left[2(w_{0,x}^{b}-w_{0,x}^{t})-\frac{h_c^3}{10}w_{0,x}^{c}\right]\right\} \tag{23k}$$

$$M_{Q1yz}^{c}=G^{c}\left\{\left(\frac{h_c}{6}\right)\left[(v_{0}^{3b}+v_{0}^{3t})-(h_{3b}\varphi_{y}^{b}+h_{3t}\varphi_{y}^{t})\right]-\frac{2}{3}h_c h_{3b}v_0+\frac{h_c^3}{12}w_{1,y}^{c}\right\} \tag{23l}$$

$$M_{Q2yz}^{c}=G^{c}\left\{\left(\frac{3h_c^3}{80}\right)\left[4(v_0^{3b}-v_0^{3t})-2(h_{3b}\varphi_y^b+h_{3t}\varphi_y^t)\right]+\frac{h_c^3}{120}v_1^c\right\} \tag{23m}$$

$$M_{1yy}^{c}=E^{c}\left\{\frac{h_c^3}{30}v_{1,y}+\left(\frac{h_c^2}{80}\right)\left[4(v_{0,y}^{3b}-v_{0,y}^{3t})-2(h_{3b}\varphi_{y,y}^b+h_{3t}\varphi_{y,y}^t)\right]\right\} \tag{23n}$$

$$M_{2yy}^{c}=E^{c}\left\{\frac{h_c^3}{30}v_{0,y}+\left(\frac{h_c^3}{80}\right)\left[2(v_{0,y}^{3b}+v_{0,y}^{3t})+(h_{3b}\varphi_{y,y}^b+h_{3t}\varphi_{y,y}^t)\right]\right\} \tag{23o}$$

$$M_{3yy}^{c}=E^{c}\left\{\left(\frac{h_c^4}{64}\right)\left[4(v_{0,y}^{3b}+v_{0,y}^{3t})-(h_{3b}\varphi_{y,y}^b+h_{3t}\varphi_{y,y}^t)\right]-\frac{3h_c^5}{160}v_{1,y}^c\right\} \tag{23p}$$

$$R_z^c=E^c h_c^2 w_1 \tag{23q}$$

$$M_z^c=E^c\left\{\left(\frac{h_c^3}{80}\right)\left[(w_0^b+w_0^t-2w_0^c)\right]\right\} \tag{23r}$$

$$Q_{xz}^c=G^c\left\{\left[(u_0^{3b}-u_0^{3t})-2(h_{3b}\varphi_x^b+h_{3t}\varphi_x^t)-\frac{3}{4}u_1h_c+w_{0,x}h_c+\frac{h_c}{12}(2w_{0,x}^b+2w_{0,x}^t-4w_{0,x})\right]\right\} \tag{23s}$$

$$Q_{yz}^c=G^c\left\{\left[(v_0^{3b}-v_0^{3t})-2(h_{3b}\varphi_y^b+h_{3t}\varphi_y^t)-\frac{3}{4}v_1h_c+w_{0,y}h_c+\frac{h_c}{12}(2w_{0,y}^b+2w_{0,y}^t-4w_{0,y})\right]\right\} \tag{23t}$$

3 DYNAMIC RESPONSES DUE TO FREE VIBRATION OF PANEL WITH SIMPLE SUPPORT

Boundary conditions for each of the 4 edges of sandwich panel were considered as simple support. The following relations, called Navier response, met the boundary conditions. Constant coefficients present in these series can be obtained by equilibrium equations:

$$\begin{gathered}\left\{u_0^{ij},u_0^c,u_1^c\right\}=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left\{u_{mn}^{ij},u_{0mn}^c,u_{1mn}^c\right\}\cos\frac{n\pi x}{a}\sin\frac{m\pi y}{b}e^{J\omega t},\quad \left\{v_0^{ij},v_0^c,v_1^c\right\}=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left\{v_{mn}^{ij},v_{0mn}^c,v_{1mn}^c\right\}\sin\frac{n\pi x}{a}\cos\frac{m\pi y}{b}e^{J\omega t}\\ \left\{w_0^j,w_0^c\right\}=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left\{w_{mn}^j,w_{mn}^c\right\}\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b}e^{J\omega t},\quad \left\{\varphi_x^j\right\}=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left\{\varphi_{xmn}^j\right\}\cos\frac{n\pi x}{a}\sin\frac{m\pi y}{b}e^{J\omega t},\\ \left\{\varphi_y^j\right\}=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left\{\varphi_{ymn}^j\right\}\sin\frac{n\pi x}{a}\cos\frac{m\pi y}{b}e^{J\omega t}\qquad (i=1,3\ \ j=t,b)\end{gathered} \tag{24}$$

By substituting Relation (24) in motion equations in (19) and considering Relations (20) and (23) and their simplification, equation of motion can be simplified as:

$$([K]-\omega^2[M])\{\Delta\}=\{0\}$$
$$\{\Delta\}=\{u_{mn}^{1t},u_{mn}^{1b},u_{mn}^{3t},u_{mn}^{3b},v_{mn}^{1t},v_{mn}^{1b},v_{mn}^{3t},v_{mn}^{3b},w_{mn}^t,w_{mn}^b,\varphi_{xmn}^t,\varphi_{xmn}^b,\varphi_{ymn}^t,\varphi_{ymn}^b,u_{0mn}^c,u_{1mn}^c,v_{0mn}^c,v_{1mn}^c,w_{0mn}^c\}^T$$

Δ Is constant vector of mode shape, K is stiffness matrix, M is mass matrix, and ω is frequency of the panel.

4 VALIDATIONS OF RESULTS

Since no similar equation exists in other articles in order to validate the derived equations in this study, therefore by omitting terms for the properties of layer of smart oil, natural frequency of the panel was compared with the results of other studies.

4.1 Validation of results for free vibration of sandwich panel with foam core

The case study of this article was a sandwich panel with simple support which had composite sheets made of glass fibers in polyester substrate and a core made of PVC foam called HEREX C70.130. Properties of composite sheets and PVC foam are presented in Table 1. [21].

Table 1
Material parameters of the sandwich plate.

Core	$E_1 = E_2 = E_3 = 0.10363$ GPa, $G_{12} = G_{13} = G_{23} = 0.05$ GPa, $\upsilon = 0.036$, $\rho = 130$ Kg / m^3
Face sheets	$E_1 = 24.51$GPa , $E_2 = E_3 = 7.77$ GPa, $G_{12} = G_{13} = 3.34$ GPa, $G_{23} = 1.34$GPa , $\upsilon_{12} = \upsilon_{13} = 0.078$, $\upsilon_{23} = 0.49$, $\rho = 1800$ kg / m^3

In order to determine validity of the model, a layer of MR was placed between the base and support composite sheets. Also, sheet layers were considered $h_{1t,1b} = 1mm, h_{2t,2b} \approx 0, h_{3t,3b} = 3mm$ and properties of MR layer (density and shear modulus) were assumed to be zero. Four dimensionless frequencies $\bar{\omega} = \omega a^2 (\rho_c / E_c)^{1/2} / h$ [21] for a square sandwich panel with dimensionless ratios of $h/a = 0.1$ and $h_c/h = 0.88$ are presented in Table 2. and then compared with the results in [15, 21, and 22]. The sandwich panel was composed of 7 layers with $(0/90/0/core/0/90/0)$ arrangement.

Table 2
Comparison of non-dimensional frequencies $\bar{\omega}$ of $(0/90/0/core/0/90/0)$ sandwich plate with $h/a = 0.1$ and $h_c/h = 0.88$.

Mode Numbers	Present	[15]	[21]	[22]
(1,1)	14.64	15.34	14.27	15.28
(1,2)	26.38	30.18	26.31	28.69
(2,1)	27.22	31.96	27.04	30.01
(2,2)	35.02	40.94	34.95	38.86

According to Table 2. , results obtained from the present method were in good agreement with the results of Rahmani et al. [21]. Little difference between the results for dimensionless frequency in the present method and the work by Rahamni et al. [21] could be due to different selections of displacement field for the core. In the present method, displacement field was considered as a polynomial with unknown coefficients based on Frostig's second model. However, in [21], displacement field for the core was obtained based on Frostig's first model assuming uniformity in shear stresses in thickness direction and using elasticity relations.

Analytical results obtained from finite element method based on Redi's higher order theory by Nayak et al. [15] are presented in Table 2. Results of the present article had maximum difference of 14% with those in [15], which was due to considering an extra degree of freedom related to core flexibility in thickness direction.

Analytical method adopted by Muniear et al. [22] was based on higher order shear theory (HSDT) and the assumption of equivalent single layer (ESL). According to the above results, frequency values of the current study were less than those of [22] and this difference increased in higher modes.

In order to study the effects of MR layer on natural frequency of the panel, properties of oil layer with thickness of 1 mm in Eq.(12) were considered as: density $\rho_{MR} = 3500 kg/m^3$ and magnetic field intensity $B = 150 Gauss$. Arrangement of layers were considered as $(0/90/MR/0/core/0/MR/90/0)$ and $h_{3t,3b} = 2, h_{1t,1b} = 1mm$. Comparison of results in this case and previous case is presented in Table 3.

Table 3
Effect of MR layer on the frequency sandwich plate.

Mode Number	$(0/90/0/core/0/90/0)$	$(0/90/MR/0/core/0/MR/90/0)$
(1,1)	14.64	20.54
(1,2)	26.38	13.72
(2,1)	27.22	33.22
(2,2)	35.02	23.31

According to Table 3. , addition of MR oil layer between composite layer increased stiffness of the sheet and, by applying magnetic field with the intensity of 150 Gauss, frequency values increased in (1,1) and (2,1) modes, while frequency values decreased in (1,2) and (2,2).

4.2 Mode shapes of sandwich panel with MR layer and foam core

Dimensionless displacement mode of the core ($w_0^c / \max w_0^c$), a square sandwich panel, with dimensionless ratios of $h/a = 0.1$, $h_c/h = 0.88$ are shown in Fig. 3. Density and intensity of magnetic field for MR oil layer with the thickness of 1 mm were $\rho_{MR} = 3500 kg/m^3$ and $B = 150 Gauss$, respectively. Arrangement of layers was considered as: $(0/90/MR/0/core/0/MR/90/0)$ and $h_{3t,3b} = 2, h_{1t,1b} = 1mm$.

4.3 Effect of magnetic field intensity on dimensionless frequencies of sandwich panel with MR layer and foam core

In order to study effects of magnetic field intensity on dimensionless frequency, a square sandwich panel with a simple support, dimensionless ratios of $h/a = 0.1$ and $h_c/h = 0.88$, and sheet and core properties similar to case 2 were considered. Then, by increasing intensity of the applied magnetic field on MR oil layer, values of dimensionless frequencies, $\omega^* = \omega a^2 (\rho_c/E_c)^{1/2}/h$, were obtained [21].

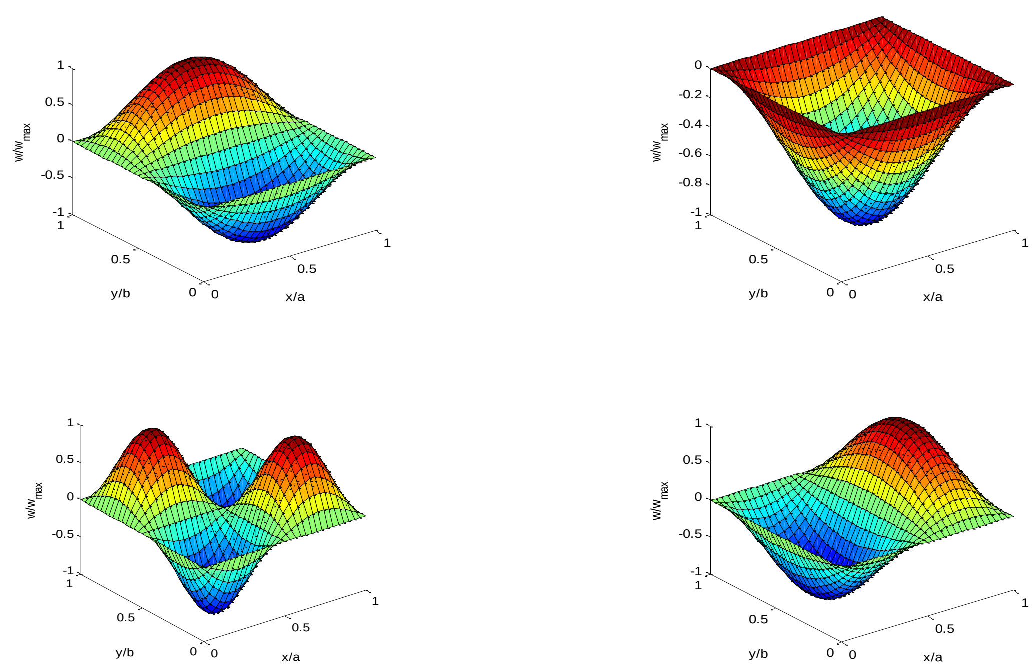

Fig.3
First four free vibration mode shapes of a sandwich plate $(0/90/MR/0/core/0/MR/90/0)$.

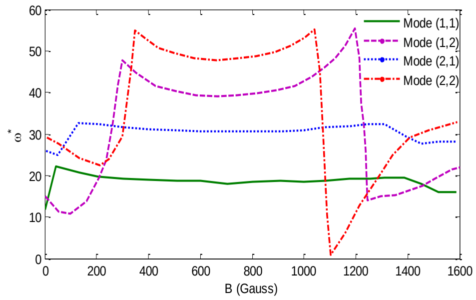

Fig.4
Dimensionless frequencies changes with increasing magnetic field.

According to Fig. 4, when MR layer was not under a magnetic field (B=0 Gauss), magnitude of dimensionless frequency, $\omega^*_{1,1}$ was the least value; as magnetic field was applied, this frequency increased to reach its maximum value $\omega^*_{1,1} = 23.39$. Then, by increasing intensity of magnetic field, $\omega^*_{1,1}$ decreased with a slight slope; when intensity of magnetic field was in the range of 400 to 1000 Gauss, $\omega^*_{1,1}$ remained almost constant ($\omega^*_{1,1} \simeq 18.5$). By further increase of intensity, $\omega^*_{1,1}$ had another increase to reach $\omega^*_{1,1} = 20.42$ and then started to decrease.

Behavior of dimensionless frequencies $\omega^*_{2,1}, \omega^*_{1,2}, \omega^*_{1,1}$ was similar to that of $\omega^*_{1,1}$ with the exception that, in diagram of $\omega^*_{1,1}$, by applying magnetic field, there would be an increase in frequency, while in $\omega^*_{2,1}, \omega^*_{1,2}, \omega^*_{1,1}$, in the presence of magnetic field, first there would be a slight decrease in frequency and then it started to increase. Also, the amount of variation of dimensionless frequency due to increased intensity of magnetic field in modes (1, 2) and (2, 2) was more extreme than other modes.

Furthermore, at a closer look, it is noted that base frequency of the system did not necessarily occur in mode (1,1) and, in specific value of magnetic field intensity, dimensionless frequency of modes (1,2) and (2,2) was less than other modes.

4.4 Effect of increase in a/b ratio on dimensionless frequencies of sandwich panel with MR layer and foam core

In order to study effect of variations of aspect ratio, a/b, on dimensionless frequency, the sandwich panel with simple support and dimensionless ratios of $h/a = 0.1$ and $h_c/h = 0.88$ sheet and core properties similar to those in case 2 was considered. Natural frequencies of the panel were non-dimensionalzed by Equation $\omega^* = \omega b^2 (\rho_c / E_c)^{1/2} / h$ [21] and variations of $\omega^*_{2,1}, \omega^*_{1,2}, \omega^*_{1,1}$ and $\omega^*_{2,2}$, due to increase in aspect ratio, was investigated. In order to compare behavior of different vibrational modes, variations of dimensionless frequencies, $\omega^*_{2,1}, \omega^*_{1,2}, \omega^*_{1,1}$ and $\omega^*_{2,2}$, due to increase in a/b ratio at B=0,100 Gauss are presented in Fig. 5.

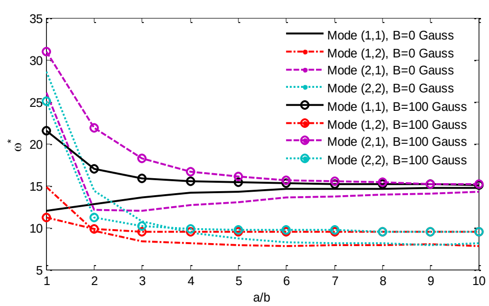

Fig.5
Dimensionless frequencies changes with increasing a/b.

According to this figure, when intensity of magnetic field was zero, dimensionless frequency $\omega^*_{1,1}$ slightly increased by increasing a/b ratio (24%) and then became constant. By increasing a/b ratio, dimensionless frequencies in other modes first decreased and then took a constant value. At a/b=1, dimensionless frequency $\omega^*_{2,1}$ had almost the same value as $\omega^*_{2,2}$, while by increasing a/b ratio, it took a constant value near $\omega^*_{1,1}$ frequency. Also, variation diagrams of $\omega^*_{1,2}, \omega^*_{2,2}$ due to increase in a/b ratio were similar to each other and they finally took the same constant value.

Furthermore, according to Fig. 5, when intensity of magnetic field was 100 Gauss, by increasing a/b ratio, dimensionless frequencies $\omega^*_{2,1}, \omega^*_{1,2}, \omega^*_{1,1}$ and $\omega^*_{2,2}$ decreased and then became constant in spite of the increase in a/b ratio. The important point is that values of frequencies $\omega^*_{1,1}, \omega^*_{2,1}$ tended to take the same constant value by increasing a/b ratio; this point was also true for frequencies $\omega^*_{1,2}, \omega^*_{2,2}$ with the exception that values of dimensionless frequencies were lower in this case.

Then, in order to study better the effect of increase in aspect ratio on dimensionless frequency, diagram of ω^* is presented in Fig. 6 for constant values of magnetic field intensity for each case.

According to Fig. 6, except for the case when magnetic field intensity was zero, as the sheet became thinner, $\omega^*_{1,1}$ decreased; then, at a/b=4 and more, $\omega^*_{1,1}$ did not increase anymore. At ratios of more than 4 (a/b>4), $\omega^*_{1,1}$ had its maximum value for B=100 Gauss, while its minimum value occurred at B=500 Gauss. At all frequencies except $\omega^*_{1,1}$, by increasing thinning ratio and when intensity was zero, frequency decreased. In other words, in the first mode, effect of applying magnetic field on natural frequency of mode (1,1) was more than other modes and this point is clearly demonstrated in Fig. 6.

At higher aspect ratios, dimensionless frequencies $\omega^*_{1,2}, \omega^*_{2,2}$ at B=500 Gauss took higher values, while dimensionless frequencies $\omega^*_{1,1}, \omega^*_{2,1}$ had higher values at B=100 Gauss for all aspect ratios.

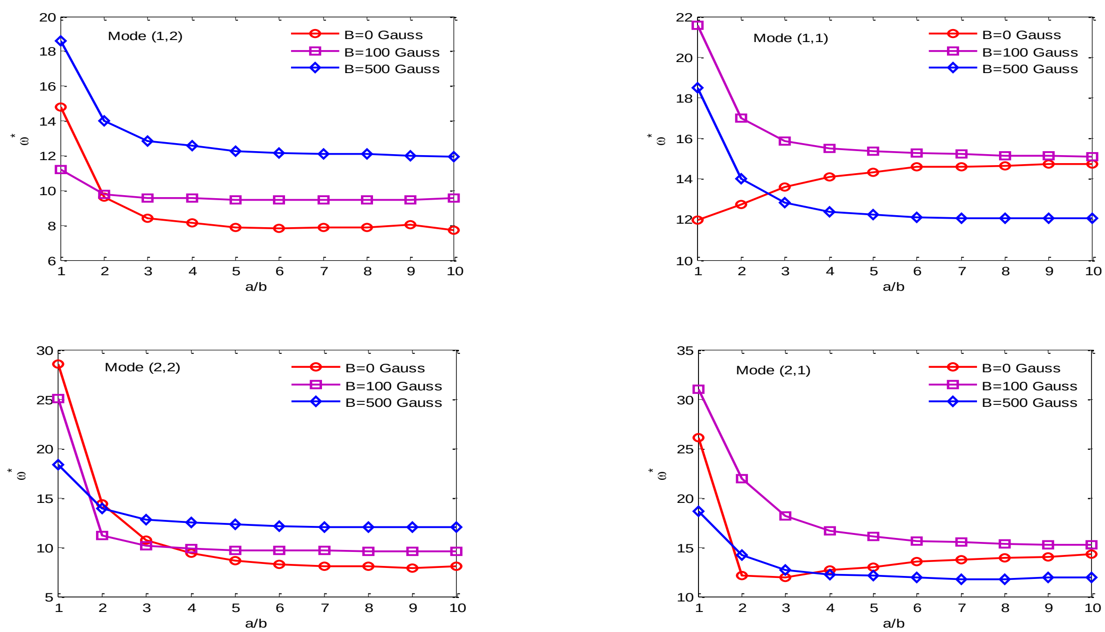

Fig.6
The effect of increasing a / b for B = 0,100,500.

4.5 Effect of h_c/h on frequency of sandwich panel with MR layer and foam core

In this section, natural frequency of the first 4 modes of sandwich panel shown in Fig. 7 is obtained for the case when core thickness ratio (h_c/h) is increased. It was assumed that magnetic field intensity was 200 Gauss. According to Fig. 7, for all h_c/h ratios, $\omega_{1,1}$ was the base frequency. By increasing h_c/h, natural frequency decreased in all four vibrational modes; however, decrease in $\omega_{1,1}$ and $\omega_{1,2}$ was less than that in $\omega_{2,1}$ and $\omega_{2,2}$.

4.6 Effect of a/h ratio on natural frequency of sandwich panel with MR layer and foam core

In this section, in addition to thinning the sheet by increasing a/h, dimensionless frequencies $\omega^* = \omega h\sqrt{\rho_c / E_c}$ are obtained for the constant values of magnetic field intensity. For B=0 and 500 Gauss, dimensionless natural frequencies $\omega^*_{2,1}, \omega^*_{1,2}, \omega^*_{1,1}$ and $\omega^*_{2,2}$ were obtained for the case when a/h increased.

According to Fig. 8, by increasing a/h ratio, which was in fact thinning of the sheet, stiffness of the panel decreased and all modes of dimensionless frequencies decreased. In Fig. 9, behavior of dimensionless frequencies of the first 4 modes was studied at B=0, 200, and 500 Gauss for the increased a/h ratio.

According to Fig. 9, for all vibrational modes, by increasing a/h ratio, frequency decreased and the important point was that, at ratios of more than 50 (a/h>50), frequencies did not vary with the variation of magnetic field intensity.

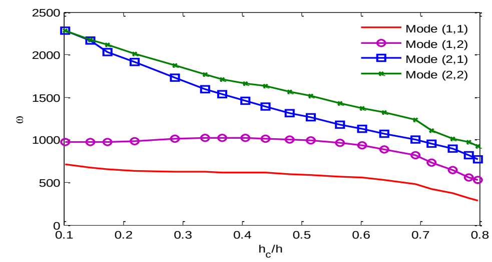

Fig.7
Natural frequencies changes with increasing h_c/h .

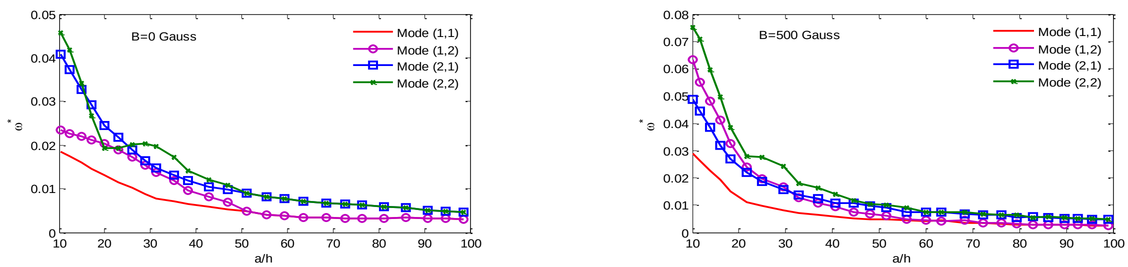

Fig.8
Dimensionless frequencies changes with increasing a/h for B=0, 500.

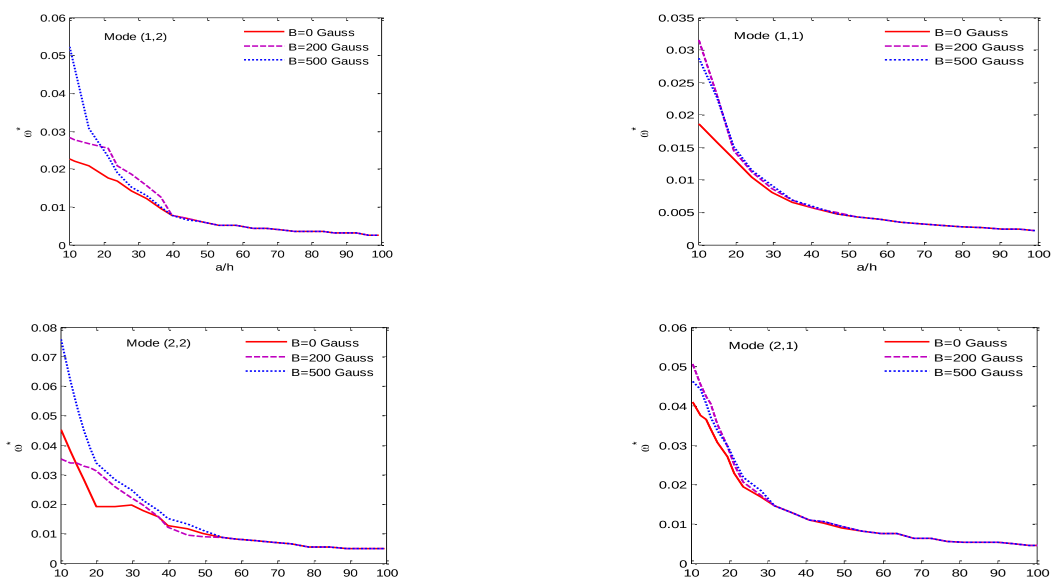

Fig.9
Dimensionless frequencies changes with increasing a/h for B=0, 200, 500**.**

4.7 Effect of MR layer thickness on natural frequency of sandwich panel with MR layer and foam core

In this section, by decreasing the ratio of oil thickness to panel overall thickness ($2h_{MR}/h$), behavior of natural frequency is investigated. The square of sandwich panel with simple support similar to the previous cases was considered. Also, behavior of its natural frequencies $\omega_{2,1}, \omega_{1,2}, \omega_{1,1}$ and $\omega_{2,2}$ was studied for constant values of magnetic field intensity, as given in Fig. 10.

According to Fig. 10, by decreasing *M* layer thickness, natural frequencies for all 4 vibrational modes decreased. When magnetic field intensity was zero, natural frequency $\omega_{1,1}$ took its minimum value and thus considered as the base frequency of the system. Also, for $2h_{MR}/h > 0.045$ and B=500Gauss, mode (1, 2) was dominant and frequency $\omega_{1,2}$ was at the base level.

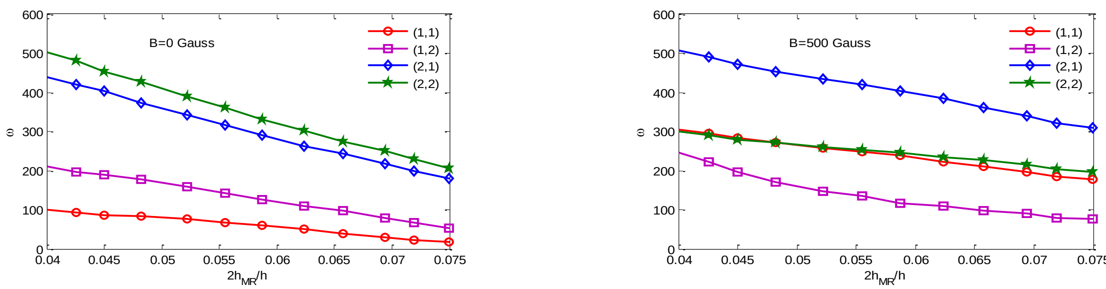

Fig.10
Natural frequencies changes with increasing $2h_{MR}/h$ for B=0, 500.

5 CONCLUSIONS

In this article, free vibrational behavior of a sandwich panel with flexible core and composite sheets in the presence of magnetorheological smart oil between the layers was investigated. Frostig's second order theory in the form of a polynomial with unknown coefficients was adopted for defining displacement of core and layers; first order shear theory was also used for displacement in layers.

Effects of ratio of sheet's length to its width, ratio of length to thickness, ratio of core thickness to overall thickness, and ratio of thickness of oil layer to overall thickness on the system frequency were also studied. Some of the major results of this study are as follows:

1. Addition of MR layer increased frequency of the first and third modes and decreases frequency of the second and fourth modes.
2. By applying magnetic field, base frequency did not necessarily occur in the first mode.
3. By increasing a/b ratio, dimensionless frequency increased for the first mode and decreased for the remaining modes.
4. By increasing h_c/h , frequency decreased for all the four modes.
5. By increasing a/h ratio, stiffness of the panel decreased and frequency of the first 4 modes decreased. For ratios of higher than 50 (a/h>50), frequency value did not change due to the variation of magnetic field.
6. By decreasing thickness of MR layer, frequency of all 4 modes increased.

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